# ASTR 3830: Problem Set 1

Due in class Wednesday February 1st

1) A spiral galaxy has a rotation curve which rises linearly from zero at the center to 200 km/s at 5 kpc from the center. The rotation curve then remains flat at a constant value of 200 km/s out to 15 kpc, beyond which it can't be measured.

(a) Make a plot of M(r), the total mass enclosed within radius r, as a function of radius. Assume that the mass distribution in the galaxy is spherically symmetric. Note: 1 kpc = $3.086 \times 10^{21}$ cm. Use grams for the y-axis on the plot.

(b) Assume that the form of the rotation curve is dominated by dark matter. Plot the density of the dark matter in g $cm^{-3}$ versus radius.

(c) If this model applied to our own Galaxy, what would be the estimated mass in dark matter interior to the Earth's orbit in the Solar System (i.e. within 1 AU of the Sun)?

2) Suppose that stars are "standard candles" with some fixed luminosity L (this unrealistic assumption is not necessary, but it simplifies things), and a number density in the Solar neighborhood n that is *independent* of position.

Show that the number of stars that have a flux at Earth greater than some value $f_0$ scales as:

$$N(f > f_0) \propto f_0^{-3/2}$$

Suppose that instead of being uniformly distributed throughout space, the stars are instead distributed uniformly in a very thin disk. Find the distribution of N in this case.

3) A galaxy has a flat rotation curve, $v(r) = v_c$, with $v_c$ a constant, out to some radius R. Interior to R the dominant contribution to the potential is dark matter, with a spherically symmetric distribution. Outside R, the mass density is zero.

Show that the escape velocity from the galaxy for $r < R$ is given by,

$$v_e^2 = 2v_c^2\left(1 + \ln\frac{R}{r}\right)$$