Gravitational Effects and the Motion of Stars

On the largest scales (galaxy clusters and larger), strong evidence that the dark matter has to be non-baryonic:

•Abundances of light elements (hydrogen, helium and lithium) formed in the Big Bang depend on how many baryons (protons + neutrons) there were.

•light element abundances + theory allow a measurement of the number of baryons

•observations of dark matter in galaxy clusters suggest there is too much dark matter for it all to be baryons, must be largely ***non-baryonic***.

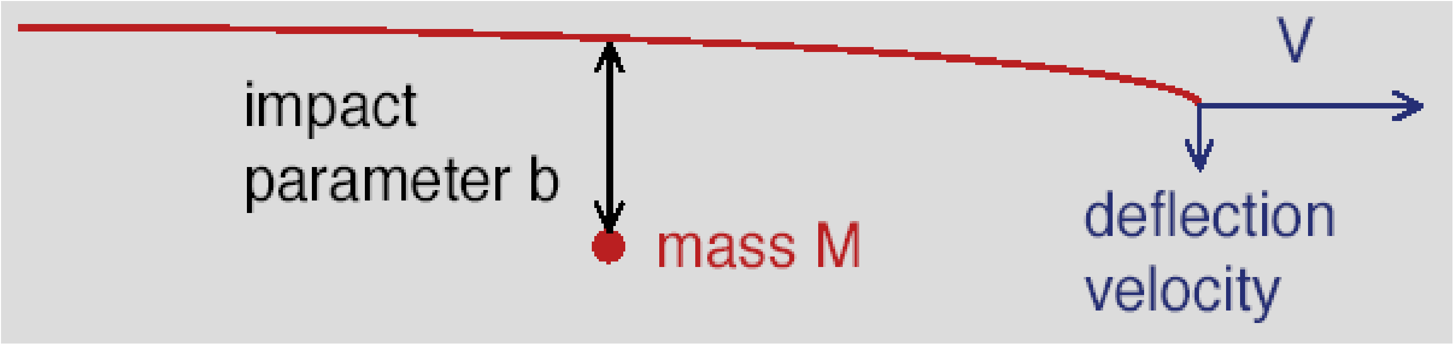
On galaxy scales no such simple argument exists. Individual types of dark matter can be constrained using various indirect arguments, but only direct probe is via gravitational lensing.

Dark Matter Candidates

- Many candidates for dark matter.
  - Dark Baryons
    - Brown dwarfs?
    - MaCHO's (Massive Compact Halo Objects)
    - Astronomer sized rocks?
    - Black Holes?
  - Non-baryonic (exotic particles)
    - WIMPS (Weakly Interacting Massive Particles)

Gravitational Lensing

- Photons are affected by gravitational fields and thus background objects can be distorted if there is a massive object in the line of sight
- If a star passes a massive body it will acquire a transverse velocity $V_{\perp}$.



- This transverse velocity can be show to be (see Section 3.2.2 eqn. 3.49)

$$\Delta V = \frac{2GM}{bV}$$

The angle that the mass is deflected is: $\alpha = \frac{2GM}{bV^2}$

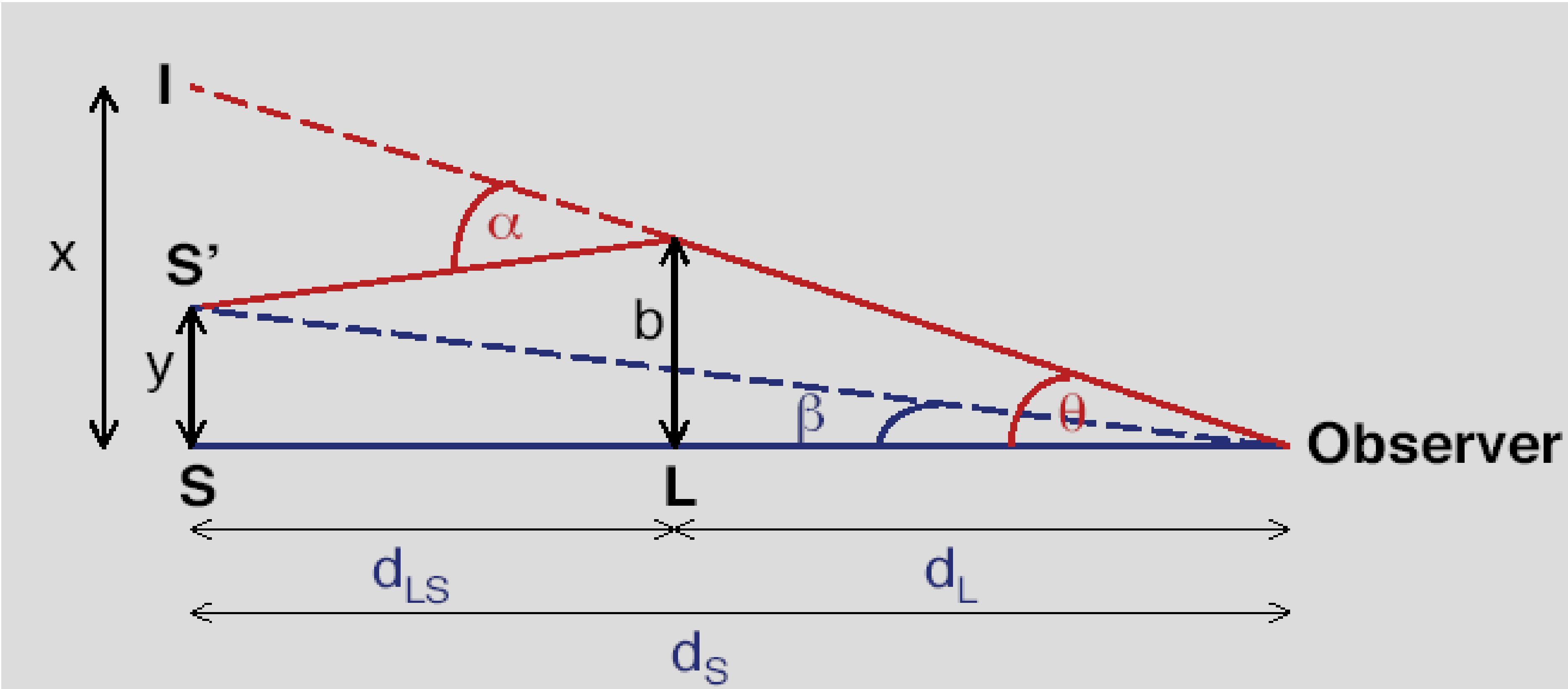
For a photon v=c and general relativity predicts that:

$$\alpha = \frac{4GM}{bc^2} = \frac{2R_s}{b}$$

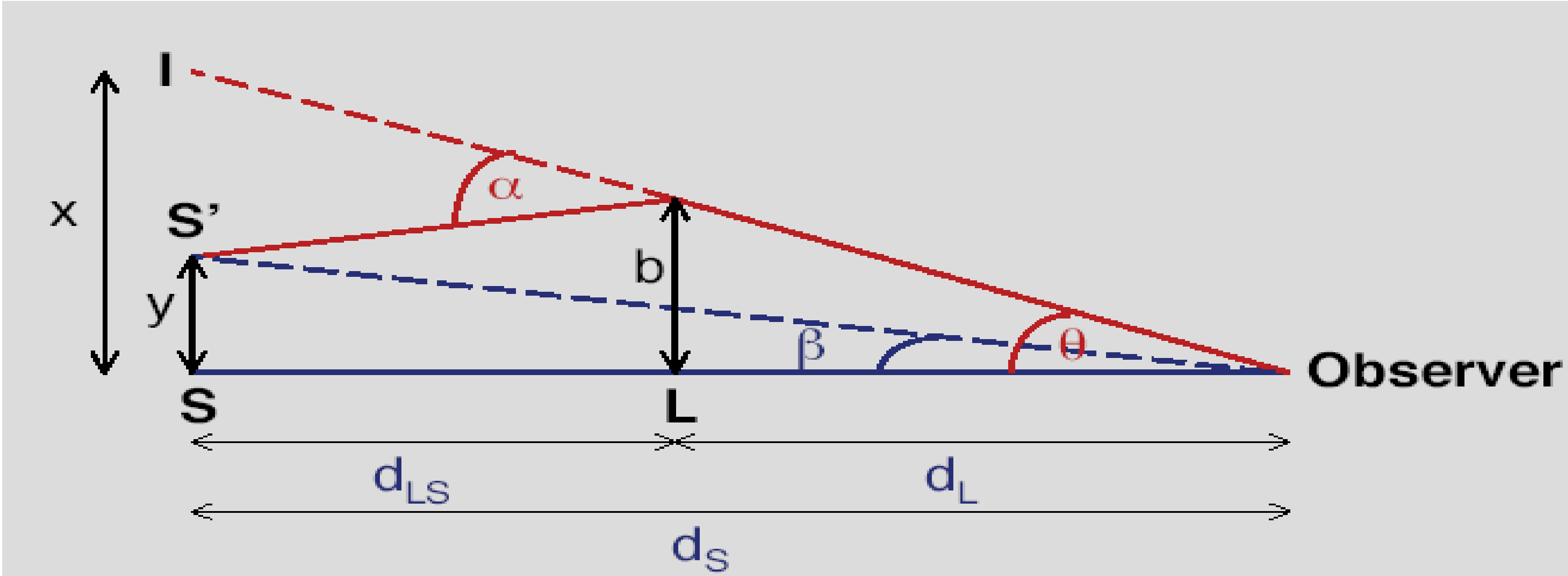
R_s is known as the Schwarzchild radius and for a solar mass object is about 3 km.

Note: the deflection angle is derived for weak encounters so b >>R_s:
implies that   is small (in radians)

For a background object at a distance of d_s from the Observer O and a point mass (the lens L) is at a distance d_L. The Observer sees and image (I) of the source S' at an angle



from the line of sight to the lens.



If $d_s >> y$, then $\approx y/d_s$, $\approx (x-y)/d_{LS}$ and $\approx x/d_s$

Recall that $\alpha = \frac{4GM}{bc^2}$ Substituting values for and

$$\theta - \beta = \frac{1}{\theta} \frac{4GM}{c^2} \frac{d_{LS}}{d_s d_L}, if\ \theta_E^2 \equiv \frac{4GM}{c^2} \frac{d_{LS}}{d_s d_L}$$

$$\theta - \beta = \frac{1}{\theta} \theta_E{}^2$$

So now the equation for the apparent position becomes:

$$\theta^2 - \beta\theta - \theta_E{}^2 = 0$$

With solutions:

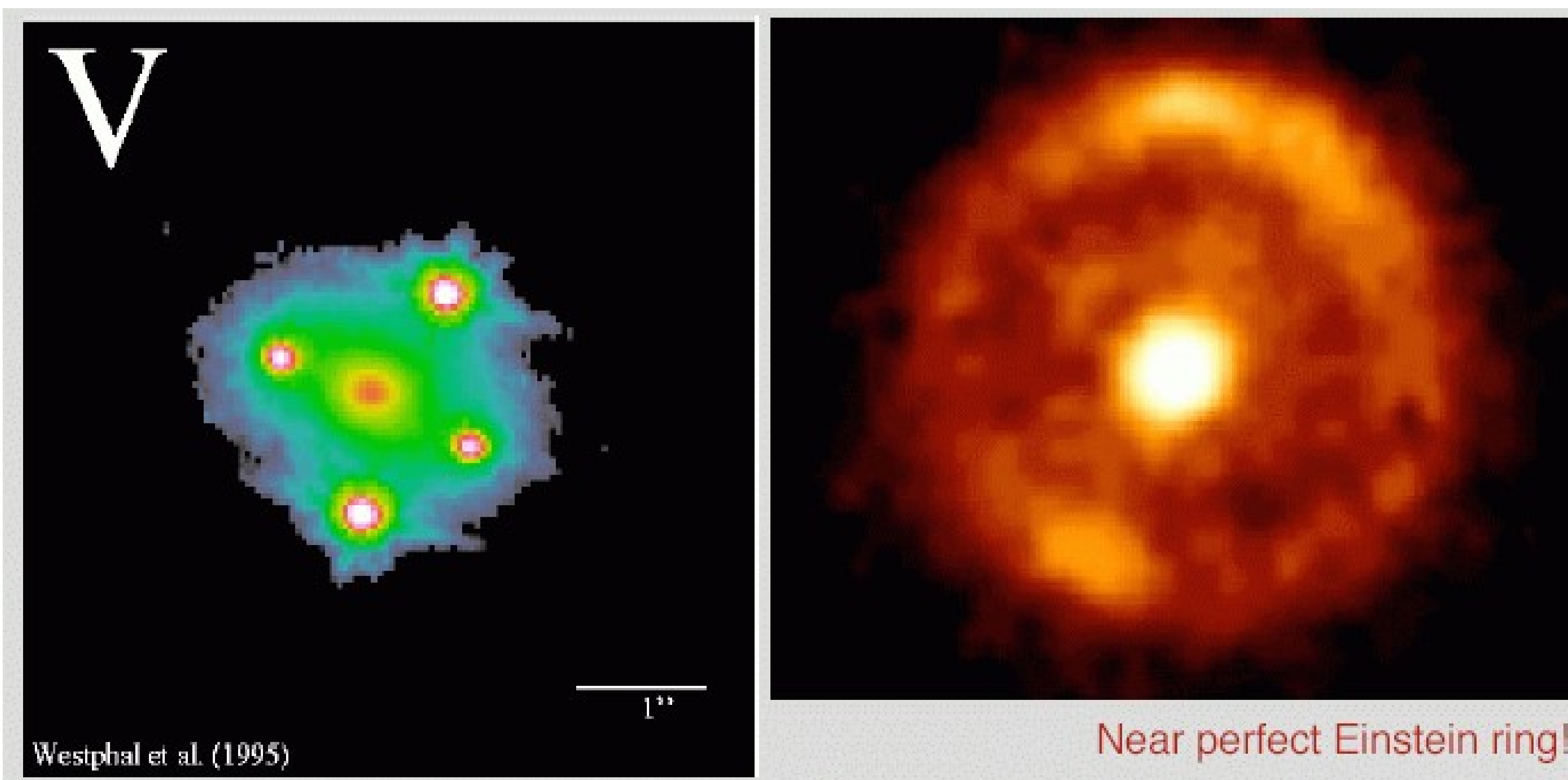
$$\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

If β is 0 (lens and source are exactly aligned) the we see a ring of light (an Einstein ring) with radius θ_E .

If $\beta > 0$ then we see two images, one inside θ_E and one outside θ_E.

Strong Lensing

This is a case where the lens is massive enough to bend light through a “large” angle.



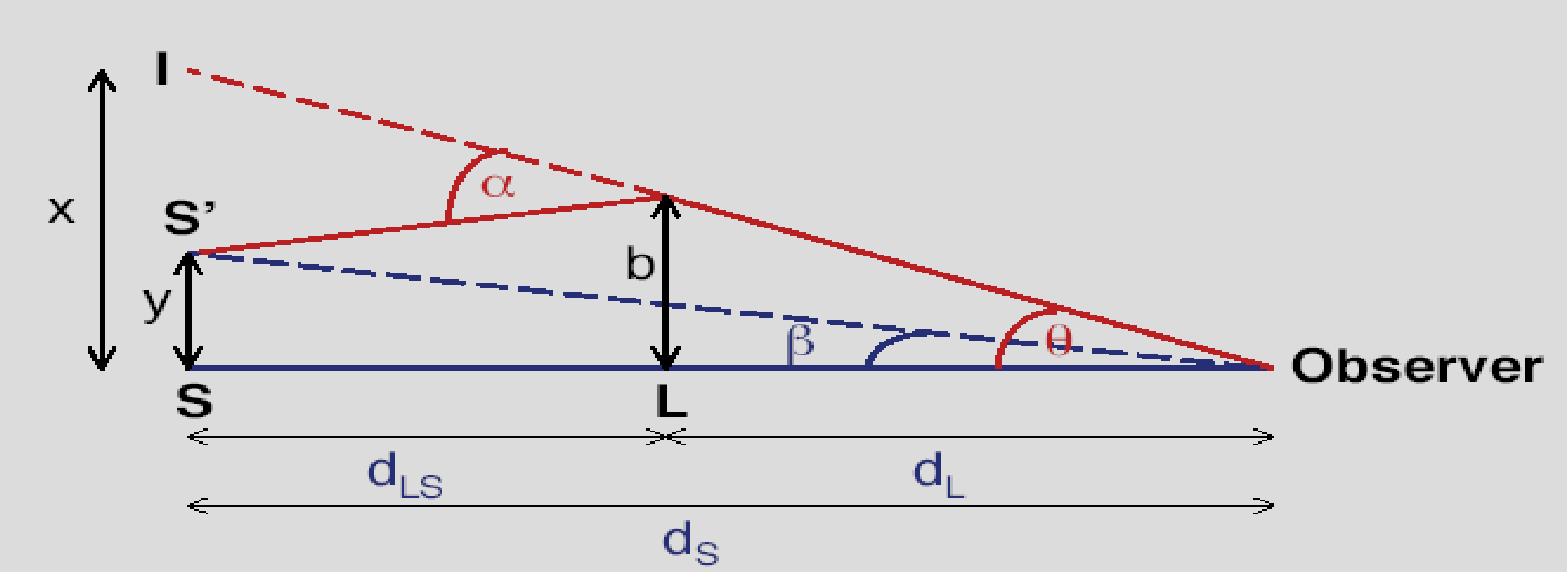
Strong lensing usually requires a lens with mass about that of a galaxy. Since galaxies are not point sources and have internal structure real lenses are more complex than our example.

What happens if you do have only a solar mass for the lens. Assume that we are observing a star in the Galactic bulge and a solar mass star passes between us and the source.

$$d_S = 8\,kpc\,, d_L = d_{LS} = 4\,kpc$$

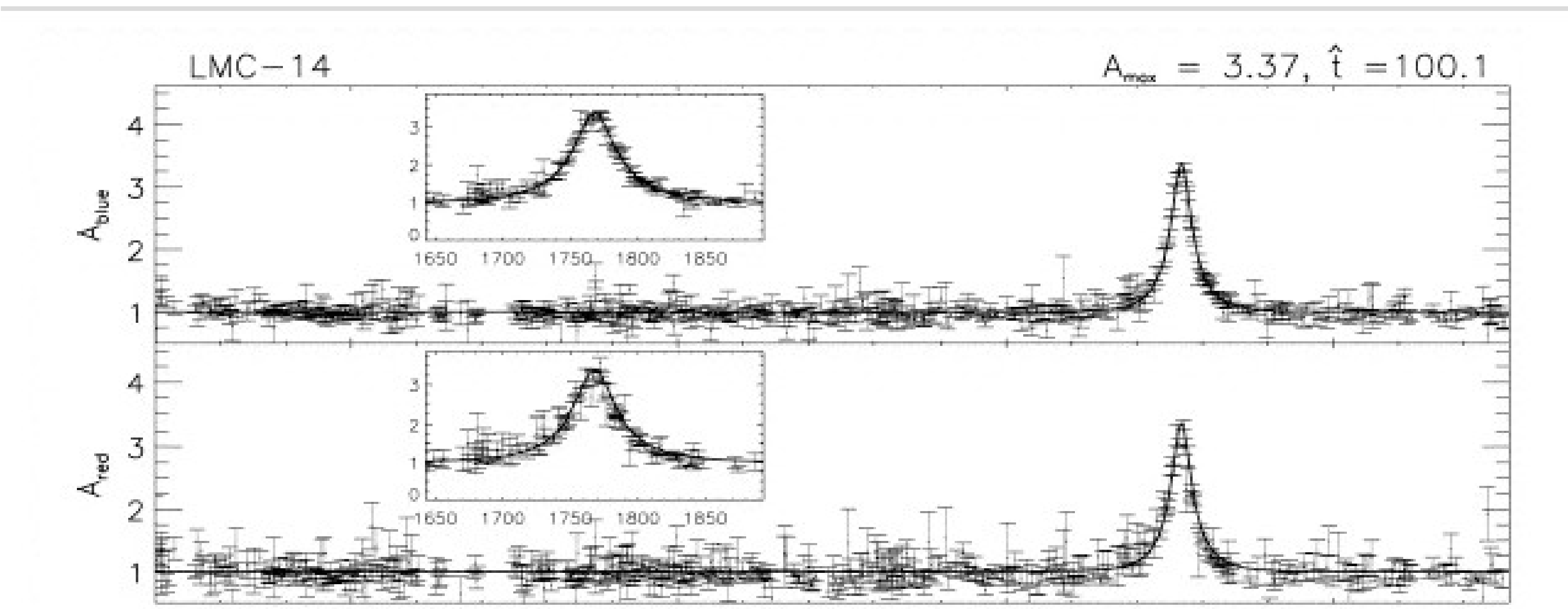
$$\theta_E = \sqrt{\frac{4\mathrm{GMd}_{LS}}{c^2 d_L d_S}} = 5\times10^{-9}\,radians = 10^{-3}\,arcsec$$

This angle is too small to measure directly even with HST



What happens if L is not stationary?
If we are observing a star as the lens L passes across out line-of-sight the star brightens when the alignment is within θ_E and then fades as the alignment is lost.

- Strong effect for $\theta<\theta_E$. Sources can be magnified my a factor of ~10 (2.5 mag) or more.
- Because the alignment must be so precise this is a very rare event, $P<10^{-6}$.
- Now routinely observed toward the Galactic bulge and the Magellanic clouds.



Motion under Gravity

Newtons law of gravity tells us that two masses attract each other with a force

$$\frac{d}{dt}(m\boldsymbol{v}) = -\frac{GmM}{r^3}\boldsymbol{r}$$

If we have a collection of masses acting on a mass m_α the force is

$$\frac{d}{dt}(m_\alpha \boldsymbol{v}_\alpha) = -\sum_\beta \frac{G m_\alpha M_\beta}{|\boldsymbol{x}_\alpha - x_\beta|^3}(\boldsymbol{x}_\alpha - x_\beta), \alpha \neq \beta$$

$$\frac{d}{dt}(m\boldsymbol{v}) = -m\nabla\Phi(\boldsymbol{x}),$$

with

$$\Phi(\boldsymbol{x}) = -\sum_{\alpha} \frac{Gm_{\alpha}}{|\boldsymbol{x} - x_{\alpha}|},\ for\ \boldsymbol{x} \neq \boldsymbol{x}_{\boldsymbol{\alpha}}$$

Is the gravitational potential. If we can approximate the discrete stellar distribution with a continuous distribution ρ.

$$\Phi(\boldsymbol{x}) = -\int \frac{G\rho(\boldsymbol{x}')}{|\boldsymbol{x} - x'|} d^3\boldsymbol{x}'$$

So the force per unit mass is

$$\boldsymbol{F}(\boldsymbol{x}) = -\nabla\Phi(\boldsymbol{x}) = \int G\,\rho(\boldsymbol{x}')\frac{(\boldsymbol{x}-\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}|^3}d^3\boldsymbol{x}'$$

To get the differential form we start with the definition of Φ and applying ∇^2 to both sides

$$\nabla^2\Phi(\boldsymbol{x}) = -\nabla^2\int\frac{G\,\rho(\boldsymbol{x}')}{|\boldsymbol{x}-x'|}d^3\boldsymbol{x}'$$

$$= 4\pi G\,\rho(\boldsymbol{x})$$

we get Poisson's equation.

In the absence of external forces a star will conserve energy along its orbit

$$\boldsymbol{v}\cdot\frac{d}{dt}(m\boldsymbol{v})=-m\boldsymbol{v}\cdot\nabla\Phi(\boldsymbol{x}),$$

$$\boldsymbol{v}\cdot\frac{d}{dt}(m\boldsymbol{v})+m\boldsymbol{v}\cdot\nabla\Phi(\boldsymbol{x})=0$$

But since $\frac{d\Phi}{dt}=\boldsymbol{v}\cdot\nabla\Phi(\boldsymbol{x})$

$$\frac{d}{dt}[\frac{m}{2}(v^2)+m\Phi(\boldsymbol{x})]=0$$

This is just the KE + PE

As a body moves far from the mass then

$$\boldsymbol{\Phi}(\boldsymbol{x}) \to 0$$

So to escape from the gravitational pull a star must have a velocity greater than

$$v^2 \geqslant 2\,\boldsymbol{\Phi}(\boldsymbol{x})$$

The escape velocity is set so that v at infinity is 0 so

$$v_{esc} = \sqrt{-2\,\boldsymbol{\Phi}(\boldsymbol{x})}$$

In addition a stars angular momentum changes according to

$$\frac{dL}{dt} = \boldsymbol{x} \times m \frac{d\,\boldsymbol{v}}{dt} = -m\,\boldsymbol{x} \times \nabla \boldsymbol{\Phi}$$

Virial Theorem

Isolated system $2\langle KE\rangle+\langle PE\rangle=0$

In General

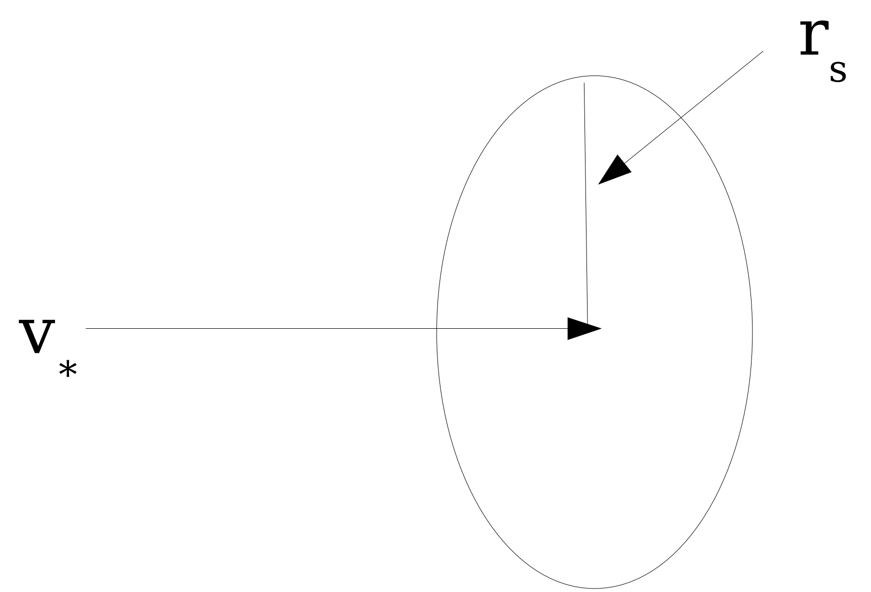
$$2\langle KE\rangle+\langle PE\rangle+\sum_{\alpha}\boldsymbol{F}^{\alpha}\times\boldsymbol{x}_{\alpha}=0$$

Used to estimate masses
Determine stability
Star formation ...

Scattering and 2 Body Encounters

For a strong encounter

$$\frac{GmM}{r} \geqslant \frac{mv^2}{2}$$



Solving for r

$$r \leqslant r_s \equiv \frac{2\mathrm{Gm}}{v^2}$$

For m=0.5 M_{sun}, v = 30 km/s then r_s = 1au

How frequently can we expect such an encounter?
To answer this we must look at the volume of space that the star sweeps out in a time t.

$$Vol = \pi r_s^{\,2} v t$$

What is the time before the 1st encounter is the star is moving through a galaxy where the stellar density is n?

$$n \pi r_s^{\,2} v t = 1$$

$$t_s = \frac{v^3}{4\pi n G^2 m^2} = 4\times 10^{12} \left(\frac{v}{10\,km/s}\right)^3 \left(\frac{m}{M_{sun}}\right)^{-2} \left(\frac{n}{1\,pc^{-3}}\right)^{-1} yr$$

$$t_s = 4\times10^{12}\left(\frac{v}{10\,km/s}\right)^{3}\left(\frac{m}{M_{sun}}\right)^{-2}\left(\frac{n}{1\,pc^{-3}}\right)^{-1} yr$$

For the solar neighborhood $n \approx 0.1$ pc^{-3} and v ~ 30 km/s so that $t_s \sim 10^{15}$ yrs. Since the age of the universe is only 1.3×10^{10} yrs this isn't a real worry for the sun.

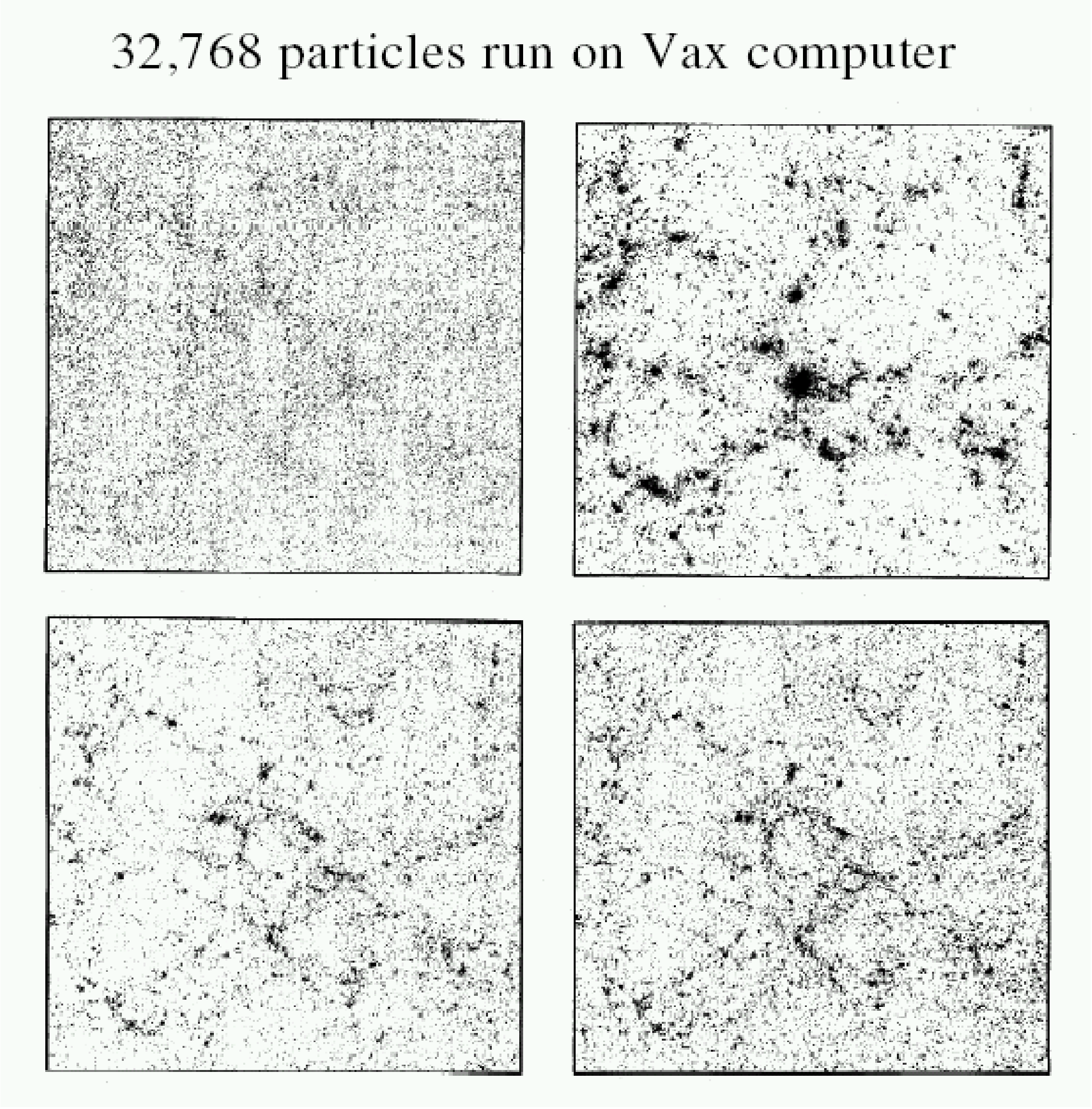
What is the density required to have at least one strong encounter in the age of the universe?

$n \approx$ few x 10^5 pc^{-3}

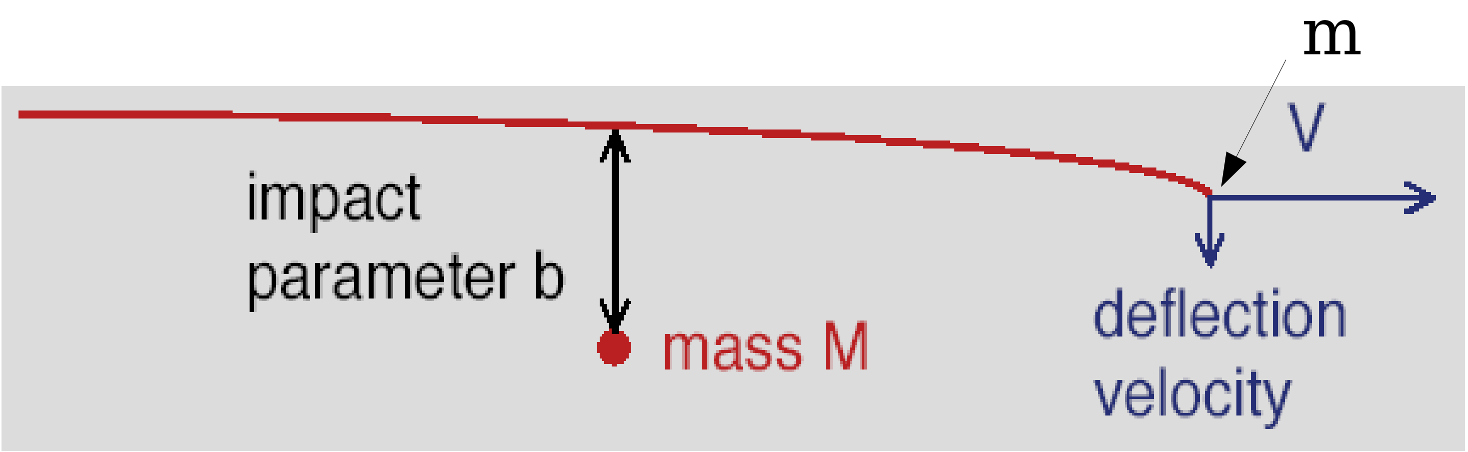
we see these densities in dense globular clusters and in the cores of galaxies.

Weak Encounters

What keeps spiral galaxies disk-like?



As we discussed last time



Now we know the force on the object m as it passes M

$$\boldsymbol{F} = \frac{GmMb}{(b^2 + v^2 t^2)^2} = M\,\frac{dv}{dt}$$

If we integrate this over the time of the encounter

$$\Delta V = \frac{1}{M}\int \boldsymbol{F}\,dt = \frac{2\mathrm{Gm}}{bv}$$

For a weak encounter $b >> r_s$, the strong encounter radius.

That is for one weak encounter but as the star orbits the galaxy there will be many of the encounter so what is the cumulative effect?
We need to add each ΔV in quadrature e.g.
$\langle \Delta V \rangle = (\Delta v_1^2 + \Delta v_2^2 + \Delta v_3^2 + \Delta v_4^2 + \ldots)^{1/2}$
and instead of summing we'll integrate

$$\langle \Delta V^2 \rangle = \int_{b_{min}}^{b_{max}} n\,v\,t \left(\frac{2Gm}{bv}\right)^2 2\pi b\,db = \frac{8\pi G^2 m^2 n t}{v} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Solve this for t

$$t_{relax} = \frac{v^3}{8\pi G^2 m^2 n \ln(\Lambda)} = \frac{t_s}{2\ln(\Lambda)}$$

We can write t_{relax} in terms of more friendly units

$$t_{relax} = \frac{2\times 10^9}{\ln(\Lambda)} \left(\frac{v}{10\, km/s}\right)^3 \left(\frac{m}{M_{sun}}\right)^{-2} \left(\frac{n}{10^3\, pc^{-3}}\right)^{-1} yrs$$

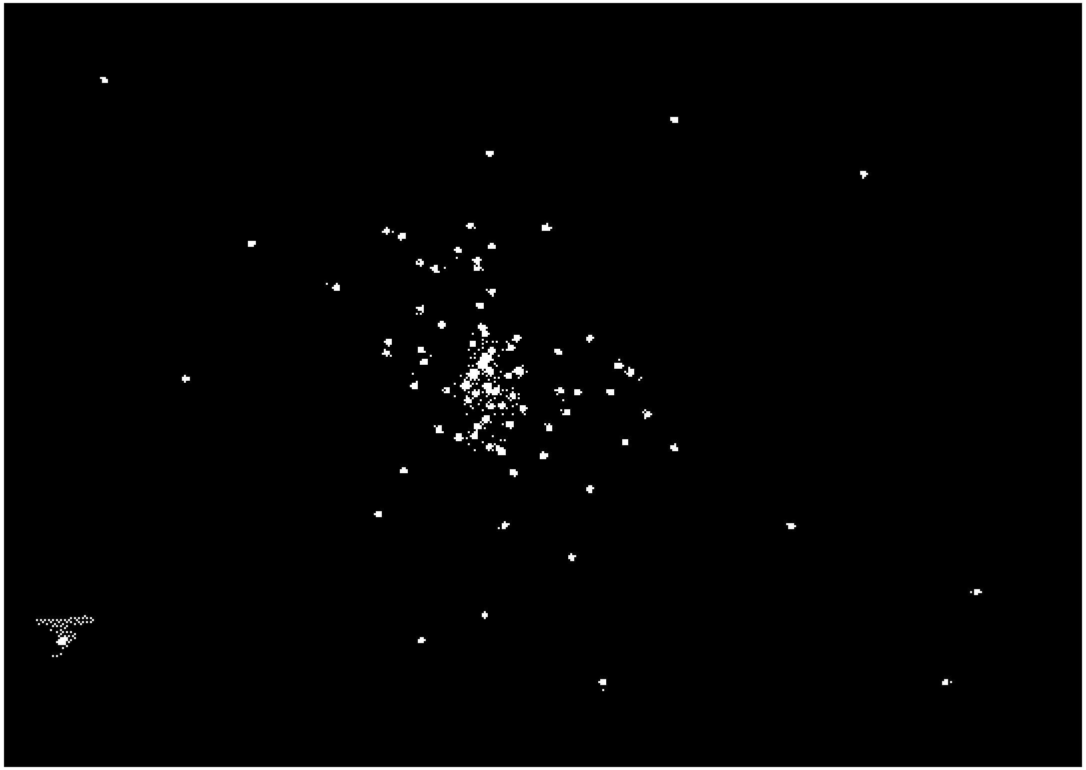
The only tricky bit is what is Λ?

Near the sun $b_{min} \approx 300$pc $b_{max} \approx 3 \times 10^4$ pc
so $\ln\Lambda$ is ln(100)= 4.61

Near the sun $t_{relax} \approx 10^{13}$ yrs so the sun acts as if it is alone in the galaxy.

What about a globular cluster?

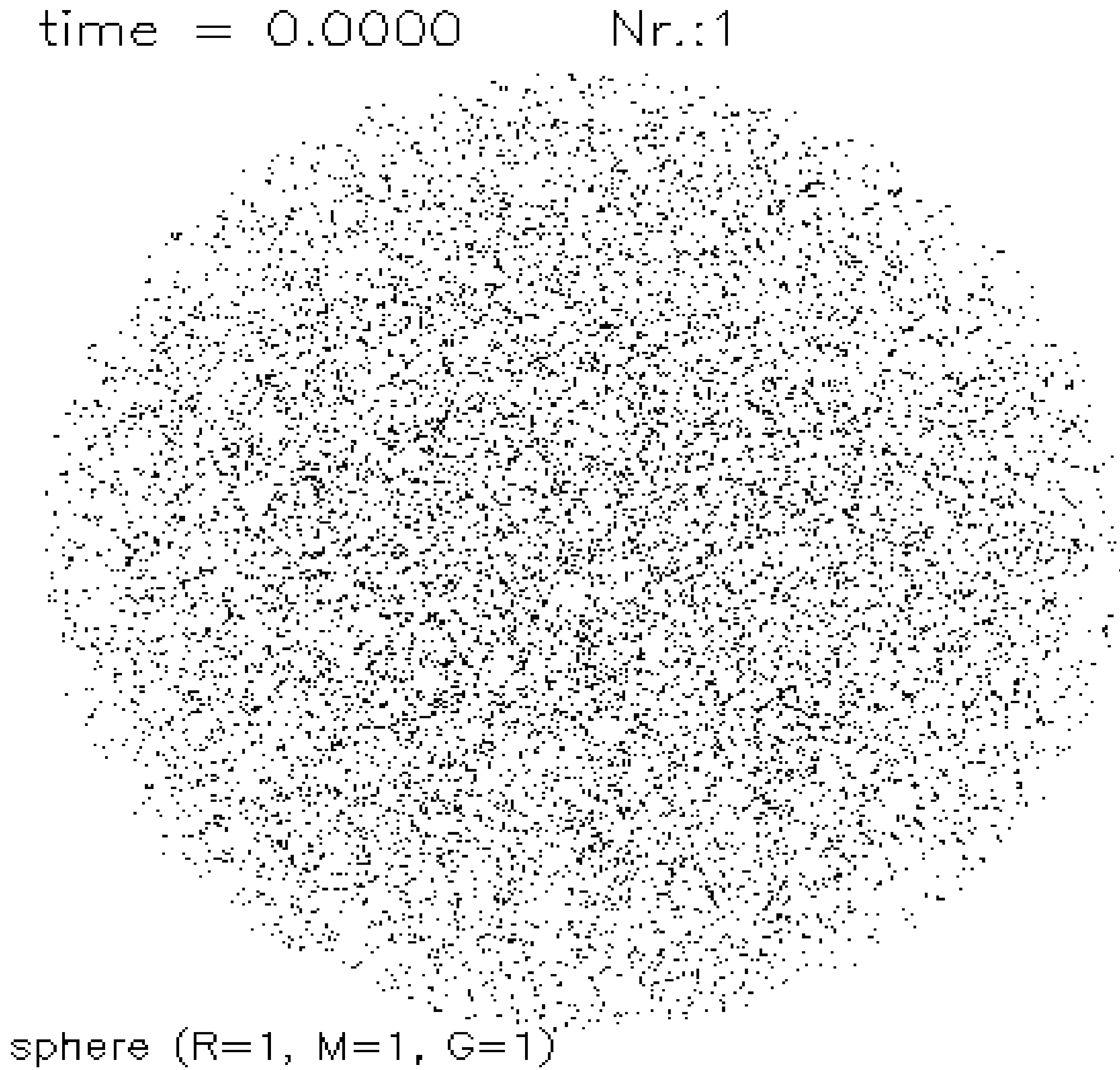
$T_{relax} \approx 0.5 \times 10^{10}$ yrs so we have to be aware of the encounters when calculating orbits



We've talked about two types of relaxation processes but both take a long time, a significant fraction of the age of the universe. So how did objects (elliptical galaxies and globular clusters get to be so smooth?

Violent relaxation!

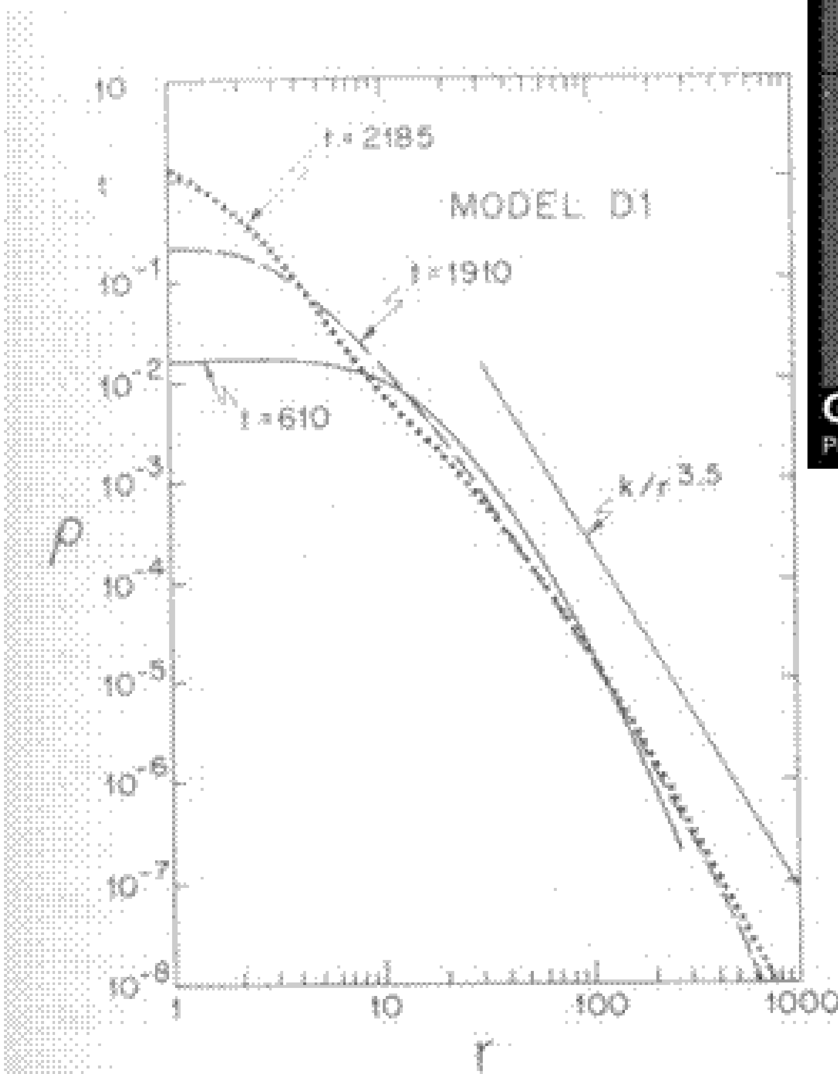
During the early phases of collapse densities can rise to large values and many strong encounters can happen in a short time.


time = 0.0000 Nr.:1
sphere (R=1, M=1, G=1)

Effects of strong encounters

- Two outcomes
  - Can form a binary star system
  - Kick one of the stars out of the system
- This leads to
  - Core collapse
  - Blue Stragglers
    - Two stars coalesce into a single star too bright, too blue, and too massive to be on the MS

- Millisecond pulsars
  - One star in a binary is a neutron star and mass transfer from the normal star spins up the neutron star
- Evaporation
  - Continuing encounters kick low mass stars out of the cluster
    - Implies mass segregation – more massive stars should be in the center
- Data shows that all of these effects are present in GC's



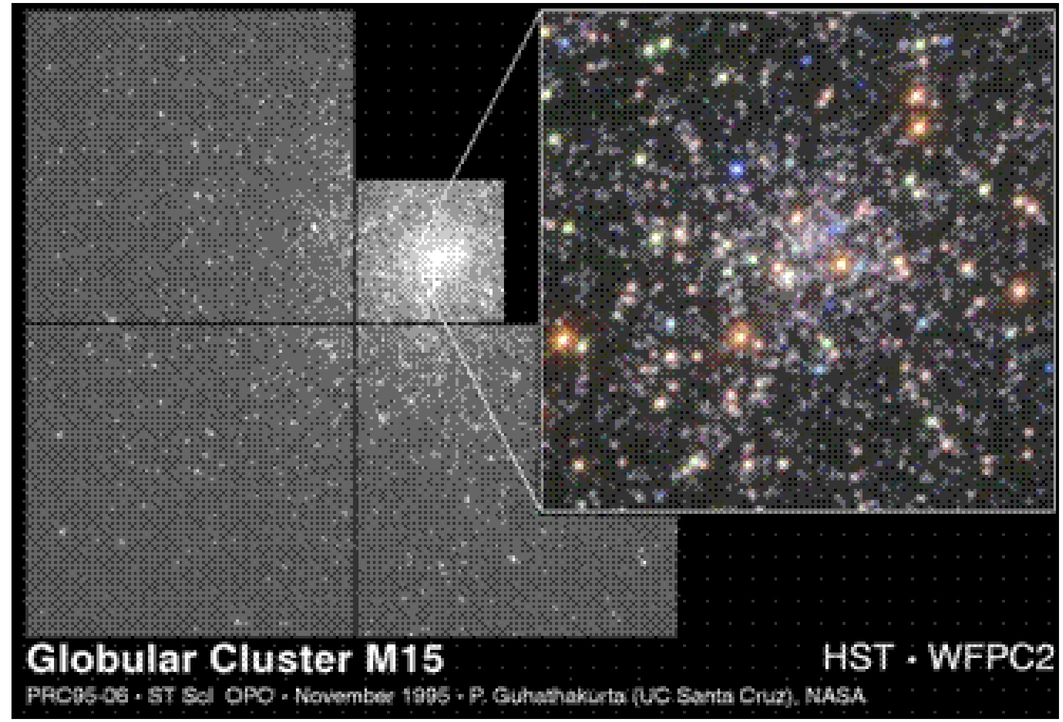


Figure 8-5. Density profiles in a model sequence computed by Spitzer and Thuan (1972). The profile of the central regions resembles that of an isothermal sphere with steadily decreasing core radius, while the profile in the outer parts is close to a power law, $r^{-7/2}$. From Spitzer (1975), by courtesy of the International Astronomical Union.