

## Homework # 2 Solutions

1. Suppose the initial mass function is the Salpeter mass function with a low-mass cutoff of  $0.1 \mathcal{M}_\odot$  and a high-mass cutoff of  $100\mathcal{M}_\odot$ . Determine the normalization from the local disk column mass density (i.e., the number density integrated vertically through the Galactic plane in units  $\text{pc}^{-2}$ ). What is the average stellar mass? Where does the mass function reach a maximum? If  $L \propto \mathcal{M}^{3.5}$  for all masses, what mass star has the average luminosity? From these results, what is the column density of stars in the solar neighborhood and what is the mean distance between stars?

The local disk column mass density is about  $55\mathcal{M}_\odot \text{pc}^{-2}$ . The total mass column density of stars is found from

$$\int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \mathcal{M} \xi(\mathcal{M}) d\mathcal{M} = \frac{\xi_o \mathcal{M}_\odot}{0.35} \left( \frac{\mathcal{M}_\odot}{\mathcal{M}_\ell} \right)^{0.35} \left[ 1 - \left( \frac{\mathcal{M}_\ell}{\mathcal{M}_u} \right)^{0.35} \right] \simeq \xi_o \frac{10^{0.35}}{0.35} [1 - 0.09] \mathcal{M}_\odot,$$

where the normalization is  $\xi_o$ . Setting these results equal, we find  $\xi_o = 9.4 \text{pc}^{-2}$ .

The average stellar mass is

$$\frac{\overline{\mathcal{M}}}{\mathcal{M}} = \frac{\int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \mathcal{M} \xi(\mathcal{M}) d\mathcal{M}}{\mathcal{M}_\odot \int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \xi(\mathcal{M}) d\mathcal{M}} = \frac{1.35 \mathcal{M}_\ell \left[ 1 - (\mathcal{M}_\ell/\mathcal{M}_u)^{0.35} \right]}{0.35 \mathcal{M}_\odot \left[ 1 - (\mathcal{M}_\ell/\mathcal{M}_u)^{1.35} \right]} = 0.35$$

The mass function reaches a maximum at  $\mathcal{M}_\ell$ .

The average luminosity is

$$\frac{\overline{L}}{L_\odot} = \frac{\int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \mathcal{M}^{3.5} \xi(\mathcal{M}) d\mathcal{M}}{\mathcal{M}_\odot^{3.5} \int_{\mathcal{M}_\ell}^{\mathcal{M}_u} \xi(\mathcal{M}) d\mathcal{M}} = \frac{1.35 \mathcal{M}_u^{2.15} \mathcal{M}_\ell^{1.15} \left[ 1 - (\mathcal{M}_\ell/\mathcal{M}_u)^{2.15} \right]}{1.15 \mathcal{M}_\odot^{3.5} \left[ 1 - (\mathcal{M}_\ell/\mathcal{M}_u)^{1.35} \right]} = 1658.$$

The star with this luminosity has a mass  $1658^{1/3.5} \mathcal{M}_\odot = 8.3\mathcal{M}_\odot$ .

The column number density is the column mass density divided by the average mass, or  $55/0.35 = 157 \text{ stars pc}^{-2}$ . From the column density and a disk thickness of 500 pc, the density of stars in the solar neighborhood is about  $n \simeq 157/500 = .31 \text{ stars pc}^{-3}$ . The mean distance between them is  $[3/(4\pi n)]^{1/3} \sim 0.9 \text{ pc}$ .

2. Do problem 2.7 in the text.

The inclination is determined by  $i = \cos^{-1}(1.10/1.62) = 47.2^\circ$  (the book is wrong, they have the complementary angle instead). The path length through A is  $d - R \sin i + R$  while the direct path length is  $d$ , the distance to SN 1987A. The path length through B is  $d + R \sin i + R$ . Thus  $t_\mp = R(1 \mp \sin i)/c$  and  $R = c(t_- + t_+)/2$ . The distance is found from  $2R/d = \tan(1.62 \text{ arcsec})$ . Observed values are  $t_- = 83$  and  $t_+ = 395$  days. Thus  $d = c(t_- + t_+)/\tan(1.62 \text{ arcsec}) = 51 \text{ kpc}$ .

With an apparent magnitude of  $m_v = 3$ , we have  $M_v = m_v - 5 \log(d/10 \text{ pc}) = -15.54$ . Thus  $L_{SN} = 10^{(4.83+15.54)/2.5} = 1.4 \times 10^8 L_\odot$ .

3. Suppose the number density of stars normal to the Galactic plane varies as  $n(z) = n_0 \exp(-z/z_h)$ . Suppose you do a survey of stars in the direction of the North Galactic Pole complete to a magnitude  $m_v = 30$ . What value would you obtain for  $\mathcal{V}/\mathcal{V}_{max}$ ?

We defer this problem to the third homework.

4. Do parts (a), (b) and (c) of problem 2.11. Without doing part (d), do you think the observable stars able in regions B and C would be more or less metal rich than what is actually in your model sky?

The regions A, B and C will have 10, 12 and 15 stars, respectively, at an average distance of 103 pc. After rolling dice and assigning absolute magnitudes to these stars, which gives them an average absolute magnitude  $\bar{M} = 4.7$ , our telescope will only see 10, 9 and 4 stars, respectively from these regions. Of the stars seen,  $\bar{M} \sim 4.5$ . The average distances of the observed stars is 95 pc. We miss the faintest stars, so our sample misrepresents the actual population.

Including the metallicity as given, re-rolling the dice gives 9, 9 and 3 observed stars in the 3 regions.  $\bar{M}$  for the stars is 4.9 but that of the observed stars is  $\bar{M} = 4.6$ . The average distance of the observed stars is now 94 pc. The original population had a mean metallicity of 0.66, but the observed stars have a mean metallicity of 0.69. Since we are more likely to observe the brighter stars, we will also see a higher mean metallicity since that leads to brighter stars.

This bias means that we will underestimate distances to an observed sample of stars compared to their true average distance. In this case, we will also overestimate their metallicities.

Another way of looking at it is: if you assume all stars in the sample have the same luminosity as the stars in the original sky, the fact we tend to observe the more luminous ones, which must be further away, means that we will overestimate the mean distances to the population.

5. Consider hydrogen gas in the galactic plane. Assume the Sun is at a distance of  $R_0 = 8$  kpc from the Galactic center with the average Galactic rotational velocity at that distance of  $V_0 = 200$  km/s. Also assume that  $V(R)$  is constant. Suppose there are 2 uniform rings of gas in the Galactic plane, one at 4 kpc radius and the other at 12 kpc radius, with negligible height and width compared to their radii. Derive an expression for the radial velocity of these rings as a function of Galactic longitude  $\ell$ . Also derive an expression for the observed intensity of each ring as a function of  $\ell$ .

For constant rotational velocity,  $V(R) = V_0 = 200$  km/s. In each case, we find the radial velocity to be  $v_r = R_0 V_0 (1/R - 1/R_0) \sin \ell$ . However, for the inner ring, the Galactic latitude has a limited range,  $R/R_0 = \sin \ell_{max}$  and  $|v_{r,max}| = V_0(1 - R/R_0)$ . This is shown in the aids to the problems at the back of the text.

The intensity of each part of a ring will vary as the inverse square of the distance, which is given in terms of a parametric angle  $\phi$ :

$$d = \sqrt{R^2 + R_0^2 - 2RR_0 \cos \phi}, \quad \sin \phi = (d/R) \sin \ell.$$

The angle *phi* is between the vectors  $\mathbf{R}$  and  $\mathbf{R}_0$  in the triangle formed by  $\mathbf{R}$ ,  $\mathbf{R}_0$  and  $\mathbf{d}$ .