ASTRO 358 (49520): HOMEWORK 2

Assigned on Th Feb 28, 2008. Due by noon on Fri Mar 07, 2008 in RLM 16.224

Instructions: In order to get full credit, you must show the method that you used to derive the answer. The number of points for each question is indicated in brackets, and the total score is 100 points.

1. In the traditional bulge-to-disk decomposition of giant spirals, the azimuthally averaged radial surface brightness profile of the bulge is fitted with a de Vaucouleurs $R^{1/4}$ profile with an effective radius R_e,

$$I(R) = I_e \; e^{-7.67[(R/R_e)^{1/4}-1]} \tag{1}$$

while the outer disk is fitted with an exponential profile with scale length R_s

$$I(R) = I_0 \; e^{-(R/R_s)} \tag{2}$$

Consider a spiral galaxy, where the best fits parameters are $I_e = 20.5$ mag arcsec^{-2} in units of μ_B, $R_e = 0.9$ kpc, $I_0 = 21.0$ mag arcsec^{-2} in units of μ_B, and $R_s = 4.0$ kpc.

(a) Calculate the bulge-to-disk (B/D) ratio, defined as the ratio of the *integrated* luminosity of the bulge to the *integrated* luminosity of the disk. **[15 pts]**

(b) Calculate the bulge-to-total luminosity (B/T) ratio, defined as the ratio of the integrated luminosity of the bulge to the integrated luminosity of the combined (bulge + disk). **[5 pts]**

2. Consider a star in the disk of a spiral galaxy, moving at a speed $v \sim 3 \times 10^5$ m s^{-1} along a circular orbit of radius $R \sim 10$ kpc. Assume for simplicity that the mass distribution can be treated as spherically symmetric.

(a) Calculate the mass $M(R)$ enclosed within radius R, expressing your answer in $M_\odot$. **[15 pts]**

(b) The total stellar luminosity of the galaxy generated within radius R is 3.3×10^9 $L_\odot$. What is the average mass-to-light ratio $(M/L)_{\rm gal}$ of the galaxy within radius R?. **[5 pts]**

(c) Assuming that the average mass-to-light ratio $(M/L)_{\rm star}$ of stellar populations in the disk is ~ 6 $M_\odot/L_\odot$, determine what fraction of the total mass $M(R)$ interior to radius R is *not* in the form of stars? **[10 pts]**

(d) The total mass $M(R)$ interior to radius R is made up of stars, dust, gas, and dark matter. Optical and radio observations show that inside the radius R, the mass of stars is 2×10^{10} $M_\odot$, while the mass of gas and dust amounts to 5×10^9 $M_\odot$. Estimate the mass of dark matter inside the radius R, expressing your answer in $M_\odot$. **[10 pts]**.

(e) Determine what fraction of the total mass $M(R)$ interior to radius R is in the form of dark matter. **[10 pts]**.

3. Assume that the Milky Way has a rotation curve given by Figure 1 (PTO), and that its mass distribution can be considered as spherically symmetric. Let R_h be the radius of the dark matter halo.

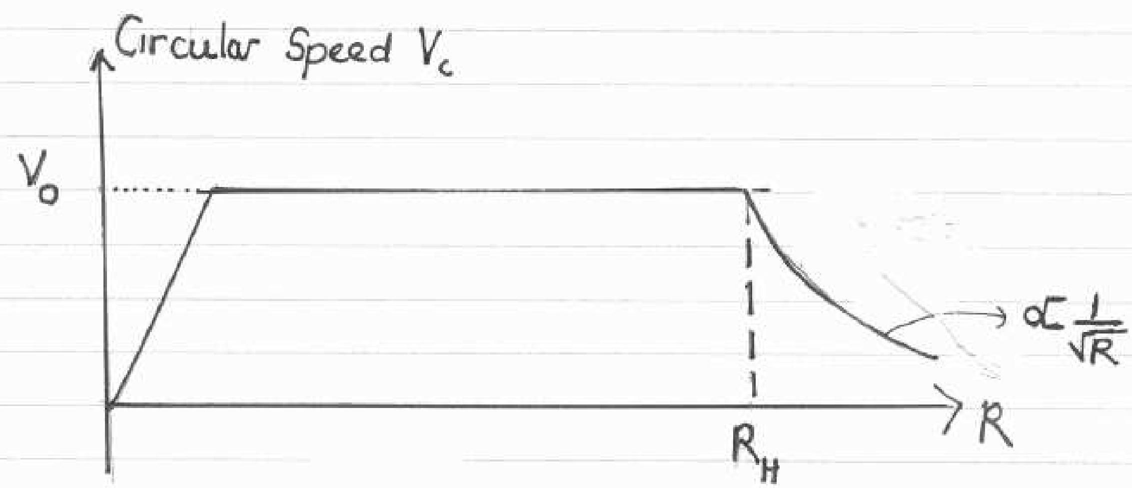
(a) Derive expressions for the total mass $M(R)$ of the galaxy interior to the radius R for two regimes: $R < R_\mathrm{h}$ and $R \geq R_\mathrm{h}$. **[2 x 5 = 10 pts]**

(b) Derive expressions for the gravitational potential $\Phi(R)$ for these two regimes ($R < R_\mathrm{h}$ and $R \geq R_\mathrm{h}$), by considering the work done in bringing unit mass from infinity to radius R. **[2 x 5 = 10 pts]**

(c) The fastest stars observed in the solar neighbourhood have speeds of 500 km s^{-1}. Taking this value as the lower limit to the escape speed at radius R, estimate a lower limit for the radius R_h of the dark matter halo, and for the total mass of the Milky Way. **[2 x 5 = 10 pts]**

Values of physical constants

1 $M_\odot = 2 \times 10^{30}$ kg.
G = Gravitational constant = 6.7×10^{-11} N m^2 kg^{-2}
Wien's constant $W = 2.9 \times 10^{-3}$ m K
Hubble's constant $H_0 = 75$ km s^{-1} Mpc^{-1}
σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}



R = radius measured from center of galaxy
R_H = radius of dark matter halo. Assume the galaxy contains no mass at $R > R_H$

V_0 = flat circular speed = 220 km s^{-1}

Fig. 1

END OF ASSIGNMENT