

### Homework # 5, due 10 Mar

1. In example given in the notes describing the observations of collapsing clouds, it is indicated in some cases that there is a locus of points for which  $v_r$  is constant.  $v_r$  is the radial velocity seen by an observer in the  $\pm z$  direction (see the relevant figure).

For the case that  $v(r) = v_0 r^{-1}$ , find equations for  $(r, \theta)$ , or  $(x, y)$  where  $r = \sqrt{x^2 + y^2}$  and  $\theta = \tan^{-1}(x/y)$ , of the locus of points for which  $v_r = 2v_0/R_i = \text{constant}$ . Here,  $R_i$  is the outer radius of the collapsing region.

Show that in the case  $v(r) = v_0 r$  there is no locus of points for which  $v_r$  is constant.

2. In a Stromgren sphere containing H and He in solar proportions, why can't the ionization front for  $\text{He}^+$  ever be larger than that for  $\text{H}^+$ ?
3. Suppose the number density of bright F dwarfs, at the Sun's radial distance from the Galactic center, varied with height above the Galactic plane as

$$n(z) = n_0 e^{-z/z_h}$$

where  $z_h = 285$  pc. Also suppose that the velocity dispersion of these dwarfs varies with  $z$  as

$$\sigma_z^2 = \sigma_{z0}^2 (1 + z/z_h), \quad \sigma_{z0} = 14 \text{ km s}^{-1}.$$

Show that the surface mass density within 1 kpc of the plane is

$$\Sigma(< 1 \text{ kpc}) \simeq 90 \mathcal{M}_\odot \text{ pc}^{-2}.$$

What factor is this larger than the surface mass density in the Galactic disc at the Sun's position?

If, instead, one assumed that the velocity dispersion was independent of height above the Galactic plane, what would you find for  $\Sigma(< 1 \text{ kpc})$ ?