THE GALAXY

Notes for Lecture Courses ASTM002 and MAS430

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Chapter 1

Introducing Galaxies

Galaxies are a slightly difficult topic in astronomy at present. Although a great amount is known about them, galaxies are much less well understood than, say, stars. There remain many problems relating to their formation and evolution, and even with aspects of their structure. Fortunately, extragalactic research currently is very active and our understanding is changing, and improving, noticeably each year.

One reason that galaxies are difficult to understand is that they are made of three very different entities: stars, an interstellar medium (gas with some dust, often abbreviated as ISM), and dark matter. There is an interplay between the stars and gas, with stars forming out of the gas and with gas being ejected back into the interstellar medium from evolved stars. The dark matter affects the other material through its strong gravitational potential, but very little is known about it directly. We shall study each of these in this course, and to some extent how they influence each other.

An additional factor complicating an understanding of galaxies is that their evolution is strongly affected by their environment. The gravitational effects of other galaxies can be important. Galaxies can sometimes interact and even merge. Intergalactic gas, for example that found in clusters of galaxies, can be important. These various factors, and the feedback from one to another, mean that a number of important problems remain to be solved in extragalactic science, but this gives the subject its current vigour.

One important issue of terminology needs to be clarified at the outset. The word "Galaxy" with a capital 'G' refers to our own Galaxy, i.e. the Milky Way Galaxy, as does the word Galactic. In contrast, "galaxy", "galaxies" and "galactic" with a lower-case 'G' refer to other galaxies and to galaxies in general.

1.1 Galaxy Types

Some galaxies (more of them in earlier epochs) have *active nuclei* which can vastly outshine the starlight. We shall not go into that here – we shall confine ourselves to *normal* galaxies and ignore active galaxies.

There are three broad categories of normal galaxies:

- elliptical galaxies (denoted E);
- disc galaxies, i.e. spiral (S) and lenticular (S0) galaxies;
- irregular galaxies (I or Irr).

Classifications often include *peculiar* galaxies which have unusual shapes. These are mostly the result of interactions and mergers.

These classifications are based on the shapes and structures of galaxies, i.e. on the *morphology*. They are therefore known as *morphological types*.

1.2 Disc Galaxies

Disc galaxies have prominent flattened discs. They have masses of $10^9 M_\odot$ to $10^{12} M_\odot$. They include spiral galaxies and S0 (or lenticular) galaxies. Spirals are gas rich and this gas takes part in star formation. S0s on the other hand have very little gas and no star formation, while their discs are more diffuse than those of spirals.

1.2.1 Spiral galaxies

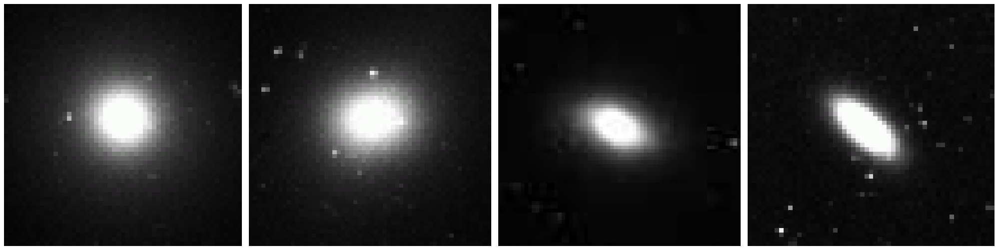
Spiral galaxies have much gas within their discs, plus some embedded dust, which amounts to 1-20% of their visible mass (the rest of the visible mass is stars). This gas shows active star formation. The discs contain stars having a range of ages as a result of this continuing star formation. Spiral arms are apparent in the discs, defined by young, luminous stars and by HII regions. Spiral galaxies have a central *bulge* component containing mostly old stars. These bulges superficially resemble small elliptical galaxies. The disc and bulge are embedded in a fainter halo component composed of stars and globular clusters. The stars of this stellar halo are very old and metal-poor (deficient in chemical elements other than hydrogen and helium). Some spirals have bars within their discs. These are called *barred spirals* and are designated type SB. Non-barred, or *normal* spirals are designated type S or SA. The spectra are dominated by F- and G-type stars but also show prominent emission lines from the gas: the spectra show the absorption lines from the stars with the emission lines superimposed. The spiral disc is highly flattened and the gas is concentrated close to the plane. The spiral arms are associated with regions of enhanced gas density that are usually caused by density waves. Discs rotate, with the stars and gas in near-circular orbits close to the plane, having circular velocities $\sim$ 200 to 250 kms^{-1}. In contrast, the halo stars have randomly oriented orbits with speeds $\sim$ 250 kms^{-1}. Spiral galaxies are plentiful away from regions of high galaxy density (away from the cores of galaxy clusters). All disc galaxies seem to be embedded in much larger dark haloes; the ratio of total mass to visible stellar mass is $\simeq 5$, but we do not really have a good mass estimate for any disc galaxy.

The disc's surface brightness I tends to follow a roughly exponential decline with radial distance R from the centre, i.e.,

$$I(R) = I_0 \exp(-R/R_0)$$

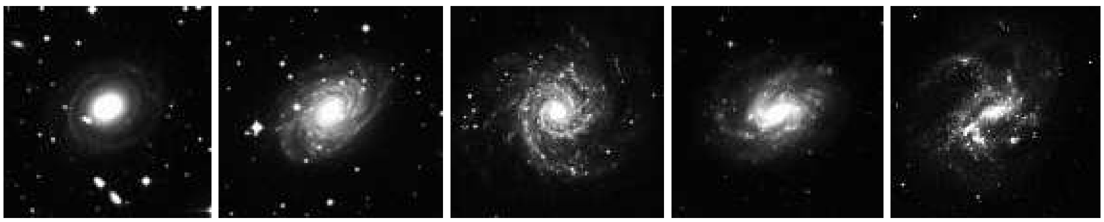
where $I_0 \sim 10^2 L_\odot\,\mathrm{pc}^{-2}$ is the central surface brightness, and R_0 is a scale length for the decline in brightness. The scale length R_0 is $\simeq 3.5\,\mathrm{kpc}$ for the Milky Way.

There are clear trends in the properties of spirals. Those containing the smallest amounts of gas are called subtype Sa and tend to have large, bright bulges compared to the discs, tightly wound spiral arms, and relatively red colours. More gas-rich spirals, such as subtype Sc, have small bulges, open spiral arms and blue colours. There is a gradual variation in these properties from subtype Sa through Sab, Sb, Sbc, to Sc.



E0	**E2**	**E4**	**E6**
NGC 1407	NGC 1395	NGC 584	NGC 4033

Figure 1.1: the sequence of morphological types of elliptical galaxies. Elliptical galaxies are classified according to their shape. [Created with blue-band data from the SuperCOSMOS Sky Survey.]



Sa	**Sb**	**Sc**	**Sd**	**Sm**
ESO286-G10	NGC 3223	M74	NGC 300	NGC 4395

Figure 1.2: the sequence of normal (non-barred) spiral types. Spiral galaxies are classified according to how tightly would their arms are, the prominence of the central bulge, and the quantity of interstellar gas. [Created with blue-band data from the SuperCOSMOS Sky Survey.]

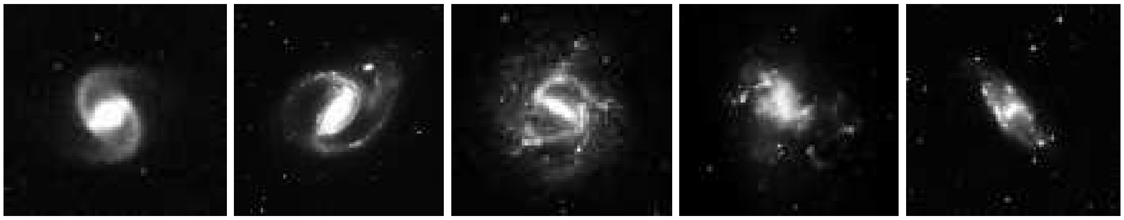
Some classification schemes include more extreme subtypes Sd and Sm. Subtypes of barred spirals are denoted SBa, SBab, SBb, SBbc, SBc, ...

Because the spiral arms mark regions of recent star formation, the stars in the arms are young and blue in colour. Therefore, spiral arms are most prominent when a galaxy is observed in blue or ultraviolet light, and less prominent when observed in the red or infrared. Figure 1.5 shows three images of a spiral galaxy recorded through blue, red and infrared filters. The spiral pattern is strong in the blue image, but weak in the infrared.

Observations show that there is a correlation between luminosity L (the total power output of galaxies due to emitted light, infrared radiation, ultraviolet etc.) and the maximum rotational velocity v_{rot} of the disc for spiral galaxies. The relationship is close to

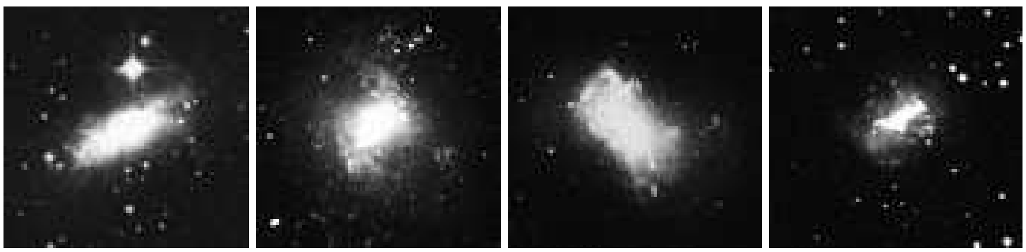
$$L \propto v_{rot}^4 \ .$$

This is known as the Tully-Fisher relationship. It is important because it allows the luminosity L to be calculated from the rotation velocity v_{rot} using optical or radio spectroscopy. A comparison of the luminosity and the observed brightness gives the distance to the galaxy.



SBa NGC 4440 | **SBb** NGC 1097 | **SBc** NGC 1073 | **SBd** NGC 1313 | **SBm** NGC 4597

Figure 1.3: the sequence of barred spiral types. [Created with blue-band data from the SuperCOSMOS Sky Survey.]



NGC 1569 | NGC 4214 | NGC 4449 | NGC 7292

Figure 1.4: examples of irregular galaxies. [Created with blue-band data from the SuperCOSMOS Sky Survey.]

Blue | Red | Infrared

Figure 1.5: the spiral galaxy NGC2997 in blue, red and infrared light, showing that the prominence of the spiral arms varies with the observed wavelength of light. The blue spiral arms are most evident in the blue image, while the old (red) stellar population is seen more clearly in the infrared image. [The picture was produced using data from the SuperCOSMOS Sky Survey of the Royal Observatory Edinburgh, based on photography from the United Kingdom Schmidt Telescope.]

1.2.2 S0 galaxies

S0 galaxies, sometimes also known as *lenticular galaxies*, are flattened disc systems like spirals but have very little gas or dust. They therefore contain only older stars. They have probably been formed by spirals that have lost or exhausted their gas.

1.3 Elliptical Galaxies

These have masses from $10^{10} M_\odot$ to $10^{13} M_\odot$ (not including dwarf ellipticals which have lower masses). They have elliptical shapes, but little other structure. They contain very little gas (and therefore very little dust), so almost all of the visible component is in the form of stars. With so little gas, there is no appreciable star formation, with the result that elliptical galaxies contain almost only old stars. Their colours are therefore red. K-type giant stars dominate the visible light, and their optical spectra are broadly similar to K-type stars, with no emission lines from an interstellar medium: the spectra have absorption lines only. Dark matter is important, and there is probably an extensive dark matter halo with a similar proportion of dark to visible matter as spirals.

Ellipticals are classified by their observed shapes. They are given a type En where n is an integer describing the apparent ellipticity defined as $n = \mathrm{int}[10(1-b/a)]$ where b/a is the axis ratio (the ratio of the semi-minor to semi-major axes) as seen on the sky. In practice we observe only types E0 (circular) to E7 (most flattened). We never see ellipticals flatter than about E7. The reason (as indicated by simulations and normal mode analyses) seems to be that a stellar system any flatter is unstable to buckling, and will eventually settle into something rounder. Note that these subtypes reflect the *observed* shapes, not the three-dimensional shapes: a very elongated galaxy seen end-on would be classified as type E0.

Luminous ellipticals have very little net rotation. The orbits of the stars inside them are randomly oriented. The motions of the stars are characterised by a velocity dispersion σ along the line of sight, most commonly the velocity dispersion at the centres σ_0. These ellipticals are usually triaxial in shape and have different velocity dispersions in the directions of the different axes. Less luminous ellipticals can have some net rotation.

Their surface brightness distributions are more centrally concentrated than those of spirals, and also have more extended outer wings than spirals. There are various functional forms around for fitting the surface brightness, of which the best known is the de Vaucouleurs model,

$$I(R) = I_0 \exp\left[-\left(\frac{R}{R_0}\right)^{\frac{1}{4}}\right] ,$$

with $I_0 \sim 10^5 L_\odot\,\mathrm{pc}^{-2}$ for giant ellipticals. (To fit to observations, one typically unsquashes the ellipses to circles first. Also, the functional forms are are only fitted to observations over the restricted range in which $I(R)$ is measurable. So don't be surprised to see very different looking functional forms being fit to the same data.)

Ellipticals have large numbers of globular clusters. These are visible as faint star-like images superimposed on the galaxies. These globular clusters have masses $10^4 M_\odot$ to few$\times 10^6 M_\odot$.

Ellipticals are plentiful in environments where the density of galaxies is high, such as in galaxy clusters. Isolated ellipticals are rare.

Observations show that there is a correlation between three important observational parameters for elliptical galaxies. These quantities are the scale size R_0, the central surface brightness I_0, and the central velocity dispersion σ_0. The observations show that

$$R_0 \, I_0^{0.8} \, \sigma_0^{-1.3} \quad \simeq \quad \text{constant} \ .$$

This relation is known as the *fundamental plane* for elliptical galaxies. (The relation is also often expressed in terms of the radius R_e containing half the light of the galaxy and the surface brightness I_e at this radius.)

An older, and cruder, relation is that between luminosity L and the central velocity dispersion:

$$L \quad \propto \quad \sigma_0^4 \ .$$

This is known as the Faber-Jackson relation. The Faber-Jackson relation, and particularly the fundamental plane, are very useful in estimating the distances to elliptical galaxies: the observational parameters I_0 and σ_0 give an estimate of R_0, which in turn with I_0 gives the total luminosity of the galaxy, which can then be used with the observed brightness to derive a distance.

1.4 Irregular Galaxies

Irregular galaxies have irregular, patchy morphologies. They are gas-rich, showing strong star formation with many young stars. Ionised gas, particularly in H II regions, is prominent around the regions of star formation. They tend to have strong emission lines in their spectra from the interstellar gas, and their starlight is dominated by B, A and F types. As a result their colours are blue and their spectra show strong emission lines from the interstellar gas superimposed on the stellar absorption-line spectrum. Their internal motions are relatively chaotic. They are denoted type I or Irr.

1.5 Other Types of Galaxy

As already noted, some galaxies have unusual, disturbed morphologies and are called *peculiar*. These are mostly the result of interactions and mergers between galaxies. They are particularly numerous among distant galaxies.

Clusters of galaxies often have a very luminous, dominant elliptical at their cores, of a type called a cD galaxy. These have extensive outer envelopes of stars.

Low-luminosity galaxies are called *dwarf galaxies*. They have masses 10^6 to $10^9 M_\odot$. Common subclasses are *dwarf irregulars* having a large fraction of gas and active star formation, and *dwarf ellipticals* which are gas poor and have no star formation. *Dwarf spheroidal galaxies* are very low luminosity, very low surface brightness systems, essentially extreme versions of dwarf ellipticals. Our Galaxy has several dwarf spheroidal satellites.

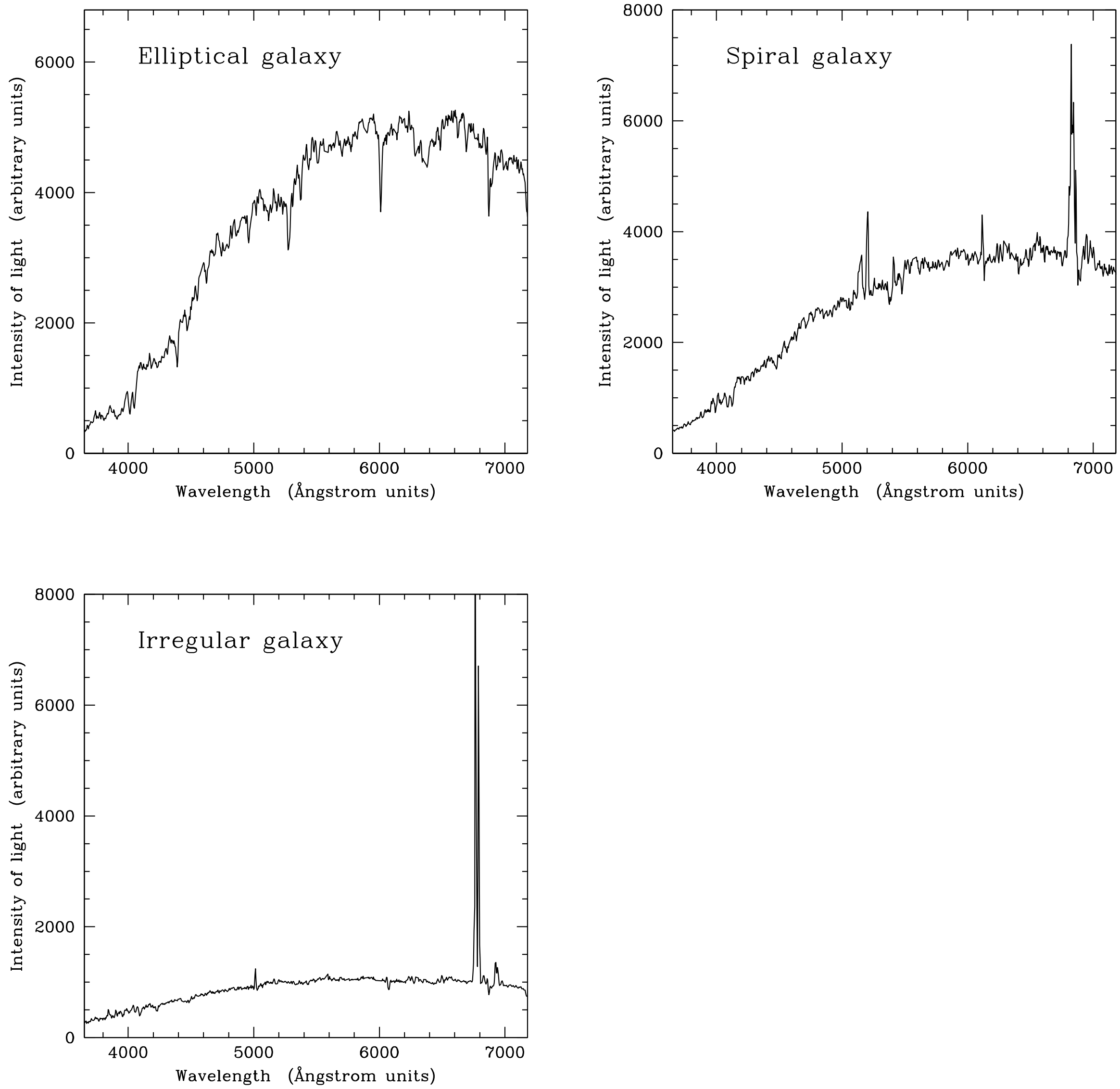


Figure 1.6: examples of the optical spectra of elliptical, spiral and irregular galaxies. The elliptical spectrum shows only absorption lines produced by the stars in the galaxy. The spiral galaxy has absorption lines from its stars and some emission lines from its interstellar gas. In contrast, the irregular galaxy has very strong emission lines on a weaker stellar continuum. [Produced with data from the 2dF Galaxy Redshift Survey.]

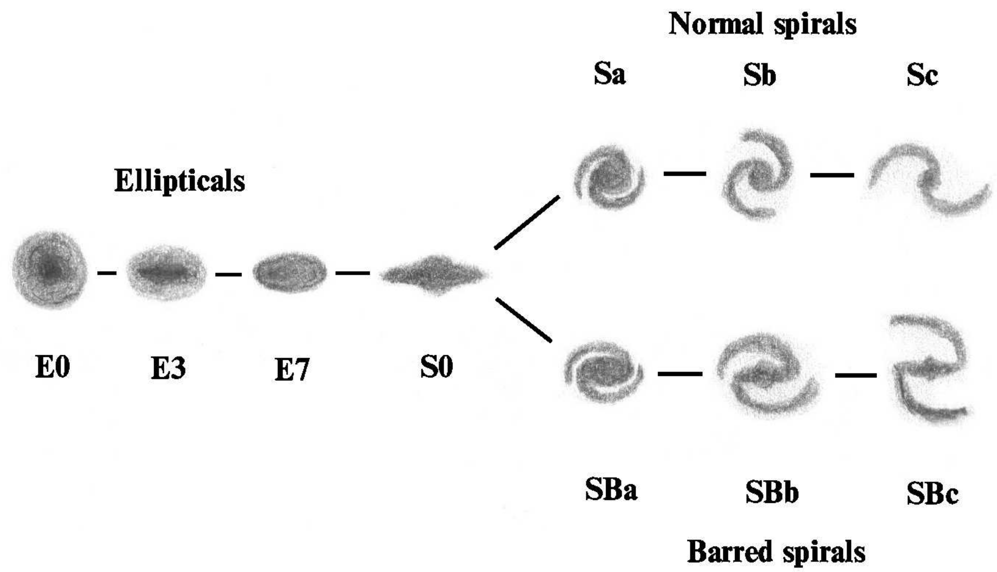


Figure 1.7: The tuning fork diagram of Hubble types. The galaxies on the left are known as early types, and those on the right as late types.

1.6 The Hubble Sequence

On the whole, galaxy classification probably should not be taken as seriously as stellar classification, because there are not (yet) precise physical interpretations of what the gradations mean. But some physical properties do clearly correlate with the so-called Hubble types.

The basic system of classification described above was defined in detail by Edwin Hubble, although a number of extensions to his system are available. Figure 1.7 shows the Hubble tuning fork diagram which places the various types in a sequence based on their shapes. Ellipticals go on the left, arranged in a sequence based on their ellipticities. Then come the lenticulars or disc galaxies without spiral arms: S0 and SB0. Then spirals with increasingly spaced arms, Sa etc. if unbarred, SBa etc. if barred.

The left-hand galaxies are called early types, and the right-hand ones late types. People once thought this represented an evolutionary sequence, but that has long been obsolete. (Our current understanding is that, if anything, galaxies tend to evolve towards early types.) But the old names early and late are still used.

Note that for spirals, bulges get smaller as spiral arms get more widely spaced. The theory behind spiral density waves predicts that the spacing between arms is proportional to the disc's mass density.

Several galaxy properties vary in a sequence from ellipticals to irregulars. However, the precise shapes of ellipticals are not important in this: all ellipticals lie in the same position in the sequence:

E (all Es)	S0	Sa	Sb	Sc	Sd	Irr

Early type	Late type
Old stars	Young stars
Red colour	Blue colour
Gas poor	Gas rich
Absorption-line spectra	Strong emission lines in spectra

Some evolution along this sequence from right to left (late to early) can occur if gas is used up in star formation or gas is taken out of the galaxies.

1.7 A Description of Galaxy Dynamics

Interactions between distributions of matter can in principle be very important over the lifetime of a galaxy, be these the interactions of stars, the interactions between clouds of gas, or the interactions of a galaxy with a near neighbour.

An important distinction between interactions is whether they are *collisional* or *collisionless*. Encounters between bodies of matters are:

- **collisional** if interactions between individual particles substantially affect their motions;
- **collisionless** if interactions between individual particles do **not** substantially affect their motions.

Gas is collisional. If two gas clouds collide, even with the low densities found in astronomy, individual atoms/molecules interact. These interactions on the atomic scale strongly influence the motions of the two gas clouds. The two clouds will therefore interact though the pressure of the gas.

Stars are collisionless on the galactic scale. If two stellar systems collide, the interactions between individual stars have little effect on their motions. The 'particles' are the stars in this case. The motions of the stars are mostly affected by the overall gravitational potentials of the two stellar systems. Interactions between individual stars are rare on the scale of galaxies. Stars are therefore so compact on the scale of a galaxy that a stellar system behaves like a collisionless fluid (except in the cores of galaxies and globular clusters), resembling a plasma in some respects.

This distinction between stars and gas leads to two very important differences between stellar and gas dynamics in a galaxy.

1. Gas with net angular momentum will tend to settle into rotating discs within galaxies. Stars will not settle in this way.

2. Star orbits can cross each other, but in equilibrium gas must follow closed paths which do not cross (and move in the same sense). Two streams of stars can go through each other and hardly notice, but two streams of gas will shock (and probably form stars). You could have a disc of stars with no net rotation (just reverse the directions of motion of some stars), but not so with a disc of gas.

The terms 'rotational support' and 'pressure support' are used to describe how material in galaxies balances its self-gravity. Stars and gas move in roughly circular orbits in the discs of spiral galaxies, where they achieve a stable equilibrium because they are *rotationally supported* against gravity. The stars and gas in the spiral discs have rotational velocities of $\simeq 250$ kms^{-1}, while the dispersion of the gas velocities locally around this net motion is only $\simeq 10$ kms^{-1}.

In contrast, stars in luminous elliptical galaxies maintain a stable equilibrium because they are moving in randomly orientated orbits. Drawing a parallel with atoms/molecules in a gas, they are said to be *pressure supported* against gravity. Random velocities of $\simeq 300$ kms^{-1} are typical. The velocities for pressure support need not be isotropically distributed. It is also possible to have a mixture of rotational and pressure support for a system of stars, with some appreciable net rotation.

Distributions of matter – such as gas clouds or systems of stars – that are not supported against gravity will collapse. We can classify the collapse as being *dissipational* or *dissipationless*. A collapse is:

- **dissipational** (or dissipative) if mechanical energy (kinetic + potential energy) is dissipated (converted into a different form, such as heat) during the collapse;
- **dissipationless** if mechanical energy is conserved during the collapse.

A large system of stars that is not supported against gravity will collapse, but because the stars are collisionless on the scale of a galaxy, the total energy (kinetic + potential) of the the system will be conserved. So the total energy will not change. However, a large gas cloud that collapses under its own gravity will be dissipative, because kinetic and potential energy can be converted into heat, and in many circumstances can be lost as electromagnetic radiation (infrared, light, etc.). Disspational collapse usually produces distributions that can be much smaller and denser than before.

1.8 A Brief Overview of Galaxy Evolution Processes

Here we consider in a descriptive way some of the processes that can drive galaxy evolution and formation. Our discussion here is brief so that we can establish some of the background that is needed for later parts of the course.

Galaxies formed early in the history of the Universe, but precisely when is not known with certainty. They were probably formed by the coalescence of a number of separate clumps of dark matter that also contained gas and stars, rather than by the collapse of single large bodies of dark matter and gas. Galaxies have evolved with time to give us the galaxy populations seen in the Universe around us today. Some of the processes driving the evolution of galaxies are briefly stated in this section. Exactly how these processes affect galaxies is not understood in precise detail at present.

Gas in a galaxy will fall into a rotationally supported disc if it has net angular momentum. Dissipation of energy will be important in this. Subsequent star formation in this gaseous disc can form the stellar disc of a spiral galaxy. Stars formed in gas clouds falling radially inwards during the formation of a galaxy can contribute to the stellar halo of a spiral galaxy. Stars in smaller galaxies falling radially inwards in a merging process can also contribute to the stellar halo of a spiral galaxy. Differential rotation in a spiral's disc will generate spiral density waves in the disc, leading to spiral arms. Spiral discs without a bulge can be unstable, and can buckle and thicken, with mass being redistributed into a bulge, giving the bulge some rotation in the process. Continuing star formation in the disc gives rise to a range of ages for stars in the disc, while bulges will tend to be older. If a spiral uses up most of its gas in star formation, it will have a stellar disc but no spiral arms.

Mergers and interactions between galaxies can be important. In a merger two galaxies fuse together. In an interaction, however, one galaxy interacts with another through their gravitational effects. One or both galaxy may survive an interaction, but may be altered in the process. If two spiral galaxies merge, or one spiral is disrupted by a close encounter with another galaxy, the immediate result can be an irregular or peculiar galaxy with strong star formation. This can produce an elliptical galaxy if and when the gas is exhausted.

If a merger between two galaxies produces an elliptical with no overall angular momentum, it will be a pressure-supported system. If a merger produces an elliptical with some net angular momentum, the elliptical will have an element of rotational support. Ellipticals, although conventionally gas-poor, can shed some gas from their stars through mass loss and supernova remnants, which can settle into gas discs and form some stars in turn.

Almost all galaxies have some very old stars. Most galaxies appear to have been formed fairly early on ($>$ 10 Gyr ago) but some have been strongly influenced by mergers and interactions since then.

These processes are not well understood at present. Understanding the evolution of galaxies is currently a subject of much active research, from both a theoretical and observational perspective. Then there is dark matter ...

1.9 The Galaxy: an Overview

1.9.1 The Structure of the Galaxy

We live in a spiral galaxy. It is relatively difficult to measure its morphology from inside, but observations show that it is almost certainly a barred spiral. The best assessment of its morphological type puts it of type SBbc (intermediate between SBb and SBc).

The Sun lies close to the plane of the Galaxy, at a distance of 8.0 ± 0.4 kpc from the Galactic Centre. It is displaced slightly to the north of the plane. The overall diameter of the disc is $\simeq 40$ kpc.

The structure of the Galaxy can be broken into various distinct components. These are: the disc, the central bulge, the bar, the stellar halo, and the dark matter halo. These are illustrated in Figure 1.8. The disc consists of stars and of gas and dust. The gas and dust are concentrated more closely on the plane than the stars, while younger stars are more concentrated around the plane than old stars. The disc is rotationally supported against gravity. The bulge and bar are found within the central few kpc. They consist mostly of old stars, some metal-poor (deficient in heavy elements). The Galactic Centre has a compact nucleus, probably with a black hole at its core. The stellar halo contains many isolated stars and about 150 globular clusters. These are very old and very metal-poor. The stellar halo is pressure supported against gravity.

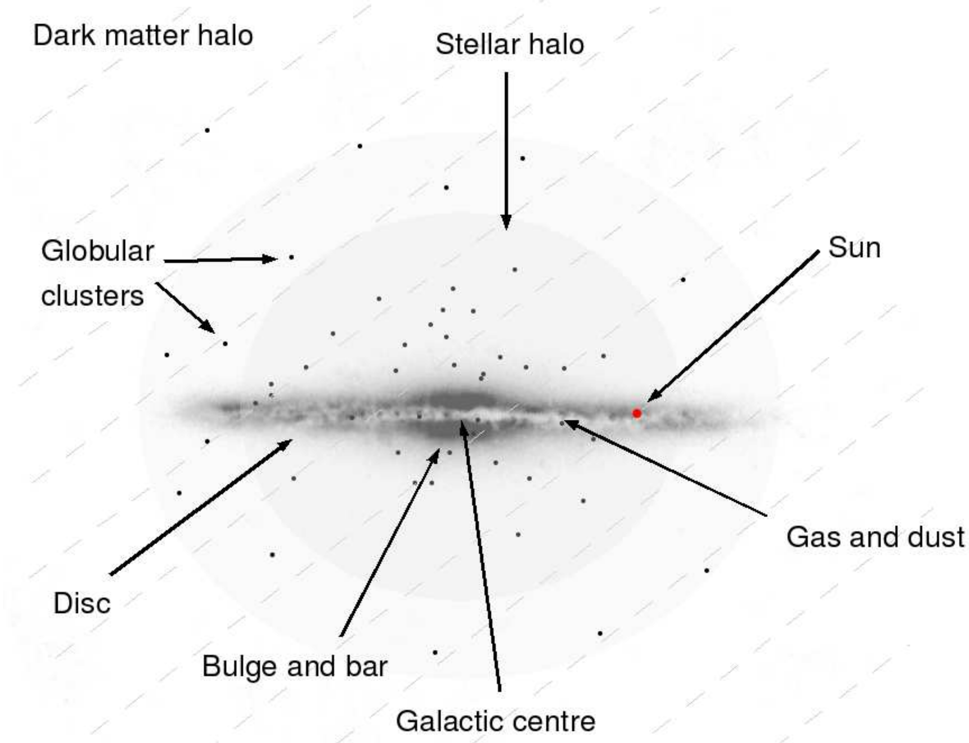


Figure 1.8: A sketch of the Galaxy seen edge on, illustrating the various components.

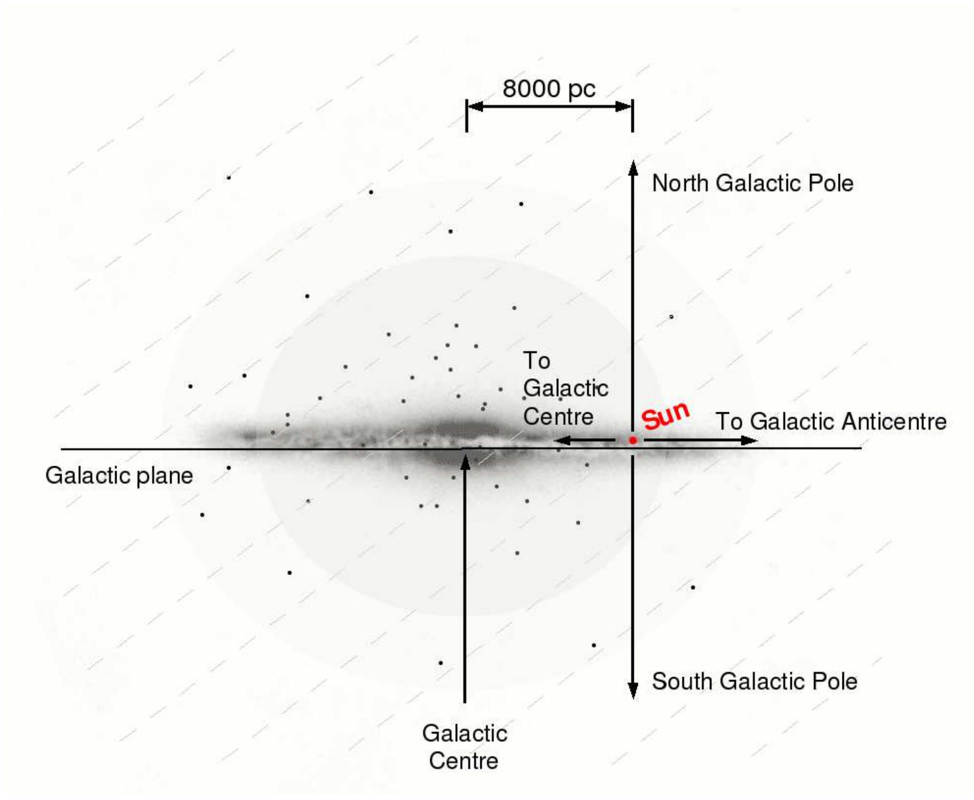


Figure 1.9: The Galaxy showing its geometry.

The entire visible system is embedded in an extensive dark matter halo which probably extends out to > 100 kpc. Its properties are not well defined and the nature of the dark matter is still uncertain.

We shall return to discuss the structure of the Galaxy and its individual components in detail later in the course.

1.9.2 Stellar Populations

It was realised in the early 20th century that the Galaxy could be divided into the disc and into a spheroid (which consists of the bulge and stellar halo). The concept of stellar populations was introduced by Walter Baade in 1944. In his picture, the stars in the discs of spiral galaxies, which contain many many young stars, were called Population I. This included the disc of our own Galaxy. In contrast, elliptical galaxies and the spheroids of spiral galaxies contain many old stars, which he called Population II. Population I systems consisted of young and moderately young stars which had chemical compositions similar to the Sun. Meanwhile, Population II systems were made of old stars which were deficient in heavy elements compared to the Sun. Population I systems were blue in colour, Population II were red. Spiral discs and irregular galaxies contained Population I stars, while ellipticals and the haloes and bulges of disc galaxies were Population II.

This picture was found to be rather simplistic. The population concept was later refined for our Galaxy, with a number of subtypes replacing the original two classes. Today, it is more common to refer to the stars of individual components of the Galaxy separately. For example, we might speak of the halo population or the bulge population. The disc population is often split into the young disc population and the old disc population.

1.9.3 Galactic Coordinates

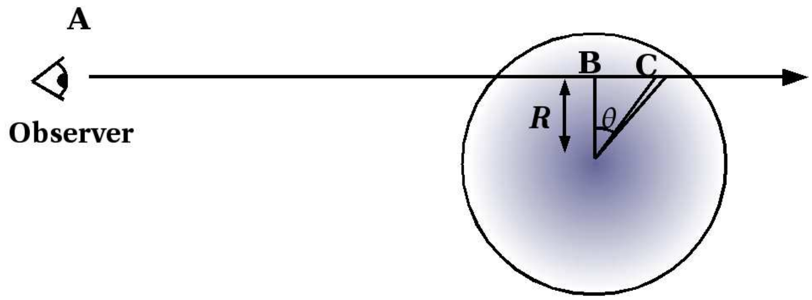
A galactic coordinate system is frequently used to specify the positions of objects within the Galaxy, and the positions of other galaxies on the sky. In this system, two angles are used to specify the direction of objects as seen from the Earth, called *galactic longitude* and *galactic latitude* (denoted l and b respectively). The system works in a way that is very similar to latitude and longitude on the Earth's surface: galactic longitude measures an angle in the plane of the *galactic equator* (which is defined to be in the plane of the Galaxy), and galactic latitude measures the angle from the equator.

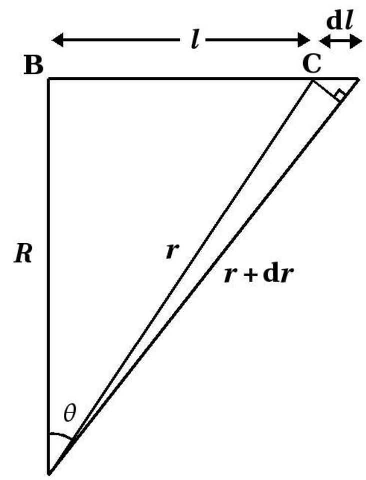
Note that the galactic coordinate system is centred *on the Earth.* Galactic longitude is expressed as an angle l between $0°$ and $360°$. Galactic latitude is expressed as an angle b between $-90°$ and $+90°$. Zero longitude is defined to be the Galactic Centre. Therefore the Galactic Centre is at $(l, b) = (0°, 0°)$. The direction on the sky opposite to the Galactic Centre is known as the *Galactic Anticentre* and has coordinates $(l, b) = (180°, 0°)$. The *North Galactic Pole* is at $b = +90°$, and the *South Galactic Pole* is at $b = -90°$. Note again that these are positions on the sky relative to the Earth, and not relative to the Galactic Centre.

1.10 Density profiles versus surface brightness profiles

The observed surface brightness profiles of galaxies are the result of the projection of three-dimensional density distributions of stars. For example, the observed surface brightness of an elliptical galaxy is well fitted by the de Vaucouleurs $R^{1/4}$ law, as was discussed earlier. This surface brightness is the projection into two-dimensions on the sky of the three-dimensional density distribution of stars in space.

Let us consider an example which demonstrates this. An observer on the Earth observes a spherically symmetric galaxy, such as an E0-type elliptical galaxy. The mean density of stars in space at a radial distance r from the centre of the galaxy is $\rho(r)$ (measured in $M_\odot\,\mathrm{pc}^{-3}$ or $\mathrm{kg\,m}^{-3}$, and smoothed out over space). Consider a sight line that passes a tangential distance R from the galaxy's centre.





Consider an element of the path length $\mathrm{d}l$ at a distance l from the point on the sight line closest to the nucleus. Let the contribution to the surface brightness from the element be

$$\mathrm{d}I(R) \;=\; k\,\rho(r)\,\mathrm{d}l \qquad (1.1)$$

where k is a constant of proportionality.

$$\therefore \quad \mathrm{d}I(R) \;=\; k\,\rho(r)\,\frac{\mathrm{d}r}{\sin\theta} \quad \text{on subs. } \mathrm{d}l \;=\; \frac{\mathrm{d}r}{\sin\theta}$$

$$=\; k\,\rho(r)\,\frac{r\,\mathrm{d}r}{\sqrt{r^2-R^2}}$$

$$\text{on subs. } \sin\theta \;=\; \frac{\sqrt{r^2-R^2}}{r} \;.$$

Integrating along the line of sight from B ($r = R$) to infinity ($r \to \infty$),

$$I_{B\infty} \;=\; \int_R^\infty k\,\rho(r)\,\frac{r}{\sqrt{r^2-R^2}}\,\mathrm{d}r \;=\; k\int_R^\infty \frac{r\,\rho(r)}{\sqrt{r^2-R^2}}\,\mathrm{d}r \;.$$

This has neglected the near side of the galaxy. From symmetry, the total surface brightness along the line of sight is $I(R) \;=\; 2\,I_{B\infty}(R)$.

$$\therefore \quad I(R) \;=\; 2k\int_R^\infty \frac{r\,\rho(r)\,\mathrm{d}r}{\sqrt{r^2-R^2}} \;. \qquad (1.2)$$

In general, the density profile of a galaxy will not be spherical and we need to take account of the profile $\rho(\mathbf{r})$ as a function of position vector $\mathbf{r}$.

The calculation of the surface brightness profile from the density profile is straightforward numerically, if not always analytically. The inverse problem, converting from an observed surface brightness profile to a density profile, often has to be done numerically.