Homework # 1, Physics 235, fall 2002.

due Friday Oct.11

problems

1) A simple model for starlight from our Galaxy. Assume the stars radiate as blackbodies, and that their radius and luminosities are given by (BM table 3.13)

$$\frac{R}{R_\odot} = min\left(\left(\frac{M}{M_\odot}\right)^{0.8}, 1.4\left(\frac{M}{M_\odot}\right)^{0.57}\right)$$
$$\frac{L}{L_\odot} = min\left(max\left(\left(\frac{M}{M_\odot}\right)^{3.9}, 0.5\left(\frac{M}{M_\odot}\right)^{2.2}\right), 10^4\frac{M}{M_\odot}\right). \quad (1)$$

For main sequence stars, the number of stars per logarithmic mass bin in the disk of our Galaxy is roughly (Shapiro and Teukolsky table 1.3)

$$\frac{dN}{d\log_{10}(M/M_\odot)} = max\left(10^{8.7}\left(\frac{M}{M_\odot}\right)^{-2.5}, min\left(10^{10.4}\left(\frac{M}{M_\odot}\right)^{-0.5}, 10^{10.2}\left(\frac{M}{M_\odot}\right)^{-5.0}\right)\right) \quad (2)$$

Make a plot of the energy radiated per unit time per logarithmic wavelength interval for main sequence stars. Compute the amount of energy radiated in the U, B, V and K filters from BM table 2.1. What mass stars emit most of the U, B, V, K band light? Which stars have most of the mass? Why might it be important to include red giant stars?

2) A simple model for infrared dust emission from a galaxy. Assume that dust grains are perfect absorbers of starlight (and consequently, by Kirchoff's law, they are perfect emitters.) Derive the equilibrium dust temperature by balancing the heating due to absorption of blackbody light from hot stars, with cooling from blackbody emission from the dust grain. Be sure to include the "dilution" factor, W, for the ultraviolet light, which measures the decline of the radiation field with distance from the stellar surface (i.e., $W \sim (R_*/\bar{D})^2$, where R_* is the radius of the star and $\bar{D}$ is the mean distance from the stars to the dust grain.) Give the grain temperature, and associated photon wavelength for thermal emission from the dust, for two cases: (a) an HI region with warm, neutral gas, which is at $\bar{D} \sim 1pc$ from stars of temperature $T_{eff} \sim 10^4 K$, and (b) an HII region with hot gas at $\bar{D} = 0.1\,pc$ from a star with $T_{eff} = 30,000K$. A more realistic assumption might be that the absorption coefficient for dust scales as λ^{-1}. Would this cause the dust temperature to go up or down relative to the perfect absorber case?

3) Black holes with masses $M_{bh} \sim 10^6 - 10^9 M_\odot$ are thought to reside in the centers of many galaxies. It has been proposed that these black holes are built up by the successive merger of galaxies. However, the black hole may not find itself smack in the center after a merger. What is the maximum initial radius for a black hole for which it has had time to spiral into the center in less than 10^{10} years. (Hint: follow the discussion in section 7.1.a of BT.)

4) It has recently been proposed that a certain globular cluster of mass $M_{gc} \sim 10^5 M_\odot$ has 10% of its mass in free-floating (not tightly bound to any star) Jupiter-sized objects. Compute the timescale over which these Jupiters will be "evaporated" from the cluster due to collisions with the cluster stars.

5) Estimate the relaxation time for the following systems: (a) a galaxy of size $R = 10\ kpc$ composed of $N = 10^{11}$ stars, (b) a dark matter halo of size $R = 100\ kpc$ total mass $M_{dm} = 10^{12} M_\odot$ composed of dark matter particles of mass $m_{dm} = 1 Tev$, (c) an open cluster of size $R = 1\ pc$ composed of $N = 10^3$ stars, and (d) the Oort cloud of size $R = 10^5\ au$ with $N = 10^{12}$ comets of mass $m_c = 10^{15}\ g$. Speculate on the future of the human race.

6) Brownian motion. A particle of velocity $\mathbf{v}$ is subject to a drag acceleration $\mathbf{a}_{drag} = -\gamma \mathbf{v}$ in addition to a random acceleration $\mathbf{f}(t)$ with the following properties: (1) $\langle \mathbf{f} \rangle = 0$, where the average is over many collisions with the background particles, and (2) the acceleration produced by different collisions is uncorrelated, so that $\langle \mathbf{f}(t) \cdot \mathbf{f}(t') \rangle = \mathcal{F} \delta(t - t')$. Here $\gamma = D(\delta v)$ is the drag coefficient, and $\mathcal{F} = D(\delta v^2)$ is is the velocity diffusion coefficient. Let these coefficients be independent of $\mathbf{v}$.

Now the questions. (a) Write down the equation of motion for the particle's velocity, including the drag force and the random force. Using an integrating factor, solve this equation for the velocity of the particle in terms of an integral over the force $\mathbf{f}$. (b) Average this velocity over long times (compared to the collision time) using the known properties of $\langle \mathbf{f} \rangle$. (c) Compute

$\langle v^2(t)\rangle$ by using the known expressions for $\langle \mathbf{f}\rangle$ and $\langle \mathbf{f}(t)\cdot\mathbf{f}(t')\rangle$. (d) At large times, the mean velocity squared $\langle v^2(t)\rangle \to 3\sigma^2$, where σ is the 1D velocity dispersion. Use this fact to relate $\mathcal{F}$ to γ and σ^2 (the *fluctuation-dissipation theorem.*) (f) Does the velocity asymptote to a finite (nonzero and not infinite) value if you set either γ or $\mathcal{F}$ to zero?

7) Galaxy formation. The Galaxy may have formed by either slow or rapid collapse compared to a star's orbit period. The orbits of old stars (identified by metallicity) and globular clusters (dated by stellar evolution) may give information about the rate of collapse if they formed early on. In this problem we will examine how an initial set of orbits changes if the Galaxy collapses slowly or quickly.

Assume the stars feel a Galactic potential $\phi = -GM(t)/r$, and express the orbits in terms of the usual semi-major axis a and eccentricity e. Questions: (a) How does the orbital time vary with position in the galaxy. (b) Assume that $M(t)$ changes slowly compared to an orbit. Starting from the conserved actions (BT eq. 3.22b and 3.177), which are adiabatic invariants, describe the change in a and e due to $M(t)$ increasing slowly. (c) Now let $M(t)$ increase much more rapidly than the orbital time. How do a and e change? For a set of orbits with a given eccentricity and random orbital phase, will the mean change be positive or negative? (d) Put parts a, b, and c together and discuss what trends might be expected in the observed eccentricities of halo objects. For disk orbits, how is this picture complicated by star formation?