Lecture 4: Newton's Laws

- Galileo (cont)
- Newton
- Laws of motion

This week: read Chapter 3 of text

Impact of Galileo's observations

- Chipping away at Aristotelian point of view:
  - Features on Sun, Moon, Saturn indicated they are not "perfect" orbs
  - Faint stars resolved in Milky Way indicates stars at many distances -- not just single sphere
  - Moons of Jupiter showed that Earth was not sole center of motion
- Crucial experiment ruling out Ptolemaic model:
  - Possible phases of Venus in Ptolemaic model are only crescent or new -- but Galileo *observed full phase*
  - Observation supported Copernican (or Tycho's) model (Venus on far side of Sun when full)
- As a result of his observations, Galileo became ardent supporter of Copernican viewpoint
- In 1632, published *Dialogue Concerning the Two Chief Systems of the World - Ptolemaic and Copernican*
- The Inquisition banned the book; Galileo was found guilty of heresy in supporting Copernican view, and sentenced to house arrest

Museum of Science History in Florence

Galileo's lens, Florence

Museo di Storia della Scienza, Florence

Galilean physics

- After 1633 trial, Galileo returned to work on physics of mechanics
- Published *Discourses and mathematical demonstrations concerning the two new sciences (1642)*
- Made experiments with inclined planes; concluded that distance d traveled under uniform acceleration a is $d=a\ t^2$
- Used "thought experiments" to conclude that all bodies, regardless of mass, fall at the same rate in a vacuum --contrary to Aristotle
  - Now known as "equivalence principle"
- Realized full principle of inertia:
  - body at rest remains at rest;
  - body in motion remains in motion (force *not* required)
- Realized principle of relative motion ("Galilean invariance"):
  - If everything is moving together at constant velocity, there can be no apparent difference from case when everything is at rest.
  - Ball dropped from top of moving ship's mast hits near bottom of mast, not behind on deck.

Experiments with Inclined plane

Tomb of Galileo in Santa Croce

Quiz: Write your name and answer in the card

- In the first lecture we talked about the characteristics of a scientific theory. This lecture we pointed out a fundamental problem (in terms of modern scientific thinking) with Ptolemy's system of epicycles to explain planetary motions.
  - What was it?

Isaac Newton in 1689, by Sir Godfrey Kneller.

Isaac Newton (1643-1727)

Father of modern physics and cosmology

- Attended Cambridge University, originally intending to study law, but reading Kepler, Galileo, Descartes
- Began to study mathematics in 1663
- While Cambridge was closed due to plague (1665-1667), Newton went home and
  - began to work out foundations of calculus
  - realized (contrary to Aristotle) that white light is not a single entity, but composed of many colors
  - began to formulate laws of motion and law of gravity
- Became professor of mathematics starting in 1669 (age 27!)
- Worked in optics, publishing "Opticks" (1704)
  - invented reflecting telescope
  - showed color spectrum from prism recombines into white light with a second prism
  - analyzed diffraction phenomenon: light is a wave

Newton's history, cont.

- In 1687, published *Philosophiae naturalis principia mathematica,* or *"Principia"*
  - publication was prompted (and paid for) by Halley
  - partly in response to claim by Hooke that he could prove gravity obeyed inverse-square law
  - included proof that inverse square law produces ellipses
  - generalized Sun's gravity law to universe law of gravitation: *all matter attracts all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them*
  - many other applications, including tides, precession, etc.
  - laid out general physics of mechanics -- laws of motion
  - showed that Kepler's laws follow from more fundamental laws
- The *Principia* is recognized as the greatest scientific book ever written!
- Retired from research in 1693, becoming active in politics and government

I: LAWS OF MOTION

Newton's first law

Newton's first law (N1) - If a body is not acted upon by any forces, then its velocity, *v*, remains constant

- N1 sweeps away the idea of "being at rest" as a natural state
- N1 includes special case with $v = 0$, i.e. a body at rest remains at rest if $F = 0$, as part of a more general law

Newton's second law

Newton's 2nd law (N2) - If a body of mass *M* is acted upon by a force *F*, then its acceleration *a* is given by $F = Ma$

- N2 defines "inertial mass" as the degree by which a body resists being accelerated by a force.
- Since momentum $p = mv$ and acceleration a = rate of change in v,

  ma = rate of change in $(m v)$
- Thus, another way of saying N2 is that force = rate of change of momentum
- Alternate form of N2 is more general, since it includes case when mass is changing

Newton's third law

<u>Newton's 3rd law (N3)</u> - If body A exerts force $F_{A \to B} = f$ on body B, then body B exerts a force $F_{B \to A} = -f$ on body A.

- N3 is often phrased in terms of "equal" (in magnitude) and "opposite" (in direction) forces
- From N3, the *total force* on a closed system is *0*, i.e.

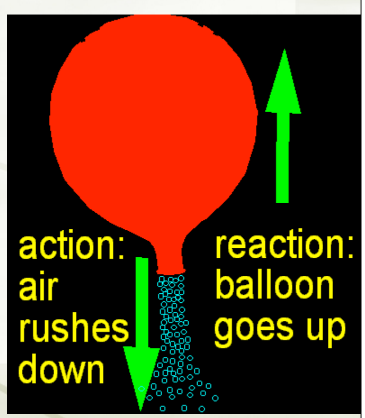
  $F_{tot} = F_{A \to B} + F_{B \to A} = f + (-f) = 0$
- Combining with N2, this implies that the total momentum of a closed system is *conserved* [does not change] if there are no external forces, i.e.

  $F_{tot} = 0 \Rightarrow$ (*rate of change of* p_{tot}) = 0 $\Rightarrow p_{tot}$ = constant
- Any momentum change of one part of a closed system is compensated for by a momentum change in another part, i.e.

  (*rate of change of* p_A)= - (*rate of change of* p_B)

Third Law

- **If a body A exerts a force F on body B, then body B exerts a force -F on body A**
- Notes
  - This is the law of "equal and opposite reaction"
  - We will see later that this law is closely tied to conservation of momentum



What pushes on what?

"Professor Goddard does not know the relation between action and reaction and the needs to have something better than a vacuum against, which to react. He seems to lack the basic knowledge ladled out daily in high schools."…

-1921 New York Times editorial on Robert Goddard's proposal that rockets could reach Moon

Blast-off!

- Rockets push against *ejecta, not air*

Apollo 11:
Launched: 16 July 1969 UT 13:32:00 (09:32:00 a.m. EDT)
Landed on Moon: 20 July 1969 UT 20:17:40 (04:17:40 p.m. EDT)

An illustration of Newton's laws

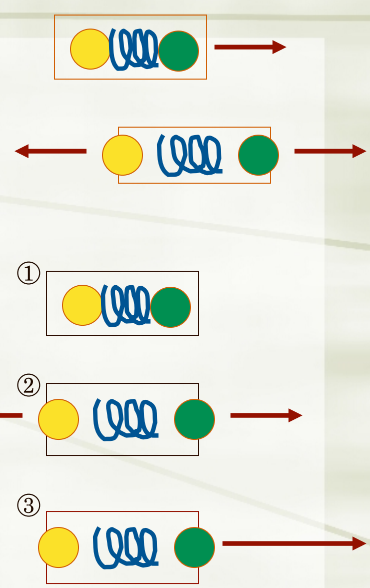
- We can see that aspects of Newton's laws arise from more fundamental considerations.
- Consider two equal masses M at rest. Initial momentum is p = 0. Masses are suddenly pushed apart by a spring… and will move apart with the same speed *V in opposite directions* (by symmetry of space!). Total momentum is p = MV-MV = 0. <u>Total</u> momentum is unchanged.

Before: $v_A = v_B = 0 \Rightarrow p_{tot} = 0$

After: $v_A = -V$, $v_B = V \Rightarrow$
$p_{tot} = Mv_A + M v_B = -MV + MV = 0$

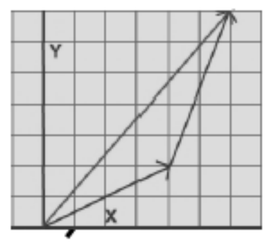
- Same situation, but masses are now both initially moving at velocity *V*. Initial momentum is $p_{tot} = 2MV$.
- Can turn into the previous situation by "moving along with them at velocity V".

1. Change of perspective [subtract V from all velocities] brings masses to rest…
2. Do same problem as before…
3. Change back to original perspective [add V to all velocities] …
4. Final velocity of one ball is *2V*; final velocity of other ball is *0*. Final total momentum is $p_{tot} = 2MV$. No change in total momentum.



Adding Vectors

- Just a reminder that vectors have direction and length and adding them is not just adding the length



Galilean relativity

- Problem in the second case was solved by "changing your frame of reference"
- The "velocity addition" rule when the reference frame changes is called a **Galilean transformation.**
- We've assumed that, after changing our reference frame and using a Galilean transformation, the laws of physics are the same. This principle is called **Galilean Relativity**.
- In either case: total momentum before = total momentum after
- Key idea: there is no absolute standard of rest in the Universe; the appearance of rest is always relative

Force and acceleration

- Forces between two bodies are equal in magnitude, but the observed reaction --the acceleration --depends on mass ($F = ma$)
- If a bowling ball and a ping-pong ball are pushed apart by a spring, the bowling ball will move very little and the ping-pong ball will move a lot

- Forces in a collision are equal in magnitude, too

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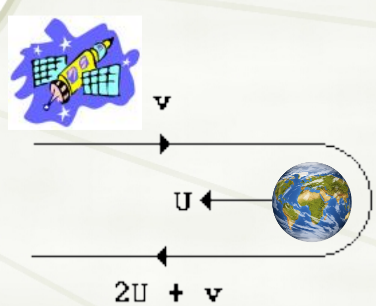
Exercise: Galilean invariance

Now: $M_A = 2$ kg, $M_B = 4$ kg

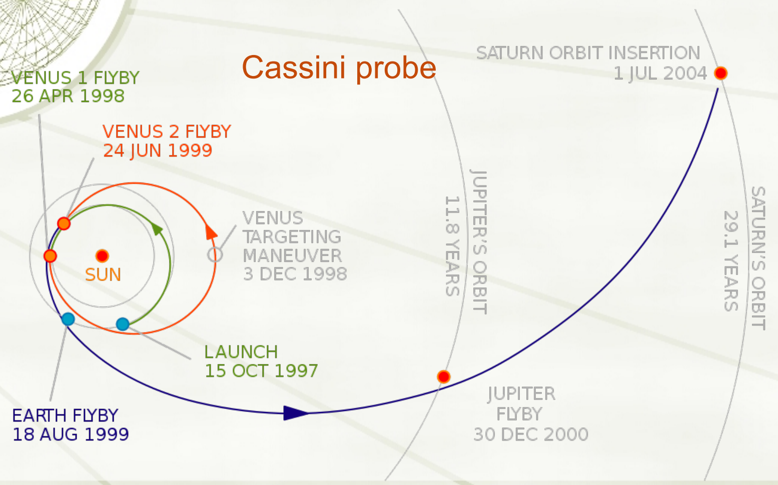
1. Start with $v_A = 0 = v_B$
   - After spring is released, $v_B = 5$ m/s
   - What is v_A? (*apply conservation of momentum*)
2. Start with $v_A = 3$ m/s $= v_B$
   - What are speeds of A and B after spring is released? (*use #1 and apply Galilean invariance*)
3. Start with $v_A = V = v_B$
   - After spring is released, $v_A = 0$.
   - What was initial V?

Gravitational slingshot



Gravity assists, commonly used to speed up interplanetary probes, are just another application of conservation of momentum

Gravitational slingshot

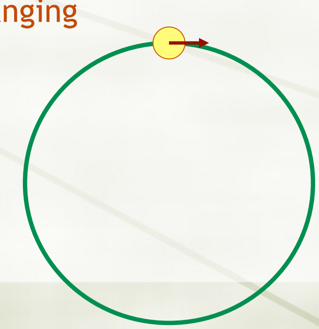


Relation of Newton's laws to symmetry and conservation principles

- N1 with v ≠ 0 comes directly from Aristotle's concept (object at rest remains at rest) by applying Galilean Relativity: change to frame with initial *v* = *0*; F = 0 so object remains at rest; change frames back and *v* = *initial v*
- N3 is exactly what's needed to make sure that the total momentum is conserved.
- So… Newton's laws are related to the symmetry of space and the way that different frames of reference relate to each other.

Speed and velocity

- **Velocity**, as used in Newton's laws, includes both a **speed** and a **direction**. ***V*** and also ***F*** and ***a*** are ***vectors***.
- Any ***change in direction***, even if the speed is constant, ***requires a force***
- In particular, motion at constant speed in a circle must involve a force at all times, since the direction is always changing



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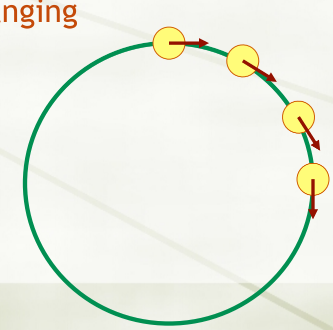

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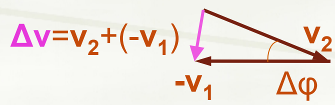
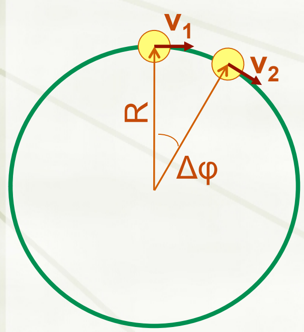
Acceleration in a circular trajectory





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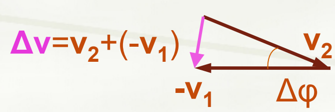
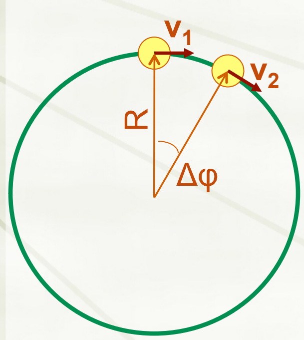


For small φ, $\Delta v \cong v\,\Delta\varphi$



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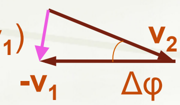
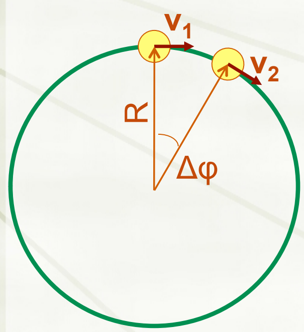
Acceleration in a circular trajectory

$\Delta \mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$





For small φ, Δv≅v Δφ

For constant speed v, the time it takes to go around is T = perimeter/v = 2πR/v

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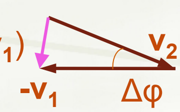

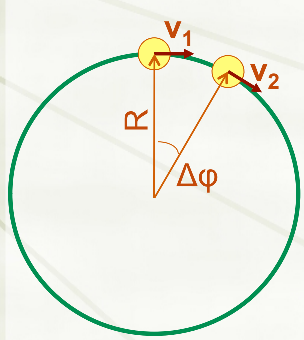

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Δφ = Δt 2π/T = Δt v/R

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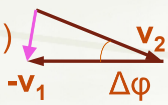

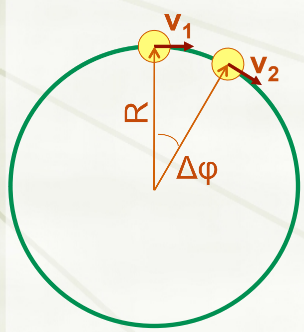

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For constant speed v, the time it takes to go around is T = perimeter/v = $2\pi R/v$

$\Delta\varphi = \Delta t\, 2\pi/T = \Delta t\, v/R$

So $\Delta v/\Delta t = v\, \Delta\varphi/\Delta t = v\, v/R$

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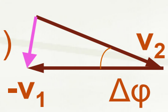

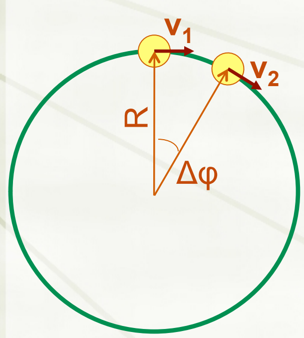

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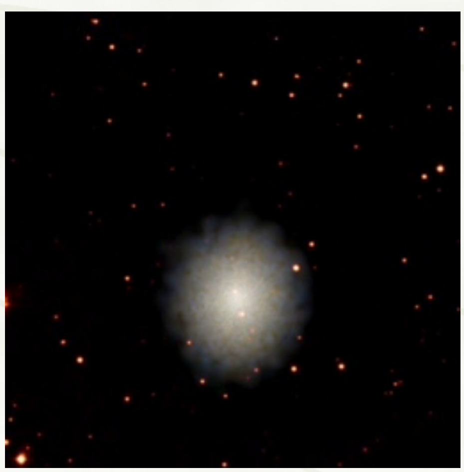
So $\Delta v/\Delta t = v\, \Delta\varphi/\Delta t = v\, v/R$

$$a = \Delta v/\Delta t = v^2/R$$

Newton's contribution in perspective

- Newton's theory of motion and gravity brought down the wall between the Earth and the Heavens built by Aristotle
  - The same phenomena happen there and here (Galileo)
  - They obey the same set of physical laws (Newton)
  - The Universe is knowledgeable!

Galaxy collision: Newton at work



Next time…

- More Newton, including gravity & orbits
- Age of the Earth
- Reference frames & fictitious fources

- HW #1 due on Sep 13th!
- *Read Chapter 3 of text*