Galactic Rotation S&G Sec 2.3

- The majority of the motions of the stars in the MW is rotational
- Prime way of measuring mass of spiral galaxies
- Map out the distribution of galactic gas
- Strong evidence for dark matter

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Galactic Rotation

- See SG 2.3, BM ch 3 B&T ch 2.6.2.7 and ch 6
- Coordinate system: define velocity vector by $\pi, \theta, z$
  - $\pi$ radial velocity wrt galactic center
  - $\theta$ motion tangential to GC with positive values in direct of galactic rotation
  - $z$ motion perpendicular to the plane, positive values toward North galactic pole
- Origin is the galactic center (center or mass/rotation)
- Local standard of rest (BM pg 536)
- Velocity of a test particle moving in the plane of the MW on a closed orbit that passes thru the present position of the Sun

B&T fig 1.3

If the galaxy is axisymmetric and in steady state then each pt in the plane has a velocity corresponding to a circular velocity around center of mass of MW

$$(\pi, \theta, z)_{\text{LSR}} = (0, \theta_0, 0)$$

with $\theta_0 = \theta_0(R)$
Local Standard of Rest

The Sun (and most stars) are on slightly perturbed orbits that resemble rosettes making it difficult to measure relative motions of stars around the Sun.

Establish a reference frame that is a perfect circular orbit about the Galactic Center.

Local Standard of Rest - reference frame for measuring velocities in the Galaxy.
Position of the Sun if its motion were completely governed by circular motion around the Galaxy.

Use cylindrical coordinates for the Galactic plane to define the Sun's motion w.r.t the Local Standard of Rest

Coordinate Systems

The stellar velocity vectors are
z: velocity component perpendicular to plane
θ: motion tangential to GC with positive velocity in the direction of rotation
π: radial velocity wrt to GC
With respect to galactic coordinates
+π= (l=180,b=0)
+θ= (l=30,b=0)
+z= (b=90)
Description of Galactic Rotation (S&G 2.3)

- For circular motion: relative angles and velocities observing a distant point
- T is the tangent point
  \[ V_r = R_0 \sin(V/R - V_0/R_0) \]
  Because \( V/R \) drops with \( R \) (rotation curve is ~flat); for value \( 0 < l < 90 \) or \( 270 < l < 360 \) reaches a maximum at \( T \)
  So the process is to find \( V_r^{\text{max}} \) for each \( l \)
  and deduce \( V(R) = V_r + R_0 \sin l \)

For \( R > R_0 \): rotation curve from HI or CO is degenerate; use masers, young stars with known distances

Galactic Rotation

- Then using a bit of trig
  \[
  R(\cos \alpha) = R_0 \sin(l) \\
  R(\sin \alpha) = R_0 \cos(l) - d \\
  \]
  so
  \[
  V_{\text{observered}}(\text{radial}) = (\omega - \omega_0) R_0 \sin(l) \\
  V_{\text{observered}}(\text{tang}) = (\omega - \omega_0) R_0 \cos(l) - \omega d \\
  \]
  then following the text expand \((\omega - \omega_0)\)
  around \( R_0 \) and using the fact that most of the velocities are local e.g. \( R - R_0 \) is small
  and \( d \) is smaller than \( R \) or \( R_0 \) (not TRUE for HI) and some more trig
  get
  \[
  V_{\text{observered}}(\text{radial}) = A \sin(2l); V_{\text{observered}}(\text{tang}) = A \cos(2l) + B d \\
  \]
  Where
  \[
  A = -1/2 \ R_0 \omega/\text{dr at} \ R_0 \\
  B = -1/2 \ ((R_0 \omega/\text{dr}) - \omega) \\
  \]
Galactic Rotation Curve- sec 2.3.1 S+G

Assume gas/star has a perfectly circular orbit

At a radius $R_0$ orbit with velocity $V_0$, another star/parcel of gas at radius $R$ has an orbital speed $V(R)$

since the angular speed $V/R$ drops with radius $V(R)$ is positive for nearby objects with galactic longitude $1 < l < 90$ etc etc (pg 91 bottom)

In terms of Angular Velocity

- model Galactic motion as circular motion with monotonically decreasing angular rate with distance from center.
- Simplest physics: if the mass of the Galaxy is all at center angular velocity $\omega$ at $R$ is $\omega = M^{1/2}G^{1/2}R^{-3/2}$
- If looking through the Galaxy at an angle $l$ from the center, velocity at radius $R$ projected along the line of site minus the velocity of the sun projected on the same line is
  $V = \omega R \sin \delta - \omega_0 R_0 \sin l$  [$l$ is galactic longitude (in figure this is angle $\gamma$)]
- $\omega$ = angular velocity at distance $R$
- $\omega_0$ = angular velocity at a distance $R_0$
- $R_0$ = distance to the Galactic center
- $l$ = Galactic longitude
- Using trigonometric identity $\sin d = R_0 \sin l/R$ and substituting into equation (1)

  $V = (\omega - \omega_0) R_0 \sin l$

Continued

- The tangential velocity $v_T = V \sin \alpha - V_o \cos l$
  and $R \sin \alpha = R_o \cos l - d$
- A little algebra then gives
  $v_T = V/R (R_o \cos l - d) - V_o \cos l$
- Re-writing this in terms of angular velocity
  $v_T = (\omega - \omega_o) R_o \cos l - \omega d$

- For a reasonable galactic mass distribution
  we expect that the angular speed $\omega = V/R$ is
  monotonically decreasing at large $R$ (most
galaxies have flat rotation curves (const $V$)
at large $R$) then get a set of radial velocities
  as a function of where you are in the galaxy
- $v_T$ is positive for $0 < l < 90$ and nearby objects-
  if $R > R_o$ it is negative
- For $90 < l < 180$ $v_T$ is always negative
- For $180 < l < 270$ $v_T$ is always positive ($S+G$
  sec 2.3.1)

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(1) $0 < l < 90$
(2) $90 < l < 180$
(3) $180 < l < 270$
(4) $270 < l < 360$
Derivation:
- for objects near to sun, use a Taylor series expansion of \( \omega - \omega_o \)
  \[ \omega - \omega_o = d\omega / dR \ (R - R_o) \]
  \[ \omega = V/R; \ d\omega / dR = d/dr(V/R) = (1/R)dV/dr-V/R^2 \]

then to first order
\[ V_r = (\omega - \omega_o)R_o \sin l = (dV/dr-V/R)(R-R_o) \sin l \]
when \( d << R_o \)
\[ R-R_o = d \cos l \]
which gives
\[ V_r = (V_o/R_o - dV/dr)d \sin l \cos l \]

using trig identity \( \sin l \cos l = 1/2 \sin 2l \)
one gets the Oort formula
\[ V_r = A \sin 2l \]

One can do the same sort of thing for \( V_T \)

Oort Constants (MBW pg 439)
- For nearby objects \( d << R \) then \( l \) is the galactic longitude
  \[ V(R) = R_0 \sin l \ (dV/dr)(R-R_0) \]
  \[ \sim d \sin(2l) - R/2(dV/dr) \sim -dA \sin(2l) \]

- \( A \) is one of 'Oorts constants'
- The other Oort constant \( B \) (pg 93 S+G) is related to the tangential velocity of an object near the sun \( V_t = d[A \cos(2l) + B] \)
- So, stars at the same distance \( r \) will show a systematic pattern in the magnitude of their radial velocities across the sky with Galactic longitude.

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\[ A = \frac{1}{2} \left[ \frac{V_o}{R_o} - \left( \frac{dV}{dR} \right)_{R_o} \right] \]
\[ B = -\frac{1}{2} \left[ \frac{V_o}{R_o} + \left( \frac{dV}{dR} \right)_{R_o} \right] \]
\[ A + B = -\left( \frac{dV}{dR} \right)_{R_o}; \ A - B = \frac{V_o}{R_o} \]

A=-1/2[Rd\omega/dr]

Useful since if know \( A \) get kinematic estimate of \( d \)
Radial velocity \( v_r \sim 2AR_0(1-\sin l) \)
only valid near \( l \sim 90 \) measure
\( AR_0 \sim 115 \text{ km/s} \)
Radial velocity components projected onto the line of sight:

\[ v_R = \theta v_c \sin l - \Delta V \sin \xi = v_c \sin l - \frac{dv}{dr} \sin l \]

\[ = r \sin l \left( \frac{v_c}{R_\alpha} \frac{dv}{dr} - A \right) \sin l \]

where \( A = \left( \frac{v_c}{R_\alpha} \frac{dv}{dr} \right)_t \) is Oort's first (shear) constant.

Transverse velocity components projected onto the sky (proper motion):

\[ v_\theta = -\theta v_c \sin l - \Delta V \cos \xi = \frac{v_c}{R_\alpha} \sin^2 l - \frac{dv}{dr} \cos^2 l \]

\[ = \left( \frac{dv}{dr} \right)_\alpha \frac{v_c}{R_\alpha} \left( \cos^2 l \sin^2 l \right) - \left( \frac{dv}{dr} \right)_\alpha \frac{v_c}{R_\alpha} \left( \cos^2 l \sin^2 l \right) + \frac{V_c}{R_\alpha} \frac{dv}{dr} \cos 2l \]

where \( B = \left( \frac{v_c}{R_\alpha} \frac{dv}{dr} \right)_t \) is Oort's second (rotation) constant.

A measures the local shear: the degree to which stars slide past each other. E.g. solid body has no shear, since \( \frac{dv}{dr} = \frac{V}{R} \), so \( A = 0 \). Note a flat rotation curve (\( \frac{dv}{dr} = 0 \)) does have shear, with \( A = \frac{V}{R} \).

B measures local rotation, or vorticity. It comes from the curl of the velocity field: \( B = \frac{1}{2} \nabla \times v \). E.g. solid body is pure rotation, at the angular velocity of the disk: \( B = -\frac{V}{R} \).

**Circular Galactic Rotation.** Derivation of Oort's constants: A & B

\[ \Delta V = V(R_\alpha) - V(R) \]

\[ = \Delta R \times \frac{dV}{dR} \]

\[ = r \cos l \times \frac{dV}{dR} \]

\[ \theta_\nu = \theta - \sin l \]

\[ \Delta V = V(R_\alpha) - V(R) \]

\[ = \Delta R \times \frac{dV}{dR} \]

\[ = r \cos l \times \frac{dV}{dR} \]

where \( R_\alpha \) is Oort's local standard of rest (sun's motion).

**Oort 'B'

- B measures 'vorticity' \( B = -(\omega = -1/2[R \omega/dr]) = -1/2[(V/R)+(dV/dr)] \) angular momentum gradient

\[ \omega = A - B = \frac{V}{R} \]; angular speed of Local standard of rest (sun's motion)

**Oort constants are local description of differential rotation**

Values

\( A = 14.8 \text{ km/s/kpc} \)

\( B = 12.4 \text{ km/s/kpc} \)

**Velocity of sun \( V_0 = R_0(A - B) \)**

I will not cover epicycles (stars not on perfect circular orbits) now (maybe next lecture): see sec pg 133ff in S&G
Onto Next Part ...

- Chapter 3 of S&G 'the orbits of stars'
- Detailed dynamics of the Milky Way