Elements of Galaxy Dynamics

I. Basic Goals

- Most galaxies are in approx. equilibrium in the present-day universe.
  - What equilibrium solutions exist and which are stable?
- A solution entails:
  - in which gravitational potential do the tracer particles move
  - on what orbits do they move?
- How much ‘formation memory’ do galaxies retain?

For MUCH more detail on any of this

Binney and Tremaine, 1996/2008

Binney and Merrifield, 2002

But, let's not forget the practical question:

How do we use observable information to get these answers?

Observables:

- Spatial distribution and kinematics of "tracer population(s)", which may make up
  - all (in globular clusters?)
  - much (stars in elliptical galaxies?) or
  - hardly any (ionized gas in spiral galaxies)

of the "dynamical" mass.

- In external galaxies only 3 of the 6 phase-space dimensions, are observable: $x_{proj}, y_{proj}, v_{LOS}$!

Note: since $t_{dynamical} \sim 10^8$ yrs in galaxies, observations constitute an instantaneous snapshot.

…the Galactic Center is an exciting exception..

Stars vs. Gas
or
Collisionless vs. Collisional Matter

How often do stars in a galaxy „collide“? (they don‘t)

- $R_{Sun} \approx 7x10^{10}$ cm; $D_{Sun-\alpha Cen} \approx 10^{19}$ cm!
  => collisions extremely unlikely!

  …and in galaxy centers?

  Mean surface brightness of the Sun is μ =-11mag/sqasec, independent. of distance. The centers of other galaxies have μ~ 12 mag/sqasec. Therefore, $(1 - 10^{-9})$ of the projected area is empty.

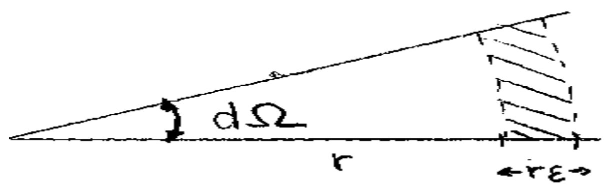
⇒ Even near galaxy centers, the path ahead of stars is empty.

Stars do not ‚feel‘ their galactic neighbours

Dynamical time-scale: t_{dyn} (=typical orbital period)

e.g. Milky Way at the solar radius:
R~8kpc v~200km/s → t_{orb}~240 Myrs
→ $t_{orb} \sim t_{Hubble}/50$ ←true for galaxies of most scales

Stars in a galaxy feel the gravitational force of other stars. But of which?
consider homogeneous distribution of stars, and force exerted on one star by other stars seen in a direction dΩ within a slice of [r , r x(1+e)]



→ $dF \sim GdM/r^2 = G\rho \times r(!) \times \varepsilon d\Omega$

→ Gravity from the multitude of distant stars dominates!

What about (diffuse) interstellar gas?

- continuous mass distribution
- gas has the ability to lose (internal) energy through isotropic radiation (no angular momentum loss through cooling)
- Two basic regimes for gas in a potential well of ‚typical orbital velocity', v
  - $kT/m \approx v^2 \rightarrow$ hydrostatic equilibrium
  - $kT/m << v^2$, as for atomic gas in galaxies
- in the second case:

  supersonic collisions $\rightarrow$ shocks $\rightarrow$ (mechanical) heating $\rightarrow$(radiative) cooling $\rightarrow$ energy loss

  For a given (total) angular momentum, what's the minimum energy orbit?

  A (set of) concentric (co-planar), circular orbits.

  → cooling gas makes disks!

II. Describing Stellar Systems in Equilibrium

Modeling Collisionless Matter: Approach I

Phase space: d$\underline{x}$, d$\underline{v}$

We describe a many-particle system by its distribution function f(x,v,t) = density of stars (particles) within a phase space element

Starting point: Boltzmann Equation (= phase space continuity equation)
It says: if I follow a particle on its gravitational path (=Lagrangian derivative) through phase space, it will always be there.

$$\frac{Df(\vec{x},\vec{v},t)}{Dt} = \frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \vec{\nabla}\Phi_{grav}\frac{\partial f}{\partial \vec{v}} = 0$$

A rather ugly partial differential equation!

Note: we have substituted gravitational force for accelaration!

To simplify it, one takes velocity moments:
i.e. $\int_{\mathbb{R}^3} \ldots v^n d^3v$ n = 0,1, ... on both sides

Moments of the Boltzmann Equation

0th Moment

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

mass conservation

r: mass density; v/u: indiv/mean particle velocity

1st Moment

$$\int \ldots v_j d^3 v$$

$$\frac{\partial}{\partial t}(\rho \vec{u}) + \vec{\nabla} \cdot \left(\rho \left(\underline{\underline{T}} + \vec{u} \cdot \vec{u}\right)\right) + \rho \vec{\nabla} \Phi = 0$$

$$\text{with } \rho \underline{\underline{T}} = \int f \cdot (v_i - u_i)(v_j - u_j) d^3 v$$

"Jeans Equation"

The three terms can be interpreted as:

momentum change $\frac{\partial}{\partial t}(\rho \vec{u})$

pressure force $\vec{\nabla}\left[\rho\left(\underline{\underline{T}} + \vec{u} \cdot \vec{u}\right)\right]$

grav. force $\rho \vec{\nabla} \Phi$

Let‘s look for some familiar ground ...

If $\underline{\underline{T}}$ has the simple isotropic form

$$\underline{\underline{T}} = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}$$

as for an „ideal gas“ and if the system is in steady state $\left(\vec{u} \equiv 0, \frac{\partial}{\partial t} \equiv 0 \right)$, then we get

$$\boxed{\vec{\nabla} p(\vec{x}) = -\rho(x) \vec{\nabla} \Phi(x)}$$

simple hydrostatic equilibrium

Before getting serious about solving the „Jeans Equation“, let‘s play the integration trick one more time ...

Virial Theorem

Consider for simplicity the one-dimensional analog of the Jeans Equation in steady state:

$$\frac{\partial}{\partial x}\left[\rho v^2\right] + \rho \frac{\partial \Phi}{\partial x} = 0$$

After integrating over velocities, let‘s now

integrate over $\vec{x}$: $\int ...d\vec{x}$

[one needs to use Gauss’ theorem etc..]

$$\boxed{-2E_{kin} = E_{pot}}$$

Application of the Jeans Equation

- Goal:
  - Avoid “picking” right virial radius.
  - Account for spatial variations
  - Get more information than “total mass”

- Simplest case
- spherical: $\rho\left(\vec{r}\right) = \rho(r)$

  static: $\vec{\upsilon} \equiv 0, \frac{\partial}{\partial t} \equiv 0$

$$\boxed{\vec{\nabla}\left(\rho\,\underline{\underline{T}}\right) = -\rho\,\vec{\nabla}\,\Phi}$$

Choose spherical coordinates: $\frac{d}{dr}\left(\rho\sigma_r^2\right) + \frac{2\rho}{r}\left(\sigma_r^2 - \sigma_t^2\right) = -\rho\frac{d\Phi}{dr}$

σ_r is the radial and σ_t the tangential velocity dispersion

$$\boxed{\frac{d}{dr}\left(\rho\sigma_r^2\right) = -\rho\frac{d\Phi}{dr}}$$

for the „isotropic“ case!

Note: Isotropy is a mathematical assumption here, **not** justified by physics!

Remember: ρ is the mass density of particles under consideration (e.g. stars), while Φ just describes the gravitational potential acting on them.

How are ρ and Φ related?

Two options:

1. $\nabla^2 \Phi = 4\pi G \rho$ „self-consistent problem“
2. $\nabla^2 \Phi = 4\pi G \underbrace{\left(\rho + \rho_{other}\right)}_{\rho_{total}}$ with

$\rho_{other} = \rho_{dark\ matter} + \rho_{gas} + \ldots \rho_{Black\ Hole}$

An Example:

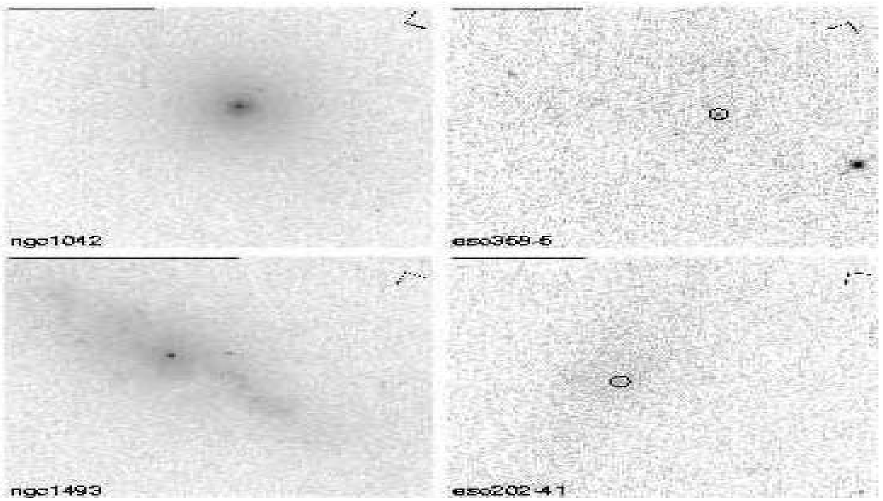
When Jeans Equation Modeling is Good Enough

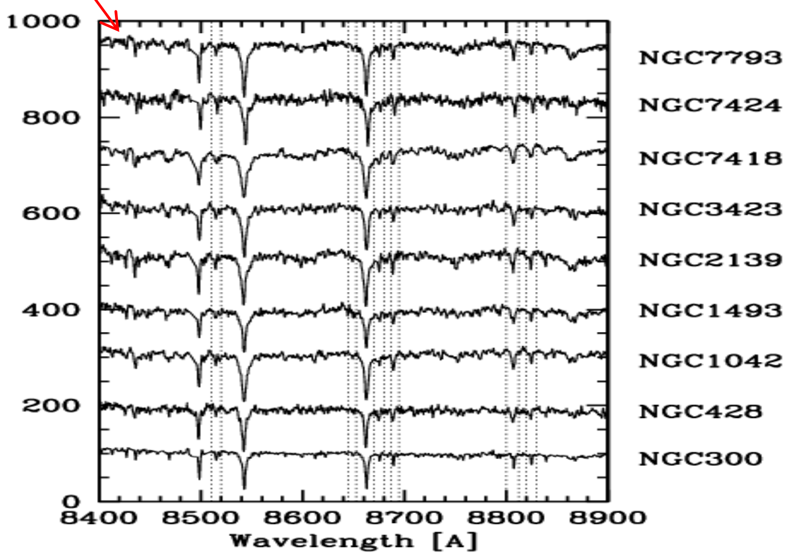
Walcher et al 2003, 2004

The densest stellar systems sitting in very diffuse galaxies..

Images → $\rho(r)$

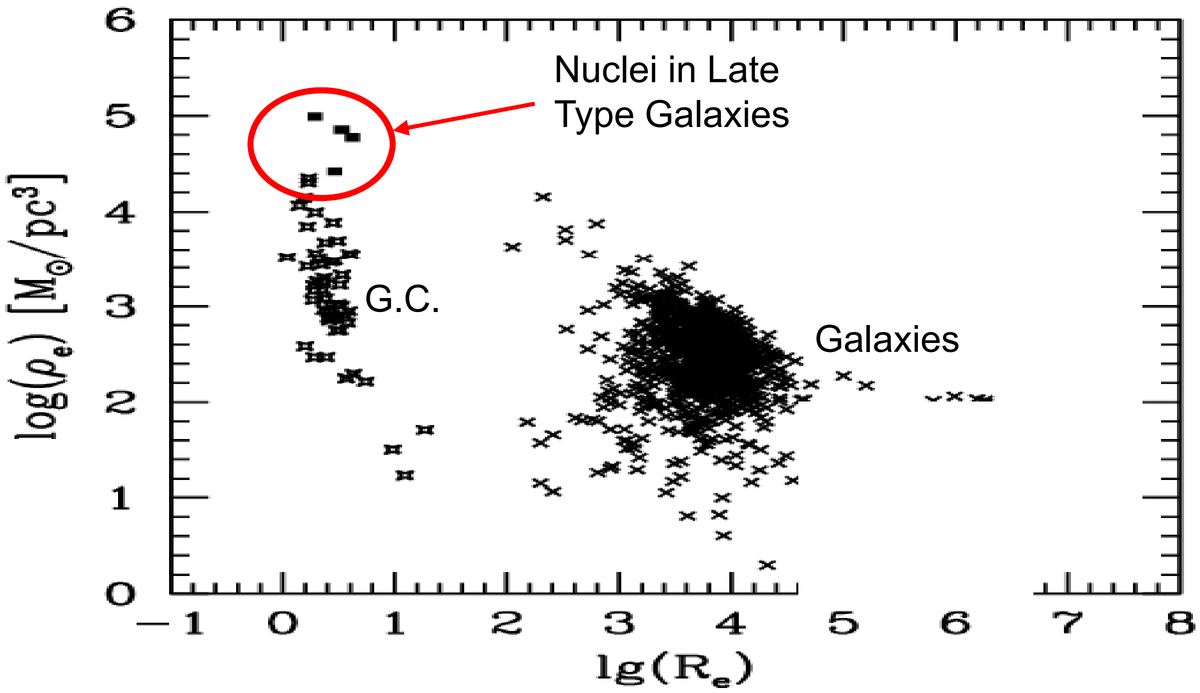
Spectra → σ (perhaps $\sigma(r)$)





Then get M from the Jeans Equation ….

Are the Nuclei of bulge-less Galaxies more like tiny bulges or like globular clusters?



- Jeans Equation is great for estimating total masses for systems with limited kinematic data

Describing Collisionless Systems: Approach II

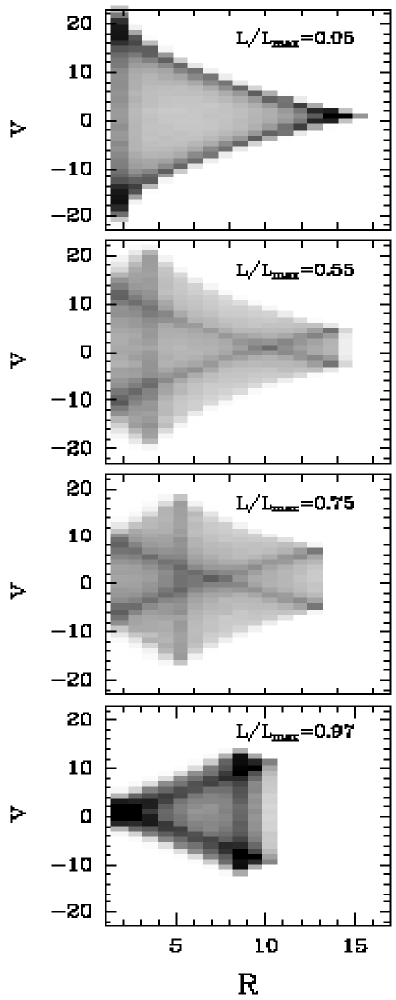
"Orbit-based" Models

Schwarzschild Models (1978)

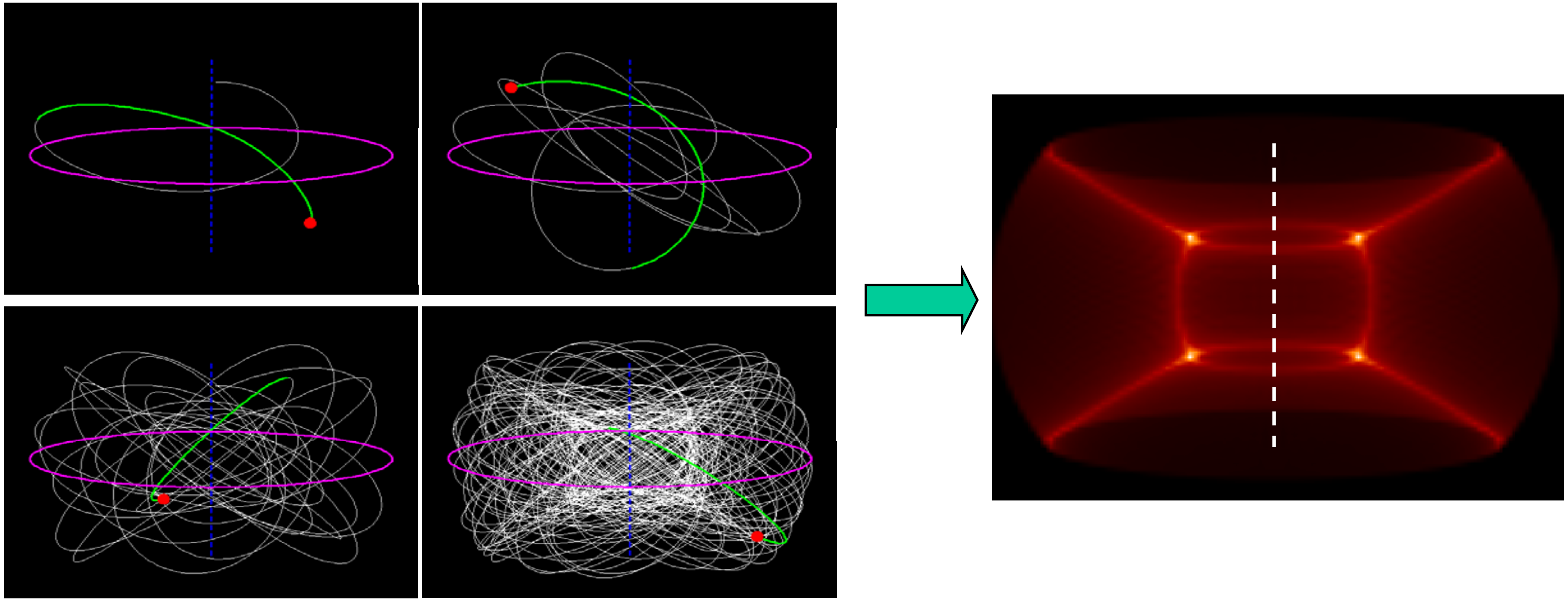
- What would the galaxy look like, if all stars were on the same orbit?
  - pick a potential F
  - Specify an orbit by its "isolating integrals of motion", e.g. E, J or J_z
  - Integrate orbit to calculate the
    - time-averaged
    - projected

    properties of this orbit

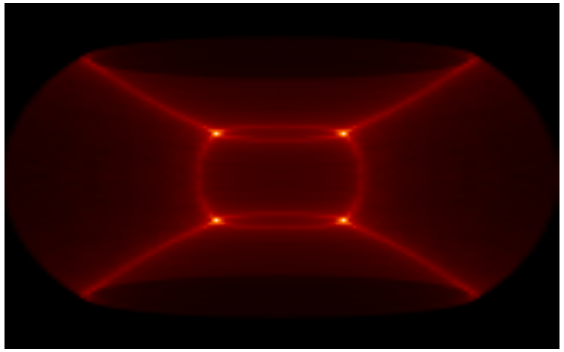
    (NB: time average in the calculation is identified with ensemble average in the galaxy at on instant)
  - Sample "orbit space" and repeat



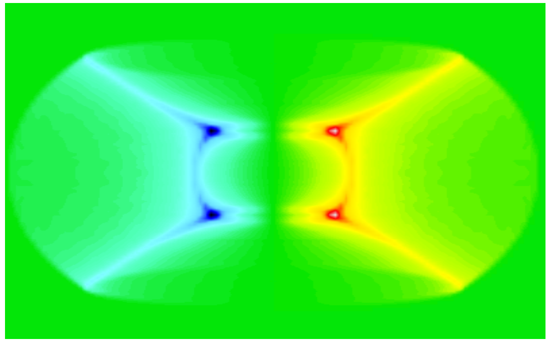
from Rix et al 1997



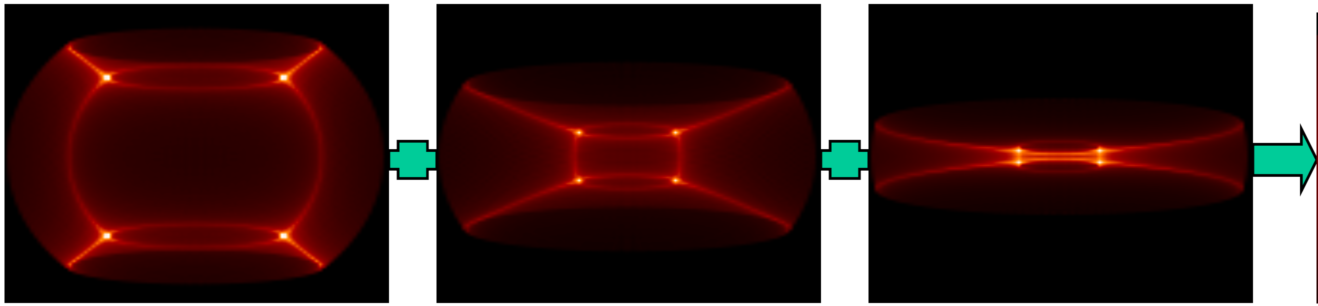
Figures courtesy Michele Capellari 2003



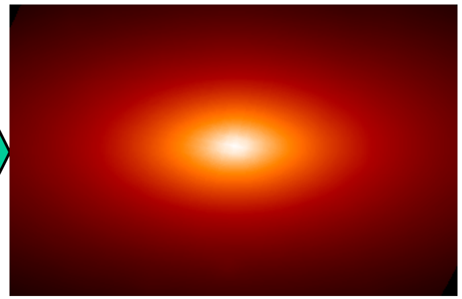
Projected density



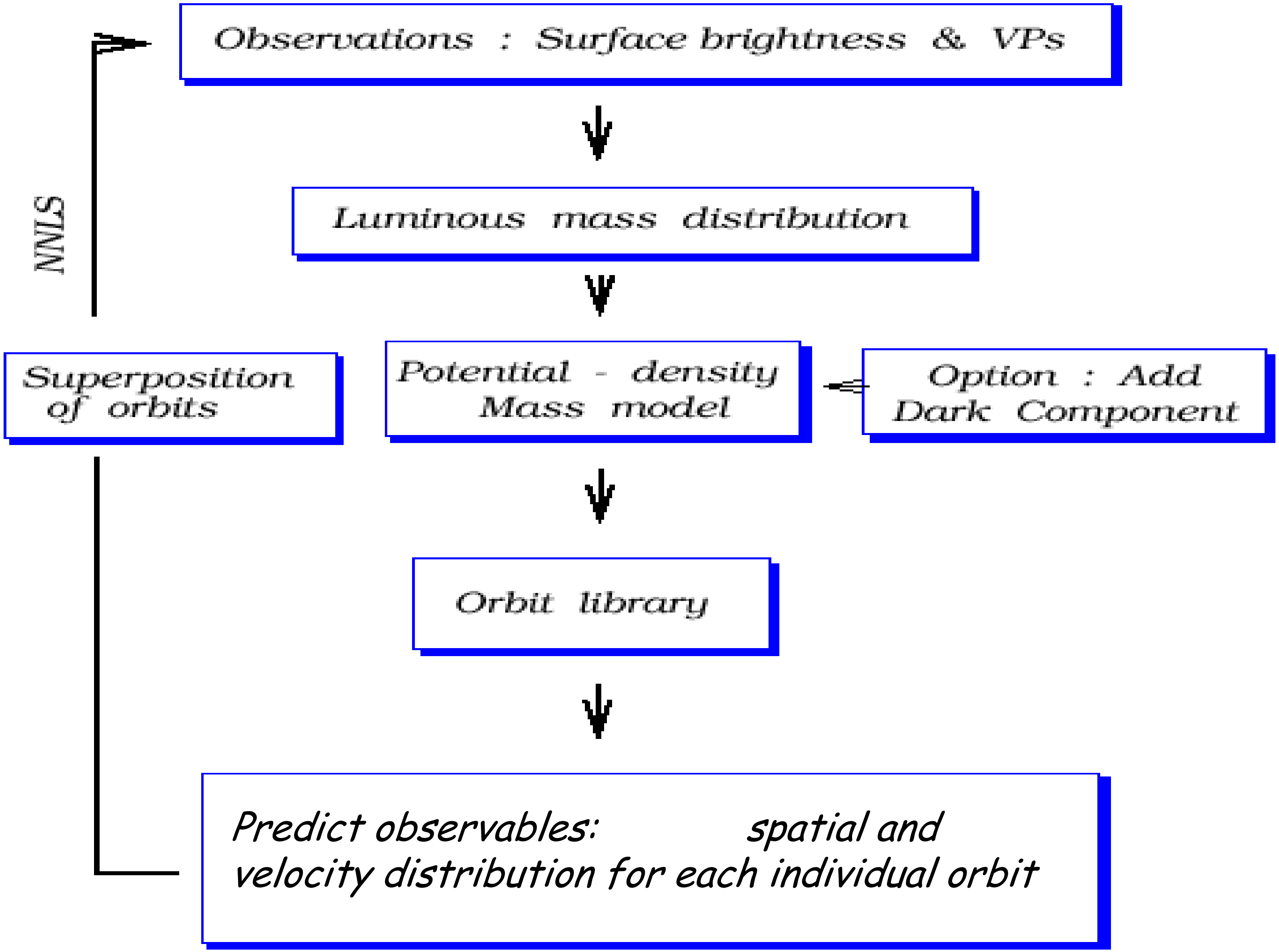
$V_{\text{line-of-sight}}$



images of model orbits



Observed
galaxy image

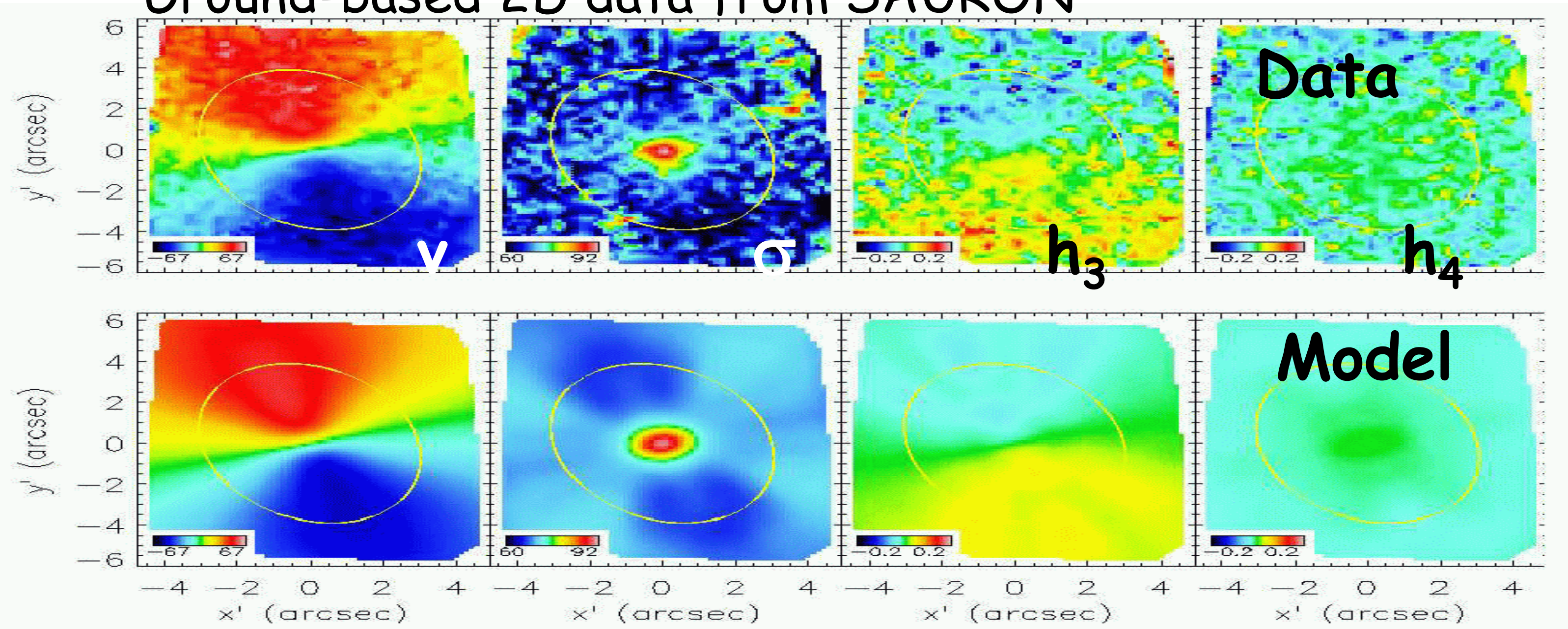

Observations : Surface brightness & VPs
Luminous mass distribution
Potential - density
Mass model
Option : Add
Dark Component
Superposition
of orbits
NNLS
Orbit library
Predict observables: spatial and
velocity distribution for each individual orbit

Example of Schwarzschild Modeling

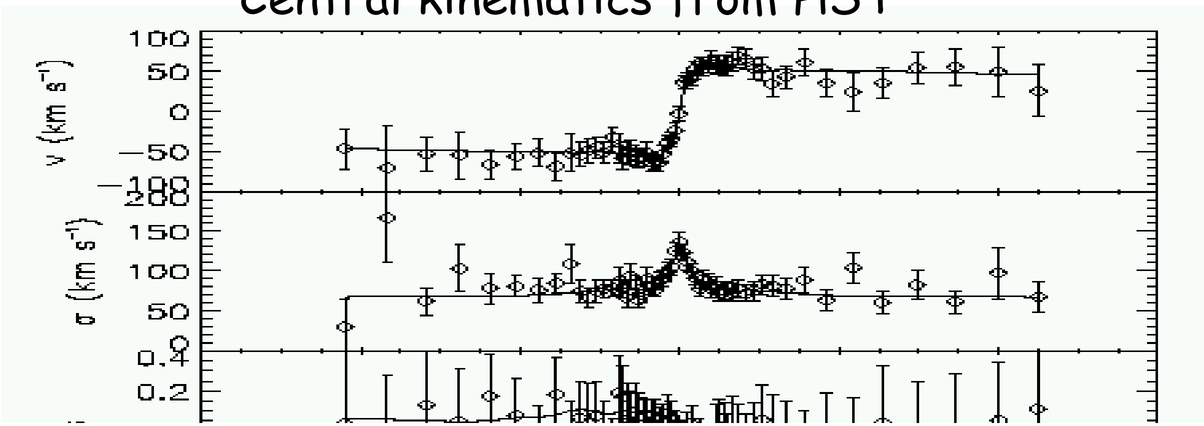
M/L and M_{BH} in M32

Verolme et al 2001

Ground-based 2D data from SAURON

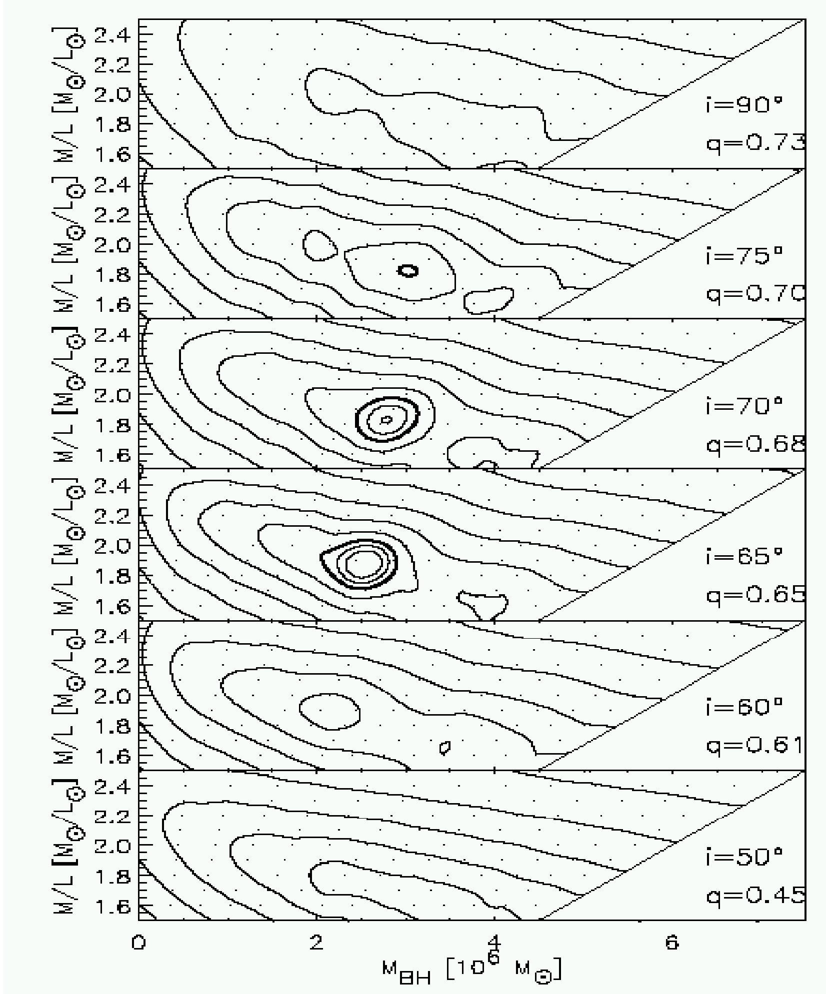


Central kinematics from HST



- Then ask:

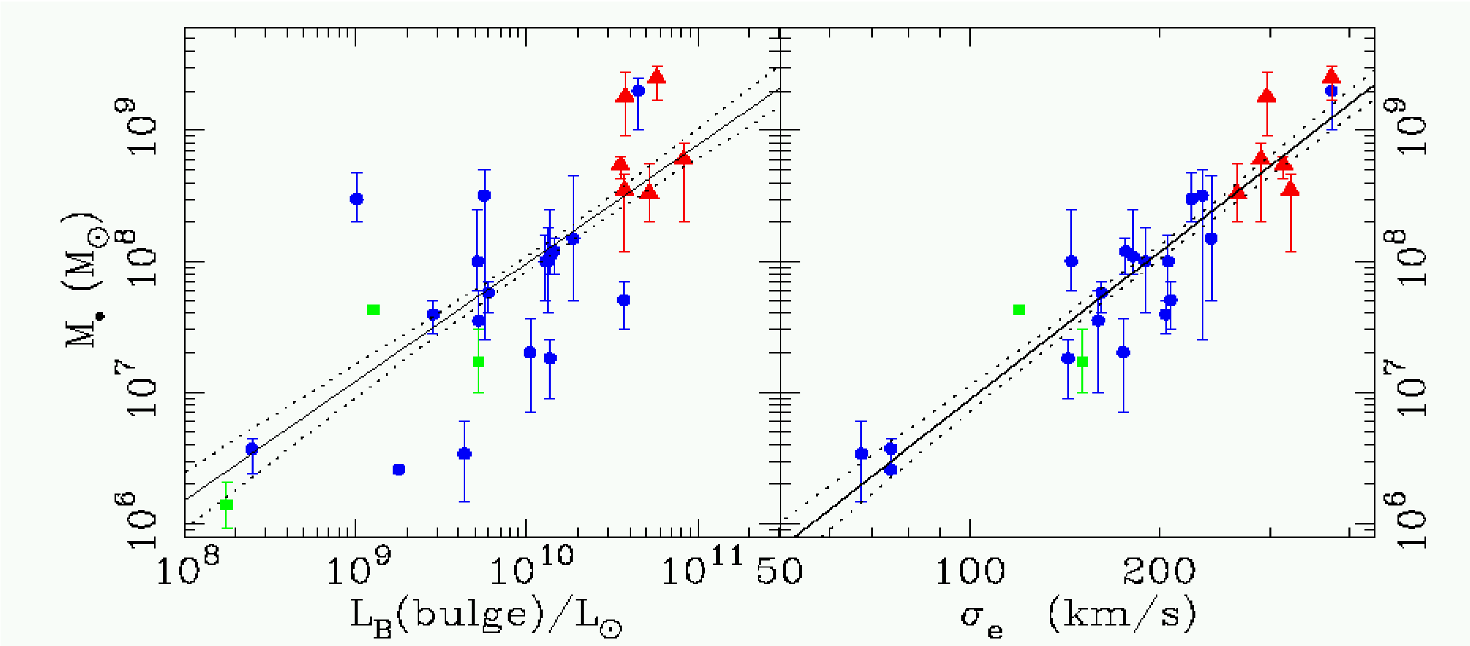
  for what potential and what orientation, is there a combination of orbits that matches the data well



Determine: inclination, M_{BH} and M/L simultaneously

NB: assumes axisymmetry

This type of modeling (+HST data) have proven necessary (and sufficient) to determine M_{BH} dynamically in samples of nearby massive galaxies



→ M_{BH} and σ_* (on kpc scales) are tightly linked

(Gebhardt et al 2001)

Stellar Kinematics and Clues to the Formation History of Galaxies

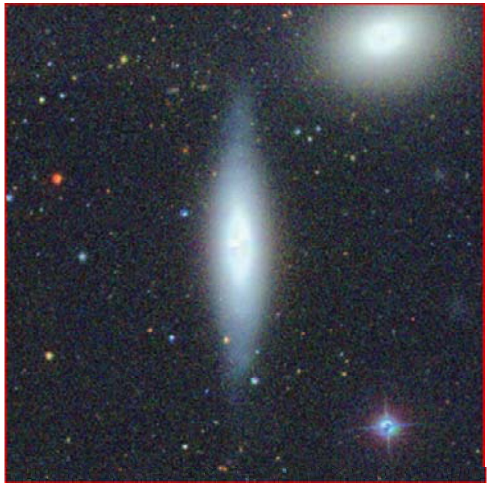
Mergers scramble the dynamical structure of galaxies, but do not erase the memory of the progenitor structures completely.

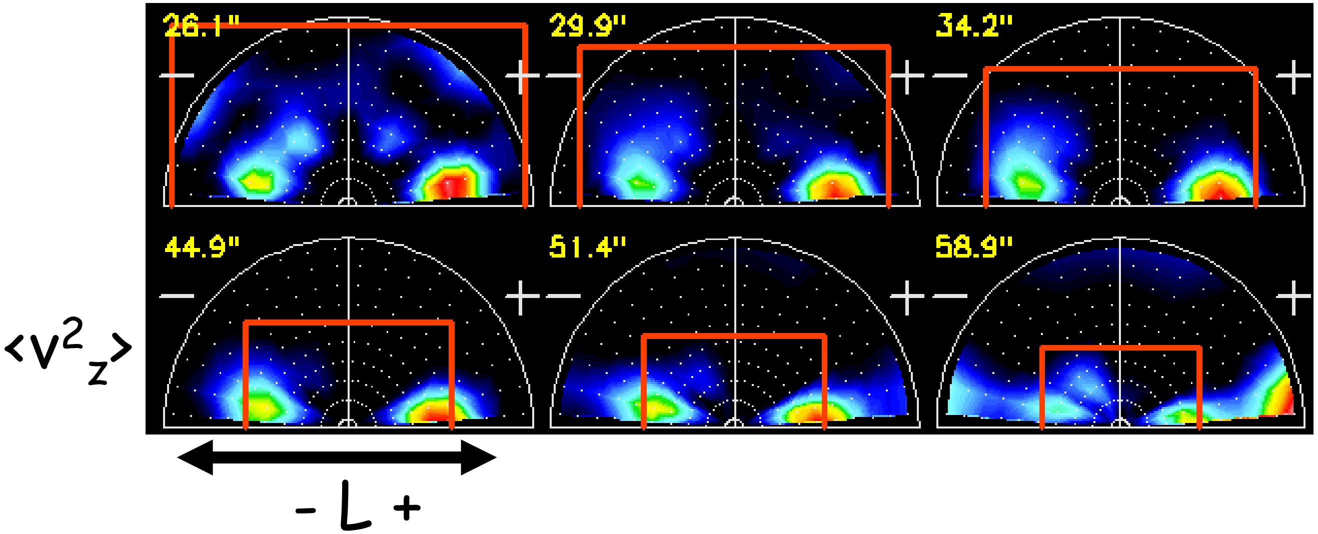
In equilibrium, phase space structure (E,J/Jz,+) is preserved.

However, observations are in $x_{proj}, y_{proj}, v_{LOS}$ space!

Connection not trivial!

NGC 4550: solve for star's phase-space distribution
(that is solution of the Schwarzschild model)





- Two counterrotating components
  - Double-peaked absorption lines (Rix et al. 1993, ApJ, 400, L5)
  - SAURON: accurate decomposition, in phase space
- Both components are disks
  - Same mass
  - Different scale height

III. The Dynamics of Gas in Galaxies (vs. Stars)

Two regimes:

- **$KT \approx V^2_{characteristic}$** **hot gas**
- **$KT \ll V^2_{characteristic}$** **warm, cold gas**
- then all gas collisions are supersonic → shocks

Dynamics of 'hot' gas

'approximate hydrostatic equilibrium'

X-ray gas, 10^6 K observable in massive galaxies

Orbits of ‘cold’ gas

To ‘avoid’ shocks, gas has to be on non-intersecting loop orbits:

- concentric circles (in axisymm. case)
- ellipses in (slightly distorted) potentials
    - E.g. weak spiral arms
- in barred potentials, closed-orbit ellipticity changes at resonances → shocks, inflow

•Observed:

•Bars drive gas inflow

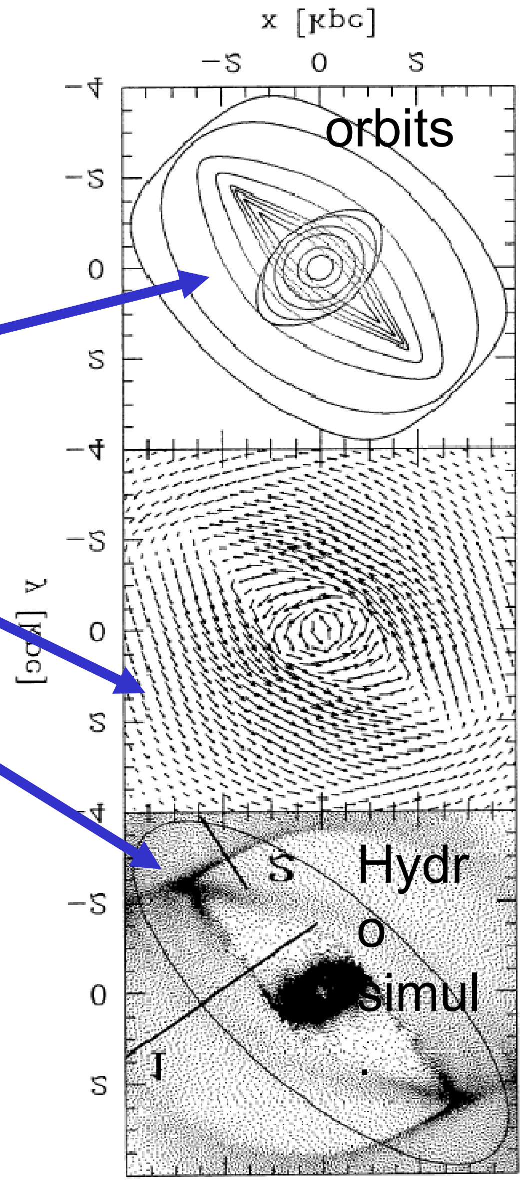
(e.g. Schinnerer et al 07)

•Whether all the way to

the black hole, unclear..

Gas Flow in Non-Axisymmetric Potential

(Englmaier et al 97)



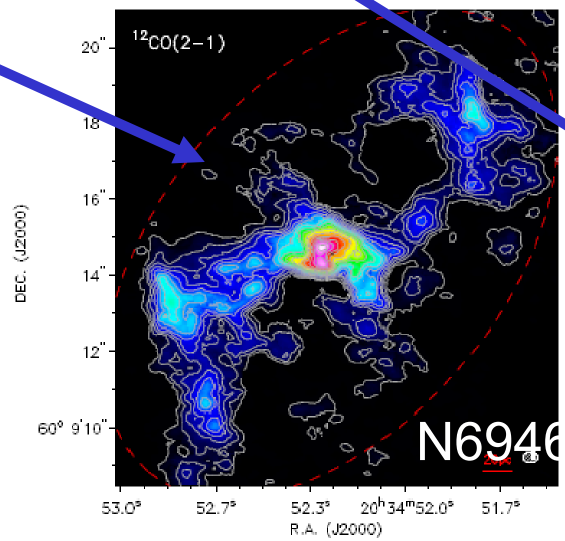
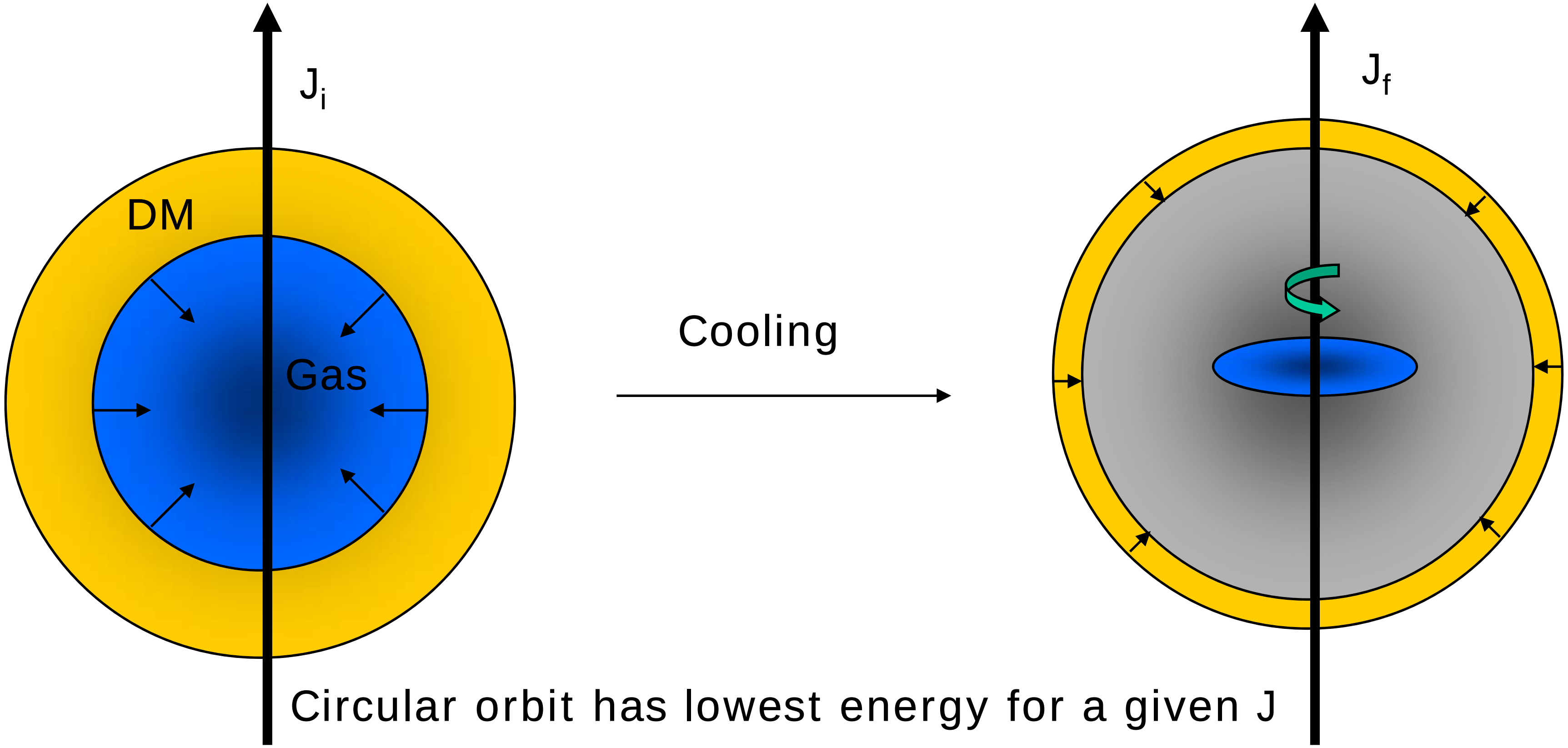


Fig. 1. Intensity map of the $^{12}CO(2\text{-}1)$ line emission at 0.4"×0.3" resolution (color and contours). The black cross marks the posi-

Why most galaxies have stellar disks



If stars form from gas in a settled disk → stellar disk

Why galaxies are disks of a characteristic size?

- Torques before the collapse induce spin λ ~0.07 (in both baryons and dark matter)

$$\lambda_{obs} \equiv \frac{J \cdot E^{1/2}}{GM^{5/2}}$$

- **Gas dissipates (by radiation) all the energy it can without violating angular momentum conservation → circular orbit**

- Fall and Efstathion (1980) showed that observed galaxy disks (*λ~0.5*) can form only in DM halos through dissipation→ central concentration (J conserved) → spin-up.

a) Presume there is no DM: $$\lambda_{obs} \equiv \frac{J \cdot E^{1/2}}{GM^{5/2}} = \lambda_{init}\sqrt{\frac{R_{init}}{R}} \quad \Rightarrow \quad \frac{R_{init}}{R} \approx 50$$

We observe $M_{disk} \approx 5\text{x}10^{10}\ M_{sun}$, $R_{disk} \approx 8\ kpc \Rightarrow R_{init} \approx 400\ kpc$

$\Rightarrow R_{turn\text{-}around} \approx 2\ R_{init} \approx 800\ kpc$

$\Rightarrow t_{collapse} \sim 50 \cdot 10^9\ years$ for $M \sim 5x10^{10}\ M_{sun}$

b) If the gas is only a small fraction of the total mass:

$\Rightarrow v_c(r)$ remains unchanged

$$\Rightarrow \quad R_{init}/R \sim \frac{\lambda_{obs}}{\lambda_{init}} \quad \Rightarrow R_{init} \sim 80 \; kpc$$

$\Rightarrow t_{dyn} \sim 10^9$ *years*

and there is enough time to form disks.

It turns out that the assumption of angular momentum conservation during the gas dissipation yield disk sizes as observed (assuming λ~0.07)

However: in (numerical) simulations much of the angular momentum is lost → modelled disks too small (unsolved)

III. Some Basics of Non-Equilibrium Stellar Dynamics

a) **Dynamical friction**

a) **Tidal disruption**

a) **Violent relaxation**

b) **Mergers,** **or how galaxies become spheroids**

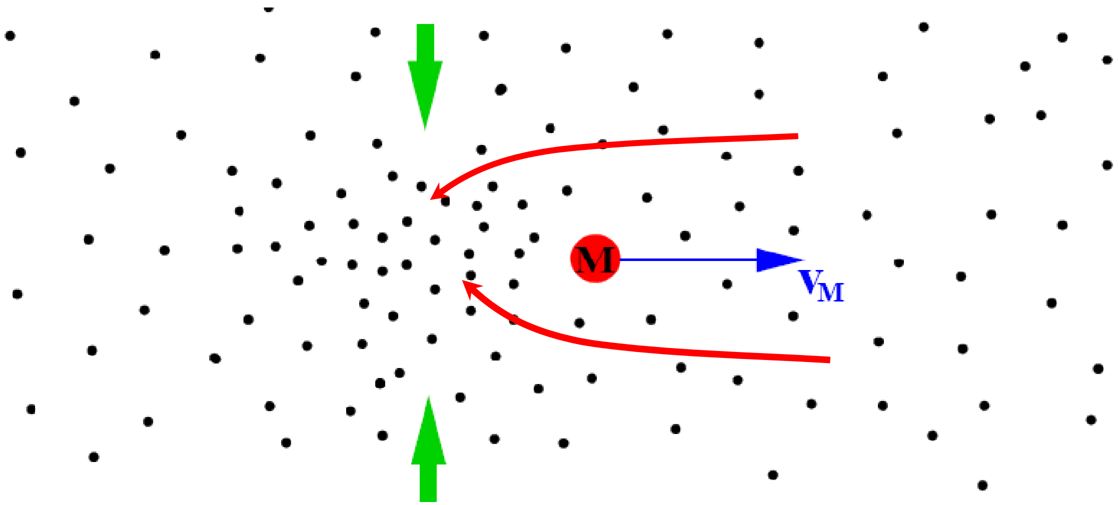
a) Dynamical friction

A "heavy" mass, a satellite galaxy or a bound sub-halo, will experience a slowing-down drag force (dynamical friction) when moving through a sea of lighter particles

Two ways to look at the phenomenon

a) A system of many particles is driven towards "equipartition", i.e.
$E_{kin}(M) \sim E_{kin}(m)$
$\Rightarrow V^2_{\text{of particle M}} < V^2_{\text{of particle m}}$

b) Heavy particles create a 'wake' behind them



$$F_{dyn.\,fric} = -\frac{4\pi G M^2}{V_M^2}\rho_m \cdot \ln\Lambda$$

Where m << M and ρ_m is the (uniform) density of light particles m, and $\Lambda = b_{max}/b_{min}$ with $b_{min} \sim \rho_M/V_2$ and b_{max} ~ size of system typically lu $\Lambda \sim 10$

Effects of dynamical Friction

a) Orbital decay: $t_{df} \sim r / (dr/dt)$

$V_{circ}\, dr/dt = -0.4 \ln\Lambda\, \rho_M/r$

or

$$t_{df} \approx \frac{1.2}{lu\Lambda}\frac{r_i^2 V_c}{\rho M}$$

Dynamical friction effective for

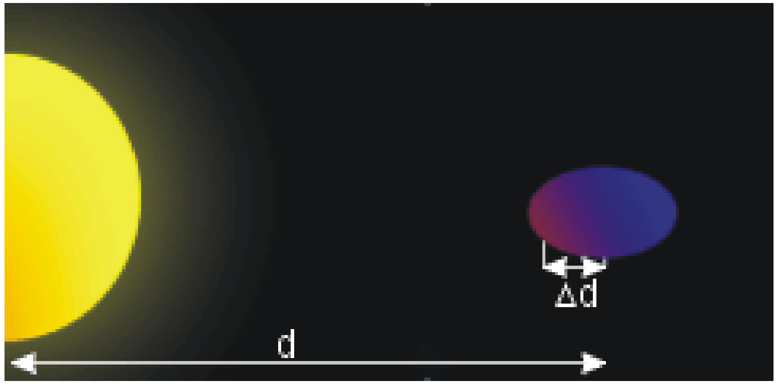
- high (host galaxy) densities
- Low mass (v_c) hosts
- small (orbital) radii
- Massive satellites (M)

Example: orbital decay of a satellite galaxy in MW Halo

$V_{cir}(MW) = 220$ km/s $M_{LMC} = 2 \times 10^{10}\ M_{SUN}$ $R_{LMC} = 50$ kpc

$\Rightarrow$ $T_{df}(LMC) = 1.2$ Gyr

b) Tidal disruption



'"Roche limit": for existence of a satellite, its self-gravity has to exceed the tidal force from the 'parent'

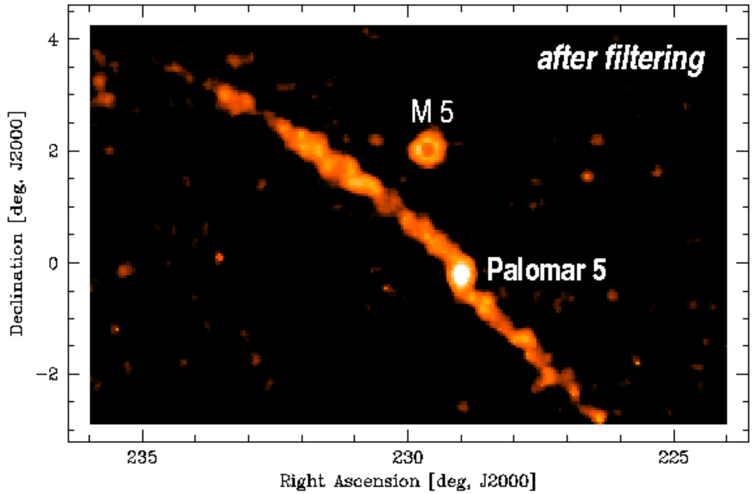
$$V_T = -\frac{3GM}{2d^3}\Delta d^2$$

Tidal radius:

$$R_{tidal}(satellite) = f\left[\frac{M_{satellite}}{M_{host}(< R_{peri})}\right]^{1/3} \times R_{peri} \quad \text{with } f \approx 2/3[1-\ln(1-e)]^{-1/3}$$

In cosmological simulations, many DM sub-halos get tidally disrupted.

- How important is it, e.g. in the Milky Way?
- The GC Pal 5 and the Sagittarius dwarf galaxy show that it happens



c) Violent relaxation

Mon. Not. R. astr. Soc. (1967) **136**, 101–121.

STATISTICAL MECHANICS OF VIOLENT RELAXATION IN STELLAR SYSTEMS

D. Lynden-Bell
(Communicated by the Astronomer Royal)

(Received 1966 December 19)

Summary

An explanation of the observed light distributions of elliptical galaxies is sought and found.

The violently changing gravitational field of a newly formed galaxy is effective in changing the statistics of stellar orbits.

Basic idea:

- (rapidly) time-varying potential changes energies of particles
- Different change for different particles

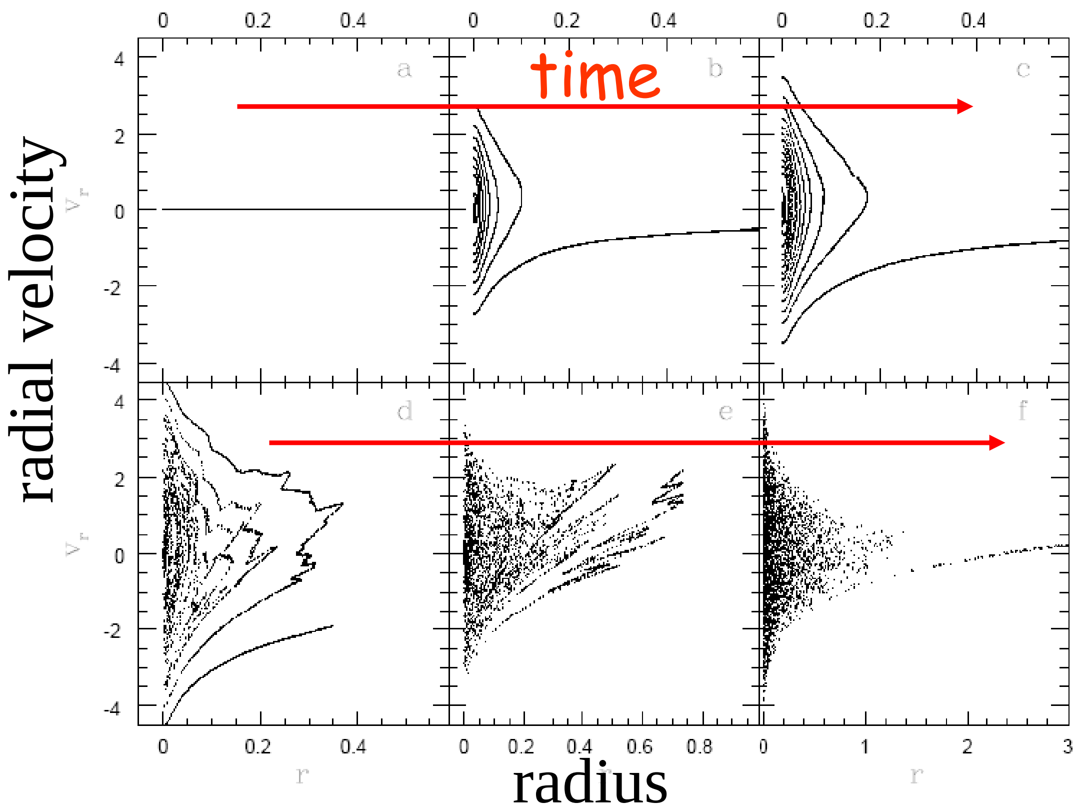
$E = \frac{1}{2}v^2 + \Phi$ and $\Phi = \Phi(\vec{x}, t)$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial E}{\partial \vec{v}} \cdot \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \frac{\partial E}{\partial \Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\partial \Phi}{\partial t}$$

The **time-scale** for violent relaxation is

$$t_{\rm vr} = \left\langle \frac{({\rm d}E/{\rm d}t)^2}{E^2} \right\rangle^{-1/2} = \left\langle \frac{(\partial\Phi/\partial t)^2}{E^2} \right\rangle^{-1/2} = \frac{3}{4}\langle \dot{\Phi}^2/\Phi^2 \rangle^{-1/2}$$

How violent relaxation works in practice (i.e. on a computer)



(from: Henriksen & Widrow 1997)

Collapse of a spherical system with $\rho_{\rm init} \propto r^{-3/2}$

Why are massive galaxies spheroids?

1. Stars form from dense, cold gas
   – either in disks
   – or from gas that is (violently) shock compressed
2. In the established cosmological paradigm larger (halos) form from the coalescence of smaller units

→ Stars in an (near) equilibrium system form from a disk and stay disk-like

→ 'Violent relaxation' shaking up stars (or stars formed during such an event) end up in spheroids

Is it plausible that in nearly all massive galaxies a (major) merger occurred after star-formation was largely complete?

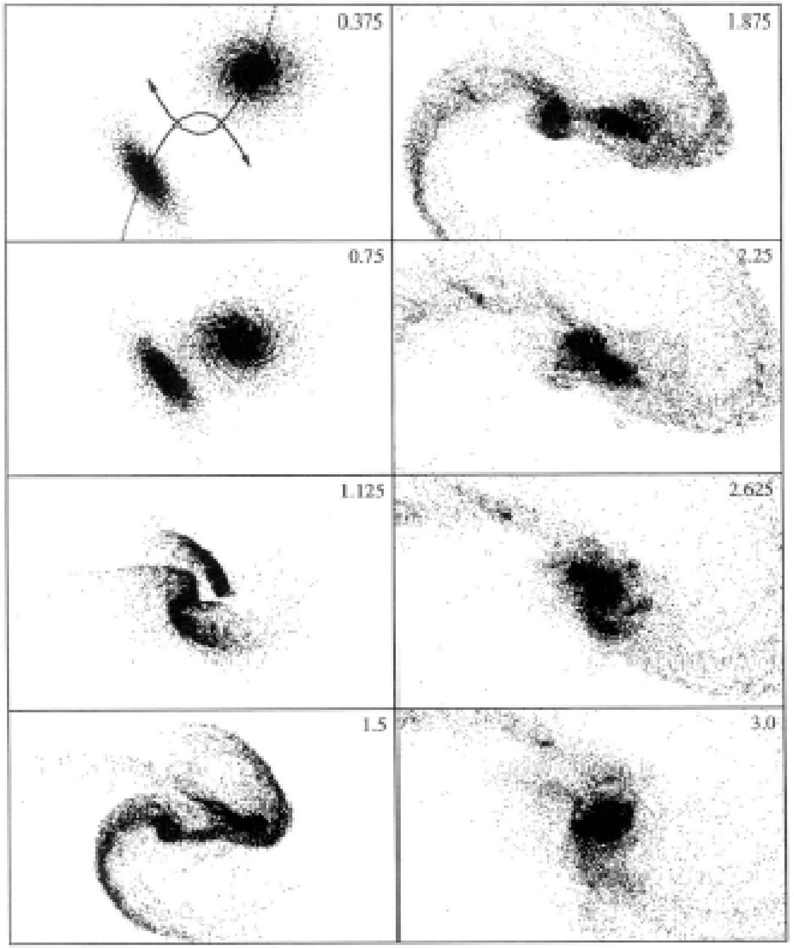


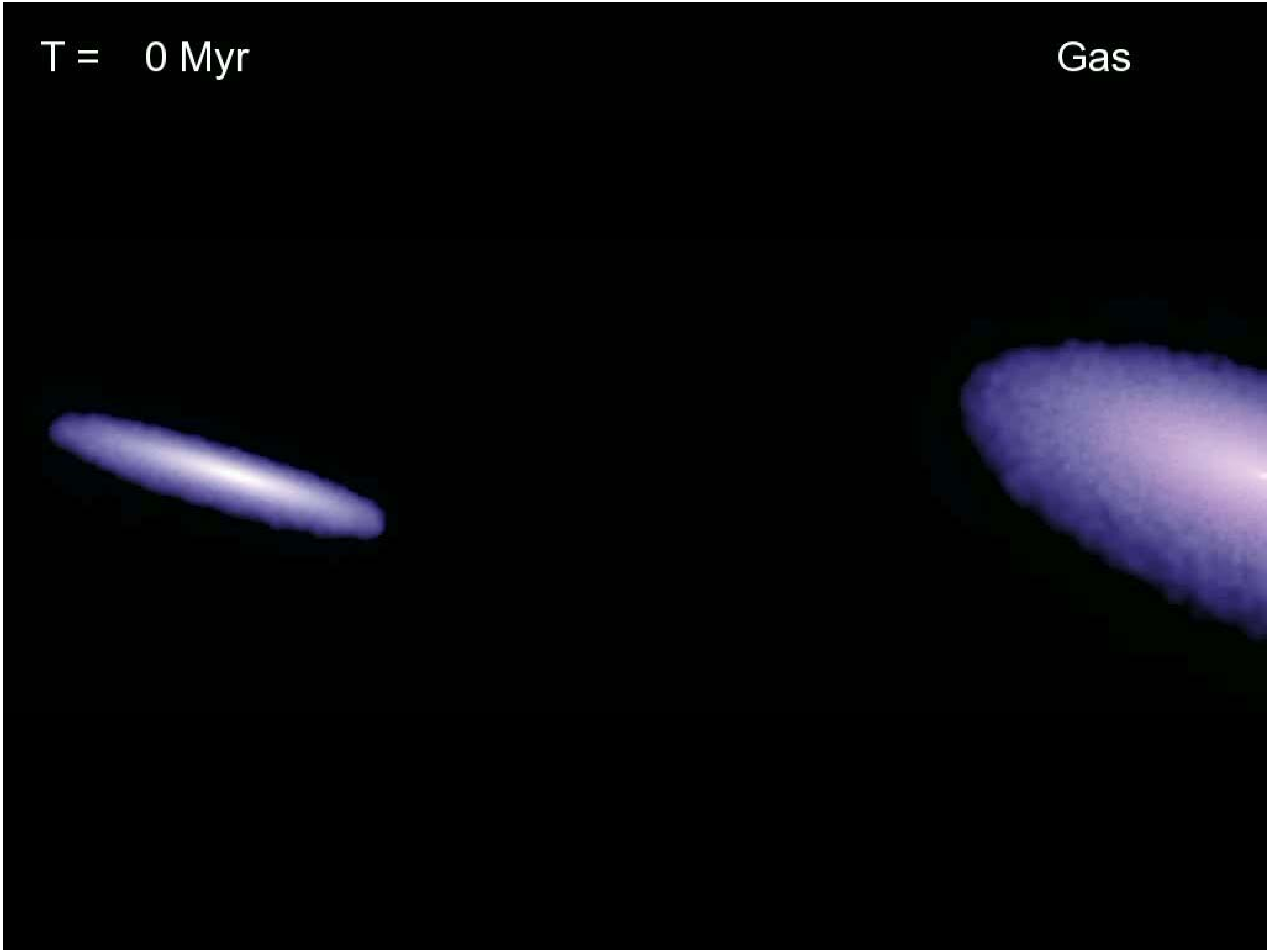
FIG. 4.—Evolution of the stellar distribution in encounter A, projected onto the orbital plane. The scale is the same as in Fig. 3.

Ultimately, dynamical friction and galaxy (or halo) merging are related:

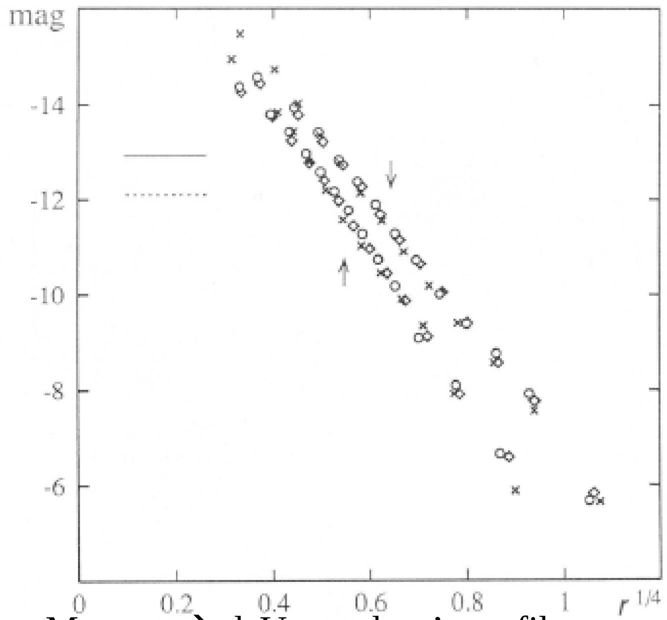
(heavy) bound part of one merger participant is transferring its orbital energy to the individual (light) particles of the other merger participant (and vice versa).

Issues:

1. Merging preserves ordering in binding energy (i.e. gradients)
2. Merging destroys disks – isotropizes
3. ‘Dry’ merging (i.e. no gas inflow) lowers (phase-space) density
4. Post-merger phase-mixing makes merger looks smooth in ~few t_{dyn}


T = 0 Myr
Gas

Some physics of mergers



Mergers → deVaucouleur's profile

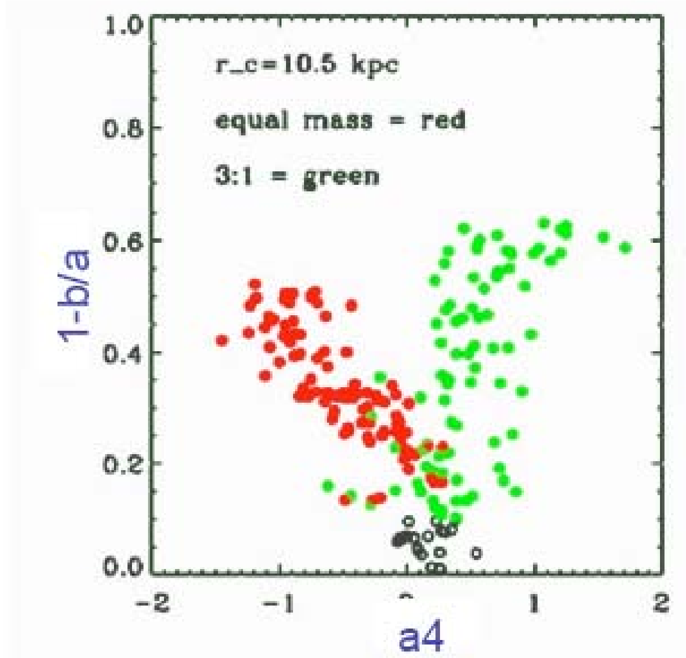
(Barnes 1989)

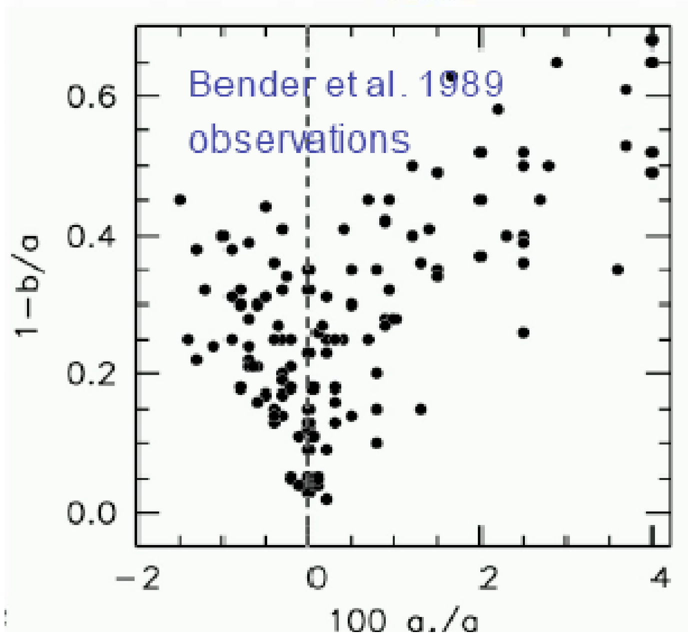
+ Some gas dissipation is needed to get the (central) densities of ellipticals 'right'

Merging moves objects 'within' the fundamental plane!

Isophote shapes

Naab&Burkert simulations





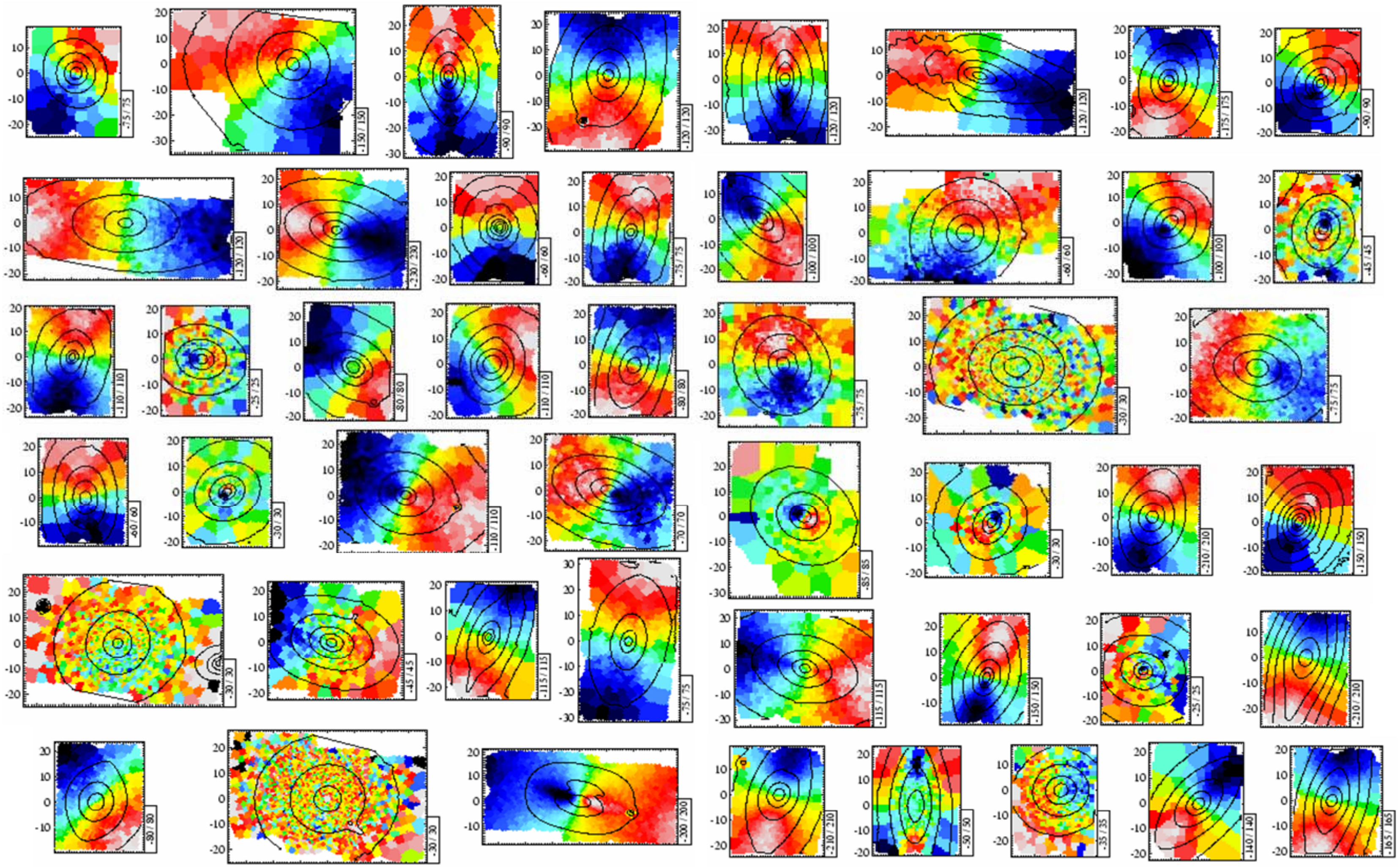
Dynamics Summary

- Collisionless stars/DM and (cold) gas have different dynamics
- "Dynamical modeling" of equilibria
  - Answering: In what potential **and** on what orbits to tracers move?
  - Two approaches: Jeans Equation vs. Orbit (Schwarzschild) modeling
  - N.B: 'kinematic tracers' need not cause the gravitational potential

    most modeling assumes random orbital phases; not true if $t_{orb} \sim t_{Hubble}$
- Phase-space density (e.g. in E,L,L_z coordinates) conserved in static or slowly varying potentials → dynamical archeology?
  - 'violent relaxation' may erase much of this memory
- (Cold) gas dynamics : dissipational, not collisionless, matter
  - Wants to form disks
  - In (strongly) non-axisymmetric potentials: shocks → inflow
  - No phase-space 'memory'

Let's look at spheroids

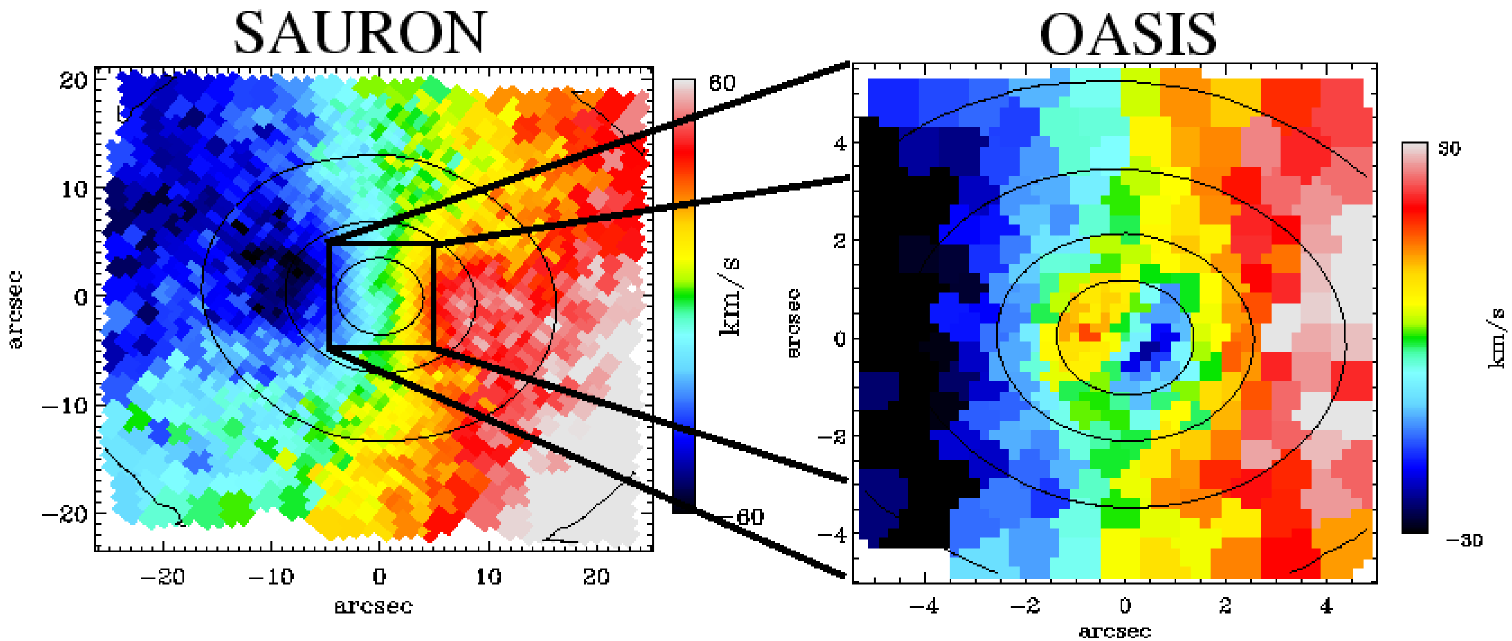
(data courtesy of the SAURON team
Co-PIs: de Zeeuw, Davies, Bacon)



Emsellem et al. 2003, MNRAS, submitted

SAURON versus OASIS

(total body vs. center)



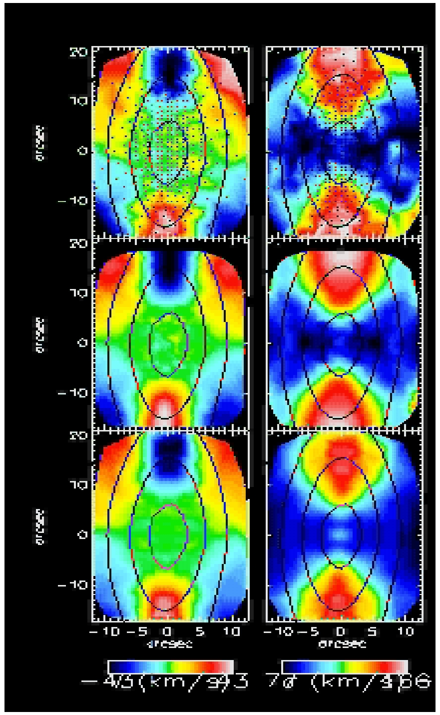
NGC 4382 McDermid et al. (2003) astro-ph/0311204

- Cores often have different (de-coupled) kinematics!

Intriguing Aside: NGC4550

a disk galaxy with ½ the stars going the wrong way?

(Rubin et al, Rix et al 1993)



2D-binned data

Symmetrized data

Axisymmetric model

M/L = 3.4 ± 0.2

V σ

- Axisymmetric dynamical model fits up to h5-h6
- M/L very accurate