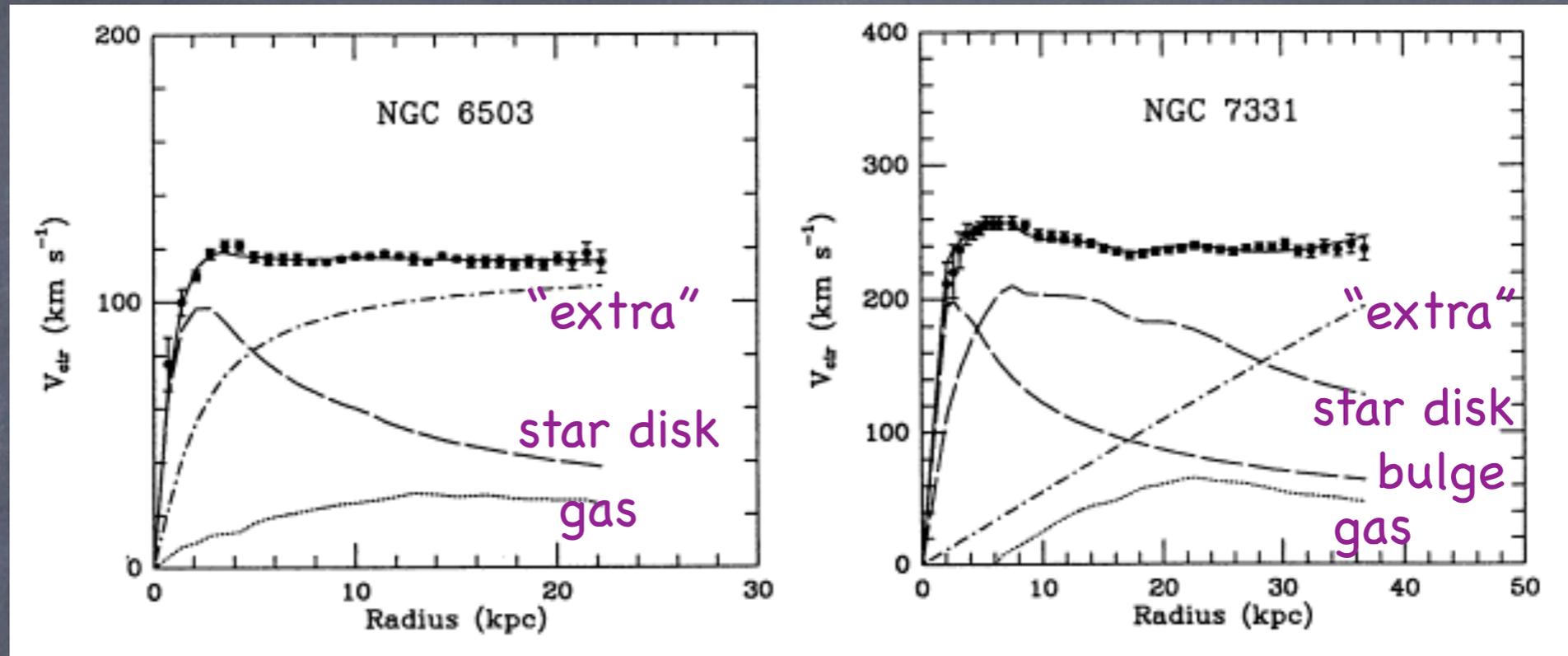


Local Dark Matter

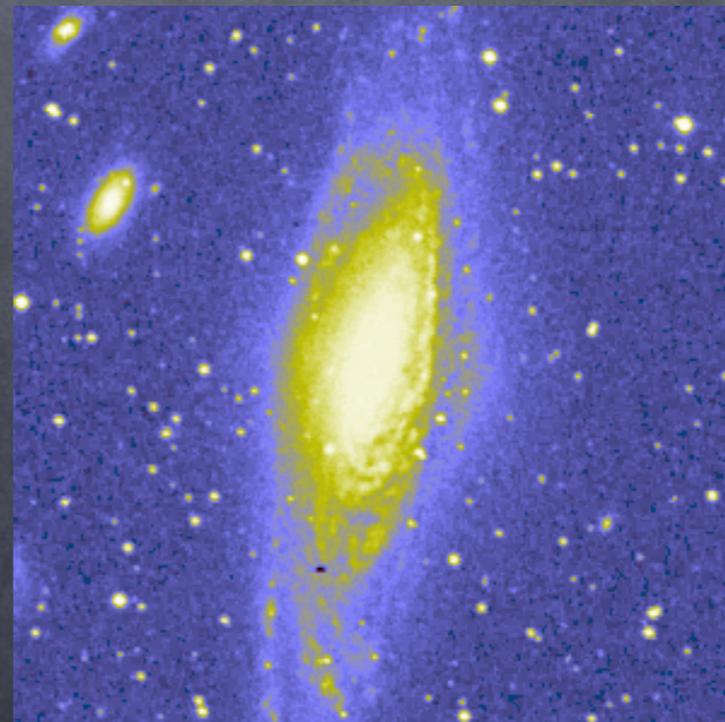
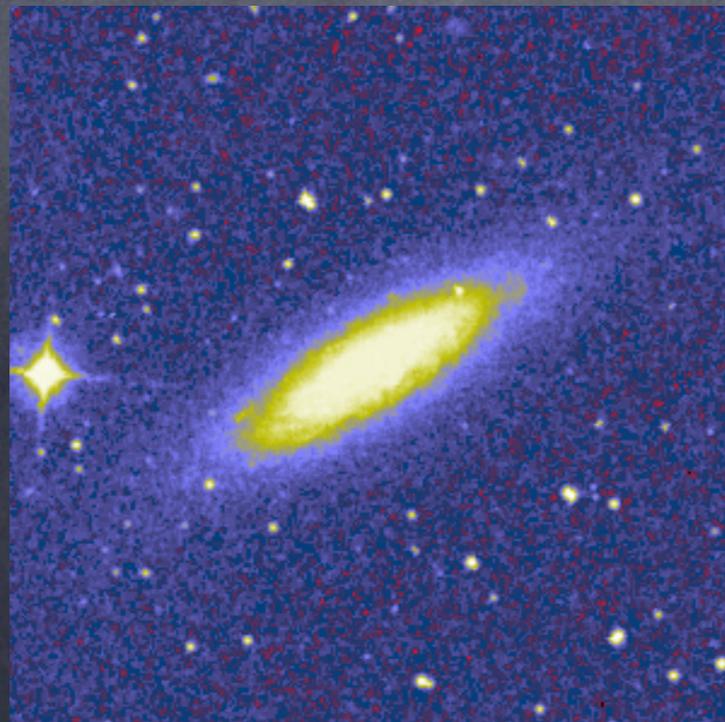
Konrad Kuijken

Leiden Observatory, Leiden University

Galaxy haloes



~ flat rotation curves: galaxies are not WYSIWYG

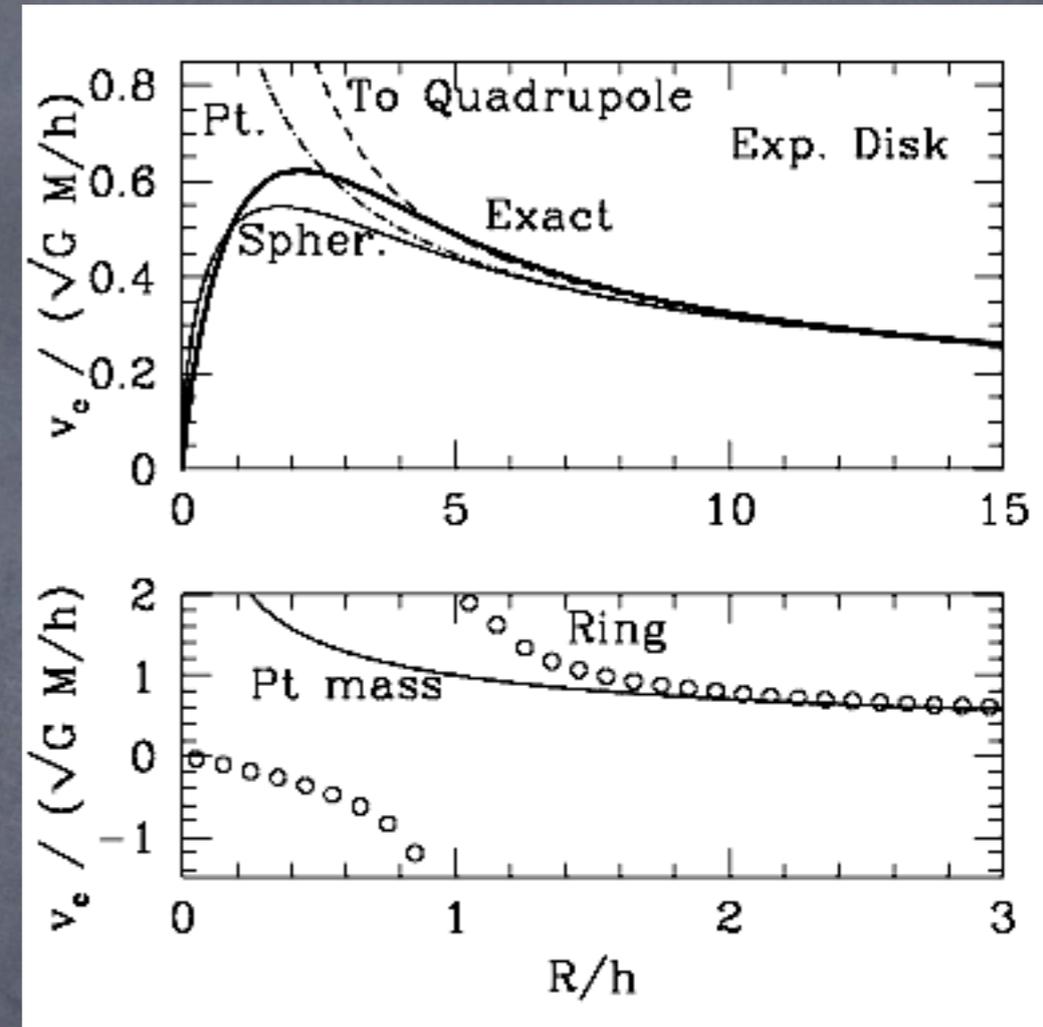


Rotation curves

$$\frac{GM(< r)}{r^2} \simeq \frac{V_{\text{circ}}^2}{r}$$

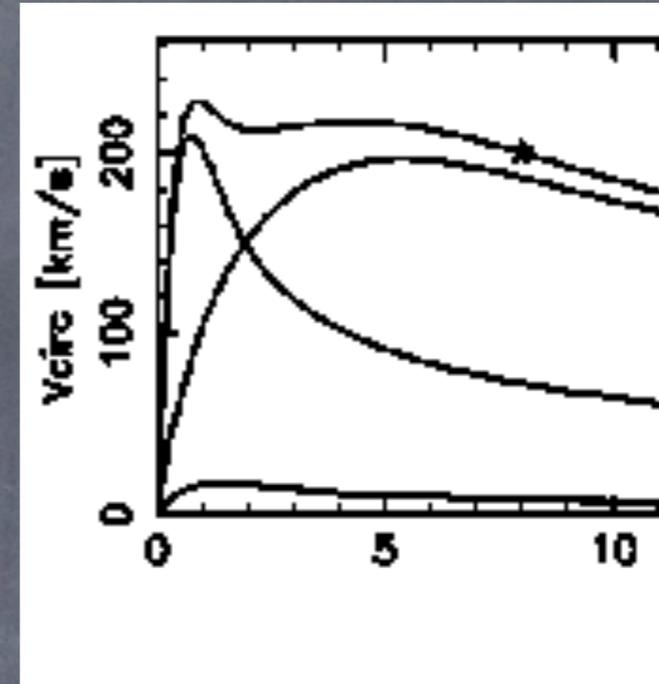
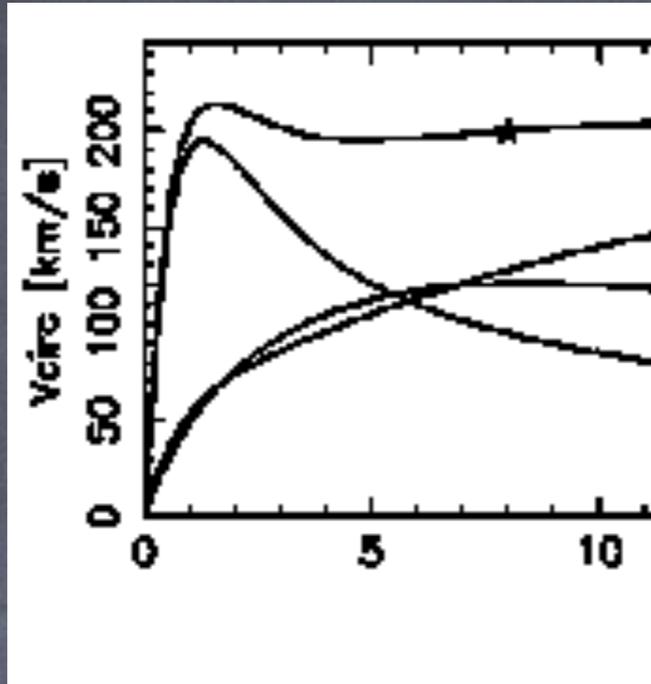
- Even in very flattened systems, the rotation curve mostly probes the enclosed mass (monopole) of the mass distribution
- example: exponential disk

$$\Sigma = \Sigma_0 e^{-r/h}$$



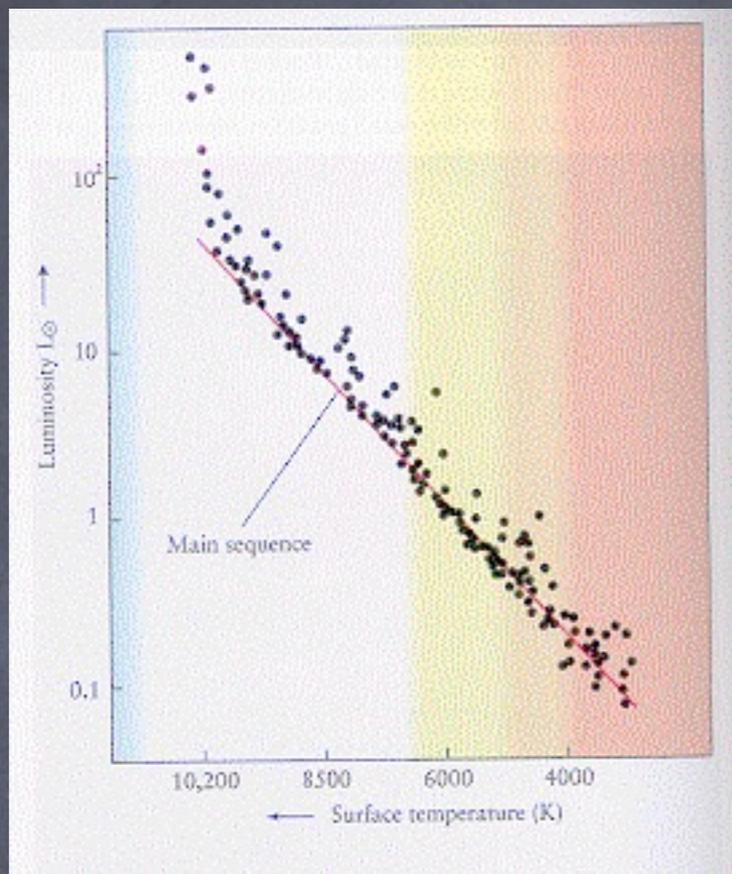
In finite systems,
Keplerian falloff
 $V=r^{-1/2}$ inevitable

Rotation curve decomposition



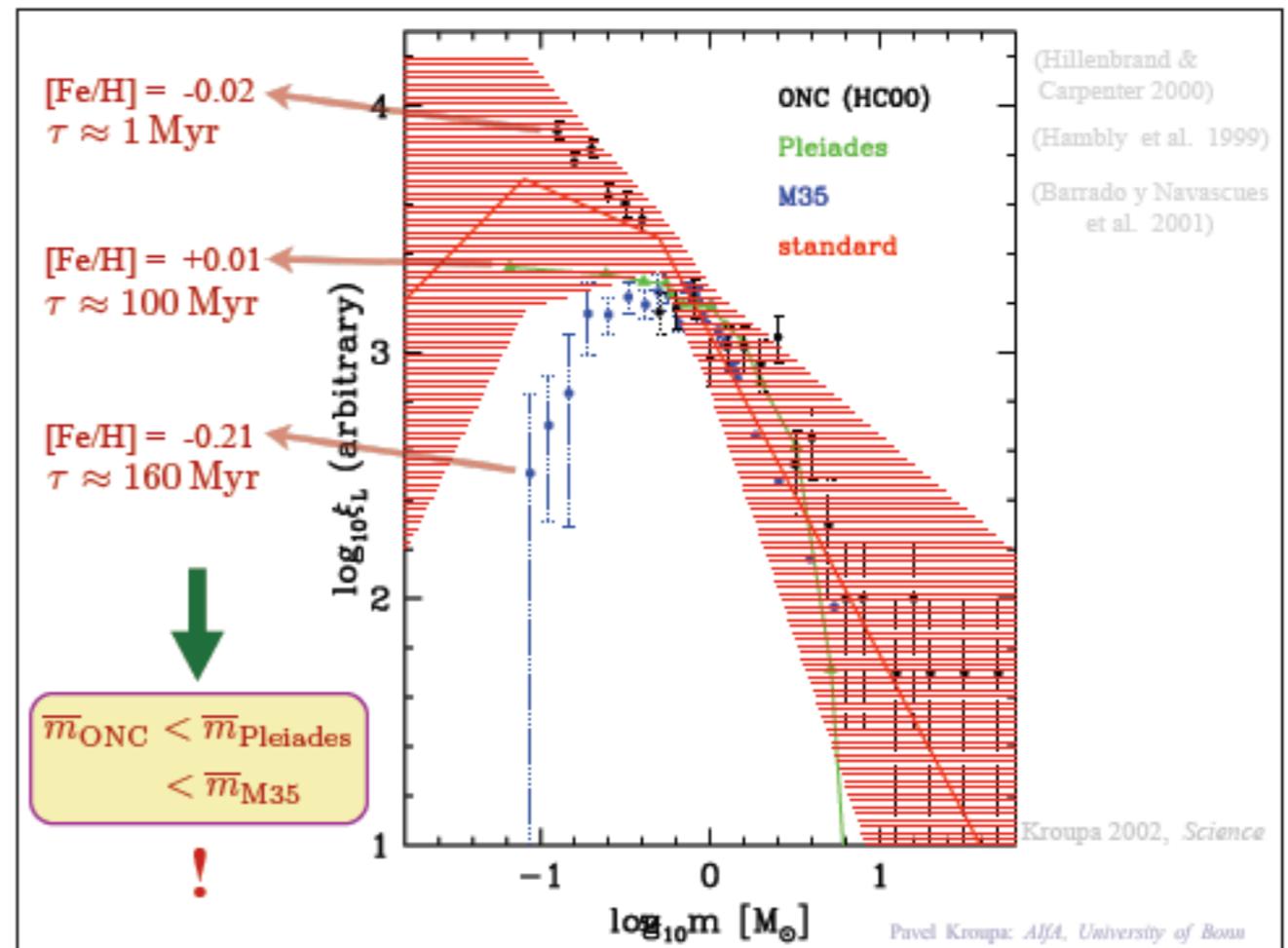
- Assign fraction of potential to 'visible' components
 - 'Maximum disk' decomposition ('minimum extra')
- Requires knowledge of Mass-to-Light ratio
 - WYSIWYG models not feasible in most cases

Stellar Mass Function



- Starlight is a poor tracer of stellar mass

- Most light emitted by most massive stars
- Most of the mass is 'invisible'



Direct measurements of M/L in disk galaxies



- Dynamics of self-gravitating thin disks
 - gravity field \sim vertical
 - vertical motions decouple from horizontal ones
- stars move in 1-D vertical potential $\psi(z)$

$$\nabla^2 \psi = 4\pi G \rho_m \Rightarrow \frac{d^2 \psi(z)}{dz^2} = 4\pi G \rho_m \Rightarrow \frac{d\psi(z)}{dz} = 2\pi G \Sigma_m (< |z|)$$

Direct measurements of M/L in disk galaxies

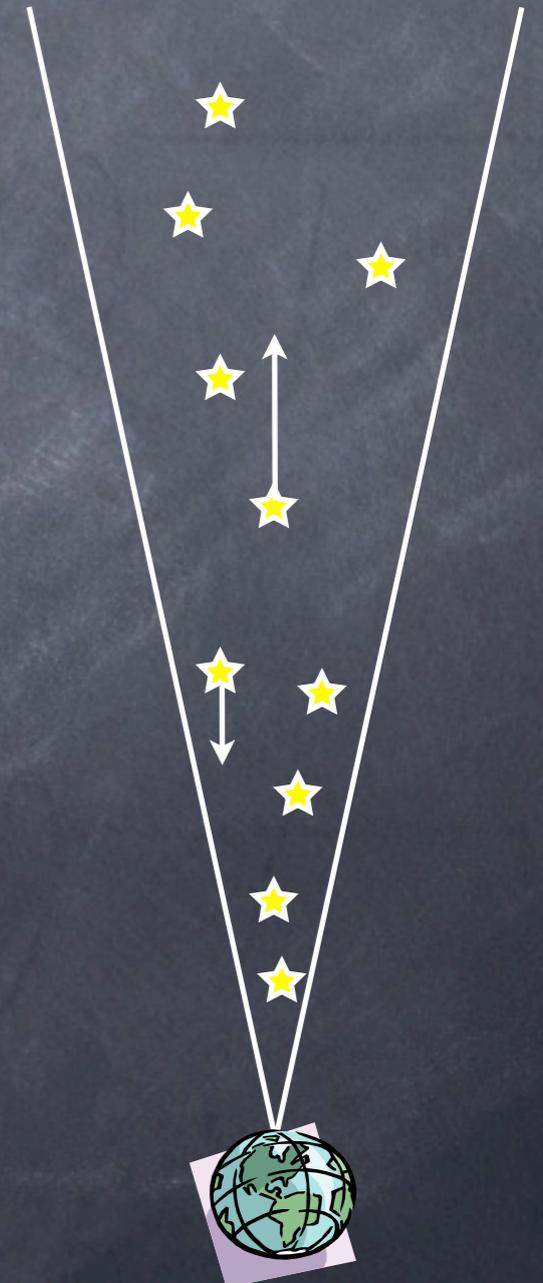


- Essentially compare mean orbital v_z^2 at $z=0$ with mean z reached
- Problem in practice:
 - need disk thickness (edge-on view)
 - velocity dispersion (face-on view)

Solar Neighbourhood



- Can measure 3D structure from star counts
- Can measure dynamics from individual stars
- Kapteyn (1922) – Oort (1932)



Kapteyn 1922

FIRST ATTEMPT AT A THEORY OF THE ARRANGEMENT AND MOTION OF THE SIDEREAL SYSTEM¹

By J. C. KAPTEYN²

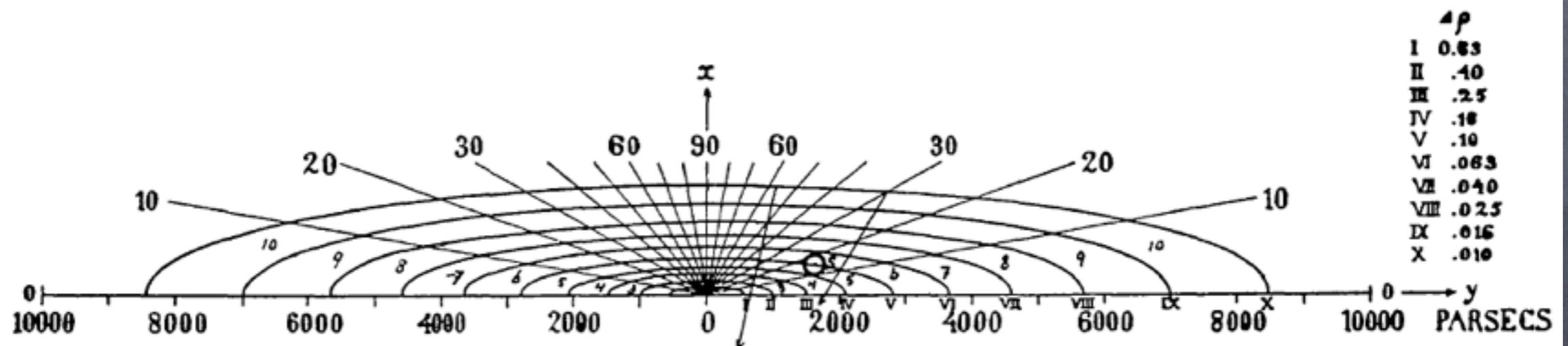


FIG. I

the relative velocity is also in the plane of the Milky way and about 40 km/sec. It is incidentally suggested that when the theory is perfected it may be possible to determine *the amount of dark matter* from its gravitational effect. (5) The *chief defects*

Oort 1932

BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1932 August 17

Volume VI.

No. 238.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems, by *J. H. Oort*.

11. *The amount of dark matter.*

From the results found for the decrease of $K(s)$ with s we may derive an approximate value of the total density of matter, Δ , in the neighbourhood of the sun. Let us suppose that we are situated inside a homogeneous ellipsoid of revolution with semi-axes a and c , and density Δ . For $s=0$ there will then be the following relation:

$$\partial K(s)/\partial s = -4\pi\gamma x\Delta \quad (14)$$

Case	Δ
I a)	·108
I b)	·093
II a)	·089
II b)	·079

total mass density (M_{\odot}/pc^3)

Stellar dynamics

- Describe stars with phase space density $f(x_i, v_j)$

$$\int f d^3 v = \rho; \quad \int v_z^2 f d^3 v = \rho \sigma_z^2;$$

- Collisionless Boltzmann equation

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \psi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0$$

- Ignore all x, y gradients

$$v_z \frac{df}{dz} = -K_z \frac{df}{dv_z}$$

- x, v_z , integrate over v_z :

$$\frac{d(\rho \sigma_z^2)}{dz} = K_z \rho$$

Stellar dynamics

- More generally, for an axisymmetric system in cylindrical polar coordinates (R, ϕ, z)

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + v_R \frac{\partial f}{\partial R} + v_z \frac{\partial f}{\partial z} + \left(K_R + \frac{v_\phi^2}{R} \right) \frac{\partial f}{\partial v_R} - \frac{v_R v_\phi}{R} \frac{\partial f}{\partial v_\phi} + K_z \frac{\partial f}{\partial v_z} = 0$$

- $\times v_i$, integrate gives Jeans equations

$$\frac{1}{R} \frac{\partial(R\rho\sigma_R^2)}{\partial R} + \frac{\partial(\rho\sigma_{Rz})}{\partial z} - \frac{\rho(v_\phi^2 + \sigma_\phi^2)}{R} = \rho K_R$$

$$\frac{\partial(\rho\sigma_z^2)}{\partial z} + \frac{\partial(\rho\sigma_{Rz})}{\partial R} = \rho K_z$$

- NB: in a thermalized system velocity dispersion is isotropic: $\sigma_{Rz}=0$; $\sigma_R=\sigma_z=\sigma_\phi$

Stellar dynamics

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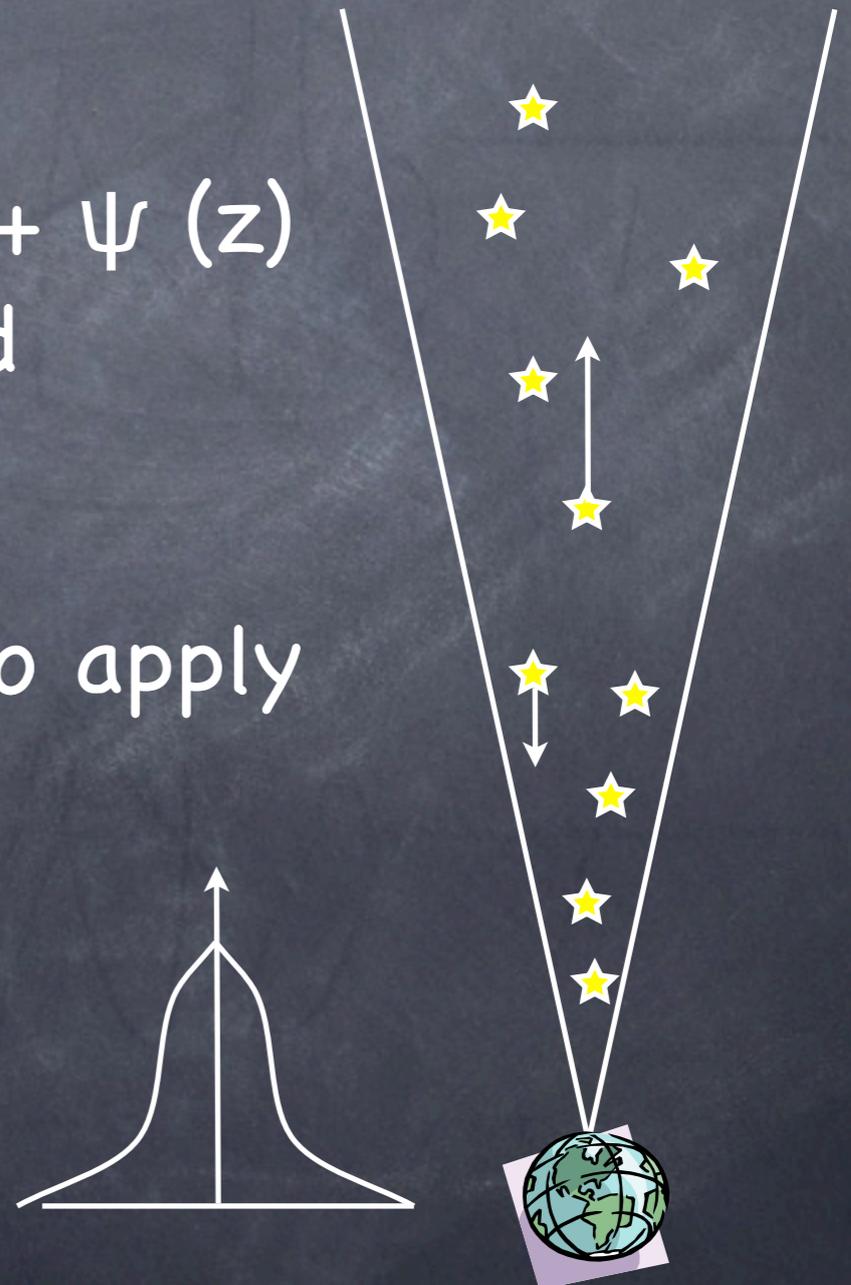
- NB: in a thermalized system velocity dispersion is isotropic: $\sigma_{Rz}=0$; $\sigma_R=\sigma_z=\sigma_\phi$

Vertical kinematics

$$v_z \frac{df}{dz} + \frac{d\psi(z)}{dz} \frac{df}{dv_z} = 0 \quad \Rightarrow \quad f(z, v_z) = f(\psi(z) + v_z^2) = f(E_z)$$

- Velocity distribution at $z=0 + \psi(z)$
==> velocity distribution (and density) at height z
- (avoids derivatives needed to apply the Jeans equation)

$$\frac{d(\rho\sigma_z^2)}{dz} = K_z \rho$$

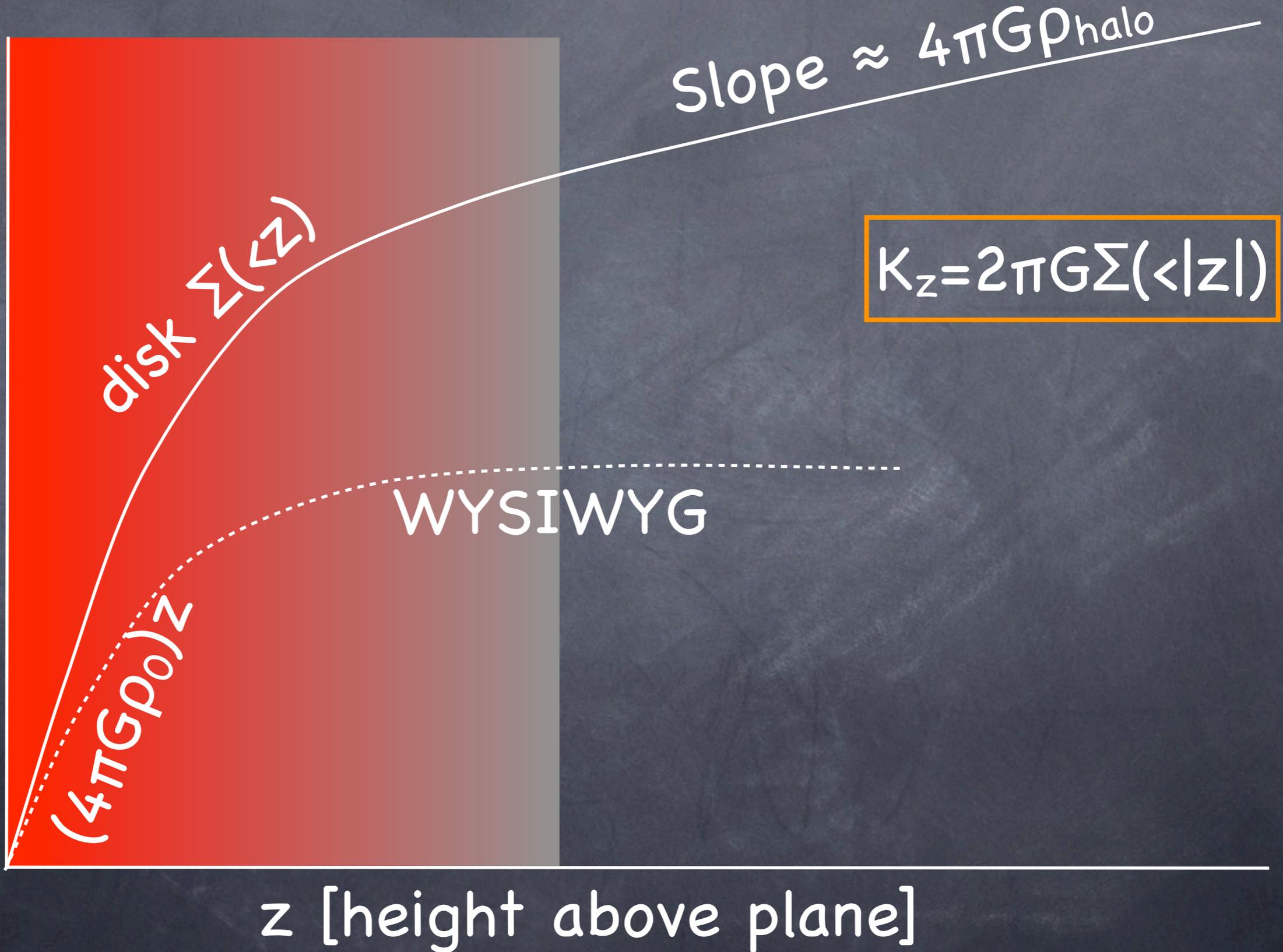


In practice

- Pick a good tracer population that probes the disk ($z < \sim 1 \text{ kpc}$)
 - numerous
 - sufficiently old, well-mixed
 - well-calibrated distances
 - good radial velocity measurements
- Lower main sequence stars (G – K dwarfs)
- Parameterize possible potentials
 - Known star populations + gas + dark disk + halo

Trial potentials

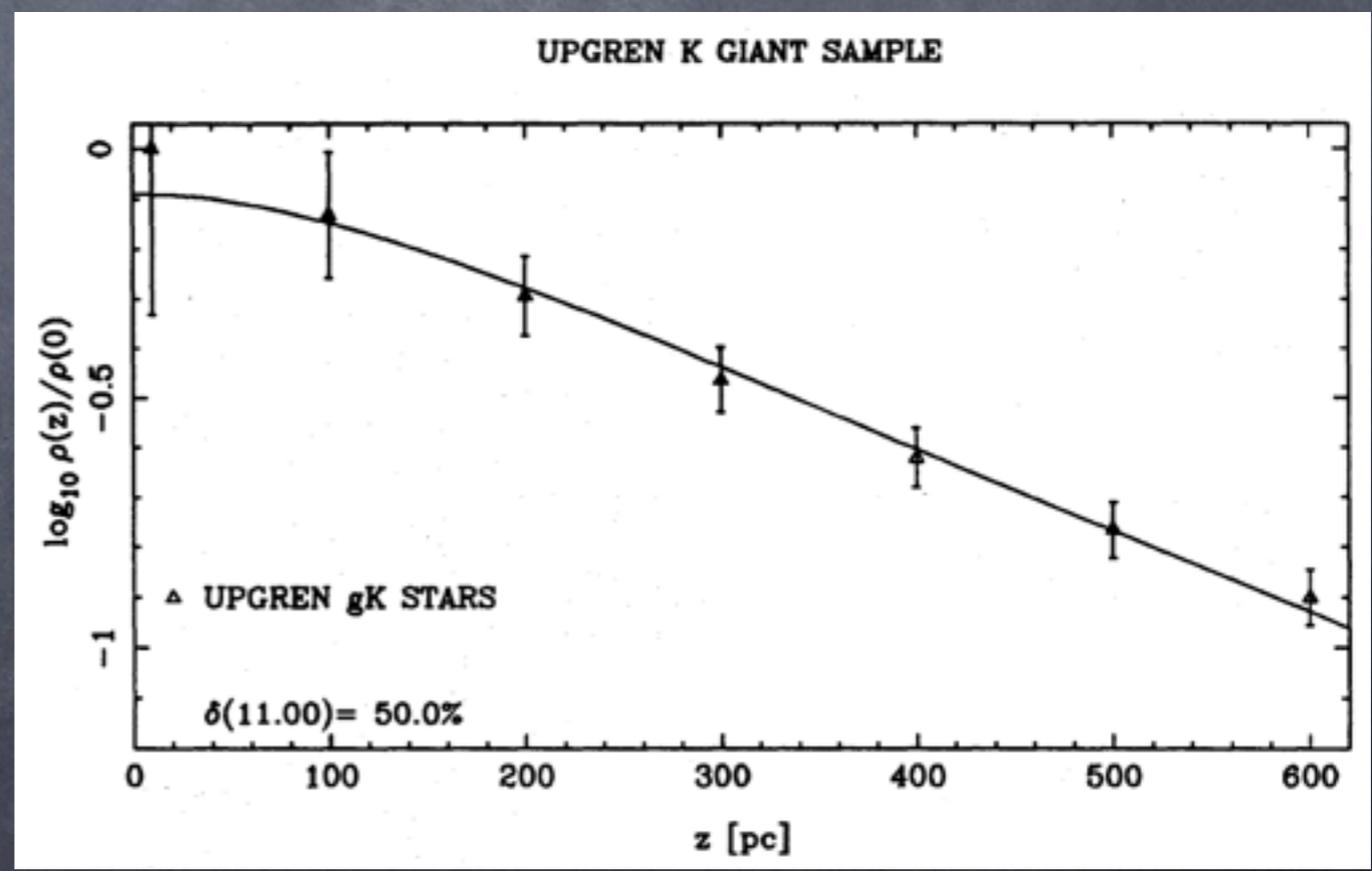
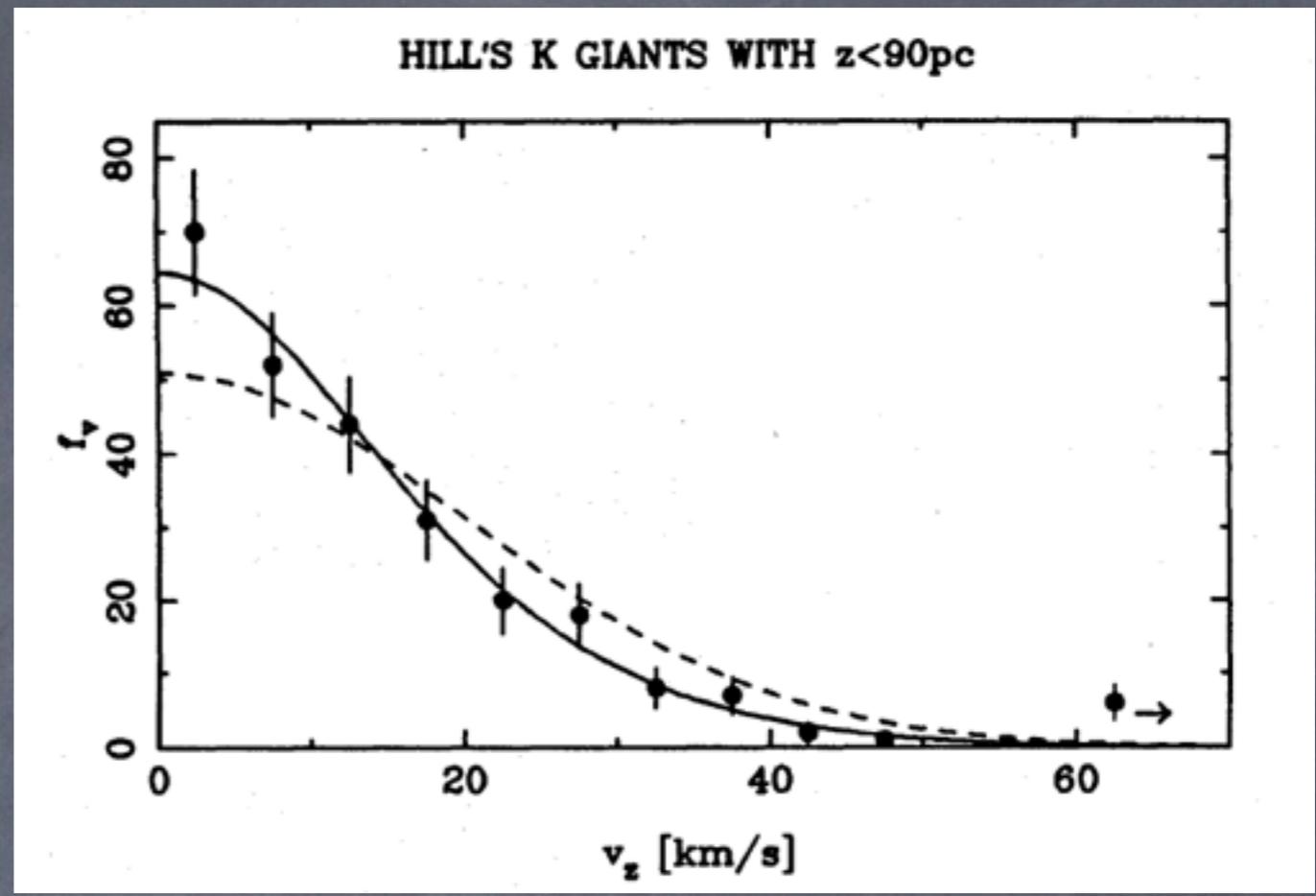
$|K_z|$ [vertical acceleration]



● Example (KK+Gilmore 1989)

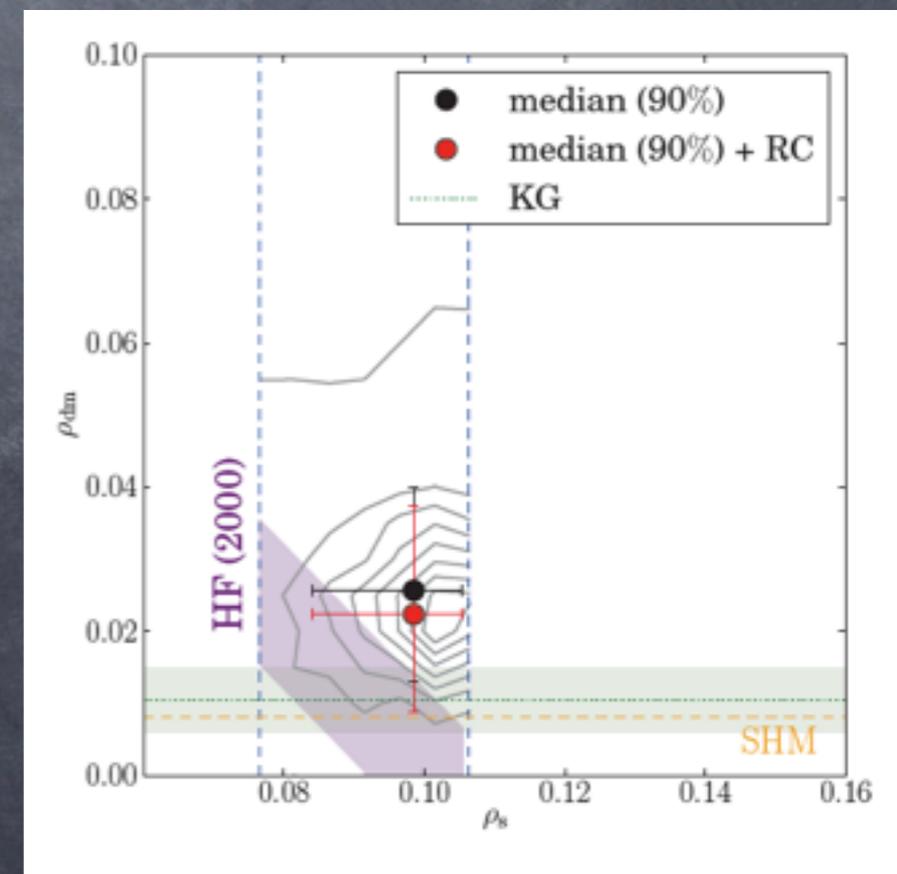
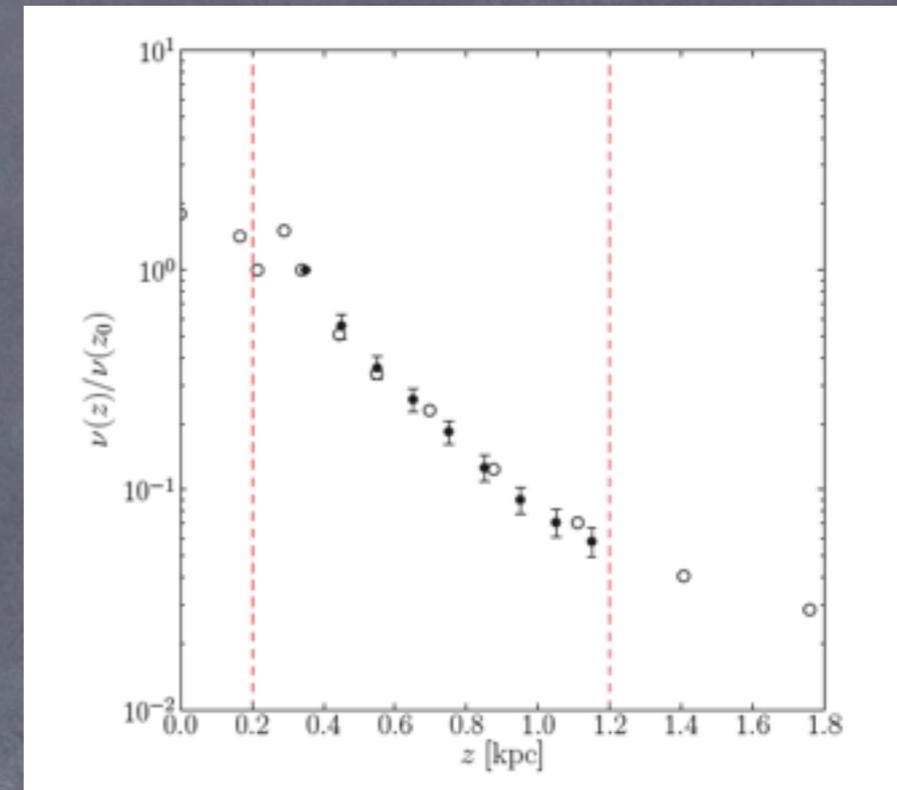
● $f(v)$ at $z=0$

● predicted $\rho(z)$ in a WYSIWYG potential



Disk dark matter?

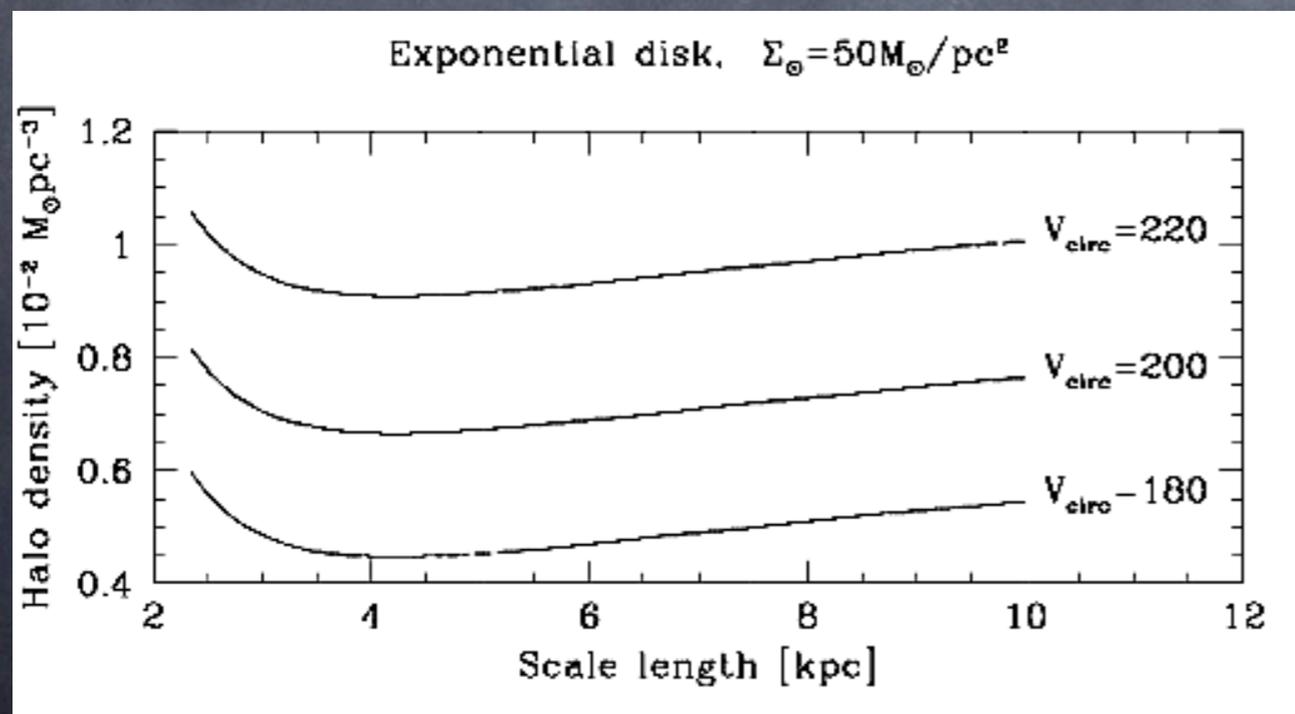
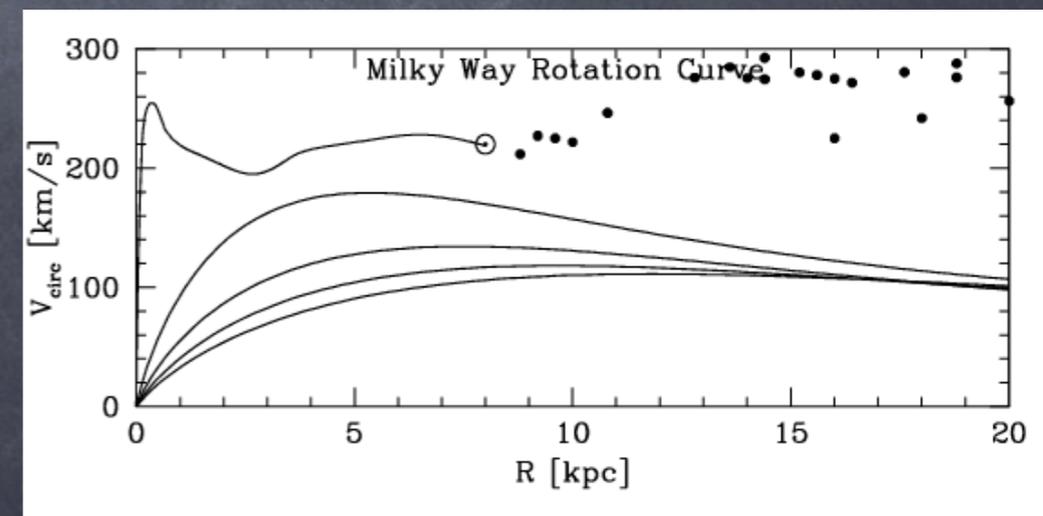
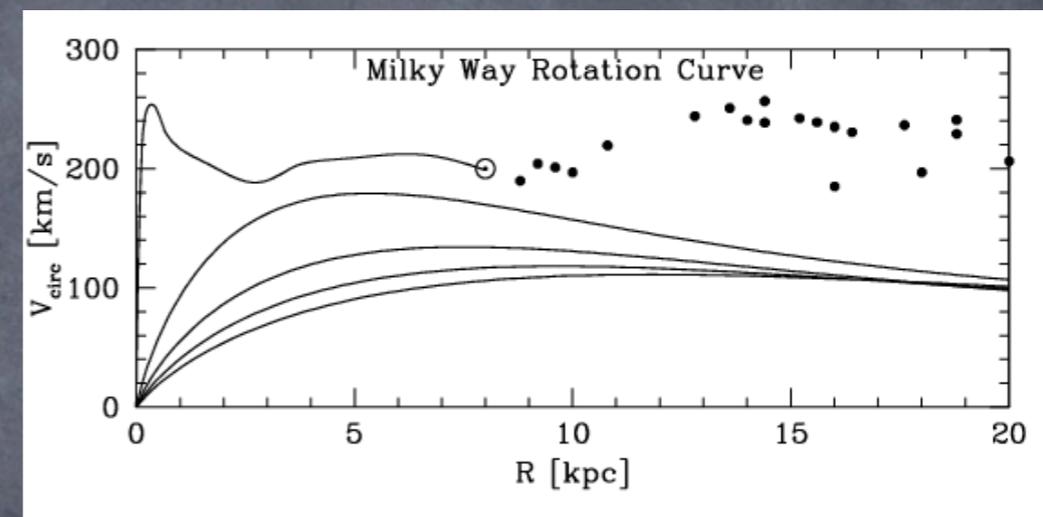
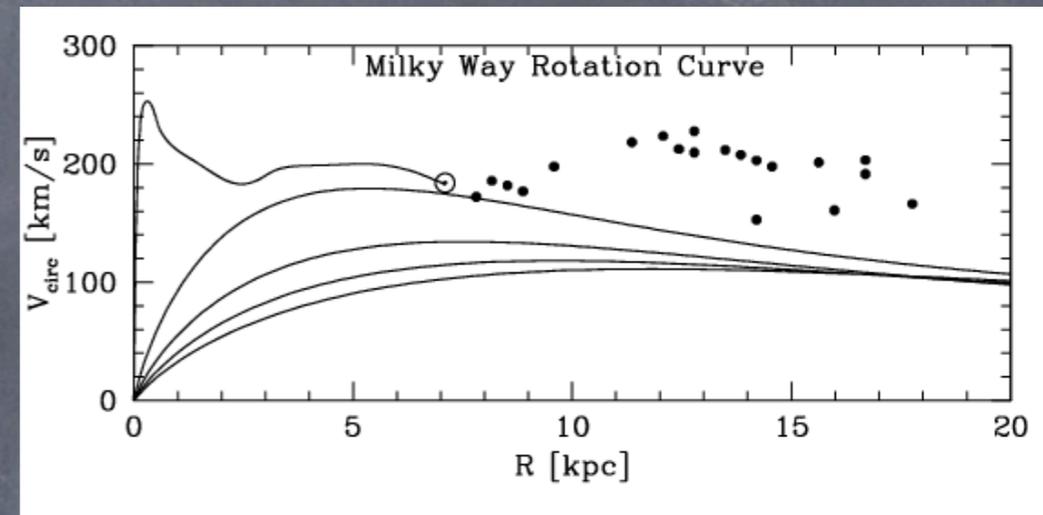
- Since 1990's, most analyses agree:
 - disk kinematics below $z=500\text{pc}$ consistent with known populations' gravity
 - disk volume density at $z=0$ is about $0.1M_{\odot}/\text{pc}^3$, 50-50 stars and gas
 - disk surf. density $50M_{\odot}/\text{pc}^2$
- Halo dark matter density?



Garbari et al 2012

Halo dark matter

- Milky Way rotation is a bit awkward to model
- Fix local disk surf. density
- Vary scale height, solar orbit speed, fit V_{circ} curve



(spherical halo model)

Using disk stars to probe halo dark matter

$$\frac{1}{R} \frac{\partial(R\rho\sigma_R^2)}{\partial R} + \frac{\partial(\rho\sigma_{Rz})}{\partial z} - \frac{\rho(v_\phi^2 + \sigma_\phi^2)}{R} = \rho K_R$$

$$\frac{\partial(\rho\sigma_z^2)}{\partial z} + \frac{\partial(\rho\sigma_{Rz})}{\partial R} = \rho K_z$$

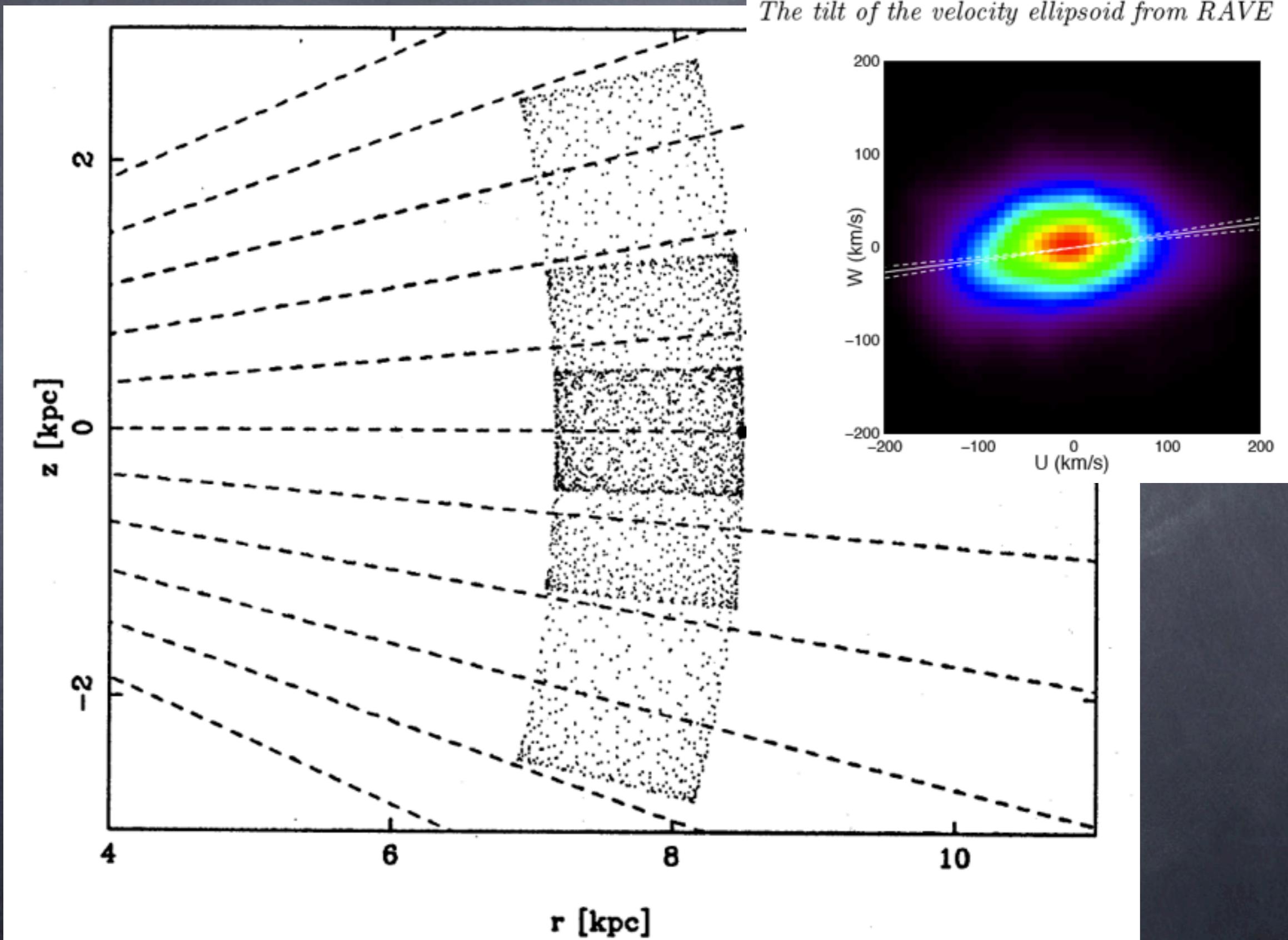
$$4\pi G\rho_m = -\frac{\partial K_z}{\partial z} - \frac{1}{R} \frac{\partial(RK_R)}{\partial R}$$

- At high z , no longer vertical-only kinematics
- Velocity ellipsoid tilts, is anisotropic
- Need to measure radial gradient of this tilt
- Also Poisson equation not separable



Orbit tilt

The tilt of the velocity ellipsoid from RAVE



Orbit tilt

$$\frac{\partial(\rho\sigma_z^2)}{\partial z} + \frac{\partial(\rho\sigma_{Rz})}{\partial R} = \rho K_z$$

- For a 2:1 radial:vertical velocity dispersion ratio

$$\sigma_{Rz} \simeq (\sigma_R^2 - \sigma_z^2) \frac{z}{R}$$

- Radial gradient set by disk scale length H

$$(\rho\sigma_{Rz})_{,R} \simeq (\sigma_R^2 - \sigma_z^2) \frac{2z}{HR}$$

- Vertical force due to halo is

$$F_h \frac{z}{R} K_R \simeq F_h \frac{z}{R^2} V_{\text{circ}}^2$$

$$\begin{aligned} \sigma_z &\approx 40 \text{ km/s} \\ V_{\text{circ}} &\approx 200 \text{ km/s} \\ H &\approx R/2 \end{aligned}$$

- Ratio halo K_z : 'fake tilt force'

$$F_h \frac{V_{\text{circ}}^2}{4\sigma_z^2} \frac{H}{2R} \simeq F_h$$

Moni Bidin et al. 2012

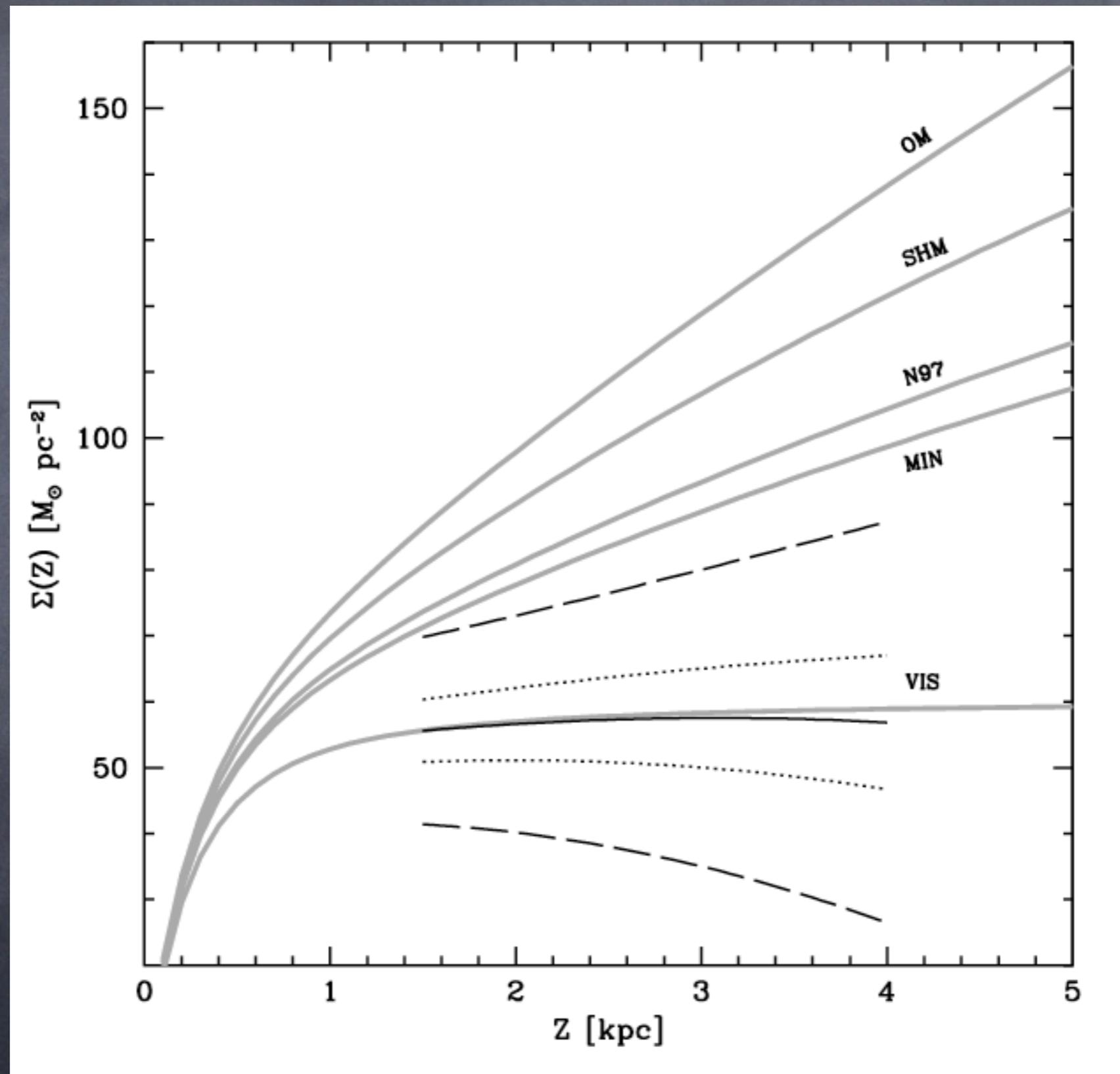
Kinematical and chemical vertical structure of the Galactic thick disk^{1,2}

II. A lack of dark matter in the solar neighborhood

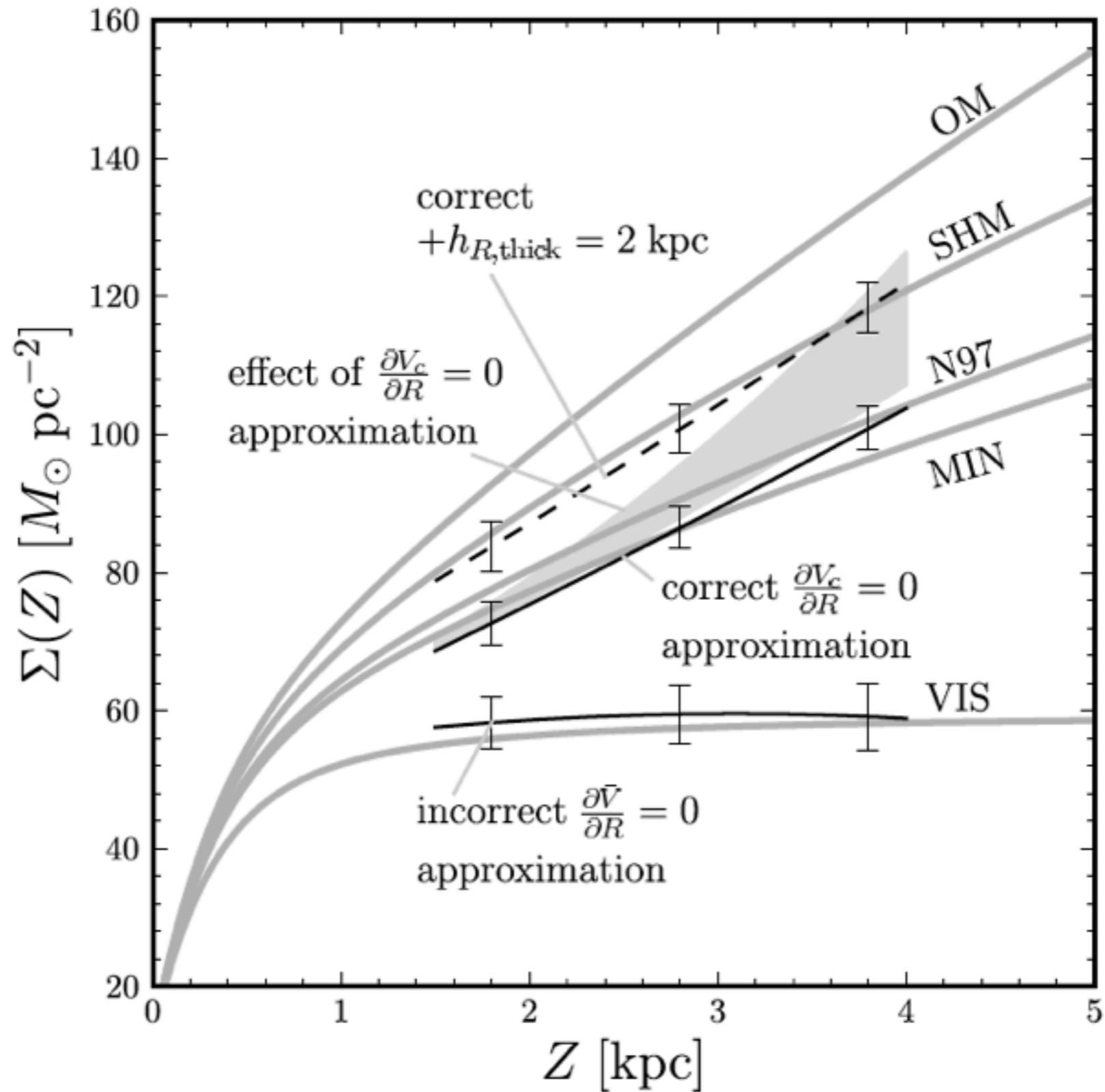
- Study of the vertical kinematics using red giant stars up to 4kpc from the disk plane.
- Large number of assumptions on missing terms; reasonable for disk population but uncertain
 - criticised by Bovy & Tremaine (2012)



Bovy & Tremaine 2012

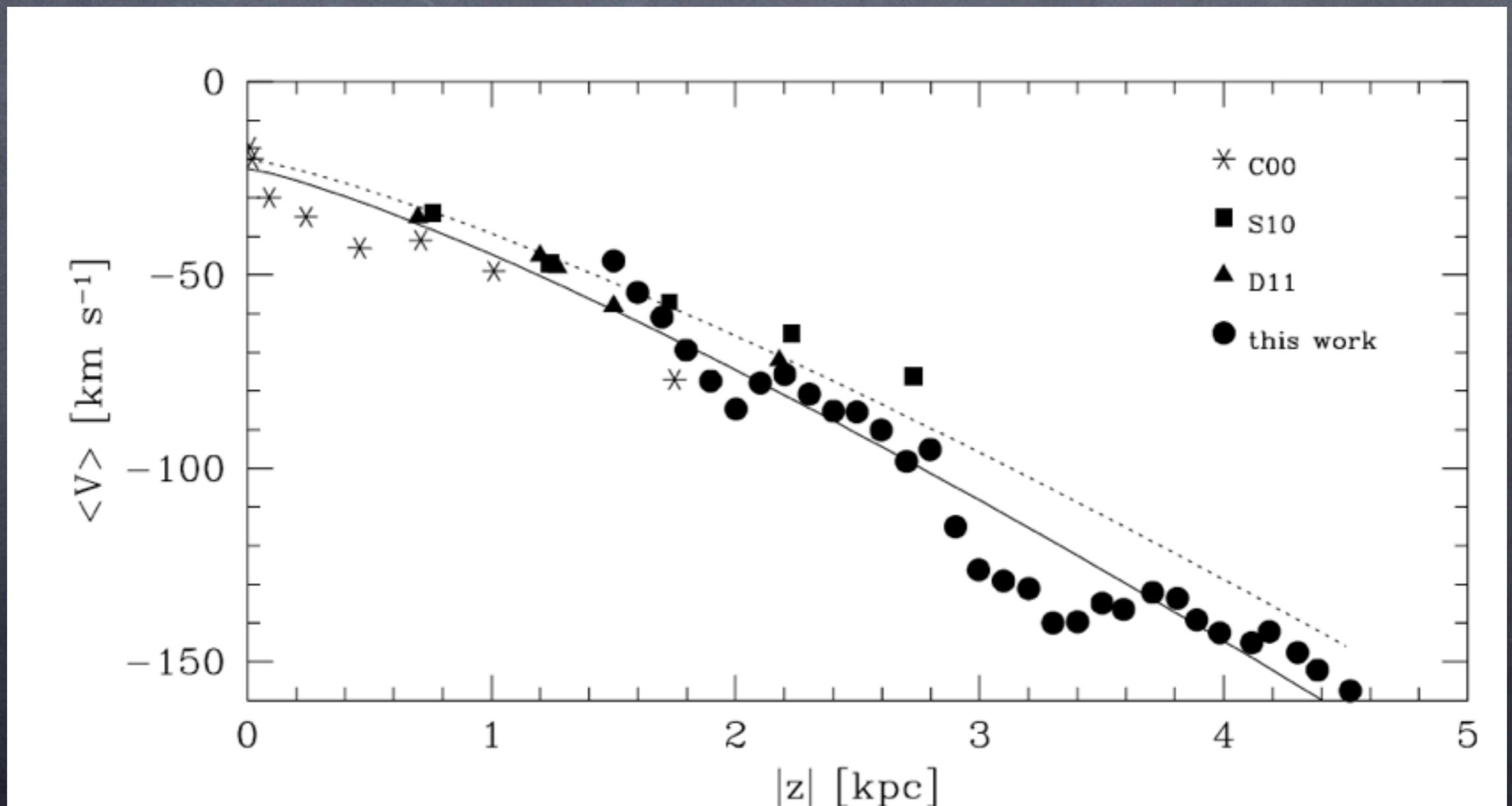


Bovy & Tremaine 2012



Moni Bidin et al. 2012

- Rotational velocity profile from MB & al Paper I
- $v_{\text{rot}} < 0.5v_{\text{circ}}$! Are these disk stars?



Not discussed here

- Halo flattening: barely affects rotation curve, but increases local density
 - prolate halo would lower it
- Local substructure: could lead to higher or lower densities
- Overall dark halo structure (size, extent, shape)

Summary

- Long history of measuring local DM density
- No convincing evidence for 'cold' DM component in the disk (as thin as a few 100 pc)
- Halo density derived from vertical kinematics unreliable:
 - velocity ellipsoids tilt, radial Poisson eq. terms
 - Needs a full picture of 3-D kinematics (Gaia!)
- Rotation curve decompositions lead to estimates of **0.005 - 0.01 M_{\odot}/pc^3 .**