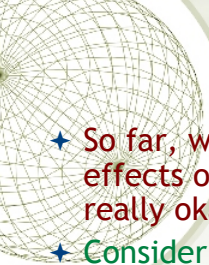


Lecture 10: General Relativity I

- ★ Recap: Special Relativity and the need for a more general theory
- ★ The strong equivalence principle
- ★ Gravitational time dilation
- ★ Curved space-time & Einstein's theory

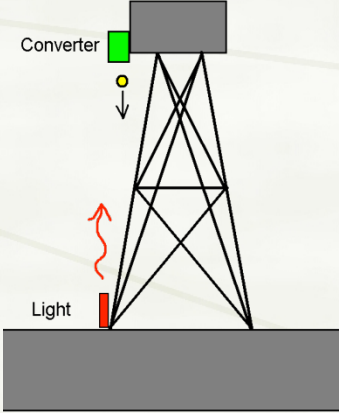
This week: read Chapter 8 of text

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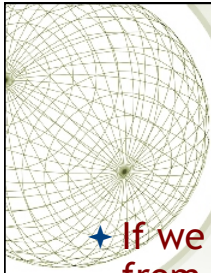


Motivation for General Relativity: Einstein's tower

- ★ So far, we have ignored the effects of gravity. Is this really okay??
- ★ Consider another thought experiment, to test whether **light can be affected by gravity**.
- ★ Consider a tower on Earth
 - ★ Shine a light ray from bottom to top
 - ★ When light gets to top, turn its energy into mass.
 - ★ Then drop mass to bottom of tower, in Earth's gravity field
 - ★ Then turn it back into energy



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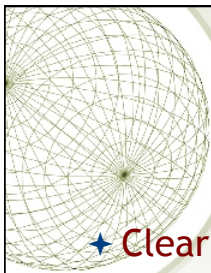


Perpetual motion?

- ✦ If we could do this, then we could get energy from nothing!
 - ✦ Original energy in light beam = E_{start}
 - ✦ Thus, mass created at top is $m = E_{\text{start}}/c^2$
 - ✦ Then drop mass... at bottom of tower it has picked up speed (and energy) due to the effects of gravitational field.
 - ✦ When we turn it back into energy, we have $E_{\text{end}} = E_{\text{start}} + E_{\text{grav}}$
 - ✦ But, we started off with only E_{start} - we have made energy! We're rich!

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Maxwell and gravity

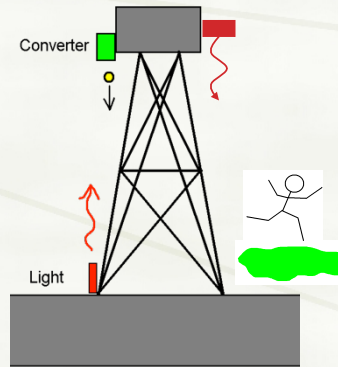
- ✦ Clearly, our assumption must be wrong...
 - ✦ light **must** be affected by gravity.
 - ✦ But gravity does not appear in Maxwell's equations, which govern light
 - ✦ Thus, Maxwell's equations are not exactly valid in the reference frame of Earth's surface, where there is gravity.
 - ✦ The Earth's surface must not be an inertial frame of reference!

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Resolving the tower problem

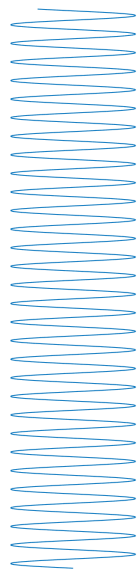
- ✦ Now consider light ray aimed from top to bottom of tower
- ✦ Free-falling (FF) observer sees light ray travel **unaffected** by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- ✦ From "Earth's" frame...
 - ✦ Free-falling (FF) observer is traveling faster and faster
 - ✦ Falling observer would see an increasing **redshift** of light source according to special relativity
 - ✦ If FF observer is **supposed** to see a constant frequency light beam, then light must get relatively **blueshifted** as it falls in gravitational field, to compensate
 - ✦ Light beam aimed upward must conversely be increasingly **redshifted** with height
 - ✦ Gravitational redshifting removes just the right amount of energy to solve the tower paradox!



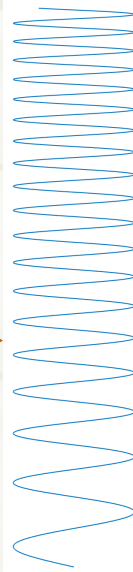
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The frequency of the photon looks like this as it moves down

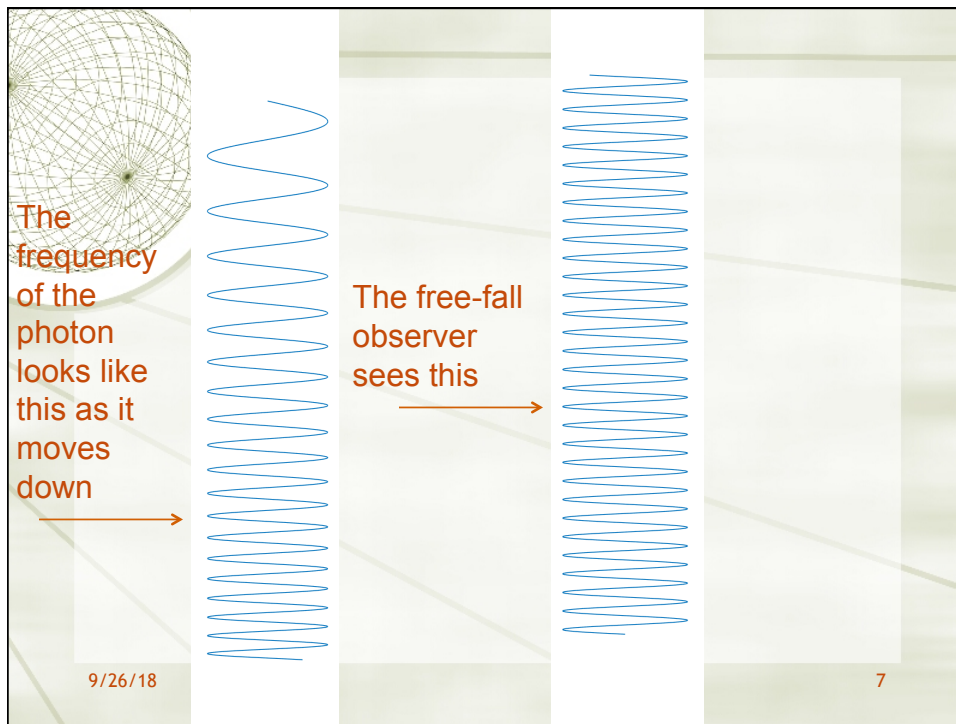


The free-fall observer sees this



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Resolving the tower problem

- Now consider light ray aimed from top to bottom of tower
- Free-falling (FF) observer sees light ray travel **unaffected** by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- From "Earth's" frame...
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First Newtonian mechanics, now his law of gravity?

- ★ Works fine in most cases, but
 - ✦ In whose frame do we measure r ?
 - ✦ Does the force depend on your reference frame?
 - ✦ Can gravity information travel (communicate) faster than c ?
If not, shouldn't there be some reference to c in the expression above?

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First Newtonian mechanics, now his law of gravity?

- ★ Works fine in most cases, but
 - ✦ In whose frame do we measure r ?
 - ✦ Does the force depend on your reference frame?
 - ✦ Can gravity information travel (communicate) faster than c ?
If not, shouldn't there be some reference to c in the expression above?
- ★ Maybe, like Newton's other laws, this is a low-speed or low-something limit to a more general theory.
- ★ Enter Einstein and a more general law of relativity than the special relativity we've been studying.

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RECAP

- ✦ Einstein's postulates for Special Relativity
 - ✦ Laws of physics look the same in any inertial frame of reference.
 - ✦ The speed of light is the same in any inertial frame of reference
- ✦ Strange consequences of Special Relativity
 - ✦ Time dilation and length contraction
 - ✦ Relativity of simultaneity and ordering of events
 - ✦ Equivalence and conversion of mass and energy
- ✦ Behavior of light in gravity field ("tower" experiment)
 - ✦ Energy of light must vary in a gravitational field to ensure that free-fall is an inertial frame

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*What makes free-fall an inertial frame?
Think back to the astronauts...*



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Apollo 10: Lots of gravity, but everything is falling together ¹²

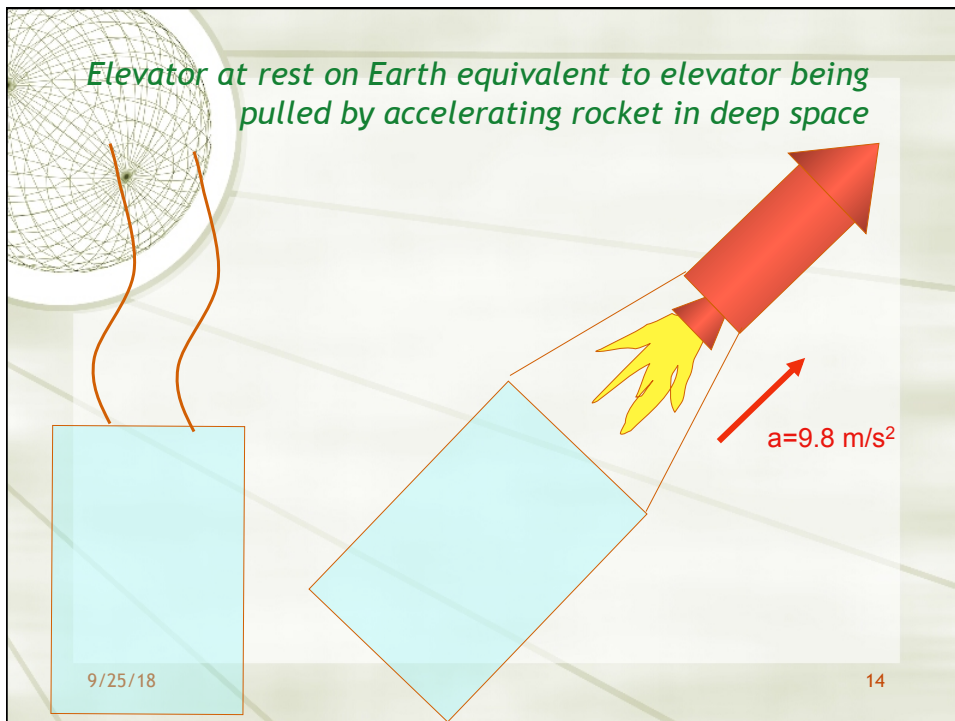
EQUIVALENCE PRINCIPLES

- ★ Recall the “weak” equivalence principle:
 - ★ All objects are observed to accelerate at the same rate in a given gravitational field.
 - ★ Therefore, the inertial and gravitational masses must be the same for any object.
 - ★ This has been verified experimentally, with fractional difference in masses $<10^{-13}$
- ★ As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot locally be distinguished

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Elevator at rest on Earth equivalent to elevator being pulled by accelerating rocket in deep space



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THE STRONG EQUIVALENCE PRINCIPLE

- ★ Einstein introduced the **strong equivalence principle** - when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- ★ More generally, all inertial and freely-falling reference frames are equivalent, and there is no (local) experiment that can distinguish them
- ★ What does this mean???

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Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space

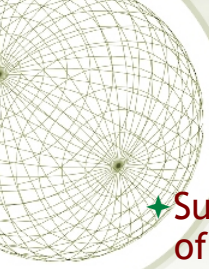


$a=9.8 \text{ m/s}^2$



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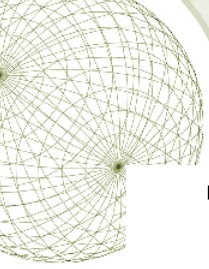
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What about gravity?

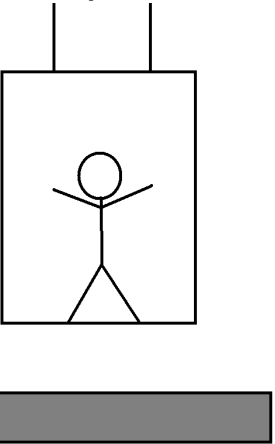
- ✦ Suppose that you decide that your frame of reference is not inertial...
 - ✦ Free bodies (no apparent forces acting) change velocity
 - ✦ Is it because of gravity or is the frame accelerating?
 - ✦ Einstein says that you cannot tell the difference!
 - ✦ Gravity is a “fictitious force” - i.e., a force which appears to exist because we are living in a non-inertial frame of reference.

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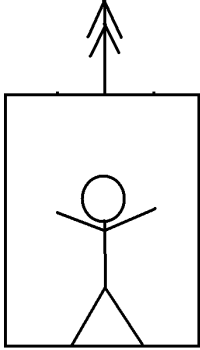


Already seen this for massive bodies...

Elevator in gravitational field



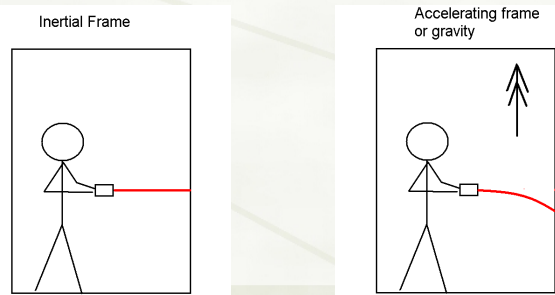
Elevator accelerating upwards



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What about light? It “falls”, too!

- ✦ Astronaut in inertial frame with flashlight
 - ✦ Inertial frame, so light goes in straight lines
 - ✦ It doesn't matter whether this is free fall or far from masses
- ✦ What if we now put flashlight in a gravitational field (accelerated frame)?
 - ✦ Light beam will bend: it must accelerate at the same rate and direction as the elevator
 - ✦ Strong equivalence principle \Rightarrow frame with gravity acts the same
 - ✦ Important conclusion - light “falls” due to gravity!



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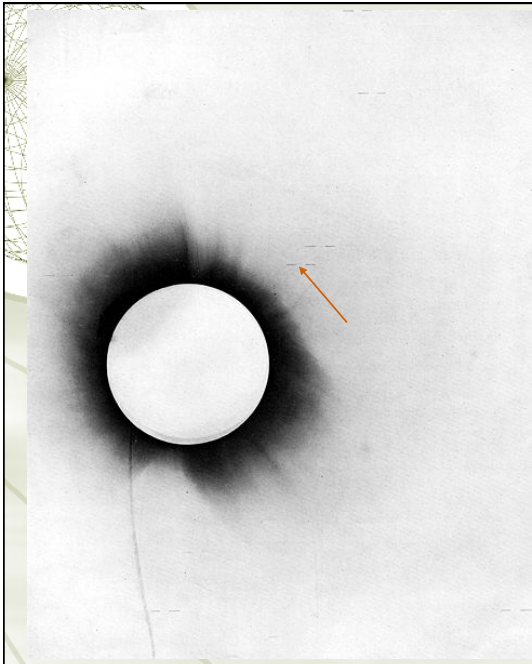
The Eddington Test

- ✦ 1919 - the first “accessible” total Solar eclipse since Einstein postulated SEP
- ✦ Arthur Eddington
 - ✦ Famous British Astronomer
 - ✦ Lead expedition to South Africa to observe eclipse
 - ✦ Was looking for effects of gravitational light bending by searching for shifts in positions of stars just next to the Sun.
 - ✦ He found them, exactly as predicted!



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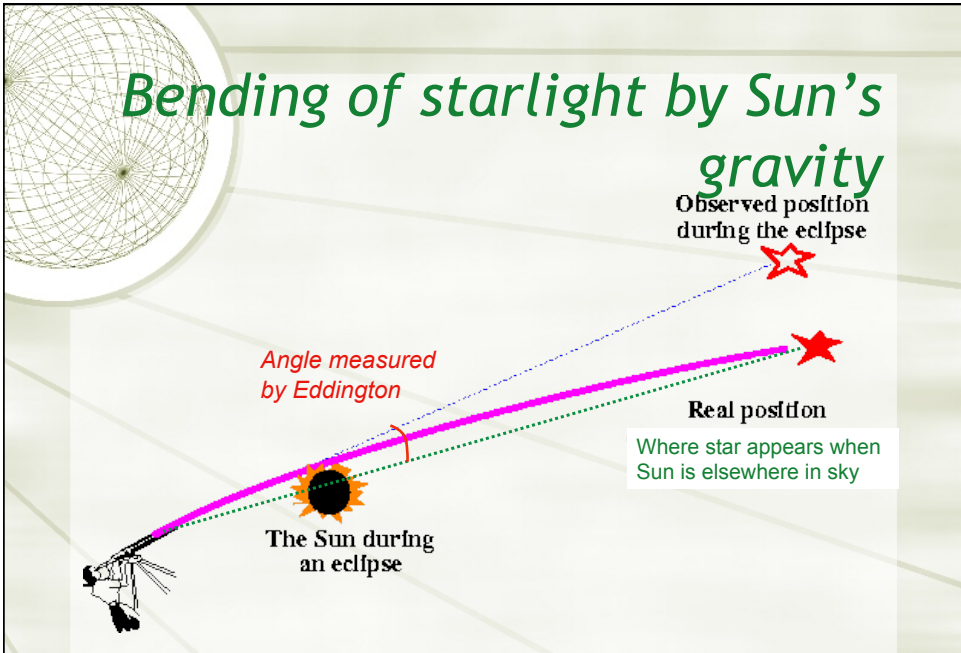
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Sobral plate

1919 Sobral plate, from 1920 article by Dyson, Eddington, & Davidson, with measured stars indicated by lines

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Bending of starlight by Sun's gravity

Observed position during the eclipse

Angle measured by Eddington

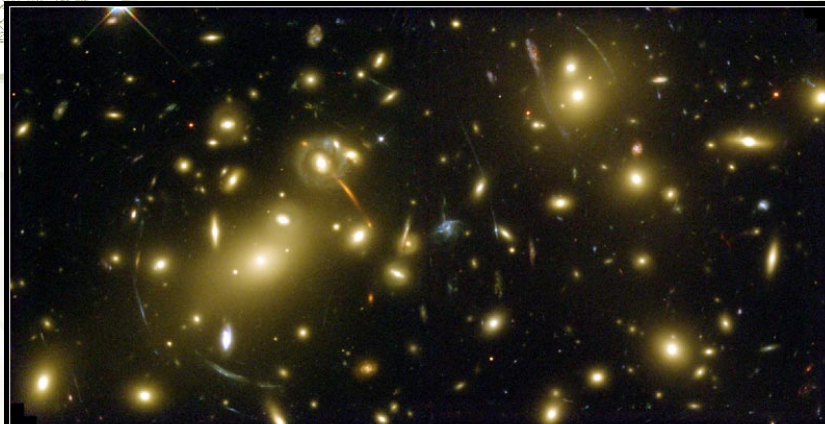
The Sun during an eclipse

Real position

Where star appears when Sun is elsewhere in sky

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Also have light bending by distant galaxy clusters: “giant lenses” in the sky



Galaxy Cluster Abell 2218

HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

GRAVITATIONAL TIME DILATION

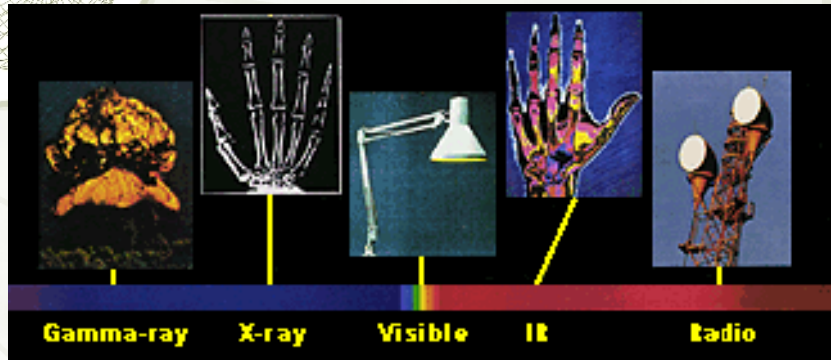
- ★ Recall properties of waves (chapter 4)
- ★ Waves are characterized by
 - ★ Wavelength (λ) = distance between crests
 - ★ Frequency (ν or f) = number of crests passing a given point per second
- ★ Speed of a crest is $c = \lambda \nu$
- ★ Energy of a wave is proportional to frequency, $E = h\nu$



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The electromagnetic spectrum



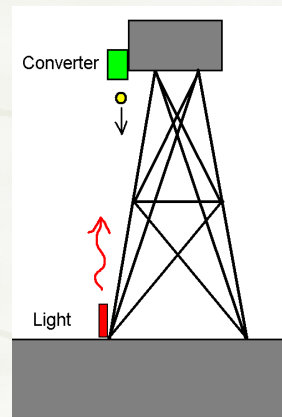
Small wavelength
High Frequency
High energy

Large wavelength
Low frequency
Low energy²⁵

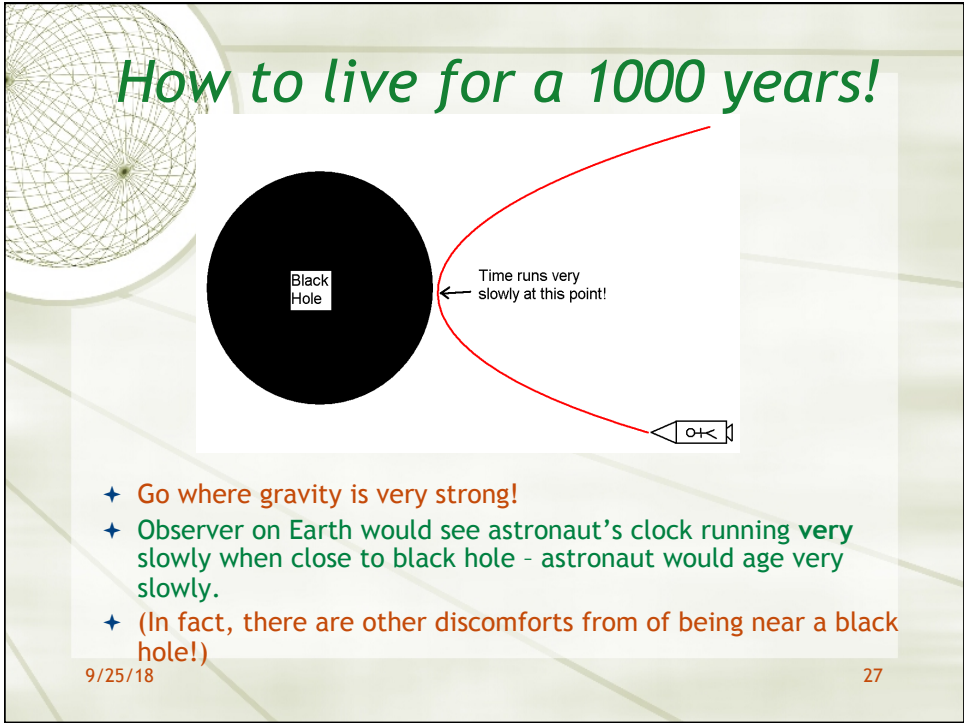
Remember the tower...

Light beam must lose energy as it climbs up

- ✦ So...frequency must decrease
- ✦ i.e., light is redshifted.
- ✦ Gravitational redshifting
- ✦ Imagine a clock based on frequency of laser light...
 - ✦ 1 "tick" = time taken for fixed number of crests to pass
 - ✦ Gravitational redshifting slows down the clock.
 - ✦ **Clocks in gravitational fields must run slowly**



How to live for a 1000 years!



A diagram illustrating time dilation near a black hole. A black circle labeled "Black Hole" is shown. A red arrow curves from the black hole towards the right, with an arrowhead pointing to a clock icon. Text next to the arrow reads "Time runs very slowly at this point!".

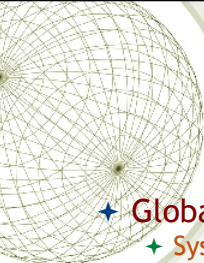
- ✦ Go where gravity is very strong!
- ✦ Observer on Earth would see astronaut's clock running very slowly when close to black hole - astronaut would age very slowly.
- ✦ (In fact, there are other discomforts from of being near a black hole!)

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A movie poster for the film "Interstellar". At the top, the names of the main cast members are listed: MATTHEW MCCONALGHEY, ANNE HATHAWAY, JESSICA CHASTAIN, and MICHAEL FASSBENDER. The central image shows an astronaut in a white space suit standing on a rocky, icy planet. Below the image, the text reads: "THE END OF EARTH WILL NOT BE THE END OF US. A FILM BY CHRISTOPHER NOLAN. INTERSTELLAR. IN THEATRES AND IMAX NOVEMBER 7." The bottom of the poster includes production credits and the website "http://interstellarmovie.com".

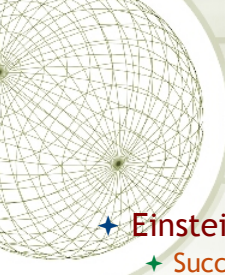
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Gravitational time dilation has practical importance!

- ★ **Global Positioning System (GPS)**
 - ★ System of satellites that emit timing signals
 - ★ Detector on Earth receives signals
 - ★ Can figure out position on Earth's surface by measuring time delay between signals from different satellite (light travel time gives distance to satellite)
 - ★ Need to measure time of signal from satellite very well!
 - ★ 10m positioning requires ~30ns time accuracy
 - ★ Satellites are at varying heights; clocks run at varying rates
 - ★ Satellite clocks drift by ~38us per day wrt Earth clocks!
- ★ **If GR effects were not included, computed GPS positions would drift from true position by kilometers per day!**

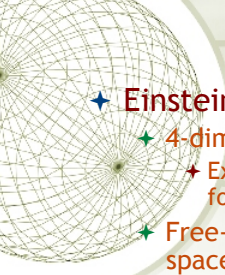
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CURVED SPACE-TIME

- ★ Einstein pondered several things...
 - ★ Success of Special Relativity showed that space & time were closely interlinked
 - ★ The “tower thought experiment” suggested that free-fall observers are (locally) free of effects of gravity: frequency of light they observe does not change as they free-fall (accelerate down)
 - ★ He wanted to say that gravity was an illusion caused by the fact that we live in an accelerating frame...
 - ★ ... but there is no *single* accelerating frame that works! Somehow, you need to stick together frames of reference that are accelerating in different directions

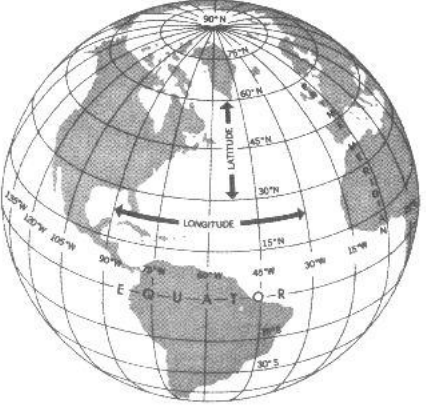
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- ★ Einstein’s proposal
 - ★ 4-dimensional space-time is “curved,” not flat
 - ★ Example: surface of sphere is curved 2D space; surface of football field is flat 2D space
 - ★ Free-falling objects move on **geodesics** through curved space-time
 - ★ The curvature (bending) of space-time is produced by matter and energy
- ★ What is a geodesic?
 - ★ The generalization of a straight line in flat space to curved space
 - ★ It is the shortest path between two points on a surface; for instance, the path flown by an aircraft between cities on the globe
 - ★ Unlike straight lines in flat space, geodesics that start as parallel can converge or diverge (or even cross)

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On Globe...



Meridian of LONGITUDE

Degrees of latitude are described as north or south of the equator (from 0° to 90°). Longitude is measured east or west of the prime meridian at Greenwich (from 0° to 180°).

Toronto, Canada
43° 40' N
79° 20' W

Equator

Parallel of LATITUDE

Cape Town, S. Africa
33° 9' S
18° 30' E

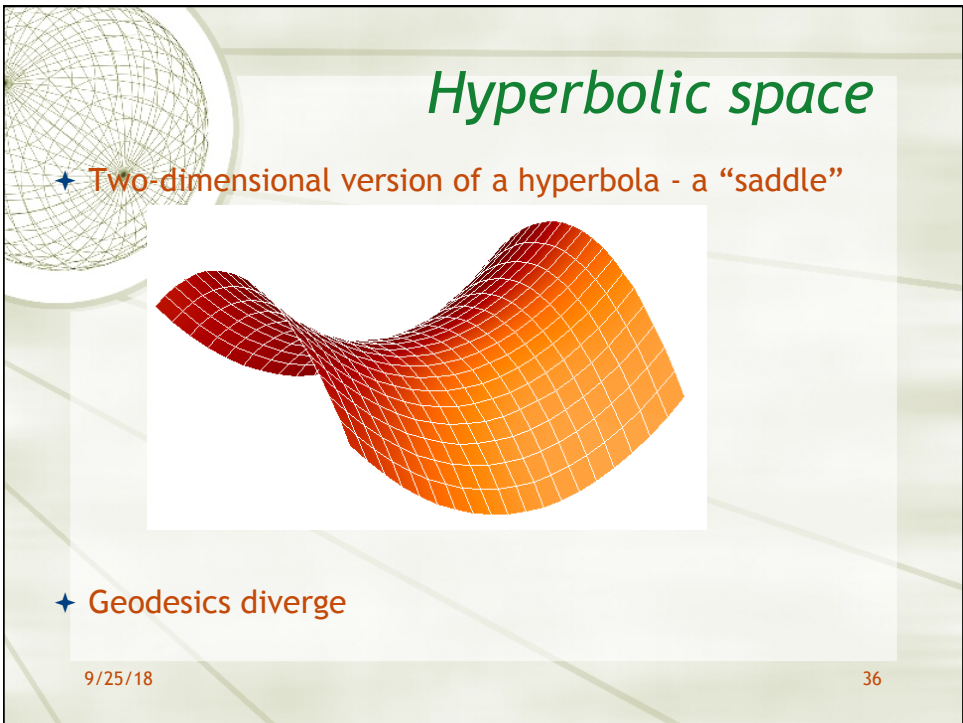
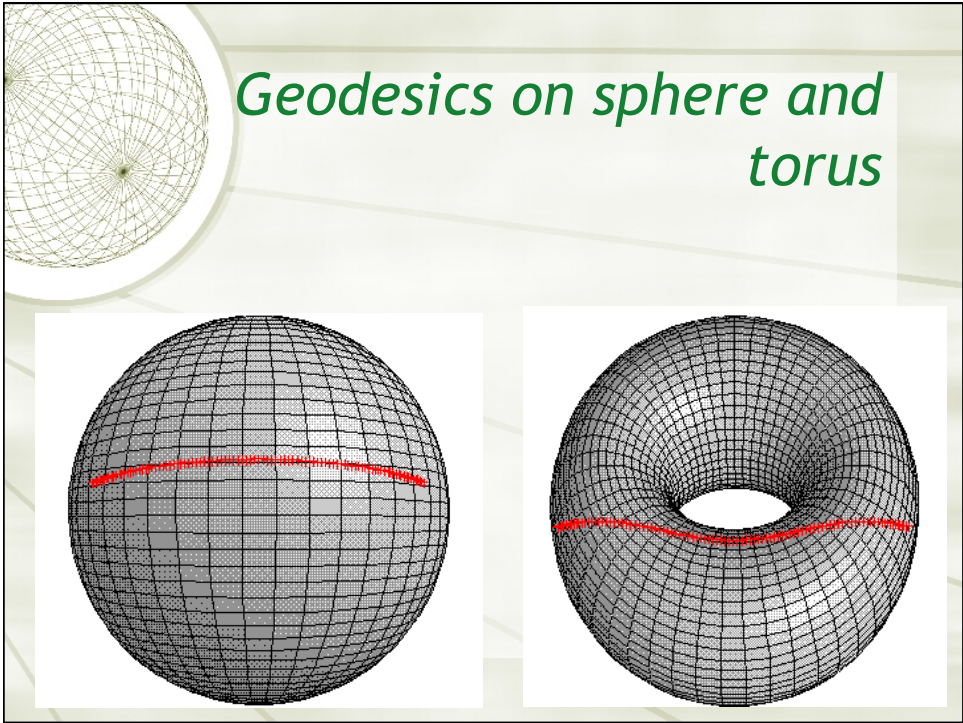
- ✦ Constant-longitude lines (meridians) are geodesics
- ✦ On the Earth, geodesics are Great Circles, the shortest distance between two points on the surface.
- ✦ Constant-latitude lines (parallels) are not geodesics

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Shortest flight paths are geodesics

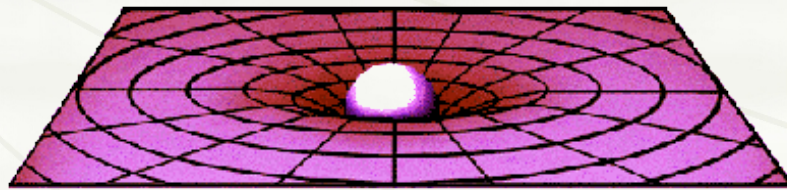



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How does matter “warp” space?

- ✦ Use two-dimensional space as an analogy: think of how rubber sheet is affected by weights
- ✦ Any weight causes sheet to sag locally
- ✦ Amount that sheet sags depends on how heavy weight is



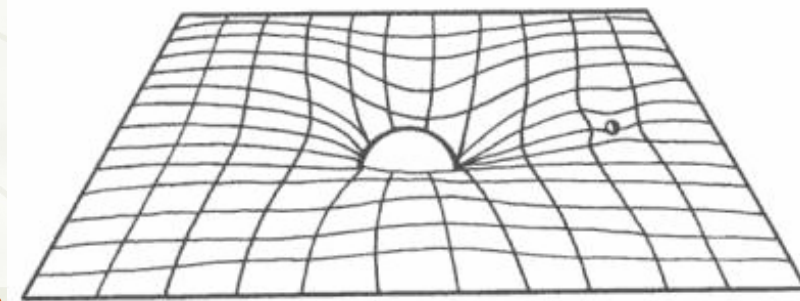
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From web site of UCSD

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Effect of matter on coordinates

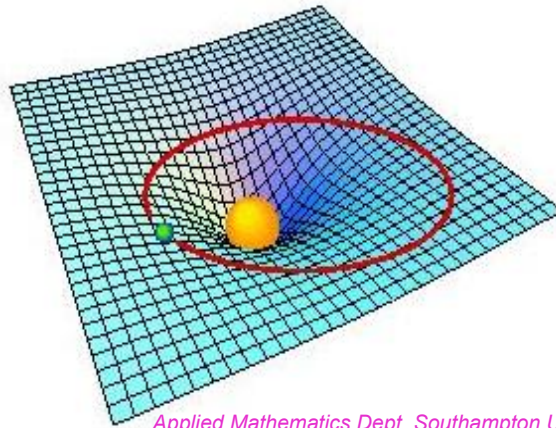
- ✦ Lines that would be straight become curved (to external observer) when sheet is “weighted”



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How are orbits affected?

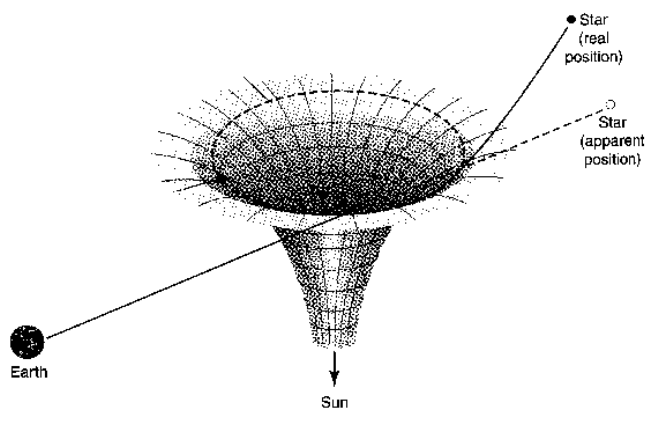
- ✦ Marble would follow straight line if weight were not there
- ✦ Marble's orbit becomes curved path because weight warps space



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Applied Mathematics Dept, Southampton University

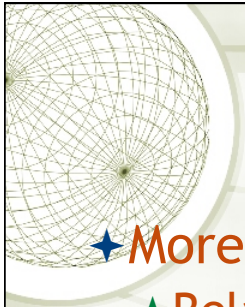
Warping of space by Sun's gravity



- ✦ Light rays follow geodesics in warped space

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Next time...

- ★ **More on General Relativity**
 - ◆ **Relativity Equations**
 - ◆ **Consequences of GR**

Read Chapter 8 of the book

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