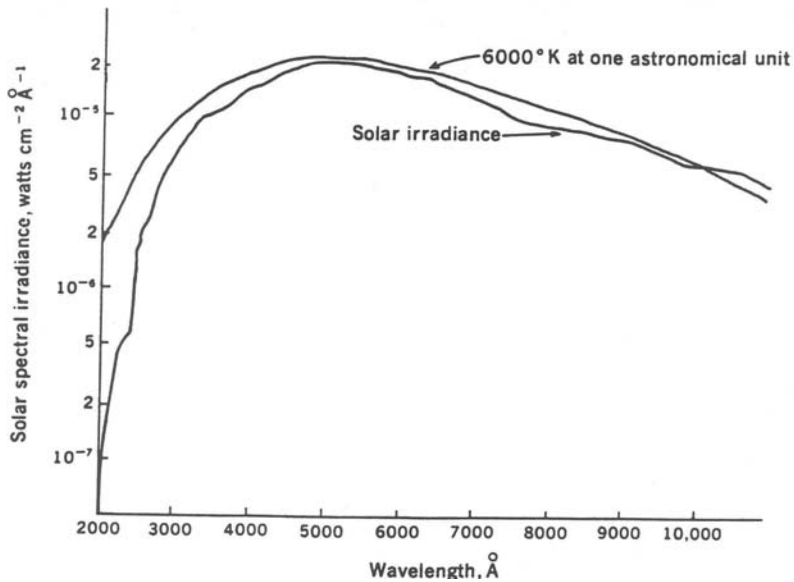
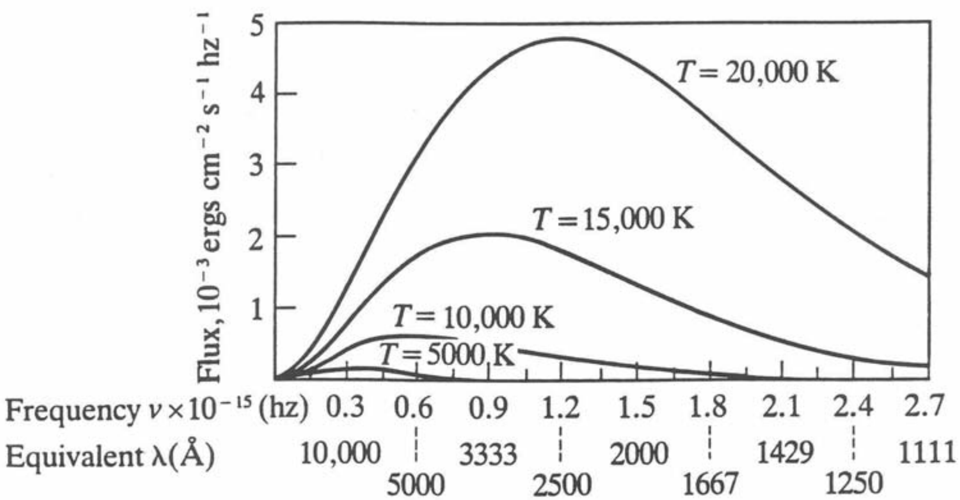
Components of Galaxies – Stars
What Properties of Stars are Important for Understanding Galaxies?

- Temperature – Determines the λ range over which the radiation is emitted
- Chemical Composition – metallicities
- Lifetimes – Determines the timescale over which a particular type of star might affect the overall properties of the galaxy
- Mass
- Luminosity

Temperature

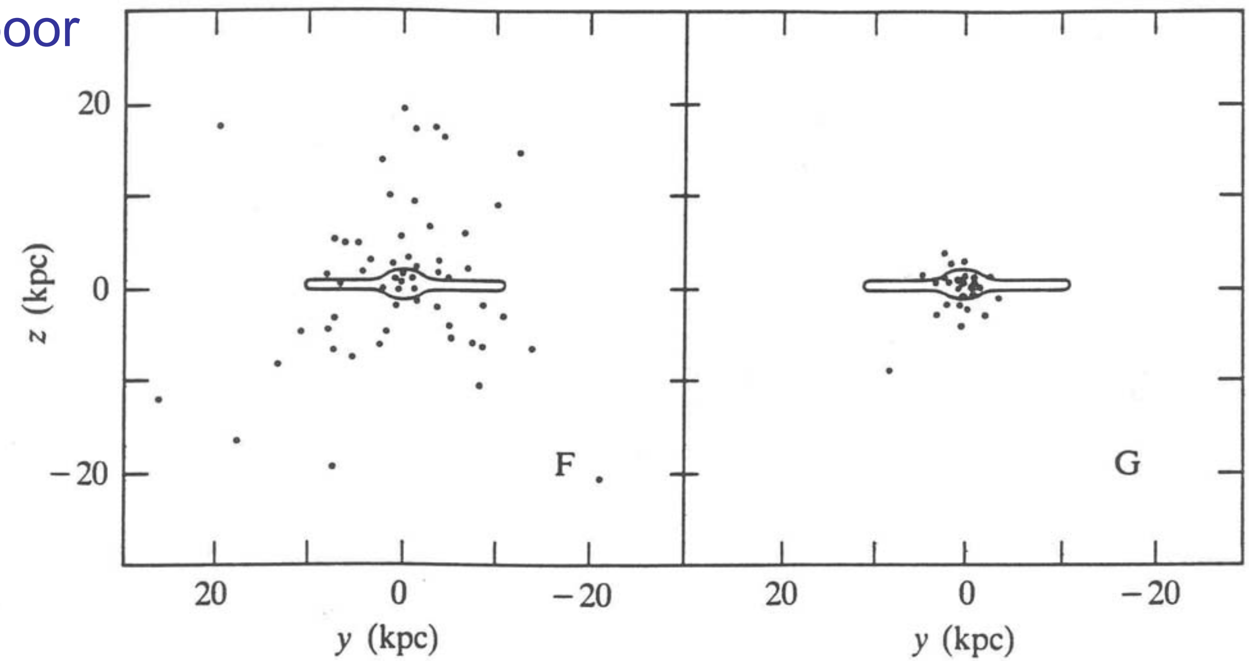




- $T^4_e = L_{star} / 4\ \pi\ \sigma\ R^2_{star}$
- T_e = effective temperature, i.e., the temperature of a blackbody having the same radiated power per area

Chemical Component - Metallicity

Metal poor
(old)



Metal Rich
(young)

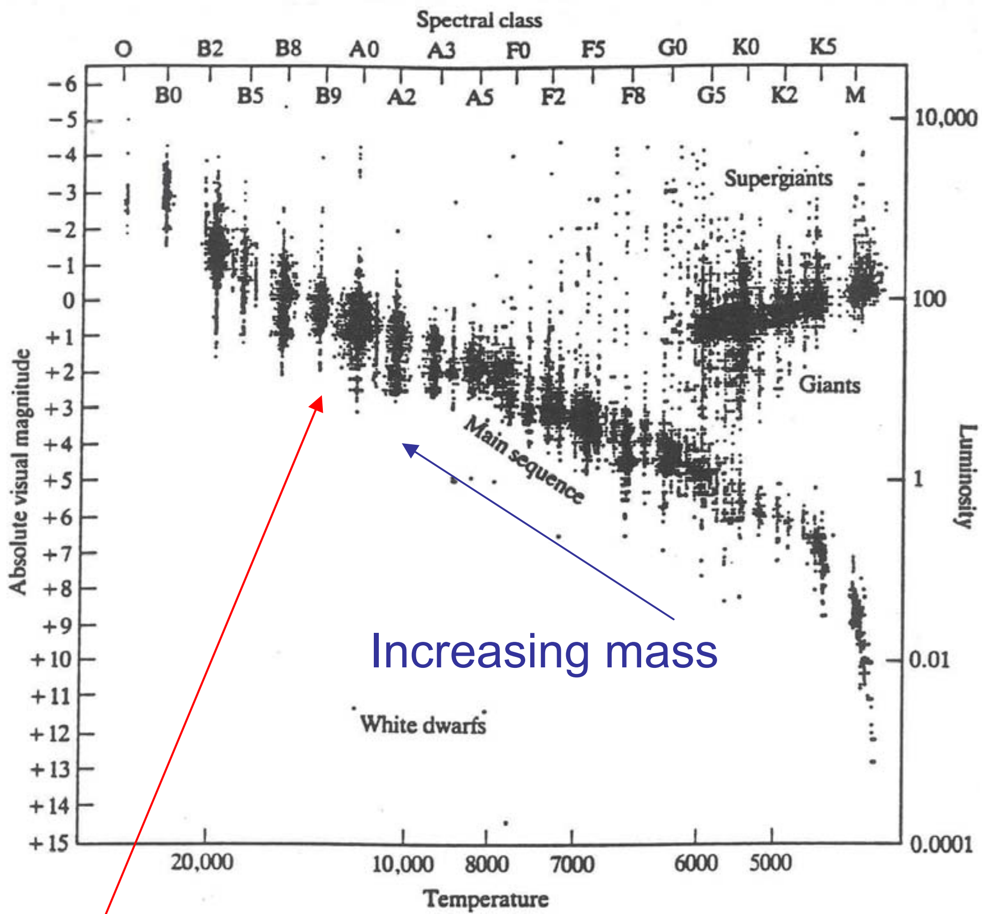
(Nomenclature)

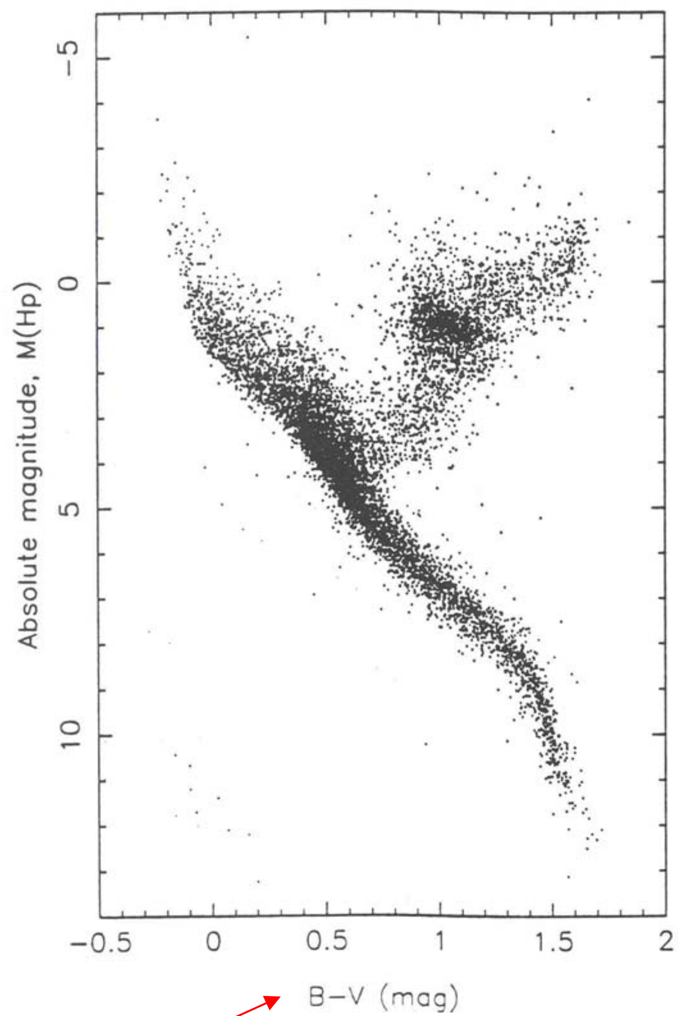
- $[Fe/H] = \log\,[n(Fe)/n(H)]_{star} - \log\,[n(Fe)/n(H)]_{solar}$

Number Density

- X = H, Y = He, Z = Others
- Typically, X = 0.70, Y = 0.28, Z = 0.02

Hertzsprung-Russell Diagram or Color-Magnitude Diagram





Stars stay on the main sequence most of
Their lives, burning H to He

i.e., Temperature

Spectral Types

Blue   Red

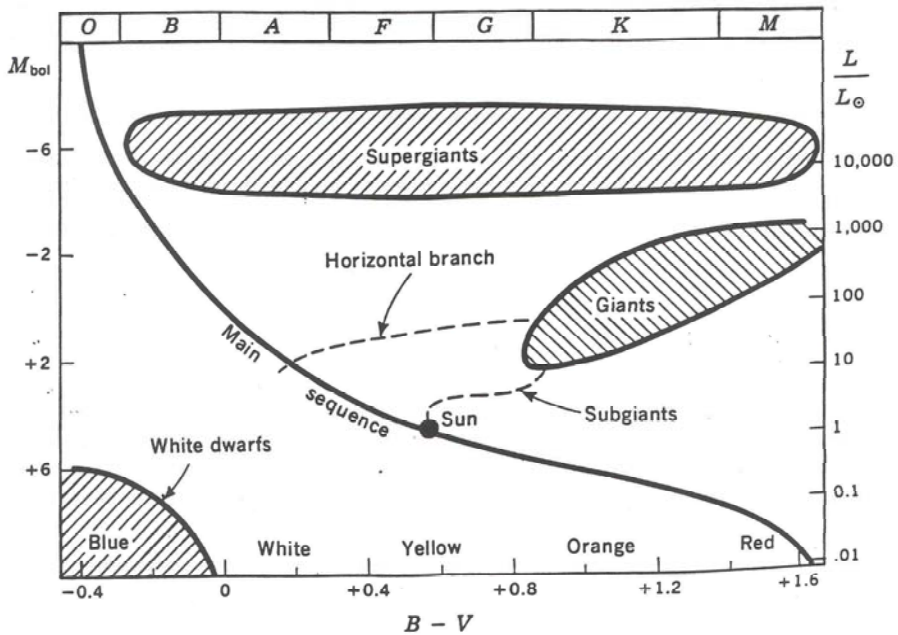


Fig. 1-12 A schematic representation of the heavily populated areas in the H-R diagram. A high percentage of stars lie near the main sequence. The next most populous groups are the white dwarfs and the giants. The subgiant and horizontal branches are conspicuous in those collections of stars having large numbers of giants, e.g., globular clusters.

Surface gravity, g, sets the Pressure Gradient of Atmosphere

$g = G\,M / R^2$

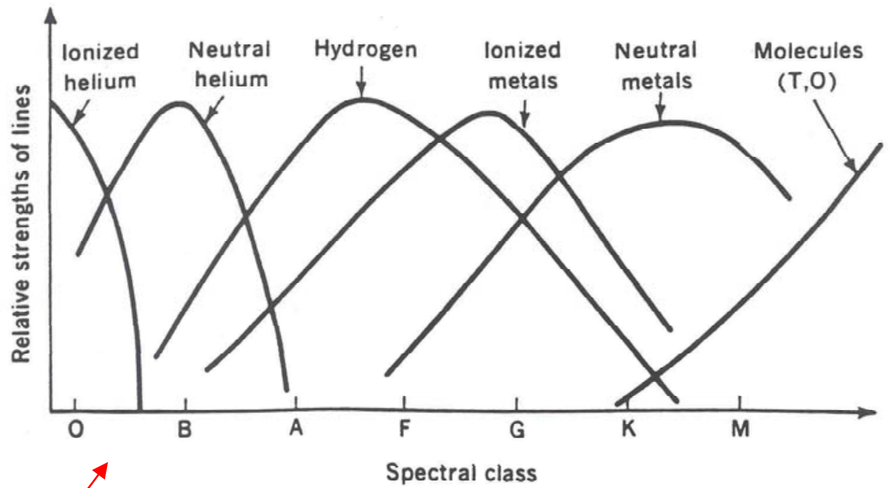


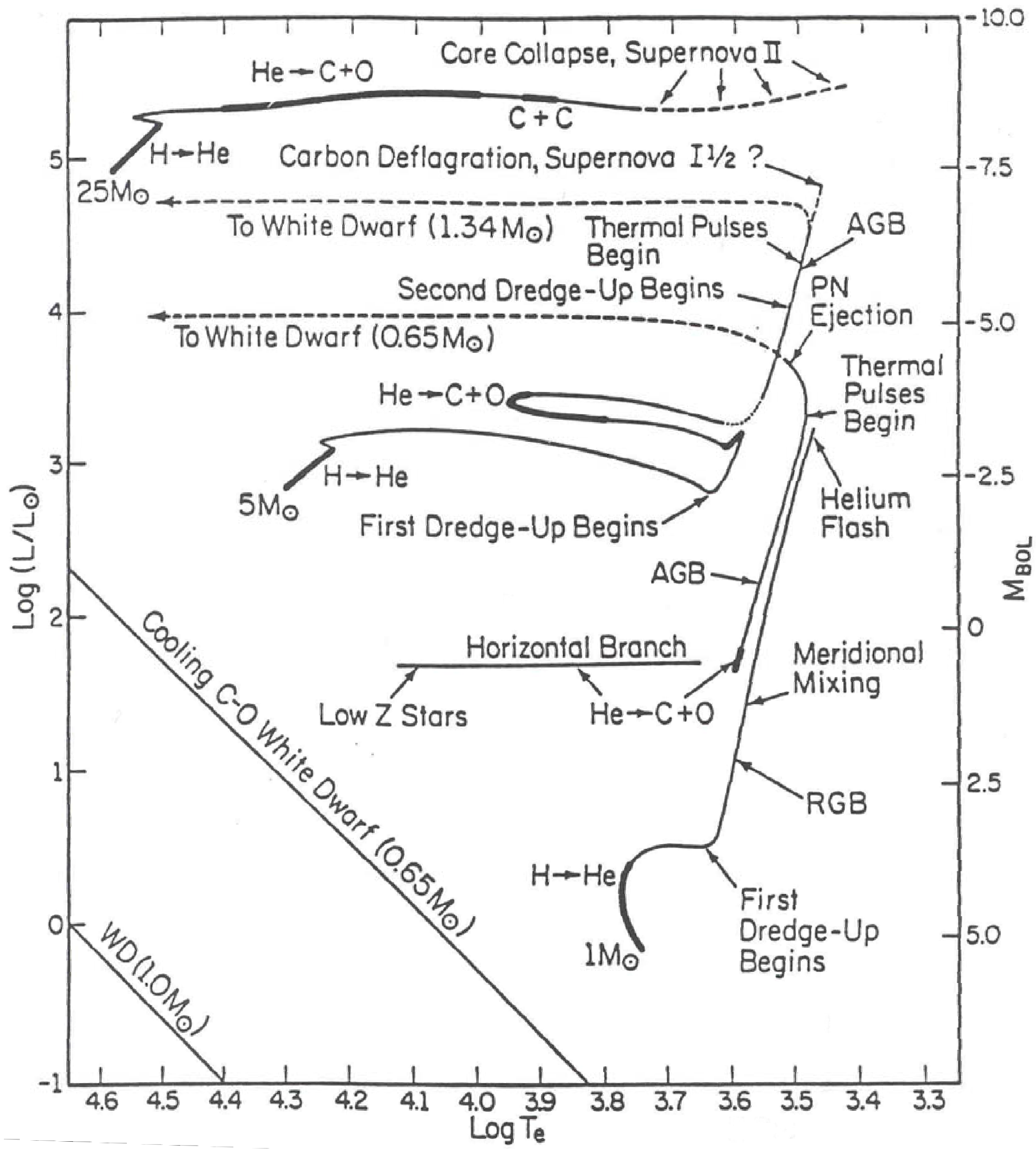
Fig. 1-11 The progression of selected spectral properties through the sequence of spectral classes. (*G. Abell, "Exploration of the Universe," Holt, Rinehart and Winston, Inc., New York*, 1964.)

Emission/Absorption Features Associated with Different S.T.s

Different Mass Stars Follow Different Tracks Along this Diagram

- 10 M_{Solar} Star – 10^7 years
- 2 M_{Solar} Star – 10^9 years
- 1 M_{Solar} Star – 10^{10} years

Evolutionary Tracks



Evolutionary Tracks, cont.

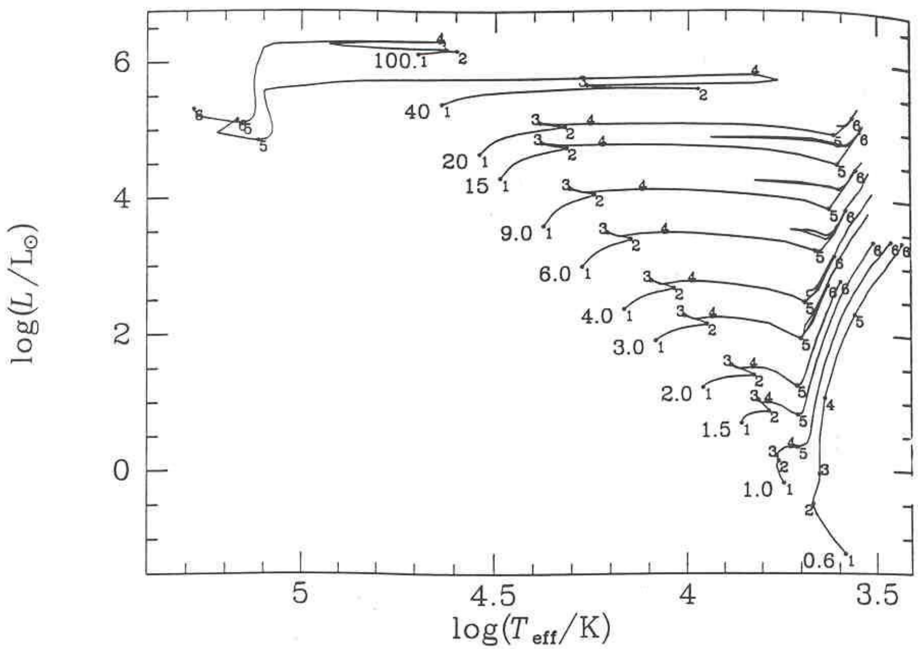


Figure 5.2 Evolutionary tracks for solar-metallicity stars $(Y, Z) = (0.28, 0.02)$ with initial masses from $0.6\,\mathcal{M}_\odot$ to $100\,\mathcal{M}_\odot$. On each track several points are marked and numbered. Table 5.2 gives the time it takes a star to reach each of these points starting from point 1. To avoid confusion tracks for $\mathcal{M} \leq 2\,\mathcal{M}_\odot$ terminate at the He flash – see Figure 5.3 for the further tracks of these stars. All models assume convective overshoot. [From data published in Bressan *et al.* (1993)]

Table 5.2 Stellar-evolution times for $Z = 0.02$

	Times (Myr) between points in Figure 5.2					
$\mathcal{M}/\mathcal{M}_\odot$	1 – 2	2 – 3	3 – 4	4 – 5	5 – 6	t_{He}
0.6	78245	4724	1591	175.3	17.70	143.7
1.0	8899	376.3	1995	178.3	907.5	133.0
1.5	2471	53.76	5.05	63.42	188.9	137.5
2.0	1030	20.26	1.700	13.82	44.75	166.7
3.0	347.4	6.785	0.492	2.199	2.558	107.1
4.0	171.1	3.119	0.254	0.551	0.683	30.14
6.0	67.92	1.057	0.070	0.088	0.147	5.94
9.0	29.45	0.414	0.023	0.020	0.041	1.94
15.0	12.37	0.136	0.008	0.008	0.011	0.82
20.0	8.46	0.085	0.005	0.006	0.006	0.61
40.0	4.43	0.138	0.007	0.541	0.032	–
100.0	1.27	1.079	0.619	0.415	0.023	–

NOTES: The final column gives the time from point 6 until the star's death.

SOURCE: From data published in Bressan *et al.* (1993)

Co-Evolving Cluster of Stars

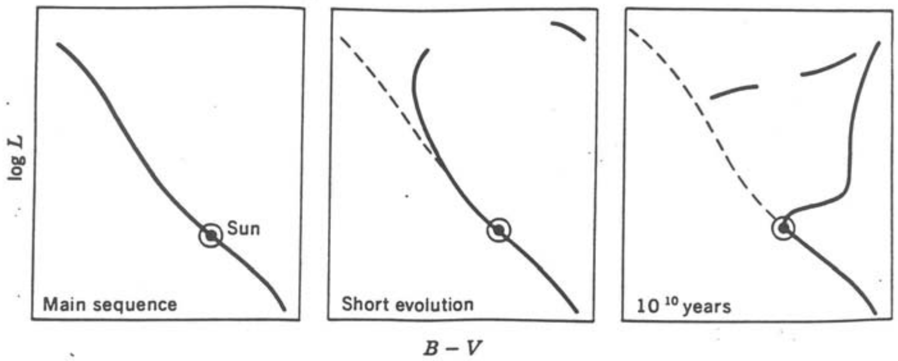


Fig. 1-20 Schematic representation of the H-R diagram of a cluster of stars at three different epochs in its history. After a short period of evolution, say about 10^8 years, the zero-age main sequence has become an evolved main sequence similar to the Pleiades. After a long period of evolution, say about 10^{10} years, the diagram resembles those of the globular clusters. Supergiants and white dwarfs have been omitted from this diagram because their participation is poorly understood.

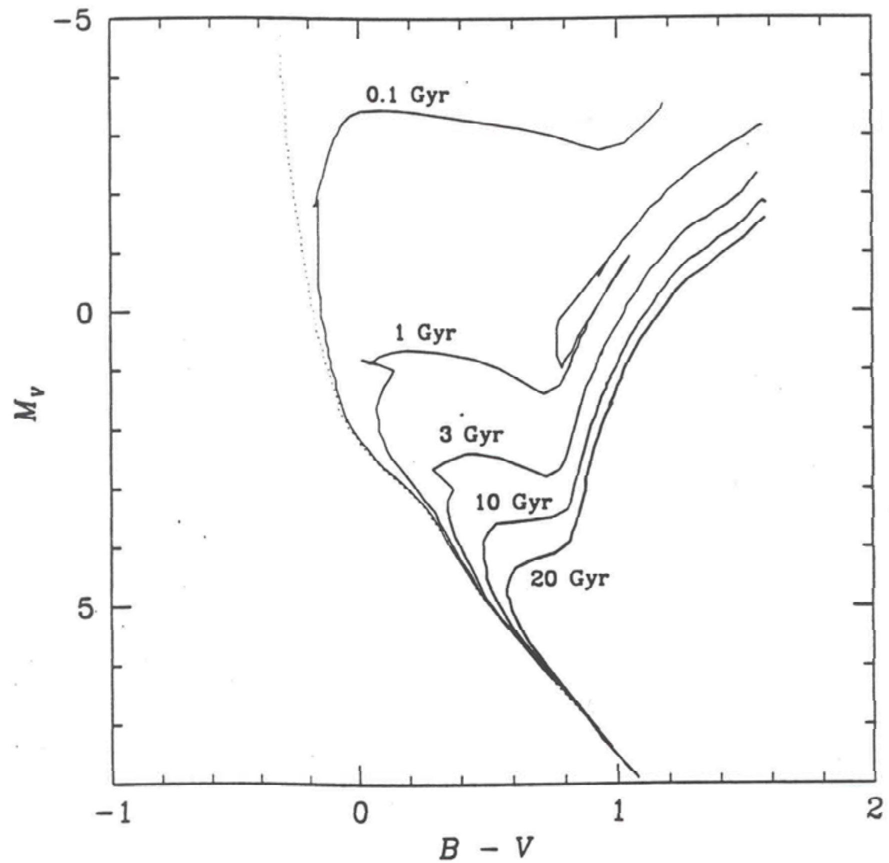


Figure 6.6 Theoretically calculated isochrones showing how a stellar population with $Z = 0.004$, $Y = 0.24$ evolves away from the ZAMS (dotted line) in the CM diagram. Each isochrones is labeled by its age. [From the calculations of Bertelli *et al.* (1994)]

- I.e., HR Diagrams can be Used to Approximate the Age of Star Clusters
- ONLY IF Star Formation is Not Continuous

Estimating Lifetimes - MS

- 26.7 MeV released every time 4H $\rightarrow$ He + ν + photons
- 26.7 MeV = 25 MeV (photons) + 1.7 MeV (ν)

- The difference in mass of 4H and He is
  $4m_{proton} - 3.97m_{proton} = 0.037m_{proton}$

- The efficiency of converting mass to energy with this process is
  0.03 / 4 = 0.007, or 0.7%

- Thus, $E = 0.0067 \Delta m_H c^2$

- *So,* $t_{ms} = (0.007\ \alpha\ M\ c^2) / L$
  where α is the total mass of H converted to He while the star is on the main sequence.

- In terms of useful units, $t_{ms} = 10^{10}\ (M / M_{solar}) / (L / L_{solar})$

Lifetime – Horizontal Branch

- The luminosity of a HB is L_{HB} = 50 L_{solar}
- During a star's life on the HB, it converts M(He core) = 0.45 M_{solar} into O & C
- About ½ He → O   &   ½ He → C
- Thus, $E = 7.2\times10^{-4} \times 0.45\, M_{solar}\, c^2 / 50\, L_{solar}$ = 0.1 Gyr

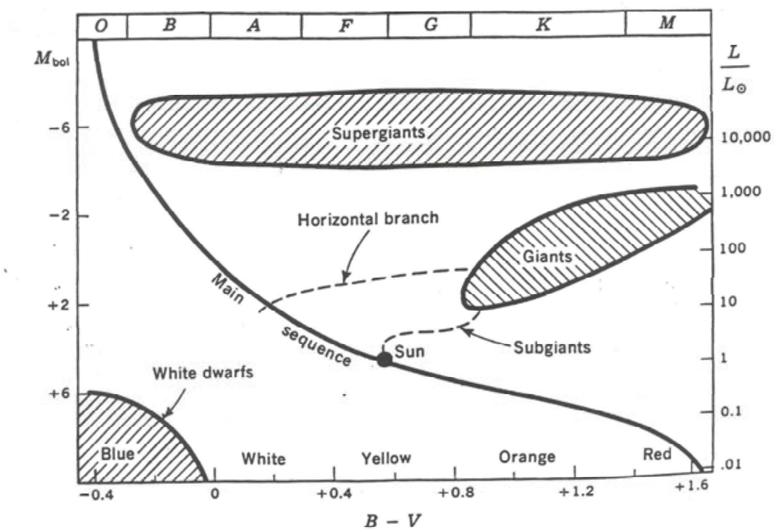


Fig. 1-12 A schematic representation of the heavily populated areas in the H-R diagram. A high percentage of stars lie near the main sequence. The next most populous groups are the white dwarfs and the giants. The subgiant and horizontal branches are conspicuous in those collections of stars having large numbers of giants, e.g., globular clusters.

Time to go Supernova

- 1.4 M_{solar} of H → Fe before exploding
- $E = 0.0085\ \Delta M\ c^2$
- If $L_3 \times 10^3\ L_{solar}$, then
  $t_{sg} = 0.0085 \times 1.4\ M_{solar}\ c^2 / 1000\ L_3\ L_{solar} = 0.18(L_3)^{-1}$ Gyr

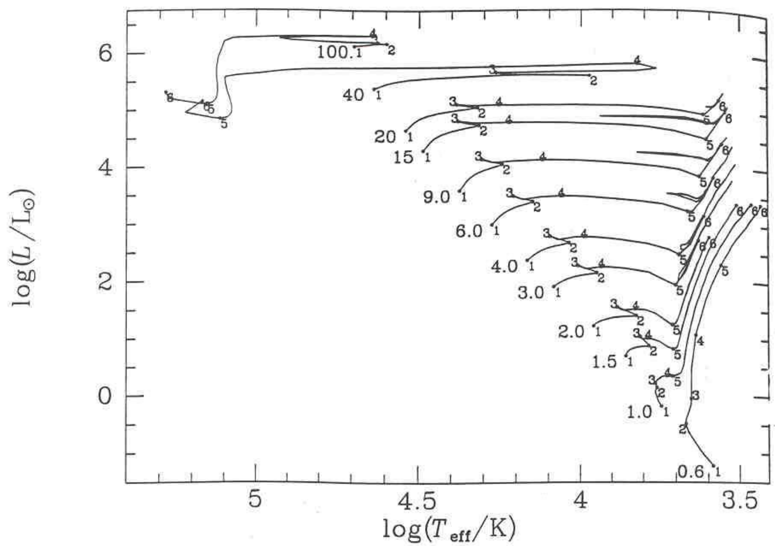
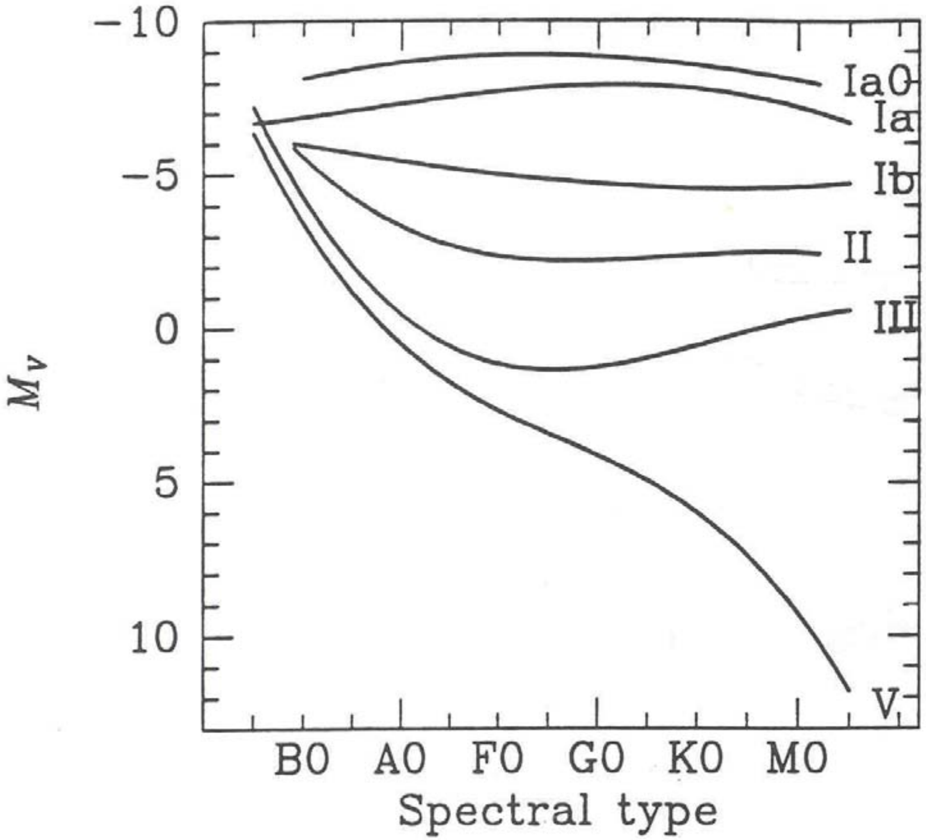


Figure 5.2 Evolutionary tracks for solar-metallicity stars $(Y, Z) = (0.28, 0.02)$ with initial masses from $0.6\,\mathcal{M}_\odot$ to $100\,\mathcal{M}_\odot$. On each track several points are marked and numbered. Table 5.2 gives the time it takes a star to reach each of these points starting from point 1. To avoid confusion tracks for $\mathcal{M} \leq 2\,\mathcal{M}_\odot$ terminate at the He flash – see Figure 5.3 for the further tracks of these stars. All models assume convective overshoot. [From data published in Bressan *et al.* (1993)]

Luminosity Class



- I = Supergiants
- II = Bright Giants
- III = Giants
- IV = subgiants
- V = dwarfs (Main Sequence)

- Luminosity is a function of *R*, i.e. $L = 4\,\pi\,R^2\,\sigma\,T^4_e$

Properties…

Table 1. Dwarfs - Luminosity Class V

Spectral Type	Mass $M_\odot$	$\log(L_{bol})$ $L_\odot$	M_V	M/L_V $M_\odot/L_\odot$	Radius $R_\odot$	T_e K
03	120	6.15	-6.0	0.01	15	52500
B3	7.6	3.28	-1.6	0.03	4.8	18700
A5	2.0	1.15	1.9	0.2	1.7	8200
G0	1.05	0.18	4.4	1.1	1.1	6030
K5	0.7	-0.82	7.4	10.7	0.72	4350
M0	0.5	-1.11	8.8	30	0.60	3850

Adapted from Table 3.13, B&M pg. 110.

Table 2. Red Giants - Luminosity Class III

Spectral Type	Mass $M_\odot$	$\log(L_{bol})$ $L_\odot$	M_V	Radius $R_\odot$	T_e K
G0	varies	1.5	···	6.0	5850
K5	varies	2.3	-0.2	25	3950
M0	varies	2.6	-0.4	40	3800

Adapted from Table 3.14, B&M pg. 110.

Remnants Condensed Matter

- For $M_{star} < 8\ M_{solar}$ $\rightarrow$ white dwarf

  Leftover after much mass loss

  $M_{WD} \approx 0.55 – 0.6\ M_{solar}$
- For $8\ M_{solar} \leq M_{star} \leq 60\ M_{solar}$ $\rightarrow$ neutron star

  Optically invisible, but visible as radio pulsars

  $M_{NS} \approx 1.4\ M_{solar}$
- For $M_{star} > 60\ M_{solar}$ $\rightarrow$ Black Hole

  Optically invisible

  $M_{BH} > 1.4\ M_{solar}$

Very Low Mass Stars

- For M = 0.0001 – 0.08 M_{solar} → black dwarfs
- Supported by electron degeneracy
- Brown Dwarfs → burning De & Li, but never H

Luminosity Functions

- It is useful to speak of stars in a galaxy collectively.

$$dN = \Phi\,(M, \boldsymbol{x})\; dM\; d^3\boldsymbol{x},$$

Where dN is the # of stars with absolute mag ($M + dM$, M) within a volume $d\boldsymbol{x}^3$.

Φ $(M, \boldsymbol{x}) = \Phi\,(M)\;\; \nu\,(\boldsymbol{x})$, such that,

$$dN = [\; \Phi\,(M)\; dM\;]\;[\;\nu\,(\boldsymbol{x})\; d^3\boldsymbol{x}\;]$$

Φ (M) = luminosity function – relative fraction of stars of different luminosities

And ν ($\mathbf{x}$) = total number density of stars at point $\mathbf{x}$.

Luminosity Function of Solar Neighborhood

$\Phi(M_v)$ = # of stars with $(M_v + ½, M_v - ½)$

Table 3.16 The general luminosity function (per 10^4 pc^3)

	M_V	$\Phi(M_V)$	$\delta L/L_\odot$	$\delta\mathcal{M}/\mathcal{M}_\odot$	M_V	$\Phi(M_V)$	$\delta L/L_\odot$	$\delta\mathcal{M}/\mathcal{M}_\odot$	
	−6	0.0001	2.6	0.005	7	29	4.0	21.3	(3)
	−5	0.0006	5.1	0.020	8	33	1.8	21.8	(3)
	−4	0.0029	9.4	0.060	9	42	0.90	24.2	(3)
	−3	0.013	17.1	0.17	10	70	0.60	35.0	(3)
	−2	0.05	28.2	0.05	11	90	0.30	36.0	(3)
	−1	0.25	53.9	1.6	12	127	0.17	36.3	(3)
	0	1	95.8	4.0	13	102	0.055	20.8	(3)
(2) →	1	3	111	7.4	14	102	0.022	16.3	← (1)
	2	5	64	8.7	15	127	0.011	16.3	
(3)	3	12	66	17.3	16	102	0.0035	10.5	
(3)	4	17	36	19.4	17	51	0.0007	4.3	
(3)	5	29	25	28.1	18	22	0.0001	1.6	
(3)	6	30	10	24.7	19	13	0.0000	0.7	
					Total	1008	532	356	

SOURCE: For $M_V \leq 0$ from data published in Allen (1973); for $M_V > 0$ from data published in Jahreiss & Wielen (1983) and Kroupa, Tout & Gilmore (1990)

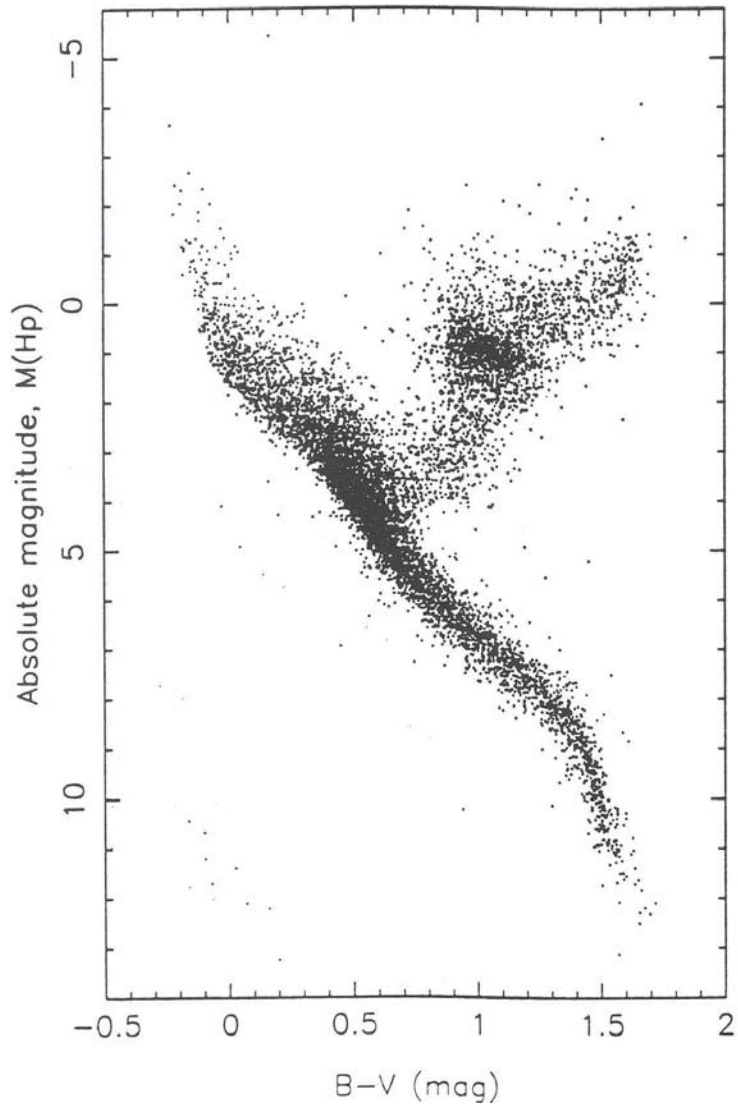
- (1) Most Stars are intrinsically faint
- (2) Intrinsically luminous stars contribute most of the light
- (3) Most of the Mass comes from Low Luminosity Stars

  0.036 M_{solar} pc^{-3}, but probably 0.039 M_{solar} pc^{-3} (missing white dwarfs)
- (4) The average $M / L \approx 1\ M_{solar} / L_{solar}$.

  This is a lower limit because we're missing remnants

Evolution of HR Diagram as a Function of Time

- Qu: The star formation in the solar neighborhood is constant. Will the HR Diagram of star in the solar neighborhood look different in the future?
- An: Yes. If the star formation is constant, the HR diagram is dependent on the number of stars of different masses formed and the time since star formation began



Evolution in HR, cont.

- Consider two Populations of stars that will live for times τ_1 & τ_2, where $\tau_2 \gg \tau_1$.
- After time $t \ll \tau_1$, the # of stars in Pop 1 & 2, N_1 & N_2 is

$$N_1 = (dN_1 / dT)\, t \qquad \& \qquad N_2 = (dN_2 / dT)\, t,$$

- Where dN_1 / dT & dN_2 / dT are the star formation rates of Pop 1 & 2, respectively.
- After time $t > \tau_1$, the oldest stars in Pop 1 will begin to disappear. Thus,

$$N_1 = (dN_1 / dT)\, t - (dN_1 / dT)\,(t - \tau_1) = (dN / dT)\, \tau_1,$$

&

$$N_2 = (dN_2 / dT)\, t.$$

- The HR diagram will continue to evolve until $t = \tau_2$, where,

$$N_1 = (dN_1 / dT)\, \tau_1 \quad \& \quad N_2 = (dN_2 / dT)\, \tau_2$$

Initial Mass Function

- IMF, $\xi\ (M)$ = distribution in mass of freshly formed stars.
- Consider a starburst

$$dN = N_0\, \xi\ (M)\ dM$$

- dN is the # of stars with mass (M, M+dM) & N_0 is the normalization constant with respect to mass (not according to #),

$$\int dM\ M\ \xi(M) = M_{solar}.$$

- Thus, N_0 is the # solar masses contained in the starburst.

Determination of ξ

- Determine Φ (Mag) for MS (solar neighborhood, or cluster)
- Correct Φ (Mag) for stellar evolution effects

If starburst, no correction needed
If SF is constant, the initial luminosity function is

$$\Phi_0 \text{ (Mag)} = \Phi \text{ (Mag) x } (t \,/\, \tau_{MS})$$

For stars with MS life less than time since the initial stars formed

$$\Phi_0 \text{ (Mag)} = \Phi \text{ (Mag)}$$

for stars with MS life longer than time since initial stars formed

- Now, convert from Magnitudes to mass

$$\xi\,(M) = [d(\text{Mag}) \,/\, dM]\ \Phi_0\ [\text{Mag}\,(M)].$$

Magnitudes to Mass

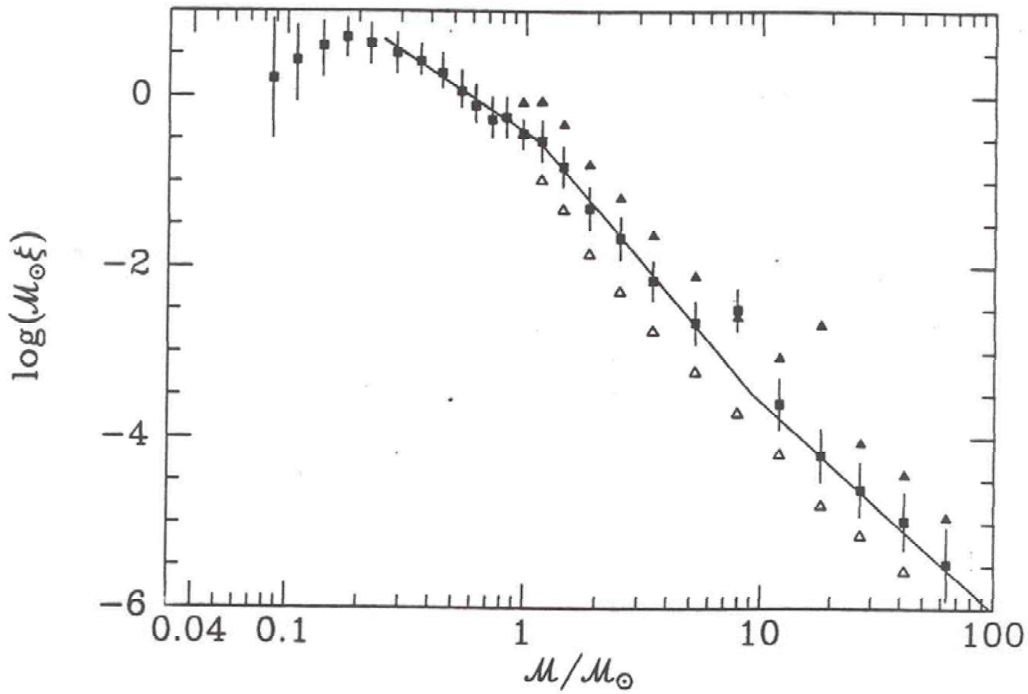


Figure 5.12 The IMF from Scalo (1986). For masses $\mathcal{M} > \mathcal{M}_\odot$ three sets of points are shown, each set being for a different assumption about the ratio of the current rate of star formation to its average over the lifetime of the solar neighborhood. The curve defined by the points that are based on the assumption of a constant star-formation rate (squares) can be approximated by the three power-law segments defined in equation (5.16).

- Determined using models, or
- Using mass determinations from binary stars

Bottom Line -

- Salpeter IMF $\xi(M) \approx M^{-2.35}$

- Scalo IMF For $M \geq 0.2\ M_{solar}$.

$\xi(M) \approx M^{-2.45}$ for $M > 10\ M_{solar}$.
$\xi(M) \approx M^{-3.27}$ for $1\ M_{solar} > M > 10\ M_{solar}$.
$\xi(M) \approx M^{-1.83}$ for $M < 0.2M_{solar}$.

Aside – Disk Gas Depletion Rate of the Milky Way

- *SFR* ≈ 3 M_{solar} yr^{-1}
- $M(H_2) \approx 2.5 \times 10^9$ M_{solar}
- Depletion time = $2.5 \times 10^9 / 3 = 8 \times 10^8$ yr

- The Sun takes 2×10^8 yr to orbit the galaxy, thus depletion time will occur in 4 revolutions
- The universe is 15 Gyr, thus depletion time takes 5% the age of the universe
- Most spiral galaxies have several $\times 10^{8\text{-}9}$ M_{solar} of molecular gas

- Why does the MW have 2.5×10^9 M_{solar} of molecular gas? Why not 0?
- Given the rapid depletion time & the age of the universe, where are the dead disks?

Aside, cont.

Possible Solutions

- The determinations of SFRs are incorrect
- The determinations of the amount of molecular gas in the MW and other Spiral Galaxies is incorrect
- Molecular Gas is replenished by infalling atomic gas.

Constructing Population Synthesis Model

- IMF (Salpeter or Scalo)
- Range in Mass of Stars Produced
- Metallicity of Stars
- Type of Star Formation (Instantaneous Burst vs. Continuous Burst)
- Filters of Interest

- Biggest Difficulty with Models – Dust

Analytic Solution of MS & Giant Luminosities from IMF

Using a power law IMF, we can determine the contribution of MS and Giant stars to the overall luminosity of the population.

Let α = *1* + *x*. Then the IMF $\xi(M)$ can be expressed as

$$\xi(M)dM = CM^{-(1+x)}dM \quad \text{for} \quad M_L < M < M_U.$$

Taking $M_U >> M_L$ & $x > 0$, the constant *C* can be normalized

$$1 = C\int_{M_L}^{M_U} M^{-(1+x)}dM = C\frac{M^{-x}}{x}|_{M_L}^{M_U},$$

$$1 = \frac{C}{x}[M_U^{-x} - M_L^{-x}],$$

$$C = xM_L^x.$$

Let $M^{\beta} = L$ and $t_{ms} = M^{-\gamma}$. Then,

$$dt_{\mathrm{ms}} = -\gamma M^{-(\gamma+1)} dM.$$

- Assume that N_0 stars are created in a single burst of star formation.
- At time t, stars with $M > M_t = t^{-\gamma}$ are no longer on the main sequence.
- The light from MS stars is thus,

$$L_{\rm ms}(t) = \int_{M_L}^{M_t} N_0 L \xi(M) dM = N_0 C \int_{M_L}^{M_t} M^{\beta} M^{-(1+x)} dM,$$

$$L_{\rm ms}(t) = N_0 x M_L^x \int_{M_L}^{M_t} M^{\beta-x-1} dM = \frac{N_0 x M_L^x}{\beta - x}[M_t^{\beta-x} - M_L^{\beta-x}],$$

$$L_{\rm ms}(t) = \frac{N_0 x}{\beta - x}[M_L^x M_t^{\beta-x} - M_L^{\beta}],$$

$$L_{\rm ms}(t) = \frac{N_0 x}{\beta - x} M_L^x M_t^{\beta-x},$$

For $M_L \ll M_t$

The number of giants at time t (for $t_g \ll t$) is

$$N_g(t) = N_0 \xi(M_t) \left[\frac{dM}{dt_{\rm ms}}\right]_{M=M_t} t_g = \frac{N_0 x}{\gamma} M_L^x M_t^{\gamma - x} t_g,$$

Where dM / dt_{ms} is the mass in giants generated per MS lifetime & t_g Is the time since the giants in question were created (note that for Post-main sequence star of a given mass M_t, $t_g = t - t_{ms}$).

In other words,

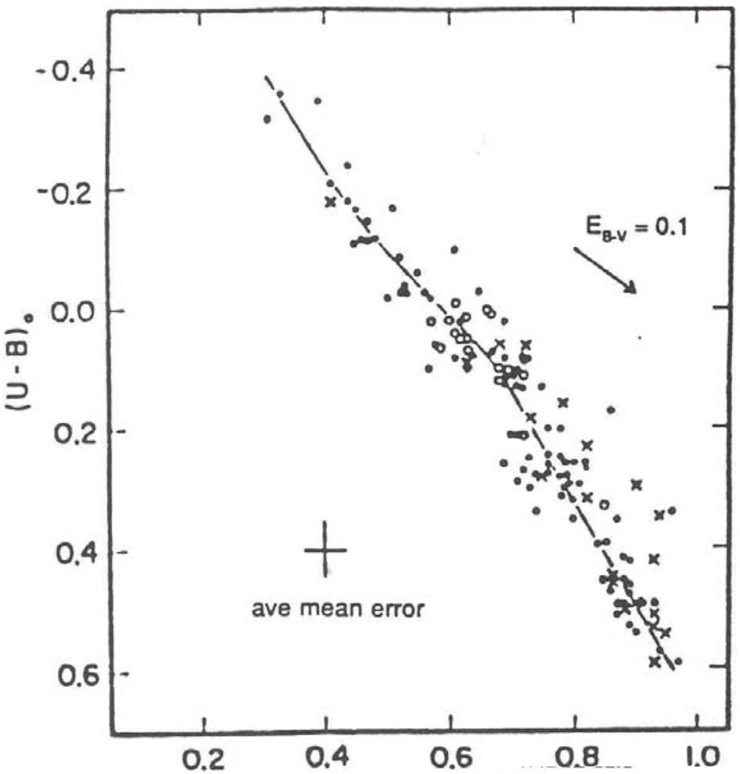
$$\left[\frac{dM}{dt_{\rm ms}}\right]_{M=M_t} t_g = \left[\frac{dM}{dt_{\rm ms}}\right]_{M=M_t} (t - t_{\rm ms}),$$

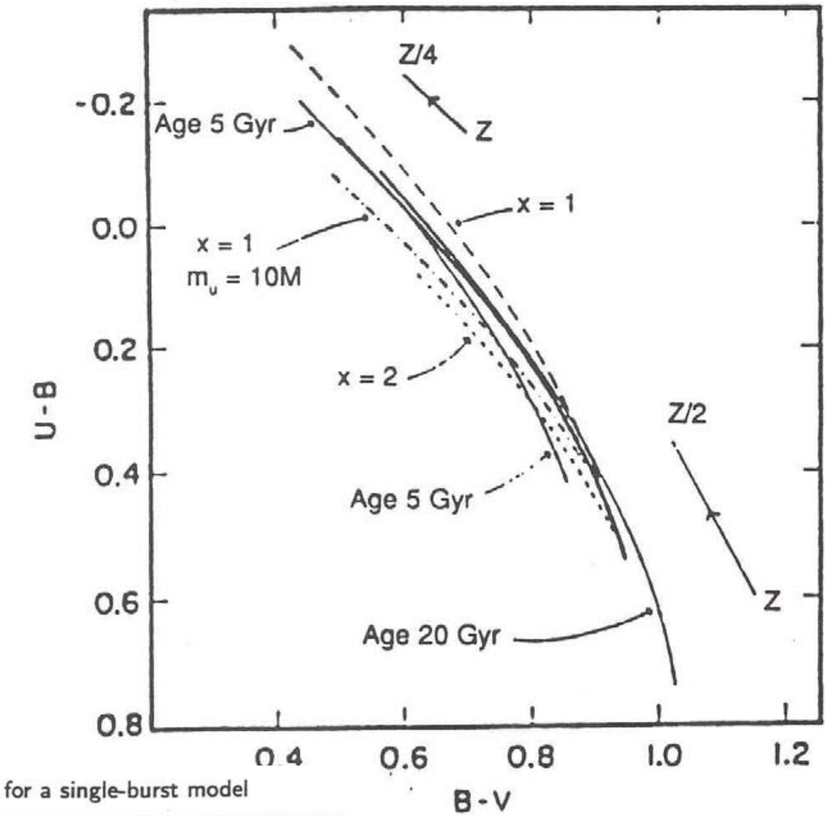
Is simply the number of giants of mass M_t that have been created. The total luminosity of a single burst is thus

$$L_{\rm SB}(t) = L_{\rm ms}(t) + N_g(t) L_g.$$

Single Burst Models

$x = 1.35$ ($\xi = M^{-2.35}$), $\gamma = 3$ ($t_{ms} = M^{-3}$), $\beta = 4.9$ (U), 4.5 (B), & 4.1 (V-band)





Colors and mass-luminosity ratios for a single-burst model

$t = 0.01$	$U - B = -0.34$	$B - V = 0.12$	$(\mathcal{M}/L)_B = 0.15$
0.03	−0.06	0.45	0.38
0.1	0.18	0.64	1.12
0.3	0.38	0.79	2.79
1	0.56	0.90	6.95
3	0.66	0.96	14.9

Other Examples of Population Synthesis Models…

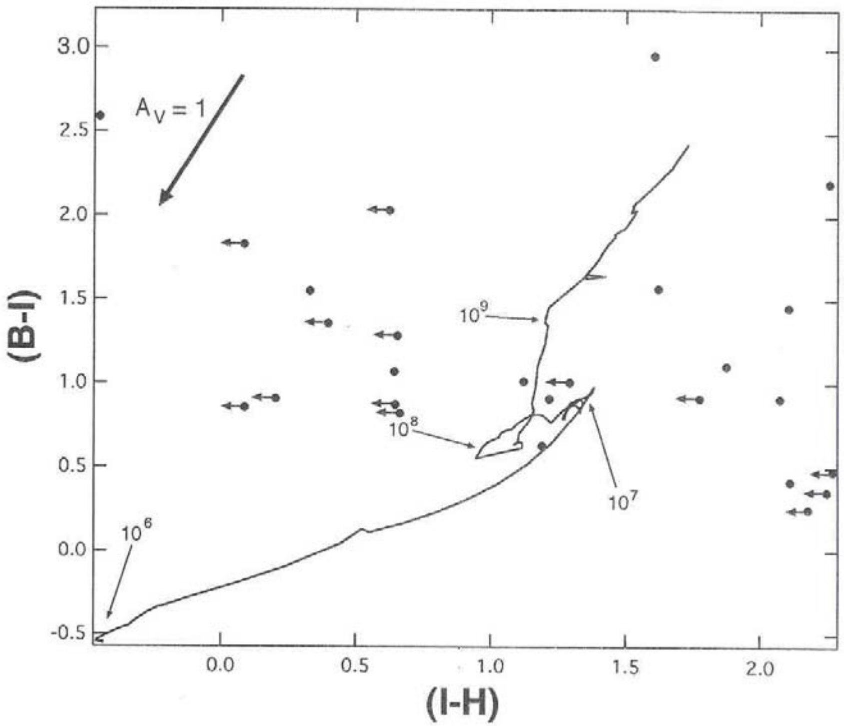
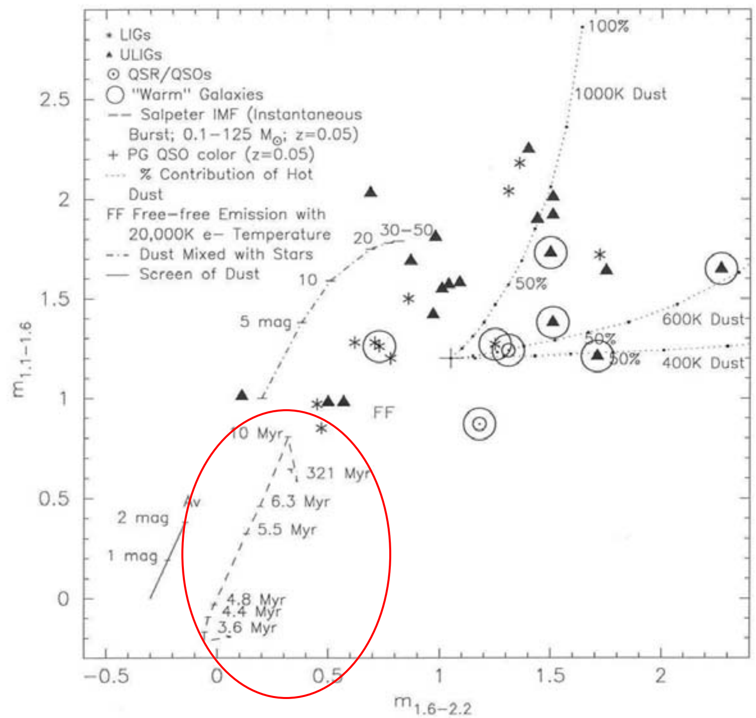
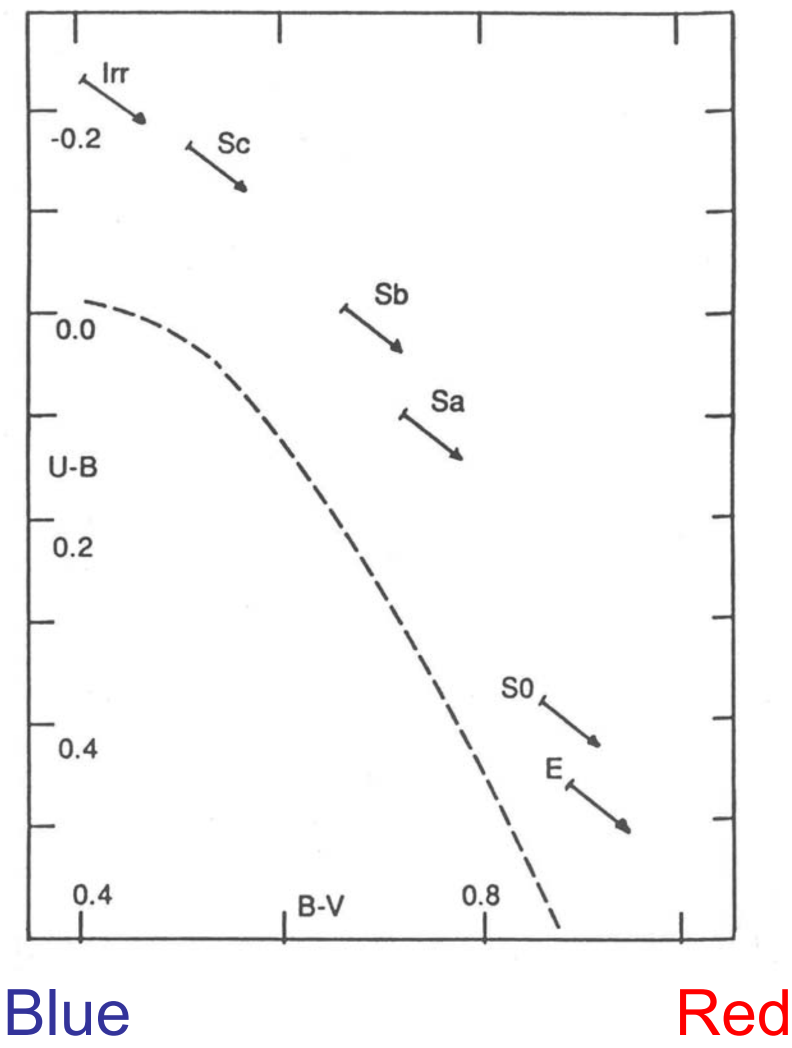
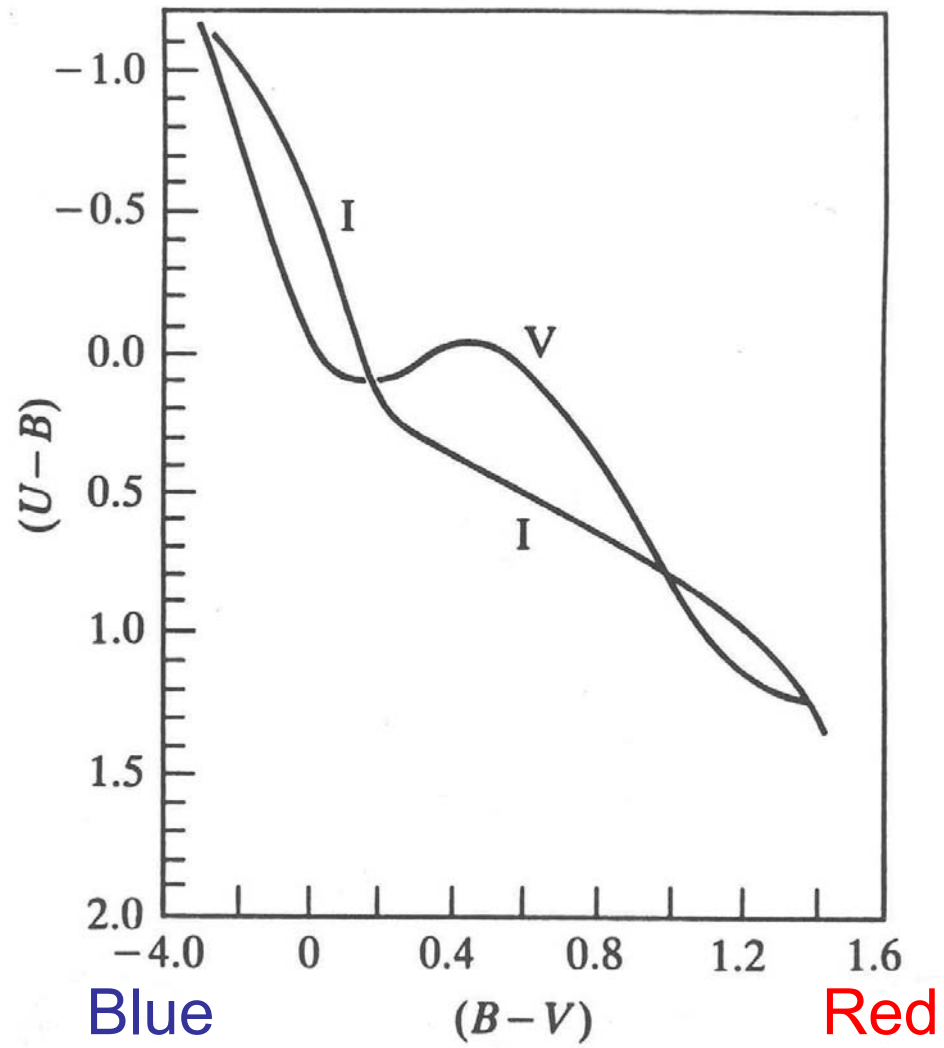


FIG. 8.—$(B-I)$, $(I-H)$ color-color diagram for an instantaneous starburst having a Salpeter IMF and a mass range from 0.1 to 125 $M_\odot$. Also shown are the colors of the star-forming knots given in Table 3.



Color-Color Diagram





Formation of Stars

What conditions are needed to induce star formation?

Jeans Mass: consider a spherical distribution of gas with density ρ undergoing collapse.
At radius *r*, the acceleration due to gravity is

$$g = -GM(r)/r^2.$$

The acceleration can be rewritten in the following fashion

$$g = d^2r/dt^2 = v(dv/dr),$$

v is the velocity at radius *r*, such that the equation of motion becomes

$$v(dv/dr) = -GM(r)/r^2.$$

During the collapse, *M(r)* is constant. Thus,

$$\int_{v=0}^{v} v dv = \int_{r=\infty}^{r} -GM dr/r^2.$$

Solving for velocity,

$$v^2/2 = GM/r \quad \rightarrow \quad v = (2GM/r)^{1/2}.$$

For gas in **free fall** collapse, the time for the free fall to occur is

$$t_{ff} = r/v.$$

Substituting in the expression for mass in terms of ρ & *r*,

$$M = (4/3)\pi\rho r^3,$$

The free fall expression becomes

$$t_{ff} \sim (3/8\pi G\rho)^{1/2} \propto (G\rho)^{-1/2}.$$

Assume an **isothermal collapse**. Collapse will occur when t_{ff} < the time taken for a sound wave to cross the cloud:

$$t_{ff} < t_{\rm sound}.$$

The sound wave crossing time can be expressed as

$$t_{\rm sound} \sim \frac{r}{v_{\rm sound}} \sim \frac{r}{(8kT/\mu m_H)^{1/2}}.$$

Thus, the criterion for collapse is,

$$\left(\frac{3}{8\pi G\rho}\right)^{1/2} < \frac{r}{(8kT/\mu m_H)^{1/2}}.$$

From above,

$$r = (3M/4\pi\rho)^{1/3}$$

Substituting & solving for mass

$$\frac{3}{8\pi G\rho} < \left(\frac{3M}{4\pi\rho}\right)^{2/3} \frac{\pi\mu m_H}{8kT}.$$

$$M_{\text{collapse}} \gtrsim \left(\frac{kT}{G\mu m_H}\right)^{3/2} \frac{1}{\rho^{1/2}}.$$

The **Jeans Mass** is the minimum mass under which collapse can occur

$$\boxed{M_J \sim \left(\frac{kT}{G\mu m_H}\right)^{3/2} \frac{1}{\rho^{1/2}}.}$$

How Big is M_J?

For neutral, molecular gas in the interstellar medium (ISM)

$$\mu = 2,\ T = 100\ \mathrm{K},\ N = 10^6\ \mathrm{m^{-3}}.$$

Thus,

$$r\ (\mathrm{kg\ m^{-3}}) = (\mu\, m_H)\, N \approx 3.3 \times 10^{-21}\ \mathrm{kg\ m^{-3}}$$

Thus $M_J \approx 10^{34}$ kg $\approx 10^4\ M_{solar}$ (i.e., mass of a globular cluster.

M_J Could be reduced to 50-100 M_{solar}, but these values are still too high (most star have masses of 1 M_{solar}).

Then Fragmentation must be Important

$$M_J \sim \left(\frac{kT}{G\mu m_H}\right)^{3/2} \frac{1}{\rho^{1/2}}.$$

- I.e., as collapse proceeds, $M_J \downarrow$ as $\rho \uparrow$, provided T = constant
- Can $T \approx$ constant? Yes

Gravitational Energy → Thermal Energy → Radiative Energy ($h\nu$[IR])

- IR photons = efficient way of cooling because they have a long mean free path

$$\ell \approx (N\, \sigma_{absorption})^{-1}$$

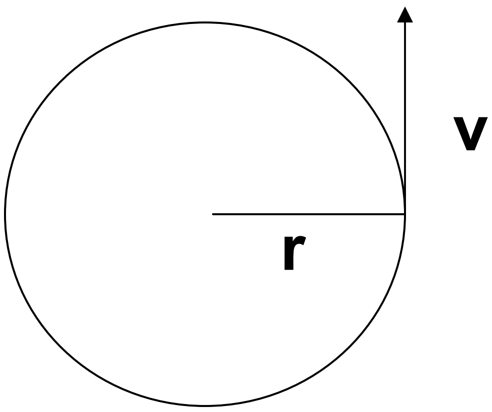
Summary of What Halts Collapse

- High ρ – the clouds becomes opaque even to IR photons. Cloud $T\uparrow$, $P\uparrow$, halt collapse.
- Rotation Effects
- Magnetic Field Effects

Angular Momentum

The angular momentum is expressed in terms of radius *r*, momentum *p*, mass *m*, & velocity *v*

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$$



$$L = |L| = mvr = m(r\omega)r = mr^2\omega,$$

Where ω is the angular velocity. For solid bodies,

$$L = kMR^2\omega,$$

Where *k* ≈ constant & is dependent on the geometry of the solid.

Now, suppose a cloud initially has,

$$r_i \approx 50 \text{ pc} \approx 10^{18} \text{ m}$$
$$w_i \approx 10^{-8} \text{ rad yr}^{-1}$$

If the cloud eventually has a radius of

$$r_f \approx 100 \text{ AU} \approx 10^{13} \text{ m}$$

We can use conservation of angular momentum

$$kM_{\text{cloud}} r_i^2 \omega_i = kM_{\text{cloud}} r_f^2 \omega_f,$$

And conclude the the angular velocity would then be

$w_f \approx 10^2$ rad yr^{-1} or a period $T = 2\pi/\omega \approx$ few days

- This is way too fast! The cloud would fly apart!
- Thus, L must be lost or converted during collapse.

Magnetic Fields

- Interstellar **B** ≈ 10^{-9} Tesla

- This **B**-field would be amplified during collapse. However, high **B**(≥ 10^4 Gauss) are not observed.

- Thus, **B**-field must be lost during collapse somehow.

Solutions

- Massive clouds will fragment into ≈ 1 M_{solar} pieces.

- Cloud spin *L* may be transported into relative *L* of fragments & removed by **B**-field.

- **B**-field may reconnect &/or be trapped in stars.

The Increased ω will lead to a Flattened Disk

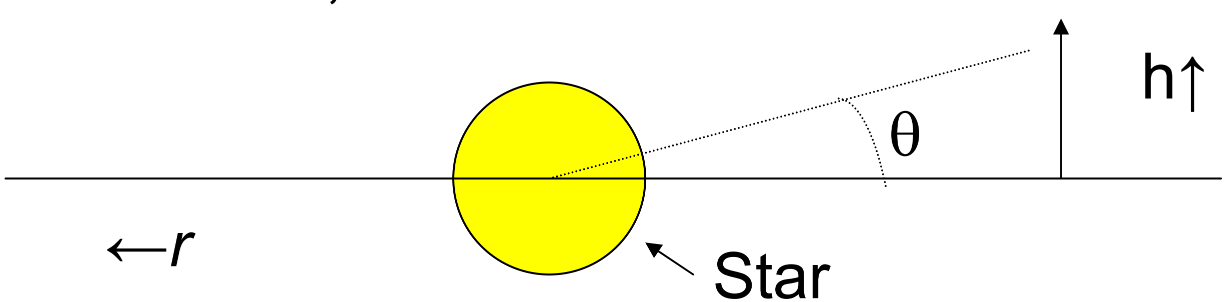
In the plane of the disk, acceleration due to gravity at a radius *r* is equal to

$$g = GM_{\rm star}/r^2.$$

Perpendicular to the disk at radius *r*, acceleration at a height *h* above the disk is

$$g_{\perp} = g\sin\theta \sim \frac{GM_{\rm star}}{r^2}\theta \sim \frac{GM_{\rm star}}{r^2}\left(\frac{h}{r}\right),$$

where sin θ = *h* / *r*, and $\theta \approx$ small.



If the gas in the disk is in **hydrostatic equilibrium**, then,

$$\frac{dp}{dh} = -pg_{\perp} = -\rho\frac{GM_{\text{star}}}{r^3}h.$$

From the **Perfect Gas Law**,

$$\frac{dp}{dh} = \frac{kT}{\mu m_H}\frac{d\rho}{dh}.$$

Setting the rhs of the above equations equal to each other and solving for ρ

$$\rho = \rho(0)exp\left[-\left(\frac{h}{H}\right)^2\right],$$

where the scale height H is,

$$H = \left(\frac{2kTr^3}{\mu m_H GM_{\text{star}}}\right)^{1/2}.$$

Thus, the gaseous component is a flaired disk.

The Dust Distribution in the Disk is Different

- Not subject to hydrostatic equilibrium, just drag force F_{drag} & gravity
- Thus, dust sinks to the midplane

$$m_{\mathrm{dust}} \frac{d^2 h}{dt^2} = F_{\mathrm{drag}} - m_{\mathrm{dust}} \frac{G M_{\mathrm{star}} h}{r^3}.$$

- Initially, the dust grains grow in size by sticking together
- When enough mass has accreted in this fashion, material can be captured via self gravity
- Accretion leads to planet formation in the midplane of the disk