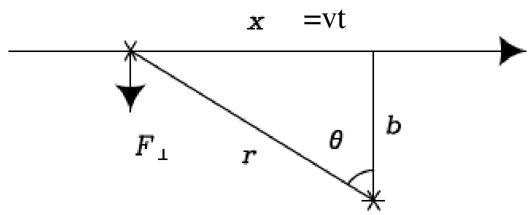
Relaxation Times

- Star passes by a system of N stars of mass m
- assume that the perturber is stationary during the encounter and that $\delta v/v<<1$- large b $>Gm/v^2$ (B&T pg 33-sec 1.2.1. sec 3.1 for exact calculation)
- Newton's Laws $m(dv/dt)=F$
- $(b^2+x^2) = r^2$
- $F=Gm^2\cos\theta/(b^2+x^2)=Gbm^2/(b^2+x^2)^{3/2}=$ $(Gm^2/b^2)(1+(vt/b)^2)^{-3/2}$ if v is constant
- Now integrate over time $\delta v = \int(F/m)dt=$

$(Gm/bv)\int(1+(vt/b)^2)^{-3/2}\,dt$ (change variables $s=(vt/b)$) ; $\delta v = 2Gm/bv$

- **In words, δv is roughly equal to the acceleration at closest approach, Gm/b^2,**

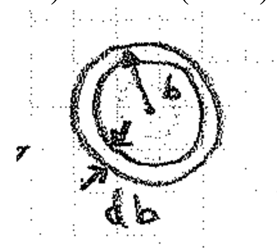
times the duration of this acceleration 2b/v.



The surface density of stars is $\sim N/\pi r^2$

N is the number of stars

let δn be the number of interactions a star encounters with impact parameter between b and δb crossing the galaxy once $\sim(N/\pi r^2)2\pi b\delta b=\sim(2N/r^2)b\delta b$



Relaxation...continued

- The net vectoral velocity due to these encounters is zero, but the mean square change is not

  $\delta v^2=(2Gm/bv)^2(2N/r^2)b\delta b$ (see B&T pg 34 eq. 1.3.2) - now integrating this over all impact parameters from b_{min} to b_{max}
- one gets $\delta v^2 \sim 8N(Gm/rv)^2 \ln \Lambda$; where r is the galaxy radius

  Λ is the Coulomb integral $\sim\ln(b_{max}/b_{min})$
- For gravitationally bound systems the typical speed of a star is roughly $v^2 \sim GNm/r$ and thus $\delta v^2/v^2 \sim 8N\ln\Lambda/N$
- For each 'crossing' of a galaxy one gets the same δv so the number of crossing for a star to change its velocity by order of its own velocity is $n_{relax} \sim N/8\ln\Lambda$
- So how long is this?? well $t_{cross} \sim r/v$; and $\sim rv^2/(Gm)$ and thus
- $t_{relax} \sim (0.1N/\ln N)t_{cross}$; if we use $N \sim 10^{11}$; t_{relax} is very very long
- In all of these systems the dynamics over timescales $t< t_{relax}$ is that of a collisionless system in which the constituent particles move under the influence of the gravitational field generated by a smooth mass distribution,rather than a collection of mass points

Relaxation

- Values for some representative systems

	<m>	N	r(pc)	t_{relax}(yr)	age(yrs
Pleiades	1	120	4	$1.7x10^7$	$<10^7$
Hydaes	1	100	5	$2.2x10^7$	410^8
Glob cluster	0.6	10^6	5	$2.9x10^9$	10^9-10^{10}
E galaxy	0.6	10^{11}	$3x10^4$	$4x10^{17}$	10^{10}
Cluster of gals	10^{11}	10^3	10^7	10^9	10^9-10^{10}

scaling laws $t_{relax} \sim (R^3/Nm)^{1/2}$

- However numerical experiments (Michele Trenti and Roeland van der Marel 2013 astro-ph 1302.2152) show that even globular clusters never reach energy **equipartition** (!) to quote from this paper 'Gravitational encounters within stellar systems in virial equilibrium, such as globular clusters, drive evolution over the two-body relaxation timescale. The evolution is toward a thermal velocity distribution, in which stars of different mass have the same energy (Spitzer 1987). This thermalization also induces mass segregation. As the system evolves toward energy equipartition, high mass stars lose energy, decrease their velocity dispersion and tend to sink toward the central regions. The opposite happens for low mass stars, which gain kinetic energy, tend to migrate toward the outer parts of the system, and preferentially escape the system in the presence of a tidal field"

So Why Are Stars in Rough Equilibrium

- It seems that another process 'violent relaxation' (MBW pg 251) is crucial.
- This is due to rapid change in the gravitational potential.
- Stellar dynamics describes in a statistical way the collective motions of stars subject to their mutual gravity-The essential difference from celestial mechanics is that each star contributes more or less equally to the total gravitational field, whereas in celestial mechanics the pull of a massive body dominates any satellite orbits
- The long range of gravity and the slow "relaxation" of stellar systems prevents the use of the methods of statistical physics as stellar dynamical orbits tend to be much more irregular and chaotic than celestial mechanical orbits-....woops.

How to Relax

- There are four different relaxation mechanisms at work in gravitational N-body systems:MBW sec 5.5.1-5.5.5
- phase mixing, chaotic mixing, Landau damping.
- Violent relaxation
  - time-dependent changes in the potential induce changes in the energies of the particles involved Exactly how the energy of a particle changes depends in a complex way on the initial position and energy of the particle (effects are independent of the mass of the particles)
  - Time scale is very fast ~free-fall time
- These processes are not well approximated by analytic calculations- need to resort to numerical simulations
  - simulations show that the final state depends strongly on the initial conditions, in particular on the initial virial ratio |2T/W|, collapse factor inversely related to virial ratio.
  - Since $T \sim M\sigma^2$ and $W \sim GM^2/r = MV^2_c$ with V_c the circular velocity at r, smaller values for the initial virial ratio $|2T/W| \sim (\sigma/V_c)^2$ indicate cold initial conditions - these come naturally out of CDM models (MBW pg 257)

Collisionless Boltzmann Eq (Vlasov eq)S+G sec 3.4

- When considering the structure of galaxies cannot follow each individual star (10^{11} of them!),
- Consider instead stellar density and velocity distributions. However, a fluid model not really appropriate since a fluid element has a single velocity, which is maintained by particle-particle collisions on a scale much smaller than the element. For stars in the galaxy this **not true**-stellar collisions are very rare, and even encounters where the gravitational field of an individual star is important in determining the motion of another are very infrequent
- So taking this to its limit, treat each particle as being collisionless, moving under the influence of the mean potential generated by all the other particles in the system $\phi(x,t)$

Collisionless Boltzmann Eq

- The distribution function is defined such that $f(\mathbf{r},\mathbf{v},t)d^3\mathbf{x}d^3\mathbf{v}$ specifies the number of stars inside the volume of phase space $d^3\mathbf{x}d^3\mathbf{v}$ centered on $(\mathbf{x},\mathbf{v})$ at time t-

At time t a full description of the state of this system is given by specifying the number of stars

$$f(x, v, t)d^3x d^3v$$

Then $f(\mathbf{x}, \mathbf{v}, t)$ is called the "distribution function" (or "phase space number density") in 6 dimensions (**x** and **v**) of the system.

$f \geq 0$ since no negative star densities

Since potential is smooth nearby particles in phase space move together-- fluid approx.

For a collisionless stellar system in dynamic equilibrium, the gravitational potential, ϕ , relates to the phase-space distribution of stellar tracers $f(\mathbf{x}, \mathbf{v}, t)$, via the collisionless Boltzmann Equation

number density of particles

$n(\mathbf{x},t)=\int f(x, v, t)d^3\mathbf{v}$

average velocity

$<v(\mathbf{x},t)>=\int f(x, v, t)\, \mathbf{v}d^3\mathbf{v}/f(x, v, t)d^3\mathbf{v}=$

$(1/n(\mathbf{x},t))\int f(x, v, t)\, \mathbf{v}d^3\mathbf{v}$

See S&G sec 3.4

- The collisionless Boltzmann equation is like the equation of continuity,
- $\partial n/\partial t+ \partial(nv)\partial x= 0$. but it allows for changes in velocity and relates the changes in f (x, v, t) to the forces acting on individual stars
- In one dimension CBE is
- $\partial f/\partial t+ v\partial f/\partial x- \partial\phi(x, t) /\partial x(x, t)\ \partial f/\partial v= 0.$

Analogy with Gas- continuity eq see MBW sec 4.1.4

- $\partial\rho/\partial t + \nabla \bullet (\rho v) = 0$ which is equiv to
- $\partial\rho/\partial t + v\nabla \bullet \rho = 0$
- In the absence of encounters f satisfies the continuity eq , flow is smooth, stars do no jump discontinuously in phase space

- Continuity equation :

define w=(x,v) pair (generalize to 3-D)

$dw/dt=(v,\nabla\phi)$

- $\partial f/\partial t + \nabla_6(f\ dw/dt)=0$

Collisionless Boltzmann Eq

- This results in (S+G pg 143)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0,$$

- the flow of stellar phase points through phase space is incompressible – the phase-space density of points around a given star is always the same

- The distribution function f is a function of seven variables, so solving the collisionless Boltzmann equation in general is hard. So need either simplifying assumptions (usually symmetry), or try to get insights by taking moments of the equation.
- Moment of an eq-- multipying f by powers of v

define n(x,t) as the number density of stars at position x

then the first moment is $\partial n/\partial t+\partial/\partial x(nv)=0$; the same eq as continuity equation of a fluid

second moment $n\partial v/dt+nv\partial v/dx=-n\partial\phi/\partial x-\partial/\partial x(n\sigma)$

σ is the velocity dispersion

But unlike fluids do not have thermodynamics to help out.... nice math but not clear how useful

Collisionless Boltzmann Eq

- .
astronomical structural and kinematic observations provide information only about the **projections of phase space distributions along lines of sight**, limiting knowledge about f and hence also about ϕ.

Therefore all efforts to translate existing data sets into constraints on involve simplifying assumptions.
• dynamic equilibrium,
• symmetry assumptions
• particular functional forms for the distribution function and/or the gravitational potential.

Jeans Equations MBW sec 5.4.3

- Since f is a function of 7 variables obtaining a solution is challenging
- Take moments (e.g. integrate over all **v**)
- let n be the space density of 'stars'

$\partial n/\partial t+\partial(n\langle v_i\rangle)/\partial x_i =0$; continuity eq. zeroth moment

first moment (multiply by v and integrate over all velocities)

$\partial(n\langle v_j\rangle /\partial t) +\partial(n\langle v_i v_i\rangle)/\partial x_i +n\partial\phi/\partial x_j=0$

equivalently

$n\partial(\langle v_j\rangle /\partial t) +n\langle v_i\rangle \partial\langle v_j\rangle/\partial x_i =-n\partial\phi/\partial x_j-\partial(n\sigma^2_{ij})/\partial x_i$

so n is the integral over velocity of f; $n=\int f\, d^3v$

$\langle v_i\rangle$ is the mean velocity in the i^{th} direction $=1/n\int f\, v_i\, d^3v$

the term $-\partial(n\sigma^2_{ij})/\partial x_i$ is like a pressure, but allows for different pressures in different directions- important in elliptical galaxies and bulges 'pressure supported' systems

Collisionless Boltzmann Eq

- Some simplifications
- $n\partial v/dt+nv\partial v/dx=-n\partial\phi/\partial x-\partial/\partial x(n\sigma)$
- assume isotropy, steady state, non-rotating
- then
- $-n\,\nabla\phi=\nabla n\sigma^2$
- using Poissons eq
- $\nabla^2\phi=4\pi\rho G$ and solve for $\sigma(r)^2$

Jeans Eq

- So what are these terms??
- $n\partial\phi/\partial x_j$:gravitational pressure
- $n\sigma^2_{ij}$ stress tensor (an anisotropic pressure) (with a bit of coordinate transform one can make this symmetric e.q. $\sigma^2_{ij}=\sigma^2_{ji}$)

Jeans Equations Another Formulation

- Jeans equations follow from the collisionless Boltzmann equation; Binney & Tremaine (1987), MBW 5.4.2. S+G sec 3.4 .

cylindrical coordinates and assuming an axi-symmetric and steady-state system, the accelerations in the radial (R) and vertical (Z) directions can be expressed in terms of observable quantities:

the stellar number density distribution ν_*

and 4 velocity components

a rotational velocity v_ϕ

and 4 components of random velocities (velocity dispersion components) $\sigma_{\phi\phi}$, σ_{RR}, σ_{ZZ}, σ_{RZ}

$$a_R = \sigma^2_{RR} \times \frac{\partial(\ln \nu)}{\partial R} + \frac{\partial \sigma^2_{RR}}{\partial R} + \sigma^2_{RZ} \times \frac{\partial(\ln \nu)}{\partial Z} + \frac{\partial \sigma^2_{RZ}}{\partial Z} + \frac{\sigma^2_{RR}}{R} - \frac{\sigma^2_{\phi\phi}}{R} - \frac{\overline{v_\phi}^2}{R},$$

$$a_Z = \sigma^2_{RZ} \times \frac{\partial(\ln \nu)}{\partial R} + \frac{\partial \sigma^2_{RZ}}{\partial R} + \sigma^2_{ZZ} \times \frac{\partial(\ln \nu)}{\partial Z} + \frac{\partial \sigma^2_{ZZ}}{\partial Z} + \frac{\sigma^2_{RZ}}{R}.$$

where a_Z, a_R are accelerations in the appropriate directions-
given these values (which are
the gradient of the gravitational potential),
the dark matter contribution can be estimated after accounting for the contribution from visible matter

Use of Jeans Eqs

- Surface mass density near sun
- Poissons eq $\nabla^2\phi=4\pi\rho G=-\nabla\bullet\mathbf{F}$
- Use cylindrical coordinates
- $(1/R)\partial/\partial R(RF_R) + \partial F_z/\partial z=-4\pi\rho G$

- $F_R=-v_c/R$ v_c circular velocity (roughly constant near sun) -

(F_R force in R direction) so

$\partial F_R/\partial R =(-1/4\pi G)\partial F_z/\partial z$; **only vertical gradients count**

since the surface mass density $\Sigma=2\int\rho dz =-F_z/2\pi G$

(integrate 0 to $+\infty$ thru plane)

Now use Jeans eq: $nF_z-\partial(n\sigma^2_z)/\partial z+(1/R)\partial/\partial R(Rn\sigma^2_{zR})$; if R+z are separable

e.g $\phi(R,z) =\phi(R)+\phi(z)$ then $\sigma^2_{zR}\sim 0$ and voila! (eq 3.94 in S+G)

$\Sigma=1/2\pi Gn\ \partial(n\sigma^2_z)/\partial z$; need to observe the number density distribution of some tracer of the potential above the plane and its velocity dispersion distribution perpendicular to the plane goes at $n\exp(-z/z_0)$

viral theorem

- The quick way
- Consider for simplicity the one-dimensional analog of the Jeans Equation in steady state:
- $\partial/\partial x[\rho v^2]+\rho\partial\Phi/\partial x=o$
- After integrating over velocities, let's now integrate over x :
- [one needs to use Gauss' theorem etc..]

- one gets $-2E_{kin}=E_{pot}$

Spherical systems- Elliptical Galaxies and Globular Clusters

- For a spherical system the Jeans equations simplify to

$(1/n)d/dr\ (n\langle v^2_r\rangle\ )+2\beta\langle v^2_r\rangle/r=-GM(R)/r^2$

- where $G(M(R)/r^2$ is the potential and n(r), $\langle v^2_r\rangle$ and $\beta(r)$ describe the 3-dimensional density, radial velocity dispersion and orbital anisotropy of the tracer component (stars)

  $\beta(r)=1-\langle v^2_\theta\rangle/\langle v^2_r\rangle$; $\beta=0$ is isotropic, $\beta=1$ is radial
- We can then present the mass profile as
- $GM(r)=r\langle v^2_r\rangle\ (d\ln n/d\ln r+d\ln\langle v^2_r\rangle/d\ln r+2\beta)$
- while apparently simple we have 3 sets of unknowns $\langle v^2_r\rangle$, $\beta(r)$, n(r)
- and 2 sets of observables I(r)- surface brightness of radiation (in some wavelength band) and the lines of sight projected velocity field (first moment is velocity dispersion)
- It turns out that one has to 'forward fit'- e.q. propose a particular form for the unknowns and fit for them.

This will become very important for elliptical galaxies

Motion Perpendicular to the Plane- Alternate Analysis-(S+G pgs140-144, MBW pg 163)

For the motion of stars in the vertical direction only-stars whose motions carry them out of the equatorial plane of the system.

$d/dz[n_*(z)\sigma_z(z)^2]=-n_*(z)d\phi(z,R)/dz$; where **$\phi(z,R)$ is the vertical grav potential**

The study of such general orbits in axisymmetric galaxies can be reduced to a two-dimensional problem by exploiting the conservation of the z-component of angular momentum of any star

the first derivative of the potential is the grav force perpendicular to the plane - call it K(z)

$n_*(z)$ is the density of the tracer population and

$\sigma_z(z)$ is its velocity dispersion

then the 1-D Poisson's eq $4\pi G\rho_{tot}(z,R)=d^2\phi(z,R)/dz^2$

where ρ_{tot} is the total mass density - put it all together to get

$4\pi G\rho_{tot}(z,R)=-dK(z)/dz$ (S+G 3.93)

$d/dz[n_*(z)\sigma_z(z)^2]=n_*(z)K(z)$ - to get the data to solve this have to determine $n_*(z)$ and $\sigma_z(z)$ for the tracer populations(s)

Use of Jeans Eq For Galactic Dynamics

- Accelerations in the z direction from the Sloan digital sky survey for

1) all matter

2) 'known' baryons only

ratio of the 2 (bottom panel

use this data + Jeans eq (see below, to get the total acceleration

(in eqs ν is the density of tracers, v_ϕ is the azimuthal velocity (rotation)

$$a_R = \sigma^2_{RR} \times \frac{\partial(\ln\nu)}{\partial R} + \frac{\partial\sigma^2_{RR}}{\partial R} + \sigma^2_{RZ} \times \frac{\partial(\ln\nu)}{\partial Z} + \frac{\partial\sigma^2_{RZ}}{\partial Z} + \frac{\sigma^2_{RR}}{R} - \frac{\sigma^2_{\phi\phi}}{R} - \frac{\overline{v_\phi}^2}{R}, \quad (1)$$

$$a_Z = \sigma^2_{RZ} \times \frac{\partial(\ln\nu)}{\partial R} + \frac{\partial\sigma^2_{RZ}}{\partial R} + \sigma^2_{ZZ} \times \frac{\partial(\ln\nu)}{\partial Z} + \frac{\partial\sigma^2_{ZZ}}{\partial Z} + \frac{\sigma^2_{RZ}}{R}. \quad (2)$$

Given accelerations $a_R(R,Z)$ and $a_Z(R,Z)$, *i.e.* the gradient of the gravitational potential, the dark matter contribution can be estimated after accounting for the contribution from visible matter.

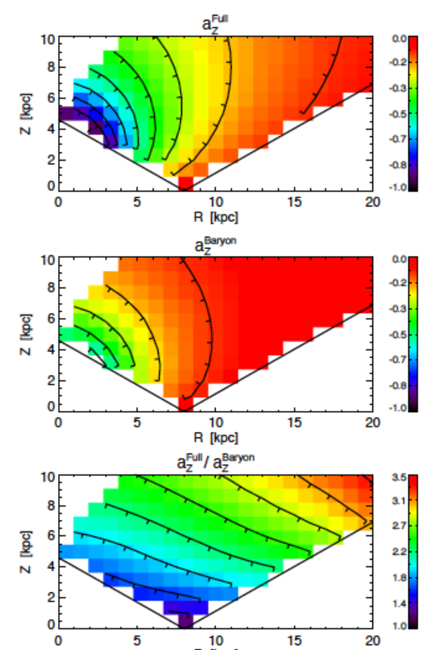


Figure 1. A comparison of the acceleration in the Z dir when all contributions are included (star, gas, and dark particles; top panel) to the result without dark matter (panel). The acceleration is expressed in units of 2.9×10^{-13} The ratio of the two maps is shown in the bottom panel. importance of the dark matter increases with the distance fr origin; at the edge of the volume probed by SDSS ($R \sim 2$

Jeans Continued

- Using dynamical data and velocity data get estimate of surface mass density in MW

Σ_{total}~70 +/- 6$M_\odot/pc^2$

Σ_{disk}~48+/-9 $M_\odot/pc^2$

Σ_{star}~35$M_\odot/pc^2$

Σ_{gas}~13$M_\odot/pc^2$

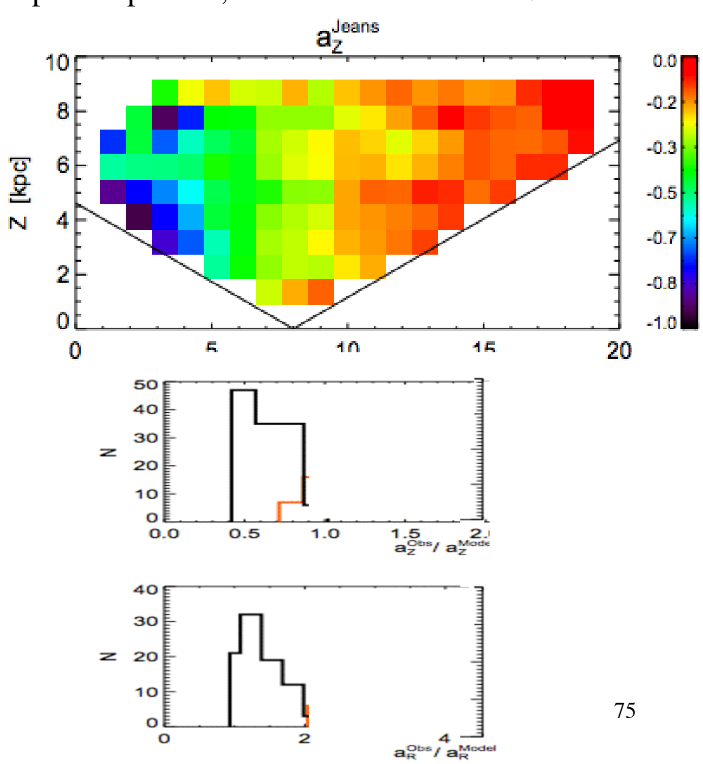
we know that there is very little light in the halo so direct evidence for dark matter

What Does One Expect The Data To Look Like

- A full-up numerical simulation from cosmological conditions of a MW like galaxy-this 'predicts' what a_Z should be near the sun (Loebman et al 2012)
- **Notice that it is not smooth or monotonic**

  and the the simulation is neither perfectly rotationally symmetric nor steady state..
- errors are on the order of 20-30%- figure shows comparison of true radial and z accelerations compared to Jeans model fits

1 kpc x 1kpc bins; acceleration units of 2.9x10^{-13} km/sec^2



Full Up Equations of Motion- Stars as an Ideal Fluid(S+G pgs140-144, MBW pg 163)

Continuity equation (particles not created or destroyed)

$d\rho/dt+\rho\nabla\cdot v=0$; $d\rho/dt+d(\rho v)/dr=0$

Eq's of motion (Eulers eq)
$dv/dr=\nabla P/\rho-\nabla\Phi$

Poissons eq $\nabla^2\Phi(r)=-4\pi G\rho(r)$
example potential

Analogy of Stellar Systems to Gases - Discussion due to Mark Whittle

- similarities :

comprise many, interacting objects which act as points (separation >> size)

can be described by distributions in space and velocity eg Maxwellian velocity distributions; uniform density; spherically concentrated etc.

Stars or atoms are neither created nor destroyed -- they both obey continuity equations- not really true, galaxies are growing systems!

All interactions as well as the system as a whole obeys conservation laws (eg energy, momentum)**if isolated**

- But :
- The relative importance of short and long range forces is radically different :
  - atoms interact only with their neighbors, however
  - stars interact continuously with the entire ensemble via the long range attractive force of gravity
- eg uniform medium : $F \sim G\,\Omega\rho/r^2 \sim r^2dr$; $r^2 \sim \rho\, dr$ **~equal force from all distances**

Analogy of Stellar Systems to Gases
- Discussion due to Mark Whittle

- The relative frequency of strong encounters is radically different :
- -- for atoms, encounters are frequent and all are strong (ie $\delta V \sim V$)
- -- for stars, pairwise encounters are very rare, and the stars move in the smooth global potential (e.g. S+G 3.2)

- Some parallels between gas (fluid) dynamics and stellar dynamics many of the same equations can be used as well as :
- ---> concepts such as Temperature and Pressure can be applied to stellar systems
- ---> we use analogs to the equations of fluid dynamics and hydrostatics
- there are also some interesting differences
- ---> pressures in stellar systems can be anisotropic
- ---> stellar systems have negative specific heat and evolve away from uniform temperature.

- Separate potential into r and z parts

$\phi(r,z)=B(R)Z(z)$

outside disk $\nabla^2\phi=0$; find $Z(z)=A\exp(-k|z|)$

eq for R dependence of potential is

$(1/R)(d/dR(RdB/dr)+k^2B(R)=0$- the solutions of this are Bessel functions J(r); but it gets even messier

Important result

- $Rd\phi/dR=v^2_c = GM(R)/R$ to within 10% for most 'reasonable' forms of mass distribution

see http://www.ast.cam.ac.uk/~ccrowe/Teaching/Handouts for lots of derivations/

Rotation Curve Mass Estimates sec 2.6 of B&T

- sec sec 11.1.2 in MBW
- Galaxy consists of a axisysmmetric disk and spherical dark matter halo
- Balance centrifugal force and gravity
- $V^2(R)=R\mathbf{F}(R)$; $\mathbf{F}(R)$ is the acceleration in the disk
- Split rotation into 2 parts due to disk and halo
  $V^2(R)=V^2_d(R)+V^2_h(R)$
- for a **spherical** system
  $V^2(R)=rd\phi/dr=GM(r)/r$
- Few analytic solutions: point mass $V_c(R)\sim r^{-1/2}$
  singular isothermal sphere $V_c(R)$=constant (see S+G eq 3.14)
  uniform sphere $V_c(R)\sim r$

for a pseudo-isothermal (S+G problem 2.20)

$\rho(r)=\rho(0)(R^2_c/R^2+R^2_c)$; $\rho(0)= V(\infty)^2/4\pi G R^2_c$ and the velocity profile is

$V(R)^2=V(\infty)^2(1-R_c/R \tan^{-1} R/R_c)$;for a **NFW** potential get a rather messy formula

Disks are Messy (MVW ch 11)

- Skipping the integrals of Bessel functions (eq 11.2 MBW) one gets
- $V^2_{c,d}(R)= 4\pi G\Sigma_0 R_d y^2[I_o(y)K_0(y)-I_1(y)K_1(y)]$

$y=R/(2R_d)$ and I and K are Bessel functions of the first and second kinds:which do not have simple asymptotic forms

Important bits: $V^2_{c,d}(R)$ depends only on radial scale length R_d and its central surface density Σ_0

Radial scale length of a spiral disk

$\Sigma(r)=\Sigma_0\exp(-R/ R_d)$**;** integrate over r to get total mass $M_d=2\pi\Sigma_0 R^2_d$

Vertical density distribution is also an exponential $\exp(-z/z_0)$ so total distribution is product of the two

$\rho(R,z)=\rho_0 \exp(-R/ R_d)\exp(-z/z_0)$

while we may know the scale length of the stars, that of the dark matter is not known.

Also the nature of the dark matter halo is not known:- disk/halo degeneracy

Nature is Cruel

$$v_c^2(R) = 2\pi G R \int_0^\infty dk\, k J_1(kR) \int_0^\infty dR'\, R' \Sigma(R') J_0(kR'). \quad (2.188)$$

- mathematics seems to be saying that it is as easy to determine a disk's surface density from measurements of its circular speed, as to obtain the circular speed from the surface density.

•Unfortunately, observational constraints destroy this symmetry.

•The key point is that the leftside of either equation (2.188) or (2.190) can be determined at any given value of R only if the variable on the right side can be measured out to radii at which its value becomes negligible.

•(2.190) $$\Sigma(R) = \frac{1}{2\pi G} \int_0^\infty dk\, k J_0(kR) \int_0^\infty dR'\, v_c^2(R') J_1(kR').$$

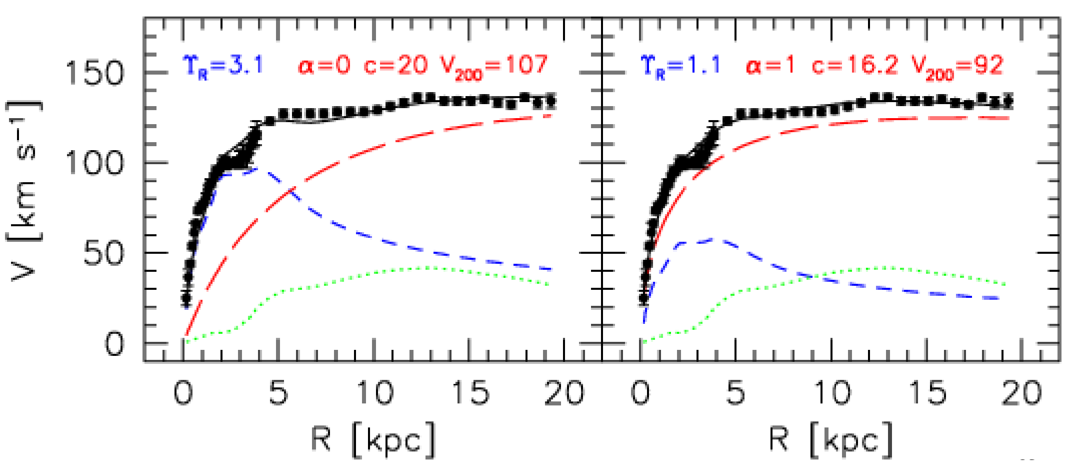
•The surface density declines rapidly with radius, so equation (2.188) can be used to derive accurate values of v_c.

•Circular speeds, by contrast, decline little if at all out to the largest observable radii. Consequently, in practice we cannot obtain the data needed to determine Σ accurately from equation (2.190)

Disk Halo Degeneracy

- MBW fig 11.1: two solutions to rotation curve of NGC2403: stellar disk (blue lines), dark matter halo - red lines.

- Left panel is a 'maximal' disk, using the highest reasonable mass to light ratio for the stars, the right panel a lower value of M/L



Potential of Spiral Galaxies B&T 2.7

- The potential of spirals is most often modeled as a 3 component system
- Bulge
- Dark halo
- Disk

as stressed by B&T usually one assumes that the potential has a certain form and is well traced by stars/gas

On pg 111 B&T give the observational constraints which models have to match.

Bulge; B&T assume $\rho(r)=\rho_B(0)(m/a_b)^{-\alpha_b} \exp(-m/r_b)^2{}_c)$; $m=\mathrm{sqrt}(R^2+z^2/q^2)$which for $q<1$ is an oblate spheroidal power-law model (no justification is given !)

They use IR star counts in the bulge (which is dominated by old stars) to get values for the parameters.

They use a similar form for the halo, but the parameters are much less well determined.

Disk: use a double exponential disk (thin and thick) for the stars and a somewhat more complex form for the gas (in the MW gas is ~25% of the mass of stars in the disk).

The most important parameter is the disk scale length :

MW Mass Model

- Notice that the mass of the bulge and the surafce mass density of the disk are highly uncertain (more later)

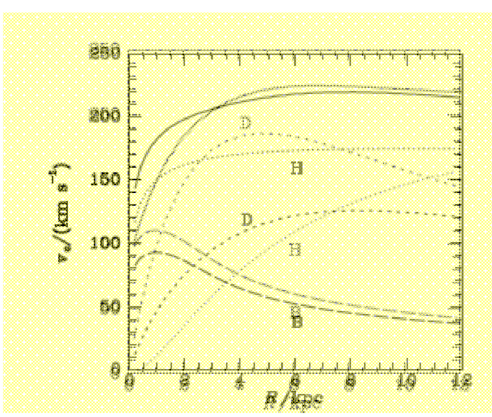


Table 2.3 Parameters of Galaxy models

Parameter	Model I	Model II
R_d/kpc	2	3.2
$(\Sigma_d+\Sigma_g)/\mathcal{M}_\odot\,\mathrm{pc}^{-2}$	1905	536
$\rho_{b0}/\mathcal{M}_\odot\,\mathrm{pc}^{-3}$	0.427	0.3
$\rho_{h0}/\mathcal{M}_\odot\,\mathrm{pc}^{-3}$	0.711	0.266
α_h	-2	1.63
β_h	2.96	2.17
a_h/kpc	3.83	1.90
$M_d/10^{10}\,\mathcal{M}_\odot$	5.13	4.16
$M_b/10^{10}\,\mathcal{M}_\odot$	0.518	0.364
$M_{h,<10\,\mathrm{kpc}}/10^{10}\,\mathcal{M}_\odot$	2.81	5.23
$M_{h,<100\,\mathrm{kpc}}/10^{10}\,\mathcal{M}_\odot$	60.0	55.9
$v_e(R_0)/\mathrm{km\,s^{-1}}$	520	494
f_b	0.05	0.04
f_d	0.60	0.33
f_h	0.35	0.63

Virial Theorem B&T 4.8.3(a)

- This fundamental result describes how the total energy (E) of a self-gravitating system is shared between kinetic energy and potential energy .
- Go to one dimension and assume steady state

Integrate over velocity and space and one finds

$-2E_{kinetic}=PE_{potential}$ (W=PE)
see S&G pgs 120-121 for full derivation

This is important for find the masses of systems whose orbital distribution is unknown or very complex and more or less in steady state (so assumptions in derivation are ok)
In general $<v^2>=W/M=GM/r_g$;r_g the gravitational radius (which depends on the form of the potential)
Many of the forms of the potential have their scale parameter ~$1/2r_g$ (pg 361 B&T)
Then if E is the total energy E=KE+PE=-KE=1/2PE (!); where does the energy go?? (radiation)

Use of Virial Theorem

- Consider a statistically steady state, spherical, self gravitating system of N objects with average mass m and velocity dispersion σ.
- Total KE=$(1/2)Nm\sigma^2$
- If average separation is r the PE of the system is $U=(-1/2)N(N-1)Gm^2/r$
- Virial theorem E=-U/2 so the total mass is $M=Nm=2\pi\sigma^2/G$ or using L as light and Σ as surface light density

  $\sigma^2\sim(M/L)\Sigma R$-picking a scale (e.g. half light radius R_e)

  $R_e\sim\sigma^\alpha\Sigma^\beta$ $\alpha=2,\beta=1$ from viral theorem

  value of proportionality constants depends on shape of potential

  For clusters of galaxies and globular clusters often the observables are the light distribution and velocity dispersion. then one measures the ratio of mass to light as

  $M/L\sim9\sigma^2/2\pi GI(0)r_c$ for spherical isothermal systems

Jeans Again

- Jeans Mass $M_J=1/8(\pi kT/G\mu)^{3/2}\ \rho^{-1/2}$

- In astronomical units this is

$M_J=0.32M_{\odot}(T/10k)^{3/2}(m_H/\mu)^{3/2}(10^6cm^{-3}/n_H)^{1/2}$

So for star formation in the cold molecular medium with T~10k and $n_H \sim 10^5$- $M_J \sim 2M$

The growth time for the Jeans instability is

$\tau_J=1/sqrt(4\pi G\rho) = 2.3x10^4 yr(n_H/10^6cm^{-3})^{-1/2}$

For pure free fall

$\tau_J= (3\pi/32G\rho)^{1/2}= 4.4x10^4 yr(n_H/10^6cm^{-3})^{-1/2}$

Jeans growth rate about 1/2 the free fall time

τ_s time scale is the sound crossing time across the Jeans Length

$c_s= sqrt(kT/m_H\mu)$ $\lambda_J= (\pi c^2_s/G\rho)^{1/2}$ $\tau_s=\lambda_J/c_s$