

# THE DISTANCE TO THE CENTER OF THE GALAXY<sup>1</sup>

*Mark J. Reid*

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street,  
Cambridge, Massachusetts 02138

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## 1. INTRODUCTION

For nearly a century astronomers have expended considerable effort to determine the size of the Milky Way. This effort is worthwhile because any change in the value of the distance from the Sun to the center of the Galaxy,  $R_0$ , has widespread impact on astronomy and astrophysics. All distances determined from observed radial velocities and a rotation model of the Galaxy are directly proportional to  $R_0$ . Most estimates of the gravitational and luminous mass of the Galaxy scale with  $R_0$ . Similarly, the mass and luminosity of objects within the Galaxy, such as giant molecular clouds and the nonthermal source at the Galactic center depend on  $R_0$ . On a larger scale, since extragalactic distances are based on Galactic calibrations, the Hubble constant and  $R_0$  are interrelated. Indeed, it may be possible to use the size of the Milky Way as an extragalactic “meter stick” and determine distances to similar spiral galaxies (e.g. de Vaucouleurs 1983a,b; van der Kruit 1986).

Historically, astronomers have measured distances to nearby stars, used these distances to calibrate their luminosities, and estimated  $R_0$  from the spatial distributions of stars and globular clusters. Recently, however, direct measurements of  $R_0$  have become feasible and the possibility of using  $R_0$  as a distance standard has emerged. For example, if one knew  $R_0$  very accurately, one could recalibrate the absolute magnitudes of Cepheid,

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RR Lyrae, and Mira variables, as well as other Galactic distance indicators. This procedure could yield more accurate stellar luminosities and help understand, for example, some X-ray sources that appear to have super-Eddington luminosities for large  $R_0$  values. In addition, recalibrating the absolute magnitudes of variable stars would result in a better extragalactic distance scale and more accurate ages for stars. Thus, an improved distance scale for the Galaxy would lead to better estimates of the ages of globular clusters,  $H_0$ , and the age of the Universe. Finally, estimates of  $R_0$  and  $\Theta_0$ , the circular rotation speed of the Galaxy, are correlated in the sense that decreasing  $R_0$  necessitates decreasing  $\Theta_0$ . Since the value of  $\Theta_0$  affects estimates of the “dark matter” in the Local Group by affecting the Andromeda infall speed (Oort & Plaut 1975, Trimble 1986), better knowledge of  $R_0$  is important for this cosmological problem.

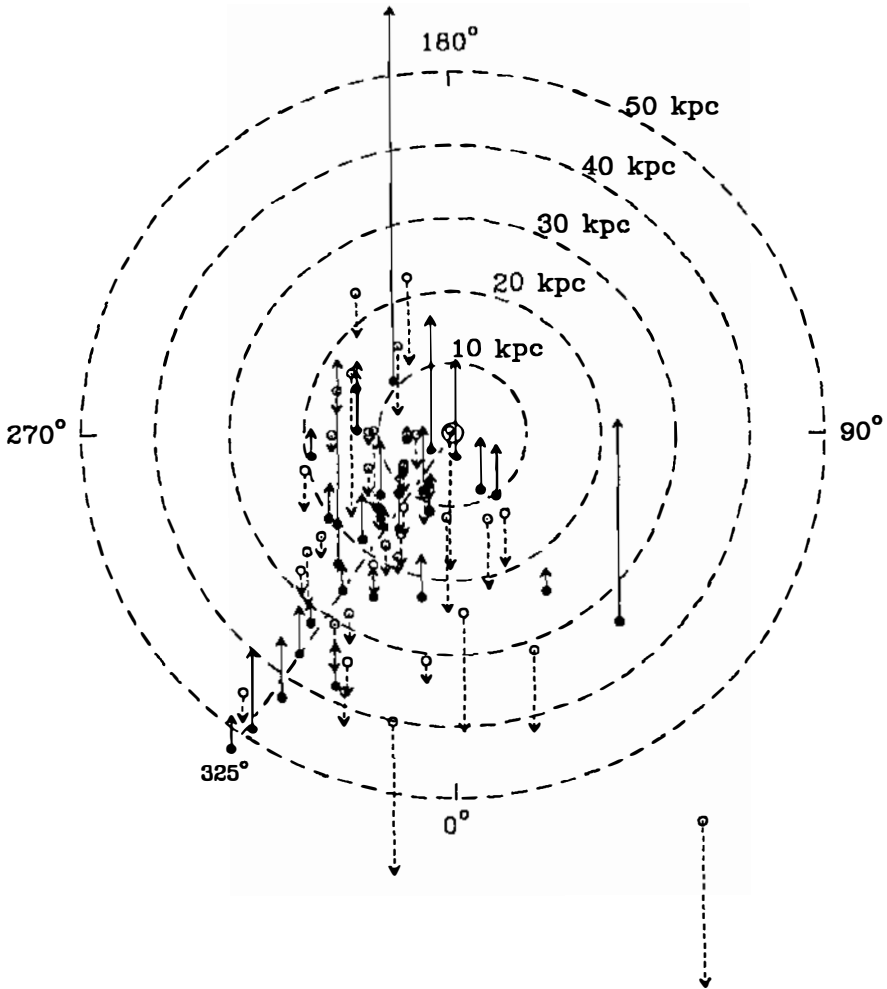
This review is primarily concerned with observations that lead to estimates of  $R_0$ . While, in principle, there can be different definitions of the center of the Galaxy, current observations indicate that the nonthermal radio source, Sgr A\*, and the infrared complex, IRS16, are within a parsec of the dynamical center of the Galaxy (see Genzel & Townes 1987). Throughout this review, the dynamical center and the location of Sgr A\* and IRS16 will be assumed to be the same.

There have been reviews of many aspects of the Galactic distance scale, including the book by Rowan-Robinson (1985) and articles by Kerr & Lynden-Bell (1986) and Feast (1987). Many details, especially of optical measurements, are reviewed in more detail in those works and, of course, in the primary references they cite. In this review, we emphasize the recent results coming out of a wide range of observations from the radio, infrared, optical, and X-ray portions of the electromagnetic spectrum. In Section 2 we briefly recount the pioneering work of Shapley in this field. In Section 3 we discuss estimates of  $R_0$  from papers published since 1974. This section includes a relatively new method of directly measuring  $R_0$  from  $H_2O$  maser proper motions. In Section 4, we continue the approach outlined by Reid (1989) and arrive at a current “best estimate” for  $R_0$ , taking into account statistical as well as systematic uncertainties. Finally, the interrelationships of the Galactic and extragalactic distance scales are discussed in Section 5.

## 2. EARLY HISTORY

In 1918 Harlow Shapley published a landmark paper on the distribution of globular clusters in the Milky Way. He estimated distances to a small number of clusters via the Cepheid period-magnitude relationship, and he filled out the remaining sample of 69 clusters with distances based on

magnitudes of the brightest stars in the clusters. When the locations of the globular clusters were plotted on a face-on view of the plane of the Milky Way, several remarkable characteristics became apparent (see Figure 1). First, clusters were not found within about 1 kpc perpendicular to the



*Figure 1* Face-on view of the Milky Way showing the distribution of globular clusters after Shapley (1918). The arrows indicate the location of clusters above (*solid*) and below (*dashed*) the plane. The Sun is located at the center; concentric circles are spaced at 10 kpc intervals and Galactic longitudes (System I) are indicated on the outer circle.

plane of the Milky Way. Second, the clusters were not symmetrically placed about the Sun, but were heavily concentrated toward Sagittarius, at a (system I) Galactic longitude of about  $325^\circ$ . Finally, Shapley deduced that globular clusters were most heavily concentrated at a distance of  $\approx 13$  kpc.

Shapley considered five hypotheses to explain the absence of globular clusters in the plane of the Milky Way. Among the discarded hypotheses was that “absorbing matter along the spine of the Milky Way analogous to the dark peripheral rings of spiral nebulae” obscured the clusters; instead, he preferred the possibility of a dynamical explanation (see Section 3.2.1 for a related effect). While the analogy between obscuration in the plane of the Milky Way and that seen toward spiral galaxies seems inescapable today, one should remember that in 1918 the concept that spiral nebulae were galaxies similar to the Milky Way was not well accepted. Indeed, at this time Shapley did not believe that spiral nebulae were outside of the Milky Way. Also, at this early date astronomers did not understand in any detail that interstellar dust grains were the absorbing matter, let alone that there was a significant interstellar medium.

While Shapley did not understand why globular clusters avoided the plane of the Milky Way, he correctly suggested that the asymmetric distribution of the clusters could be interpreted as originating from a symmetric one concentrated at the center of the Milky Way and observed from its periphery. Thus, the distribution of globular clusters “pointed a finger” at the location (near  $\alpha = 17^{\text{h}}30^{\text{m}}$ ,  $\delta = -30^\circ$ ) and distance of the Galactic center. The “finger” did not point at the apparent maximum of the general stellar distribution, which later was understood to be strongly affected by extinction.

Shapley’s estimate of 13 kpc for  $R_0$ , the distance to the Galactic center, is within a factor of two of the current best value (see Section 4). This level of agreement is somewhat fortuitous, since it is partially the result of compensating systematic errors. The absolute magnitudes of the Cepheid variables adopted by Shapley ( $M_v = -2.4$  at  $P = 6$  days) were too dim by about 1 mag. On the other hand, Shapley included “Population II Cepheids” (W Virginis stars) when establishing his distance scale. Since W Virginis stars typically are 2 mag dimmer than “Population I Cepheids,” the net effect was to have a distance scale approximately 1 mag too bright, leading to an overestimate of distance by about a factor of 1.6.

Over the past 75 years, much has been learned about the nature and populations of pulsating stars. Attention has focused on the effects of absorption and metallicity, and considerable effort has gone into understanding and correcting for systematic errors. Also, new approaches to measuring distance have emerged in the past decade. For example, obser-

vations in the infrared of the classical distance indicators (e.g. RR Lyraes and Cepheids) are much less sensitive to the effects of absorption and metallicity corrections than observations in the visible. In addition, totally new and direct methods of measuring distance from the proper motions of H<sub>2</sub>O masers at radio frequencies have been applied to the problem of estimating  $R_0$ .

### 3. DETERMINATIONS OF $R_0$

Broadly speaking, we categorize methods of measuring the distance to the center of the Galaxy as follows: primary measurements, secondary measurements, and indirect measurements. We define a primary measurement of  $R_0$  as a distance measured directly, without a secondary calibration (e.g. not using a “standard candle” calibration or Galactic rotation model), to a source at or very near to the Galactic center. Presently this has been done only for the H<sub>2</sub>O maser sources in Sgr B2. Secondary measurements use “standard candle” distances to objects whose distributions are assumed to be symmetrical about the Galactic center. This category includes the method used by Shapley of finding the centroid of the distribution of globular clusters, calibrated by Cepheid or RR Lyrae variables. Indirect determinations of  $R_0$  combine a variety of observations with either a model of the Galaxy or some other theoretical constraints. For example, one approach is to assume a fixed (e.g. Eddington) luminosity for a type of source and determine distances from observed fluxes.

#### 3.1 *Primary Measurements*

3.1.1 H<sub>2</sub>O PROPER MOTIONS Interstellar masers occur at the periphery of newly-formed, massive stars. Water vapor (H<sub>2</sub>O) is a trace constituent of the molecular material associated with these stars. Population inversion of the molecular energy levels, followed by coherent de-excitation, causes the appearance of masing “spots” of emission  $\sim 10^{13}$  cm in size with brightness temperatures as high as  $10^{15}$  K. Because of the small sizes and high brightnesses of interstellar maser spots, they are amenable to precise astrometric measurements which allow their proper motions to be determined. Very Long Baseline Interferometry (VLBI) techniques have achieved a relative positional accuracy of  $\sim 10$  micro-arcsec ( $\mu$ as) across fields of size  $\sim 3$  arcsec. This is sufficient to determine proper motions and estimate distances throughout the Galaxy and possibly to other galaxies.

The first proper motions of H<sub>2</sub>O masers were measured by Genzel et al (1981a). They found that the H<sub>2</sub>O maser spots in the Orion-KL region were expanding in an energetic stellar wind from a newly formed massive star, possibly IRc2. The maser spots were observed to move along straight

lines on the sky, and the three measured motions (two dimensions of proper motion and the line-of-sight motion) were modeled as an expanding, spherical source. A distance parameter was needed to transform the proper motions from angular to linear speeds, and independent information about (linear) expansion speeds came from the (Doppler shifted) line-of-sight motions. A least-squares fit of this model to the motion data allowed Genzel et al to estimate a distance to the Orion-KL region of  $480 \pm 80$  pc. This result was in good agreement with optical distance estimates of between 400 and 500 pc based upon luminosities of associated O-type stars.

Genzel et al (1981b) and Schneps et al (1981) measured proper motions of two  $\text{H}_2\text{O}$  masing regions in the W51 region and obtained distances of  $7 \pm 1.5$  and  $8.3 \pm 2.5$  kpc. Given a kinematic model for the Galaxy and the local standard of rest (LSR) velocity of W51, one can determine a kinematic distance to this region. This kinematic distance can be scaled by varying the value of  $R_0$  until the model distance matches the measured distance. W51 has an LSR velocity of about  $57 \text{ km s}^{-1}$  and a Galactic longitude of  $49^\circ$ ; thus, it lies near the Galactic tangent point. Unfortunately, for sources near the tangent point, small changes in LSR velocity imply large changes in the model distance. Assuming  $\Theta_0 = 220 \pm 20 \text{ km s}^{-1}$  and  $10 \text{ km s}^{-1}$  random motions, the more accurate W51 distance estimate yields  $R_0 = 10.8 \pm 4.6$  kpc (Reid et al 1988a). This large uncertainty precludes the use of the W51 distance to significantly constrain  $R_0$ .

Proper motion studies of the Sgr B2(North) water masers in the Galactic center region by Reid et al (1988a) reveal an expanding source qualitatively similar to the Orion-KL source. A least-squares fit to the data, as done for Orion-KL, yields an estimate of the distance to Sgr B2(North) of  $7.1 \pm 1.5$  kpc, where the uncertainty is from approximately equal contributions of statistical noise and systematic (modeling) uncertainty. Sgr B2(North) is almost certainly within 0.3 kpc of the Galactic center (see discussion in Reid et al) and, thus, the Sgr B2(North) distance can be directly used as an estimate of  $R_0$ .

The accuracy of distances to  $\text{H}_2\text{O}$  maser sources is primarily limited by two factors. First, the motions of individual maser spots exhibit a random component,  $\approx 15 \text{ km s}^{-1}$  in each coordinate, which exceeds a typical measurement uncertainty of a few  $\text{km s}^{-1}$ . For a given number of measured spot motions, this usually limits the precision of the technique. Second, the maser spots are not usually distributed uniformly around the exciting star. In Sgr B2(North), the masers appear to be distributed only over a hemisphere. This nonuniform distribution, coupled with the need to estimate the line-of-sight distance from the central star for each maser spot,

leads to correlations among model parameters, especially the expansion speed and the source distance. For Sgr B2(North), Reid et al (1988a) estimate a systematic modeling uncertainty of about 1.1 kpc.

Sgr B2(North) is only one of several H<sub>2</sub>O maser complexes in the giant molecular cloud Sgr B2, reflecting the contemporaneous formation of massive stars at several sites in that cloud. Proper motions of the Sgr B2(Middle) H<sub>2</sub>O masers have been presented by Reid et al (1988b). The maser spots in this source appear to have similar kinematic properties to those in Sgr B2(North), and preliminary analyses of the proper motions suggest a distance to Sgr B2(Middle) of between 6 and 7 kpc, with statistical and systematic uncertainties each of about 1 kpc. For the purposes of combining this result with others, we will adopt  $6.5 \pm 1.5$  kpc as the measured distance to Sgr B2(Middle).

Recently, Gwinn et al (1992) determined proper motions (see Figure 2) of the most luminous H<sub>2</sub>O maser in the Galaxy, W49(North). They measured motions of 105 maser spots and estimated a distance of  $11.4 \pm 1.2$  kpc. Even though this source is not near the Galactic center, it can be used to obtain a fairly precise estimate of  $R_0$ . As described above for W51, one can calculate a kinematic distance and adjust  $R_0$  to match the measured

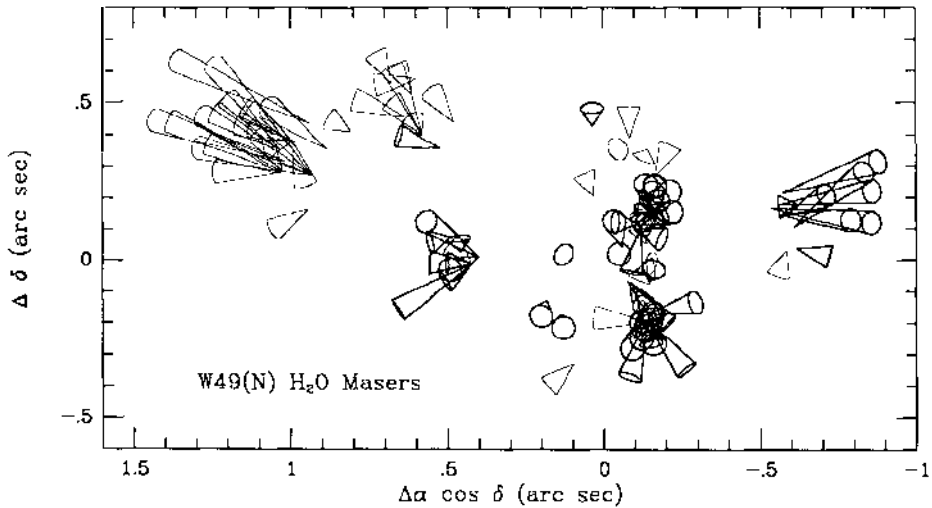


Figure 2 Proper motions of the H<sub>2</sub>O masers in W49(North) from Gwinn et al (1992). Cone apices indicate the positions of maser spots; cone lengths and orientations indicate the motion over 150 years; heavy and light cones are for red and blue shifted spots, respectively. The motions indicate a global expansion from a common center near the origin of the plot.

distance. Whereas W51 was unfavorably located near the Galactic tangent point, W49 is favorably located near the solar circle, and transforming the source distance to an estimate of  $R_0$  is not sensitive to the assumed rotation speed of the Galaxy ( $\Theta_0$ ). Gwinn et al conclude that  $R_0 = 8.1 \pm 1.1$  kpc. Strictly speaking, this estimate of  $R_0$  requires a kinematic model of the Galaxy. However, since the value of  $\Theta_0$  assumed does not significantly affect the uncertainty of  $R_0$ , we include the result here along with the Sgr B2 H<sub>2</sub>O maser estimates in Table 1.

**3.1.2 OH/IR STARS NEAR THE GALACTIC CENTER** The distance to late-type stars that have OH maser emission (OH/IR stars) can be estimated by measuring 1. the angular diameter of the OH maser shell directly by radio interferometry and 2. the light-travel time across the shell by determining the time lag of the variations of the redshifted emissions from the far side relative to the blueshifted emission from the near side of the shell. This method was pioneered by Schultz et al (1978) and Jewell et al (1980). Since there are many OH/IR stars near the Galactic center, it was hoped that one could directly determine  $R_0$  by measuring distances to these stars. Unfortunately, VLBI observations by van Langevelde & Diamond (1991) show that the angular sizes of OH/IR stars near the Galactic center are strongly affected by scattering due to electrons in the intervening interstellar medium. Since, intrinsic shell sizes cannot be measured, they conclude that OH/IR stars near the Galactic center cannot be used to measure  $R_0$  directly. However, one can still use distances to OH/IR stars near the solar circle to estimate  $R_0$  (see Herman et al 1985, Moran 1993, Section 3.3.1).

## 3.2 Secondary Measurements

**3.2.1 GLOBULAR CLUSTERS *Centroid of distribution*** This technique, pioneered by Shapley, assumes that globular clusters are symmetrically distributed about the Galactic center. Given the distance to each cluster in a large sample, one can project the cluster locations onto the line joining the Sun and the Galactic center. The distance to the center can then be estimated by finding the location of greatest density via the mean (or

**Table 1** H<sub>2</sub>O masers

Reference	$R_0$ (kpc)	Calibration	Comments
Reid et al 1988a	$7.1 \pm 1.5$	...	Sgr B2 (North)
Reid et al 1988b	$6.5 \pm 1.5$	...	Sgr B2 (Middle)
Gwinn et al 1992	$8.1 \pm 1.1$	Solar Circle	W49 (North)



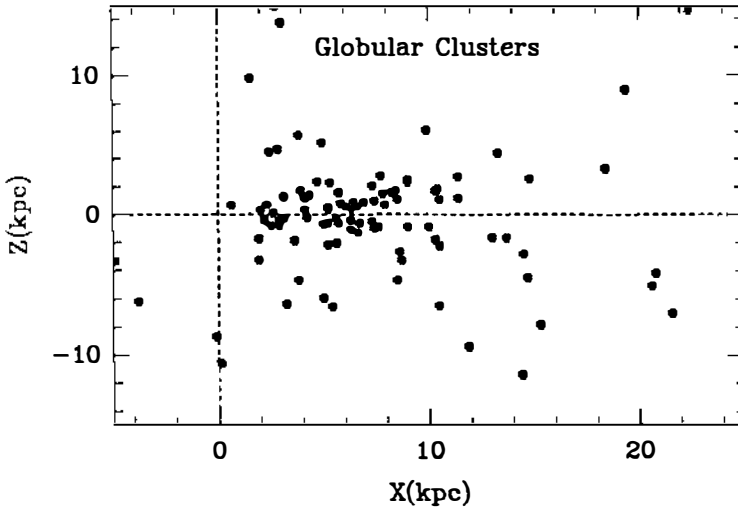


Figure 3 Distribution of globular clusters toward the Galactic center ( $X$ ) and perpendicular to the Galactic plane ( $Z$ ) after Harris (1976). The Sun is at the origin and distances are based on RR Lyrae stars, assuming  $M_v(\text{RR}) = 0.6$  mag.

median) of the distribution or by binning the data by distance and finding the peak of the resulting histogram. Figure 3 displays the projected positions of globular clusters toward the Galactic center and perpendicular to the Galactic plane for the sample of Harris (1976). Their distance scale is based on the apparent visual magnitude of the horizontal branch,  $M_v(\text{HB})$ , of 0.6 mag. Table 2 lists recent estimates of  $R_0$  from the distribution of globular clusters.

There are some difficulties in obtaining precise estimates of  $R_0$  from cluster distributions. First, different statistical estimators yield different

Table 2 Globular clusters

Reference	$R_0$ (kpc)	Calibration	Comments
Harris 1976, 1980	$8.5 \pm 1.6$	$M_v(\text{HB}) = 0.6$	Using means, $Z_{\text{lim}}$
Racine & Harris 1989	$7.5 \pm 0.9$	$M_v(\text{RR}) = 0.6$	Estimate absorption
de Vaucouleurs & Buta 1978	7.0	$M_v(\text{RR}) = 0.86$	Harris's method
Frenk & White 1982	$6.2 \pm 0.9$	$M_v(\text{HB}) = 0.6$	Low metallicity
	$7.2 \pm 1.1$	$M_v(\text{HB}) = 1.1$	High metallicity
Sasaki & Ishizawa 1978	$9.2 \pm 1.3$	$M_v(\text{HB}) = 0.5$	Cone of avoidance
Surdin 1980	$10.1 \pm 0.7$	$M_v(\text{RR}) = 0.6$	Metallicity distrib.

results. For example, a portion of the difference between the  $R_0$  values of Harris (1976, 1980) and Frenk & White (1982), which are based on essentially the same data set, stem from differences between using mean and median values of distributions. Second, crowding of stellar images in these very densely populated fields can lead to inaccurate measures of apparent magnitude. Finally, perhaps the most important problem is extinction. Clusters at low Galactic  $Z$ s (i.e. in the Galactic plane), and distant clusters at moderate  $Z$ s, will appear dimmer and reddened, or may not be detected. Indeed, few globular clusters are found close to the Galactic plane.

One method to minimize extinction uncertainties is to exclude clusters near the Galactic plane from analysis. Harris (1976, 1980) evaluated the globular cluster data and compiled estimates of  $R_0$  with varying  $Z$ -cutoffs. This technique reduces systematic uncertainties in the estimate of  $R_0$  but, by reducing sample sizes, it increases the statistical uncertainties. Recently, Racine & Harris (1989) took the alternative approach of using all the globular cluster data and trying to correct for the effects of extinction. This method of estimating  $R_0$  (see Table 2) has a precision of  $\pm 0.5$  kpc. Systematic errors in the zero point of the cluster distance scale and in the ratio of total-to-selective absorption become the dominant sources of uncertainty.

There is considerable controversy over the question of the absolute magnitude of the horizontal branch,  $M_v(\text{HB})$ , as a function of metallicity. Differences in  $M_v(\text{HB})$  of up to 0.5 mag, or a distance factor of about 25%, are involved. In particular, the analysis of Frenk & White (1982) suggests that significant differences in estimates of  $R_0$  appear between subsamples of globular clusters grouped by metallicity. These authors favor a combination of a fainter  $M_v$  with increasing metallicity and  $R_0 \approx 7$  kpc. Note that the calibration of  $M_v(\text{HB})$  is closely linked to the calibration of the absolute magnitude of RR Lyrae stars (see Section 3.2.2).

*Cone of avoidance* Wright & Innanen (1972) noted that the density of globular clusters diminishes in a cone whose axis is aligned with the Galactic rotation ( $Z$ ) axis and has an opening angle of  $\sim 15^\circ$ . Sasaki & Ishizawa (1978) suggest that tidal interactions near the Galactic center will preferentially disrupt clusters located along the Galactic rotation axis. They claim that setting  $R_\bullet = 9.2 \pm 1.3$  kpc maximizes the cone angle and that this procedure indicates the distance of the Galactic center. This distance estimate is based upon the catalog of Peterson & King (1975), which has a variety of calibrations, including  $M_v(\text{HB}) = 0.5$  mag and no correction for cluster metallicity.

*Metallicity distribution* The metallicity of globular clusters decreases with distance,  $R$ , from the Galactic center. Surdin (1980) points out that if

the globular cluster distribution is axially symmetric about the Galactic rotation axis, then  $R_0$  can be estimated by adjusting its value (and rescaling cluster distances) until the cluster metallicity is *uncorrelated* with galactocentric azimuth. Metallicity estimates are not strongly affected by extinction corrections, thus minimizing this source of systematic error. Surdin estimates a value of  $R_0 = 10.1 \pm 0.7$  kpc, averaging analyses of the catalogs of Harris (1976), based on  $M_v(\text{RR}) = 0.6$  mag, and Kukarkin (1974), based on a metallicity dependent  $M_v(\text{RR})$ . The quoted statistical error seems to be considerably underestimated, since it is based on the scatter in a plot of the correlation coefficient (of metallicity with galactocentric azimuth) versus  $R_0$ . Because the same data set is used for each point in that plot, the points are correlated, leading to an underestimate of the variation in  $R_0$  that would arise were an ensemble of globular cluster data sets available.

**3.2.2 RR LYRAE VARIABLES** Individual RR Lyrae variables can be seen across the Galaxy and toward the Galactic center through fortuitous “windows” of low extinction such as Baade’s Window. Figure 4 is a

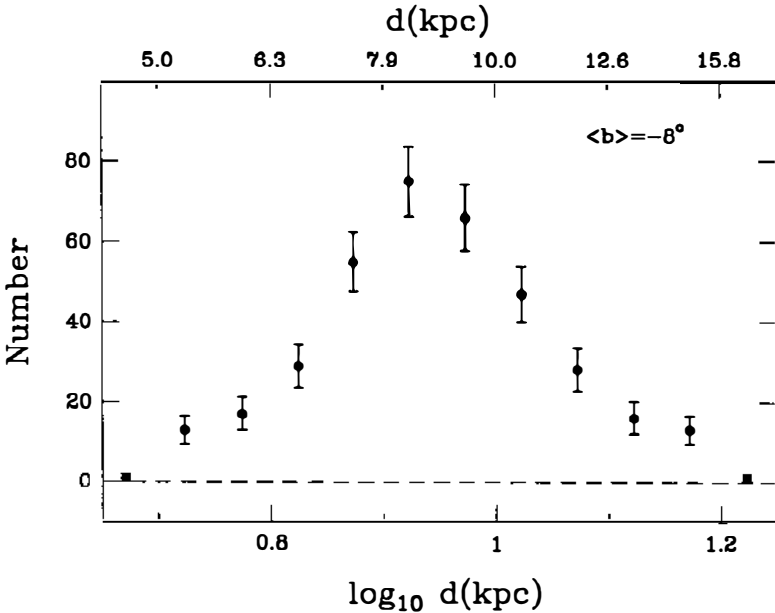


Figure 4 Histogram of the number of RR Lyrae stars in logarithmic bins of projected distance toward the Galactic center (after Oort & Plaut 1975). The distance scale is based on  $M_{pg} = 0.7$  [or  $M_v(\text{RR}) \approx 0.63$ ], independent of metallicity.

histogram of the number of RR Lyrae variables in the most populated “window” at  $l^{\text{II}} = 0^\circ$  and  $b^{\text{II}} = -8^\circ$  from the work of Oort & Plaut (1975).  $R_0$  can be estimated from the distance corresponding to the peak of the distribution of RR Lyrae variables toward the Galactic center (similar to the method used for globular clusters). Table 3 summarizes recent RR Lyrae results.

Since  $M_v(\text{HB})$  and  $M_v(\text{RR})$  are nearly identical, globular cluster distances are *correlated* with those of RR Lyrae variables. As for globular clusters,  $M_v(\text{RR})$  as a function of metallicity is not well known. This problem has been hotly debated for more than a decade, with claims of  $M_v(\text{RR})$  ranging from about 0.3 to 1.1 mag and arguments for no metallicity sensitivity to ratios of  $\Delta M_v / \Delta[\text{Fe}/\text{H}] \approx 0.4$ .

Feast (1987) provides an extensive discussion of the metallicity dependence of  $M_v(\text{RR})$  and more complete bibliographic references than will be given below. Some recent determinations of the absolute magnitude of RR Lyrae stars as a function of metallicity are displayed in Figure 5. The most persuasive arguments for a significant metallicity dependence come from the explanation of the Oosterhoff effect by Sandage (1982, 1992) and from Baade-Wesselink method results (e.g. Liu & Janes 1990a, Longmore et al 1990, Carney et al 1992). Some studies report finding “no difference” in  $M_v(\text{RR})$ , when dividing their samples into high and low metallicity groups; however, most of these studies do not have sufficient sample sizes and/or internal precision to claim significance at levels of interest. While the precise value for the change in  $M_v(\text{RR})$  with  $[\text{Fe}/\text{H}]$  is not yet agreed upon, an examination of Figure 5 suggests that it probably lies between a  $+0.1$  and  $+0.3$  mag dex $^{-1}$ .

In addition to the debate over metallicity corrections, the “canonical value” of  $M_v(\text{RR}) = 0.6$  mag has been questioned by many workers. Over the last half-dozen years a number of studies were published that suggest an absolute magnitude near 0.7 mag or dimmer. Hawley et al (1986) and

**Table 3** RR Lyrae variables

Reference	$R_0$ (kpc)	Calibration	Comments
Oort & Plaut 1975	$8.7 \pm 0.6$	$M_{\text{pg}}(\text{RR}) = 0.7$	
Clube & Dawe 1980	$7.0 \pm 1.0$	$M_v(\text{RR}) = 1.0$	
Blanco & Blanco 1985	$8.0 \pm 0.7$	$M_v(\text{RR}) = 0.6$	all metallicities
	$6.9 \pm 0.6$	$M_v(\text{RR}) \propto [\text{Fe}/\text{H}]$	mean $M_v = 0.82$
Walker & Mack 1986	$8.1 \pm 0.4$	$M_v(\text{RR}) = 0.6$	
Fernley et al 1987	$8.0 \pm 0.6$	$M_v(\text{RR}) \approx 0.6$	infrared
Walker & Terndrup 1991	$8.2 \pm 1.0$	$M_v(\text{RR}) = 0.85$	$[\text{Fe}/\text{H}] \approx -1$

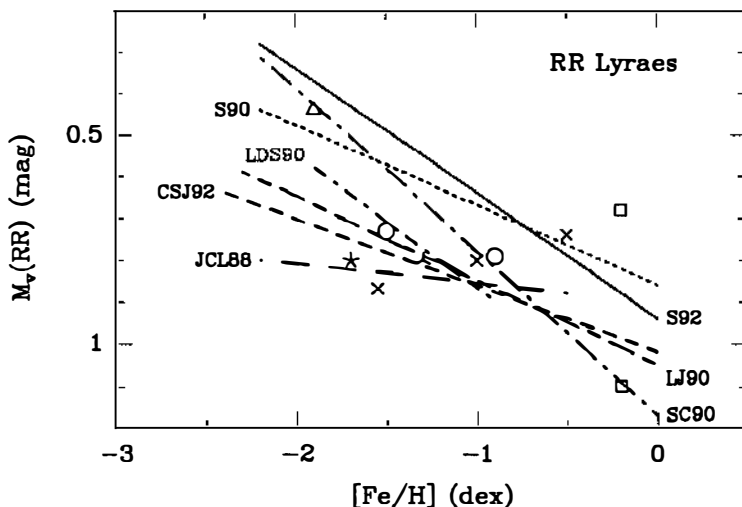


Figure 5 Recent estimates of the absolute visual magnitude of RR Lyrae stars,  $M_v(\text{RR})$ , as a function of metallicity,  $[\text{Fe}/\text{H}]$ . References for different lines are as follows: CSJ92—Carney et al (1992); JCL88—Jones et al (1988); LDS90—Longmore et al (1990); LJ90—Liu & Janes (1990a); S90—Sandage (1990); S92—Sandage (1992); SC90—Sandage & Cacciari (1990). Studies of stars with a limited range of metallicity are indicated with symbols as follows: Walker (1992, *triangle*); Hawley et al (1986, *circles*); Jones et al (1992, *squares*); Strugnell et al (1986, *4-pt crosses*); Liu & Janes (1990b, *3-pt cross*); Sandage (1982, *5-pt cross*).

Barnes & Hawley (1986), from proper motion data, suggest  $M_v(\text{RR}) \approx 0.7$ . Longmore et al (1990) determined  $M_v(\text{RR}) \approx 0.74$  (evaluating their metal-dependent relation at  $[\text{Fe}/\text{H}] = -1.3$ ) from distances determined from observations at infrared wavelengths. Strugnell et al (1986), analyzing stars with statistical parallaxes, favor  $M_v(\text{RR}) = 0.75$ . Finally, Jones et al (1988) and Liu & Janes (1990a,b), using distances from the Baade-Wesselink method, argue for  $M_v(\text{RR}) \approx 0.85$ .

On the other side of the debate one finds recent papers that argue for a  $M_v(\text{RR})$  brighter than 0.7 mag. For the references cited below, we quote  $M_v(\text{RR})$  values at  $[\text{Fe}/\text{H}] = -1.3$  when a metal-dependent relation is given. Sandage & Cacciari (1990) arrive at  $M_v(\text{RR}) = 0.66$  for double-mode pulsators, and the study by Sandage (1990) of field RR Lyrae stars yields  $M_v(\text{RR})$  between 0.61 and 0.76. Also, Sandage (1992) obtains  $M_v(\text{RR}) = 0.55$  (at  $[\text{Fe}/\text{H}] = -1.3$ ) from an analysis of the pulsation equation (i.e. luminosity versus period, temperature, and mass) with parameters evaluated along the fundamental blue edge of the instability strip

in the HR diagram. Finally, additional support for a bright  $M_v(\text{RR})$  is found by Walker (1992) and Saha et al (1992) for RR Lyrae stars in other galaxies (see Section 5), assuming distances calibrated by Cepheid variables and other techniques.

For the purposes of this review we require a method of re-normalizing the estimates of  $R_0$  that are calibrated by  $M_v(\text{RR})$  or  $M_v(\text{HB})$ . While a metal-dependent normalization would be optimal, this would be a formidable task to implement, since the effective metallicity of many samples is not given. Instead, we adopt a calibration at the “midpoint” of the results shown in Figure 5,  $M_v(\text{RR}) = 0.7$  at  $[\text{Fe}/\text{H}] \approx -1.3$ , and use this value to re-normalize all appropriate  $R_0$  estimates. The effect of neglecting a metallicity correction should be diminished by averaging over the results based on different samples, and the effect is probably much smaller than the uncertainty in the “average”  $M_v(\text{RR})$  value adopted.

One further level of complication has recently emerged: Corrections to absolute magnitudes for metallicity may be sensitive to the adopted value of the ratio of total-to-selective absorption,  $A_v/E(B-V)$ . Because heavily reddened clusters tend to have high metallicities, calibrations may be complicated by correlations between reddening and metallicity corrections (Longmore et al 1990). Walker & Terndrup (1991) point out that their  $R_0 = 8.2$  kpc assumed  $A_v/E(B-V) = 3.1$ , but find  $R_0 = 7.7$  kpc for  $A_v/E(B-V) = 3.35$ .

Because of the controversy over metallicity and extinction corrections, observers have looked toward infrared observations as offering considerably less sensitivity to these effects. Compared to observations in the visible, infrared observations seem preferable for many reasons. First, the light curves of stars in the instability strip have smaller amplitude excursions at infrared ( $\approx 0.2$  mag) compared to visible ( $\approx 1$  mag) wavelengths. Thus, one can determine the *mean* apparent magnitude of a star from a small number of observations much more accurately at infrared compared to visible wavelengths. Second, infrared absorption corrections are considerably smaller than visible corrections. Third, because stellar atmospheric opacities due to molecular blanketing are smaller at longer wavelengths, metallicity corrections to absolute magnitudes are smaller in the infrared. Finally, there are advantages in the infrared for distance measurements using the Baade-Wesselink method. The Baade-Wesselink method relates the change in apparent size of the star, inferred from the apparent magnitude and effective temperature, to the change in size predicted from the radial velocities of photospheric lines integrated over time. Contributions to the apparent magnitude from temperature changes (as opposed to stellar size changes) complicate the analysis. Such temperature corrections are greatest near the peak of the blackbody spectrum, and are

minimized in the Rayleigh-Jeans portion of the spectrum in the infrared.

Recently, efforts have been made to determine period-luminosity relations for RR Lyrae stars at infrared wavelengths of 1.6 and 2.2 $\mu$ . From 1.65 $\mu$  photometry of 70 RR Lyrae stars in Plaut's Field #3 (at  $l^{\text{II}} = 0^\circ$ ,  $b^{\text{II}} = 12^\circ$ ), Fernley et al (1987) claim  $R_0 = 8.0 \pm 0.35$  kpc, but with an additional systematic error of  $\pm 0.55$  kpc to account for a  $\pm 0.15$  mag uncertainty in the zero point of the period-infrared magnitude relation. However, their calibration of the period-infrared magnitude relation uses only three stars whose distances have been measured via the Baade-Wesselink method (assuming  $M_v(\text{RR}) = 0.79 + 0.15[\text{Fe}/\text{H}]$  mag), and the calibration should be improved with more data.

**3.2.3 GIANTS AND MIRAS** Bright stars, other than RR Lyrae variables, can be seen through interstellar windows. For example, Mira variables (Glass & Feast 1982) are a particularly attractive class of stars for estimating  $R_0$ , since they are luminous and can be observed with moderately sized telescopes. In addition, they are bright at infrared wavelengths where the effects of extinction are greatly reduced. However, the complex molecular opacity sources in the atmospheres of these cool stars makes calibration of absolute magnitudes less certain than for the shorter period variables such as RR Lyrae. Table 4 gives estimates of  $R_0$  from red giant stars.

van den Bergh & Herbst (1974) estimate  $R_0$  from the apparent visual magnitude at which a histogram of all stars in the window at  $l^{\text{II}} = 0^\circ$ ,  $b^{\text{II}} = 12^\circ$  shows a peak. They assume that the peak density occurs for stars at the main-sequence turn-off. Adopting an absolute visual magnitude for the turn-off,  $M_v(\text{TO})$ , a 3.85 mag, they obtain an estimate of  $R_0 = 9.2 \pm 2.2$  kpc. This window is one of those used by Oort & Plaut (1975) in their study of RR Lyrae stars, and they find  $R_0 = 8.4$  kpc for this subset of their data. While the difference between  $R_0$  estimates from these two studies, based on different stellar populations, is well within their uncertainties, Oort & Plaut warn that the maximum in the van den Bergh & Herbst histogram might correspond to stars up to one magnitude beyond the turn-off, which could reduce the van den Bergh & Herbst estimate of  $R_0$ .

**Table 4** Giant stars

Reference	$R_0$ (kpc)	Calibration	Comments
van den Bergh & Herbst 1974	$9.2 \pm 2.2$	$M_v(\text{TO}) = 3.85$	Main seq. turn-off
Glass & Feast 1982	7.9	$M_{\text{bol}}(0) = 0.76$	Galactic calibration
	8.8	$M_{\text{bol}}(0) = 0.54$	LMC = 18.69 mag

Glass & Feast (1982) observed 70 Mira variables at infrared wavelengths toward two Galactic center windows. They estimate  $R_0$  at 7.9 or 8.8 kpc, based on two different calibrations of the bolometric magnitude,  $M_{\text{bol}}$ , of Miras. They point out that the zero point,  $M_{\text{bol}}(\log P = 0)$ , in the period-luminosity calibration of Miras differs by 0.2 mag (see Table 4) depending on whether one adopts a Galactic calibration, based on solar neighborhood stars (using a combination of methods, including statistical parallaxes), or a calibration based on the Large Magellanic Cloud (LMC) distance [assuming  $(m - M)_0 = 18.69$  mag] from Cepheid variables. Note, however, that recent estimates of the distance to the LMC, including those based on the expanding ring around SN 1987A (Panagia et al 1991) and on SN photospheric models (Schmidt et al 1992), indicate a distance modulus near 18.50 mag which would bring the LMC calibration of Glass & Feast into agreement with the Galactic calibration (see Section 5 for further discussion) and reduce the 8.8 kpc estimate for  $R_0$  of Glass & Feast to about 8.0 kpc.

Even though bolometric magnitudes of red giant stars are less sensitive to absorption than those of hotter stars such as RR Lyrae variables, differences in absorption estimates allow for an uncertainty of  $\approx 0.1$  magnitude. The estimates of  $R_0$  by Glass & Feast (1982) assume  $A_v = 2.03$  for the Mira variables in Baade's Window. Perhaps greater uncertainty comes from the use of solar neighborhood (Pop. I) Miras to calibrate the absolute magnitudes of Galactic center (Pop. II) Miras. While Glass & Feast point out that the two populations of Miras have similar emission lines and colors, little quantitative data exist regarding absolute magnitude differences. Indeed, Frogel & Whitford (1987) studied M-type giants in Baade's Window and found that bulge giants have bluer colors, and can be up to 2 mag fainter, than solar neighborhood giants of the same spectral type.

### 3.3 *Indirect Measurements*

There are a great variety of objects and analysis techniques in the literature that lead to indirect estimates of  $R_0$ . Table 5 summarizes some of the recent results; the reader should consult the original papers for more details of the distance calibrations for the various objects.

**3.3.1 ROTATION MODELS OF THE GALAXY** Given a kinematic model of the Galaxy, stars and (atomic, molecular, and ionized) clouds that partake in the Galactic rotation and have measured distances can be used to estimate  $R_0$ . For example, radial velocity measurements for a sample of stars can be used with a kinematic model for the Galaxy to derive "kinematic" distances. These distances can be compared with luminosity distances, and they can be brought into agreement by adjusting  $R_0$ , since



**Table 5** Cepheids, OB stars, HII regions, etc

Reference	$R_0$ (kpc)	Calibration <sup>a</sup>	Comments
Cruz-González 1974	$8.9 \pm 0.5$	$A = 15$	Nearby stars
Toomre 1972; Rybicki et al 1974	9	$A - B = 25$	Modelling
Bolona & Feast 1974	9.0	$H_\beta$	OB's; $\odot$ -circ.
Crampton et al 1976	8	$H_\gamma$	OB's; $\odot$ -circ.
Byl & Ovenden 1978	$10.4 \pm 0.5$		Mostly OB's
Caldwell & Coulson 1987	$7.8 \pm 0.7$	LMC = 18.45	188 Cepheids
Caldwell et al 1992	$8.5 \pm 0.5$	LMC = 18.55	212 Cepheids
Quiroga 1980	8.4		HI vs OB's
Brand 1986; Blitz & Brand 1988	$8.0 \pm 0.5$	$\Theta_0 = 220$	HII regions
Rohlfs et al 1986	$7.9 \pm 0.7$		HII regions
Herman et al 1985	$9.2 \pm 1.2$	$\Theta_0 = 250$	OH/IR stars
Moran 1993	$8.8 \pm 0.9$	$\Theta_0 = 220$	OH/IR stars
Backer & Sramek 1986, 1992	$7.7 \pm 0.9$	$\Theta_0 = 220$	Sgr A*
Caldwell & Ostriker 1981	$9.1 \pm 0.6$	$M_r(\text{RR}) = 0.6$	Modelling

<sup>a</sup> Units of Oort's  $A$  and  $B$  constants are  $\text{km s}^{-1} \text{kpc}^{-1}$ ;  $\Theta_0$  is in  $\text{km s}^{-1}$ ; LMC distance modulus is in magnitudes.

kinematic distances scale directly with  $R_0$ . Similarly, one can use the magnitude of Oort's  $A$  parameter, the local velocity shear, Galactic distance scale. Given radial velocities and distances for stars near the Sun, one can adjust  $R_0$  in a model to match the observed velocity shear with the assumed  $A$  parameter. For a flat rotation curve for the Galaxy, the smaller the Galaxy the larger the apparent shear. Thus, independent measurements of Oort's  $A$  and  $\Theta_0$ , yield

$$2AR_0 = \Theta_0.$$

While Cepheid variables play a crucial role in the *extragalactic* distance scale, their impact on  $R_0$  is not that great, primarily because Cepheids are not as abundant as RR Lyrae stars. This severely limits the accuracy of  $R_0$  estimates using the centroid of distribution approach. However, one can combine radial velocities and distances of Cepheid variables, in the context of a rotation model for the Galaxy,

Caldwell and collaborators (see Table 5). Here the Galactic and extragalactic distance scales are interrelated, since these workers adopt an extragalactic distance calibration for Galactic Cepheids.

Some progress in avoiding calibration problems will likely result from efforts to observe Cepheids in the infrared, for similar reasons as given above for RR Lyrae stars. Also, Madore (1985) has noted that results of B and V photometry can be combined to produce tighter period-luminosity relationships than from V alone. This procedure recovers some of the benefits of infrared approaches for the vast body of visual observations

of Cepheids. Unfortunately, reddening effects are not minimized via the method of Madore as they are for observations in the infrared.

Thackeray (1972) and Crampton et al (1976) point out that there is a considerable difference in the kinematic properties of OB-type stars in the northern and southern portions of the Galaxy. Northern stars tend to yield  $R_0$  values about 3 to 4 kpc smaller than southern stars. Byl & Ovenden (1978) claim to reconcile some of this difference by accounting for noncircular motions associated with spiral structures. It is important to remember that most of these methods are sensitive to local deviations from noncircular motions and/or to sizable extinction corrections. Also, Bolona & Shobbrook (1984) question the calibrations of absolute magnitudes of early type (OB) stars and suggest that giant and supergiant OB stars are 0.3 mag dimmer than previously assumed. Adopting the calibrations of Bolona & Shobbrook would reduce OB-type star distances by about 15%.

Some of the  $R_0$  estimates cited in Table 5 are sensitive to the assumed value for  $\Theta_0$ , the circular rotation speed of the Galaxy at the position of the Sun. The current IAU recommendation for  $\Theta_0$  is  $220 \text{ km s}^{-1}$  (cf Knapp 1983, Kerr & Lynden-Bell 1986). However, there is considerable controversy over the correct value for  $\Theta_0$ . Recently, for example, Rohlfs et al (1986) suggest  $\Theta_0 = 184 \text{ km s}^{-1}$ , Alvarez et al (1990) argue for  $\Theta_0 = 209 \text{ km s}^{-1}$ , while other estimates range to  $250 \text{ km s}^{-1}$  or higher (e.g. Shuter 1983). An additional caveat comes from Blitz & Spergel (1991) who model the Galaxy as a rotating triaxial spheroid and find that circular speeds may be uncertain by up to 20%.

The measurement of the proper motion of the compact, nonthermal radio source at the Galactic center, Sgr A\*, by Backer & Sramek (1986, 1992) is of particular importance for constraining  $R_0$ . This work uses the Very Large Array (VLA) operating at 6-cm wavelength in the largest (A) configuration to determine the apparent shift in the position of Sgr A\*, caused primarily by the orbit of the Sun around the Galactic center. They obtain an estimate of apparent motion of Sgr A\* of  $-0''.00639 \pm 0''.00074 \text{ yr}^{-1}$  in Galactic longitude and  $-0''.00011 \pm 0''.00060 \text{ yr}^{-1}$  in Galactic latitude. After correcting for the peculiar motion of the Sun, the Galactic longitude result yields a direct observational estimate of the quantity  $\Theta_0/R_0$ . For  $\Theta_0 = 220 \text{ km s}^{-1}$ , this implies  $R_0 = 7.7 \text{ kpc}$ . Also, for a flat rotation curve,  $2AR_0 = \Theta_0$ , and hence this result implies  $A = 14.2 \text{ km s}^{-1} \text{ kpc}^{-1}$ . These interpretations require that there is no significant motion of Sgr A\* with respect to the dynamical center of the Galaxy; the "null result" for apparent motion in Galactic latitude supports this assumption.

3.3.2 LUMINOSITY LIMITS *X-ray sources* Ebisuzaki et al (1984) estimated the luminosity of a sample of X-ray bursters. Assuming that the

emission is associated with a  $1.4 M_{\odot}$  compact object (e.g. a contact binary containing a neutron star) and that the emission is at the Eddington limit, they derive “luminosity distances” for the sample. The distribution for 27 bursters peaks toward the Galactic center at a distance of  $\approx 7$  kpc (Table 6). This can be taken as an estimate of (or an upper limit to)  $R_0$ , provided the emission is at (or below) the Eddington limit. More recently, Ebisuzaki (1987) modeled the X-ray luminosity of a neutron star with attention to the opacity from chromium and iron. This study indicates the X-ray luminosity models, and hence distance estimates, can vary systematically by at least 15%.

Cyg X3 is a contact binary containing one and possibly two compact objects. It is a strong, periodic X-ray and radio source. HI (21-cm wavelength) studies (Dickey 1983) indicate that all Galactic HI emission lines are seen in absorption against the radio continuum emission of Cyg X3, implying a distance of at least  $1.16R_0$ . If the (X-ray) emission from Cyg X3 is at the Eddington limit for a  $1.4 M_{\odot}$  object, then Molnar (1985) finds a distance of 9.0 kpc, suggesting  $R_0 = 7.7$  kpc (Table 6).

The critical assumptions used to estimate (or limit)  $R_0$  from the X-ray bursters and Cyg X3 are that the emissions are at (or below) the Eddington limit and that the compact objects ultimately responsible for the emissions have typical neutron star masses of  $1.4 M_{\odot}$ . While the former assumption seems reasonable, the possibility that more massive objects, perhaps black holes, are involved should be considered. Increasing the stellar mass increases the limiting luminosity and, for an observed flux density, increases the distance to the source.

*Planetary nebulae* Observations of luminosity functions of planetary nebulae have been used to obtain distance estimates to the Magellanic Clouds and other galaxies. This method involves comparing the maximum luminosity observed in a large sample with a theoretically determined maximum luminosity. Dopita et al (1992) apply this method to the bulge-population planetary nebulae and estimate a “sample distance” of between 7.5 and 9.1 kpc, for nebulae ages of 5.0 and 0.8 Gy, respectively. They favor the greater nebulae age and adopt  $R_0 = 7.6 \pm 0.7$  kpc (Table 6).

**Table 6** Luminosity limits

Reference	$R_0$ (kpc)	Calibration	Comments
Ebisuzaki et al 1984	7	$L_{\text{Edd}}$	X-ray bursters
Molnar 1985	$7.7 \pm 1$	$L_{\text{Edd}}$	Cyg X-3
Dopita et al 1992	$7.6 \pm 0.7$	Lum. Funct.	Planetary Nebulae

In addition to a strong dependence of luminosity with age, metallicity assumptions affect calculated luminosities, as well as the assumption that hydrogen (not helium) burning is appropriate.

#### 4. A “BEST VALUE” FOR $R_0$

It is not possible to combine all existing estimates of  $R_0$  to form a “best value” in a statistically rigorous manner, as this would require knowledge of the full variance-covariance matrix for the set of  $R_0$  estimates. Unfortunately, we do not have reliable values of the uncertainty for each estimate because *systematic* sources of error are poorly known and often not given. In addition to not knowing the variances, we have only a qualitative understanding of the *covariances* among the different  $R_0$  estimates. For example, a change in the RR Lyrae absolute magnitudes directly affects the calibration of absolute magnitudes for globular clusters and to some extent for other stars such as red giants. Thus, the covariances among different methods of determining  $R_0$  are substantial.

In Figure 6, we plot  $R_0$  versus publication date for the results cited in this review. Based upon this plot, a case could be made for a statistically significant decrease in estimates of  $R_0$  with time until about 1990. Since we have rescaled the  $R_0$  estimates to common calibrations (see below), this effect is probably not a result of the “evolution” of the calibrations. One could speculate that a significant “bandwagon effect” is operative here. Statistical analyses of (usually incomplete) astronomical data are not straightforward. Unfortunately, it is all too easy to allow current wisdom as to the “correct answer” to subtly affect judgements, for example, of how to edit data. Faced with these problems Kerr & Lynden-Bell (1986) adopted the simplest approach to finding a “best value” for  $R_0$  and calculated an *unweighted* average of  $R_0$  values published between 1974 and 1986, without rescaling to common calibrations. In this manner they arrived at the IAU recommended value of  $R_0 = 8.5$  kpc. We will adopt a different approach, trying to account (in an admittedly crude manner) for statistical and systematic errors as well as for the covariances among different methods.

Table 7 presents eight groupings of  $R_0$  estimates that have *nearly independent* calibrations. Each group is further subgrouped by the stars or sources used to estimate  $R_0$ . Each entry in the table contains an unweighted mean value of  $R_0$  for that subgroup (from data given in Tables 1 to 6) and a *statistical* uncertainty that approximately reflects the precision of the technique. We have adopted  $M_v(\text{RR}) = 0.70$  mag,  $\Theta_0 = 220 \text{ km s}^{-1}$ , and  $(m - M)_0 = 18.47$  mag (49.4 kpc) for the LMC, and, before combining results, we rescaled the  $R_0$  estimates accordingly.

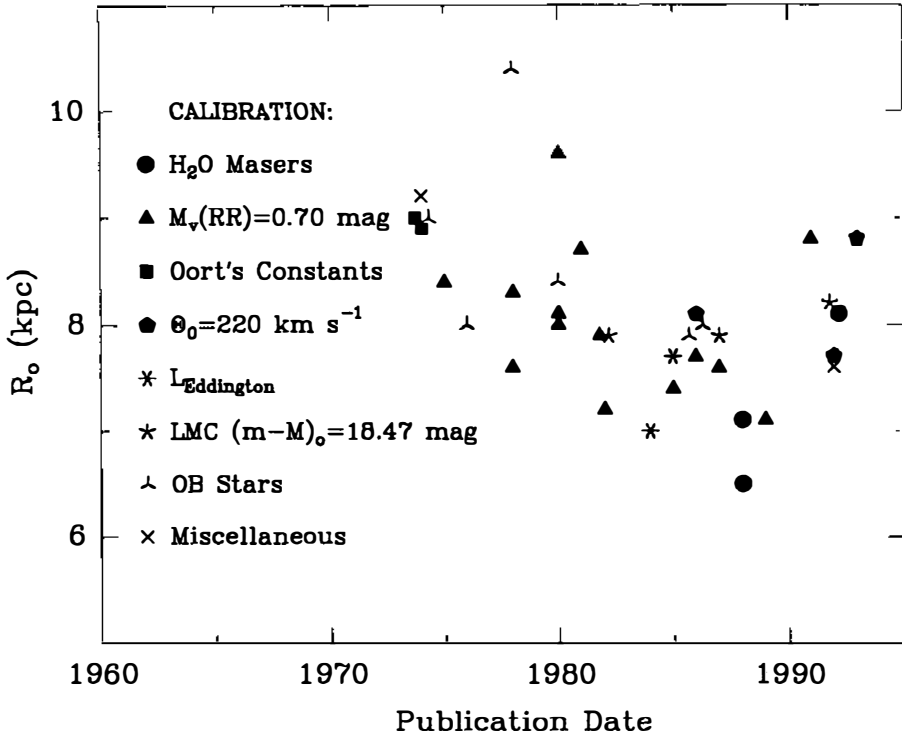


Figure 6 Estimates of the distance to the Galactic center,  $R_0$ , versus publication date since 1974. The eight symbols correspond to the eight groupings of  $R_0$  estimates using the calibration methods given in Table 7. The  $R_0$  estimates based upon RR Lyrae magnitudes have been rescaled to  $M_v(\text{RR}) = 0.70$  mag, those based upon the circular rotation speed of the Galaxy have been scaled to  $\Theta = 220$  km s<sup>-1</sup>, and those based on the distance to the LMC have been scaled to  $(m-M)_0 = 18.47$  mag.

Table 8 is a compilation of the results presented in Table 7 by group. For each group, the variance-weighted average of  $R_0$  is indicated along with its statistical uncertainty ( $\sigma_{\text{stat}}$ ). In addition to the statistical error, Table 8 contains an estimate of the *systematic* error ( $\sigma_{\text{sys}}$ ) likely for the group value. For example, the 0.8 kpc systematic error associated with the  $R_0$  estimates that are calibrated by  $M_v(\text{RR})$  (e.g. globular clusters, RR Lyrae variables, and red giants) is primarily due to an uncertainty of  $\approx 0.2$  mag in  $M_v(\text{RR})$ . We combine the statistical and systematic errors in quadrature and calculate a weighted average of the values of  $R_0$  for the eight groups. This approach yields

**Table 7**  $R_0$  estimates grouped by calibrations

Calibration group	$R_0 \pm \sigma_{\text{stat}}$ (kpc)
Primary Measurements:	
H <sub>2</sub> O Proper Motions	$7.2 \pm 0.7$
Scaled by $M_v(\text{RR}) = 0.70$ mag:	
Globular Clusters	$8.0 \pm 0.8$
RR Lyrae Variables	$8.0 \pm 0.5$
Red Giants	$7.9 \pm 1.0$
Galaxy Modelling	$8.7 \pm 0.6$
Scaled by $\Theta_0 = 220$ km s <sup>-1</sup> :	
Sgr A* Proper Motions	$7.7 \pm 0.9$
OH/IR Stars	$8.1 \pm 1.1$
Using Oort's Constants:	
Nearby Stars	$8.9 \pm 1.0$
Disk Modelling	$9.0 \pm 1.0$
OB Star Calibration:	
OB Stars	$9.1 \pm 1.0$
HI & HII Regions	$8.1 \pm 0.5$
Scaled by LMC $(m-M)_0 = 18.47$ mag:	
Cepheids	$8.0 \pm 0.5$
Miras	$7.9 \pm 1.0$
Eddington Luminosity ( $1.4 M_\odot$ ):	
X-ray Sources	$7.4 \pm 1.0$
Miscellaneous:	
Planetary Nebulae	$7.6 \pm 0.7$
M-S Turn-off	$9.2 \pm 2.2$

**Table 8** "Best Value" for  $R_0$ 

Method	$R_0 \pm (\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2)^{1/2}$ (kpc)
Primary Measurements	$7.2 \pm (0.7^2 + 1.1^2)^{1/2}$
Calibrated by $M_v(\text{RR})$	$8.2 \pm (0.3^2 + 0.8^2)^{1/2}$
Using $\Theta_0$	$7.9 \pm (0.7^2 + 0.8^2)^{1/2}$
Using Oort's Constants	$9.0 \pm (0.7^2 + 1.6^2)^{1/2}$
OB Star Calibration	$8.3 \pm (0.5^2 + 2.0^2)^{1/2}$
Calibrated by LMC	$8.0 \pm (0.5^2 + 1.0^2)^{1/2}$
Eddington Luminosity	$7.4 \pm (1.0^2 + 2.0^2)^{1/2}$
Miscellaneous	$7.8 \pm (0.7^2 + 2.0^2)^{1/2}$
Weighted Average	

$$R_0 = 8.0 \pm 0.5 \text{ kpc.}$$

(This value for  $R_0$  does not contain any estimate of possible bias from the “bandwagon effect” mentioned above.)

The “best value”  $R_0$ , as determined above, is not strongly weighted toward any single result or calibration technique and, as such, it is statistically robust. Changing the relative weights of the different “methods” listed in Table 8 has little effect on  $R_0$ ; for example, weighting them all equally also yields nearly the same result. Similarly, adopting a new calibration for one method has only a limited effect on this estimate of  $R_0$ . The method with the highest weight employs RR Lyrae calibrations, and changing  $M_v(\text{RR})$  by  $\pm 0.2$  mag leads to an adjustment in the “best value”  $R_0$  of only  $\mp 0.2$  kpc.

## 5. EXTRAGALACTIC DISTANCES

The Galactic and the extragalactic distance scales are interrelated, since most extragalactic distance measurements are based on Galactic calibrations. Thus, for example, adjusting the absolute magnitudes of pulsating stars, impacts both  $R_0$  and extragalactic distances. Of course, Cepheid variables play a major role in establishing distances to other galaxies, but they are of lesser significance for Galactic distances. Conversely, RR Lyrae stars are important for determining  $R_0$ , but due to their limited brightness they are not easily detected in galaxies beyond the Local Group and, hence, do not figure prominently in the extragalactic distance scale. However, questions related to the absolute magnitude of RR Lyraes as a function of metallicity bear critically on the ages of globular clusters, which in turn constrain values of  $H_0$ .

One approach to measuring distances to galaxies, which depends directly on  $R_0$ , is to use the Milky Way as a length and/or luminosity standard for similar galaxies. This has been attempted by de Vaucouleurs (1983a,b) who uses  $R_0$  and  $\Theta_0$  as parameters of models that predict the total absolute magnitude of the Milky Way. These values can be used to establish the zero points for the Tully-Fisher relations, and thus yield distances to spiral galaxies. Similarly, de Vaucouleurs argues that one can recalibrate the Faber-Jackson relationship and relations involving the luminosity index, absolute magnitude, and isophotal diameter of a galaxy. Following such an approach, de Vaucouleurs argues that the recently adopted “shorter” distance scale for the Milky Way ( $R_0 = 8.5$  kpc and  $\Theta_0 = 220 \text{ km s}^{-1}$ ) is consistent with the “shorter” extragalactic distance scale of  $H_0$  near  $90 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . On the other hand, van der Kruit’s (1986) analysis of the disk luminosity of the Galaxy from *Pioneer 10* data favors a longer

distance scale corresponding to  $H_0$  of  $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . It is important to remember that using the Milky Way as a standard spiral galaxy assumes that we know its morphologic type, that it is an “average” galaxy for its type, and that the models employed to estimate luminosities that would be observed from outside the Milky Way are sufficiently accurate.

A direct test of the calibration of absolute magnitudes of different types of pulsating stars comes from observations of these stars in external galaxies. Since stars in external galaxies are at nearly the same distance, a discrepancy in distance moduli for different types of stars suggests a corresponding discrepancy in their absolute magnitudes. Walker (1992, and references therein) measured magnitudes of 182 RR Lyrae variables among nine LMC clusters. He adopted a LMC distance modulus of 18.5, based primarily on Cepheids, and obtained  $M_v(\text{RR}) = 0.44$  at a mean  $[\text{Fe}/\text{H}] = -1.9$ . This absolute magnitude is 0.26 mag brighter than the average  $M_v(\text{RR})$  adopted in this review. Saha et al (1992a) also concluded that RR Lyrae absolute magnitudes may need to be brighter to conform with currently accepted Cepheid magnitudes, based on their observations of RR Lyrae variables in the nearby dwarf irregular galaxy IC 1613. These results can be interpreted as evidence for 1. brighter RR Lyrae variables, 2.  $M_v(\text{RR}) \approx 0.7$  for  $[\text{Fe}/\text{H}] \approx -1$  and a strong dependence of magnitude on metallicity, or 3. dimmer Cepheids. The results are, however, possibly consistent within measurement uncertainties.

In Table 9, we expand the comparison of distance moduli from different methods to five local galaxies that have measured RR Lyrae stars. We have followed Feast & Walker (1987) and assumed an LMC distance modulus of 18.47 mag, consistent with an absolute magnitude for Cepheids given by  $M_v(\delta \text{ Cep}) = -2.78 \log P - 1.35$  mag. To facilitate comparison of the RR Lyrae data, we have adjusted the published distance moduli to a metal-independent  $M_v(\text{RR}) = 0.70$ , as done in Section 4. Also included in Table 9 are distance moduli based on Mira variables [assuming  $M_{\text{bol}}(0) = 0.76$  from Glass & Feast 1982] and the tip of the giant branch as calibrated by Frogel et al (1983). Note that the tip of giant branch calibration is based on  $M_v(\text{HB}) \approx 0.7$ , and as such it is not independent of the RR Lyrae calibration.

Clearly, for each galaxy in Table 9, there is agreement among the distance moduli for different stars at about the 0.3 mag level (15% in distance). The approximate equality of the Cepheid and RR Lyrae distance moduli for M31 weakens the case (based on the LMC and IC 1613) for a simple calibration offset between these two types of pulsators. Saha et al (1992a), noting this, suggested that a strong dependence of  $M_v(\text{RR})$  on metallicity might explain the Cepheid and RR Lyrae distance moduli data. While precise metallicity information is not available for all cases, mean



**Table 9** Extragalactic distance comparisons

Galaxy	Distance Modulus		
	Cepheids <sup>a</sup>	RR Lyraes <sup>b</sup>	Giants <sup>c</sup>
LMC	18.47 <sup>d</sup>	18.24 <sup>e</sup>	18.47 <sup>f</sup>
IC 1613	24.39 <sup>g</sup>	24.17 <sup>h</sup>	...
M 31	24.31 <sup>i</sup>	24.30 <sup>j</sup>	24.40 <sup>k</sup>
NGC 205		≥ 24.72 <sup>l</sup>	24.3 <sup>m</sup>
NGC 147		23.99 <sup>n</sup>	24.0 <sup>o</sup>

<sup>a</sup> Adjusted to an LMC  $(m-M)_o = 18.47$  (see Feast & Walker 1987).

<sup>b</sup> Assuming  $M_v(\text{RR}) = 0.70$  mag for all metallicities.

<sup>c</sup> Assuming  $M_{\text{bol}}(0) = 0.76$  mag for Mira variables, or  $M_{\text{bol}} = -3.56$  mag at  $[\text{Fe}/\text{H}] \approx -1$  for the tip of the giant branch from Frogel, Cohen & Persson (1983).

<sup>d</sup> Feast & Walker (1987) who assume  $M_v(\text{Cepheid}) = -2.78 \log P - 1.35$  mag and average data of Caldwell & Coulson (1987) and Walker (1987).

<sup>e</sup> Walker (1992);  $M_v(\text{RR}) = 0.44$  for  $(m-M)_o = 18.5$  at  $[\text{Fe}/\text{H}] = -1.9$ .

<sup>f</sup> Glass & Feast (1982); Mira  $M_{\text{bol}}(0) = 0.54$  for  $(m-M)_o = 18.69$ .

<sup>g</sup> Madore & Freedman (1991);  $(m-M)_o = 24.42$  for LMC  $(m-M)_o = 18.50$ .

<sup>h</sup> Saha et al (1992a);  $(m-M)_g = 24.90$  for  $A_g = 0.07$ ,  $M_g(\text{RR}) = 0.73$  and  $(g-V) = -0.04$ .

<sup>i</sup> Mean of Freedman & Madore (1990), who obtain  $(m-M)_o = 24.44$  for three fields assuming an LMC  $(m-M)_o = 18.50$ , and Welch et al (1986), who obtain  $(m-M)_o = 24.26$  assuming an LMC  $(m-M)_o = 18.52$ .

<sup>j</sup> Pritchett & van den Bergh (1987, 1988);  $(m-M)_B = 25.68$  for  $A_B = 0.31$ ,  $M_B(\text{RR}) = 1.14$  and  $(B-V) = 0.37$  at  $[\text{Fe}/\text{H}] = -1.0$ .

<sup>k</sup> Mould & Kristian (1986); tip of giant branch at  $[\text{Fe}/\text{H}] = -0.6$ .

<sup>l</sup> Saha, Hoessel & Krist (1992b);  $(m-M)_g \geq 25.5$  for  $A_g = 0.12$ ,  $M_g(\text{RR}) = 0.73$ , and  $(g-V) = -0.04$ .

<sup>m</sup> Mould, Kristian & Da Costa (1984); tip of giant branch at  $[\text{Fe}/\text{H}] = -0.9$ .

<sup>n</sup> Saha, Hoessel & Mossman (1990);  $(m-M)_g = 25.25$  for  $A_g = 0.6$ ,  $M_g(\text{RR}) = 0.73$ , and  $(g-V) = -0.04$ .

<sup>o</sup> Mould, Kristian & Da Costa (1983); tip of giant branch.

values for  $[\text{Fe}/\text{H}]$  are likely to be near  $-1.1$  for M31,  $-1.6$  for IC 1613, and  $-1.9$  for the LMC RR Lyraes. If the slope of the magnitude-metallicity relation is near  $0.3 \text{ mag dex}^{-1}$ , then the RR Lyraes in IC 1613 and in the LMC would be expected to be intrinsically brighter by 0.15 and 0.24 mag, respectively, compared to RR Lyraes in M31. Subtracting  $-0.15$  and  $-0.24$  mag from the RR Lyrae distance moduli for IC 1613 and the LMC, respectively, would bring all distance indicators into agreement at better than the  $\pm 0.05$  mag level (except for NGC 205). This level of agreement is perhaps better than one should expect from measurement uncertainties, and, in any event, more evidence is needed to corroborate this calibration procedure. Of course, throughout this discussion, we are only testing the *relative* agreement of zero-point calibrations; in principle, the whole system might be shifted (albeit in a complicated manner) without upsetting these findings.

More generally, as  $R_o$  becomes increasingly better determined, especially

by direct measurement, one might be able to use  $R_0$  as a calibration standard. Conceptually, this involves reversing the procedures normally used to obtain  $R_0$ . For example, consider the study of globular clusters by Racine & Harris (1989). They assume a distance calibration [based on  $M_v(\text{RR}) = 0.6 \text{ mag}$ ] and, from the distribution of clusters, infer a distance to the Galactic Center of 7.5 kpc. The *precision* of the measurements is given as  $\pm 0.5 \text{ kpc}$ , owing to uncertainties in magnitude measurements and extinction corrections. Racine & Harris note that the *accuracy* is probably limited by the uncertainty in  $M_v(\text{RR})$  of  $\pm 0.2 \text{ mag}$  ( $\pm 10\%$  in distance). Now, were we to have a very accurate, independent estimate of  $R_0$ , say 8.00 kpc, and force their  $R_0$  estimate of 7.5 kpc to 8.00 kpc, this would require a revised calibration of  $M_v(\text{RR}) = 0.73$ . The accuracy of this revised calibration would be limited by the original  $R_0$  precision ( $\pm 0.5 \text{ kpc}$ ) and would be  $\pm 0.13 \text{ mag}$ .

Continuing along the same lines, if we assume that  $R_0 = 8.0 \text{ kpc}$ , based on the results in Table 8, we could obtain distances to the LMC based upon a Galactic calibration. Given the study of Glass & Feast (1982), we would recalibrate the zero point of the absolute magnitudes of Miras to  $M_{\text{bol}}(0) = 0.74$ , in order to obtain  $R_0 = 8.0 \text{ kpc}$ . This would change their distance modulus for the LMC [Glass & Feast used a modulus of 18.69 for  $M_{\text{bol}}(0) = 0.54$ ] to 18.49 mag. Similarly, scaling the  $R_0$  estimates from Cepheid variables of Caldwell & Coulson (1987) and Caldwell et al (1992) to 8.0 kpc would require a recalibration of their Cepheid magnitudes and lead to LMC distance moduli of 18.50 and 18.43 mag, respectively. These values for the distance to the LMC are in agreement with recent results from SN 1987A (see Section 2.2.3) and suggest that the Mira and Cepheid distance scales are consistent for Galactic and extragalactic sources at better than the 10% level.

## 6. CONCLUSIONS

Seventy-five years after Shapley published his estimate of  $R_0$ , there is a reasonable consensus as to its value. Nearly all methods of determining  $R_0$  now yield values between 7 and 9 kpc. Based upon the works cited in this review a best estimate of  $R_0$  is 8.0 kpc, with a standard error of about 0.5 kpc. This level of confidence in distance measurements is in contrast to measurements of  $H_0$ , where values of between 40 and 90  $\text{km s}^{-1} \text{ Mpc}^{-1}$  are still reported.

The accuracy of  $R_0$  estimates should certainly improve with time. For example, *Hipparcos* and the *Hubble Space Telescope* should lead to improved distance calibrations through better proper motions and the resolution of individual stars in distant clusters. In addition, radio fre-

quency observations necessary to obtain a distance to Sgr A\*, via the straightforward technique of trigonometric parallax, are being attempted with the new Very Long Baseline Array. It is reasonable to expect that, by the end of the decade, we can look forward to knowing the value of  $R_0$  to better than 3% uncertainty.

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