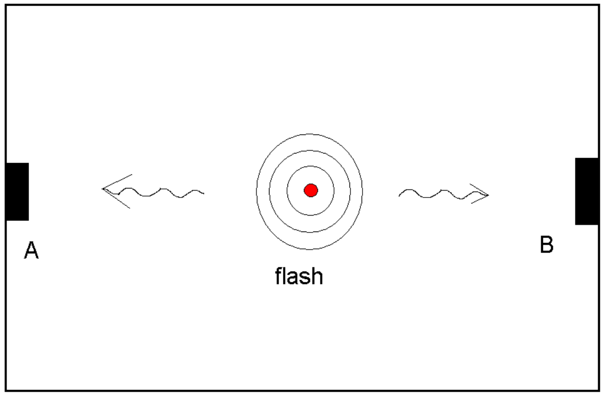
Lecture 8: *Special Relativity II*

- Simultaneity and causality
- Space-time diagrams
- Reciprocity and the twins paradox

Please continue reading Ch. 7 of text

I: SIMULTANEITY

- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



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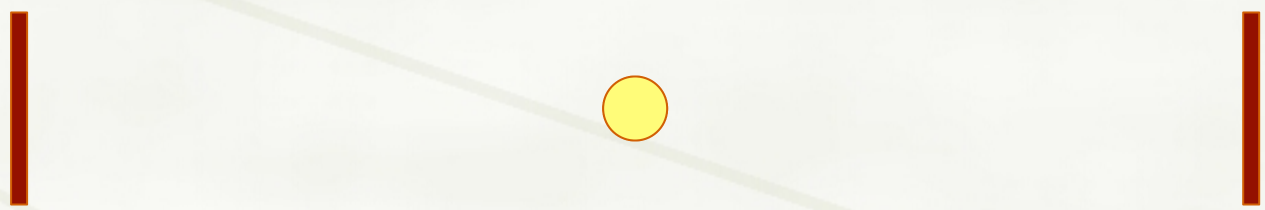

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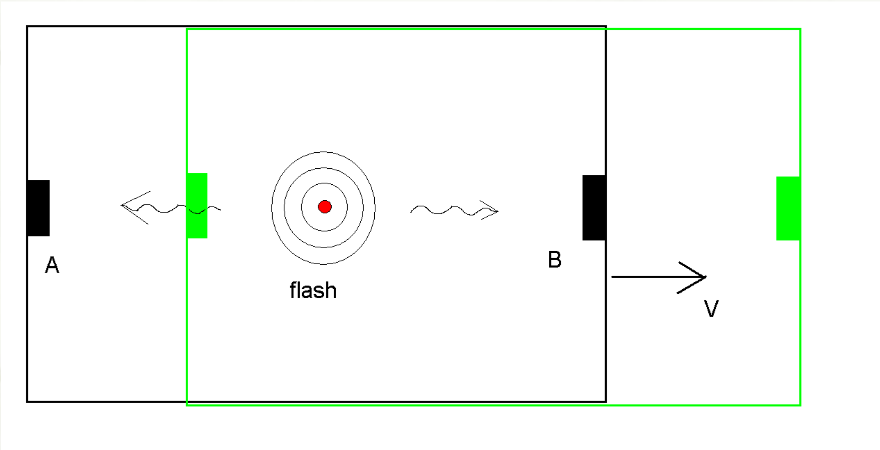
Change frames…

- Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.



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- Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

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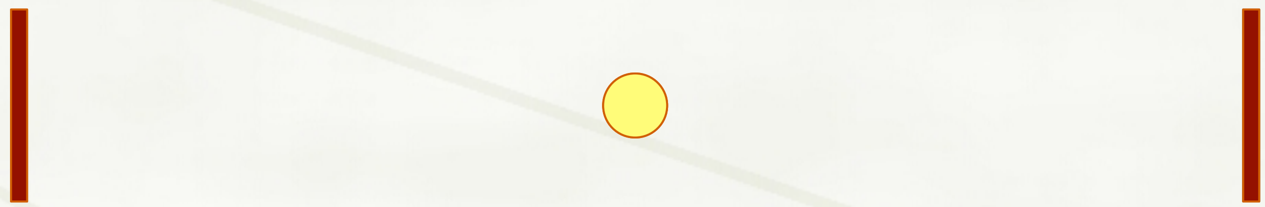

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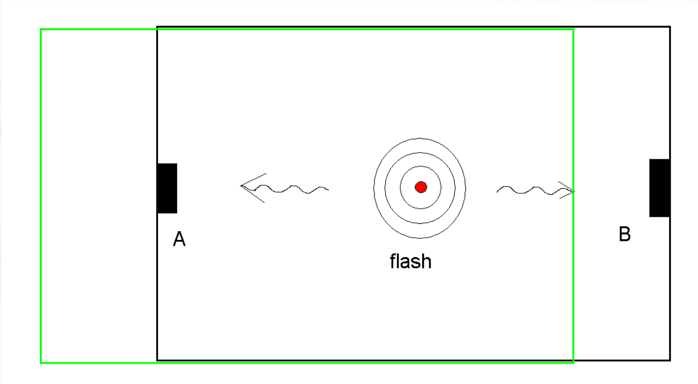
Change frames again!

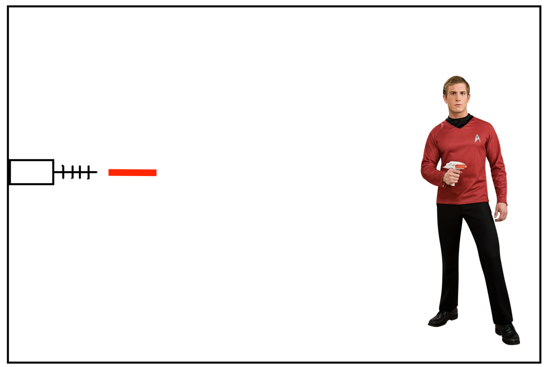
- What about perception of a 3rd observer who is moving faster than spacecraft?



- 3rd observer sees event B before event A
- So, order in which events happen can depend on the frame of reference.

The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.

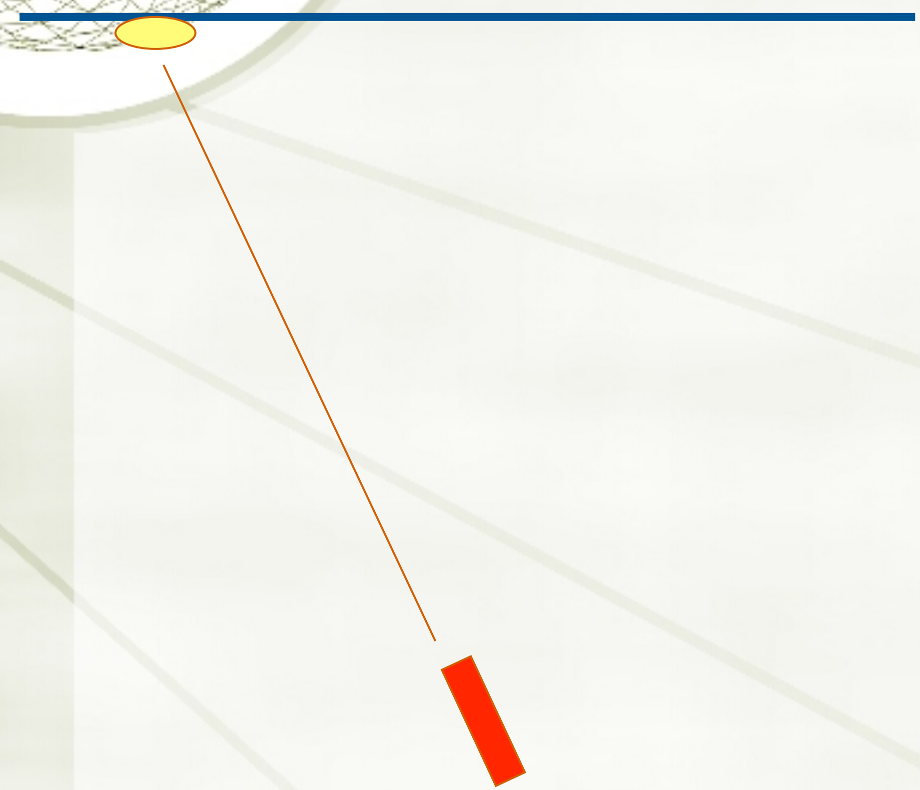


- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

- This is a question of **causality**.
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- In fact, it is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.

- This is true provided that
  - The laser blast does not travel faster than the speed of light
  - We do not change to a frame of reference that is going faster than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information.**

Some things move faster than light



- But they transmit no information
- E.g., light spot on a distant screen

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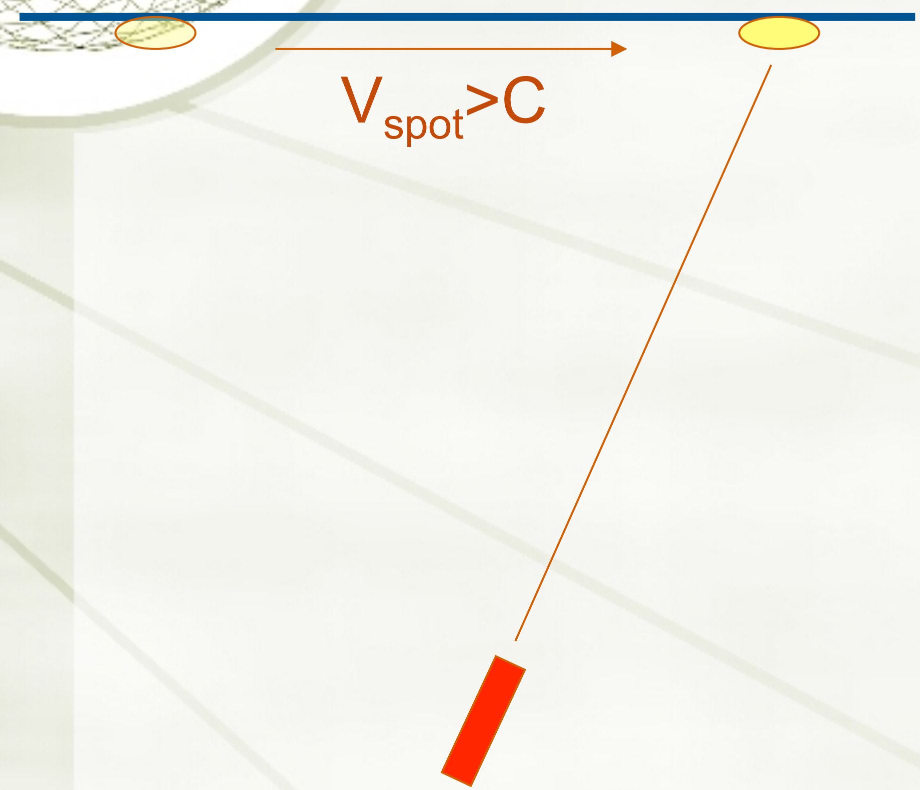

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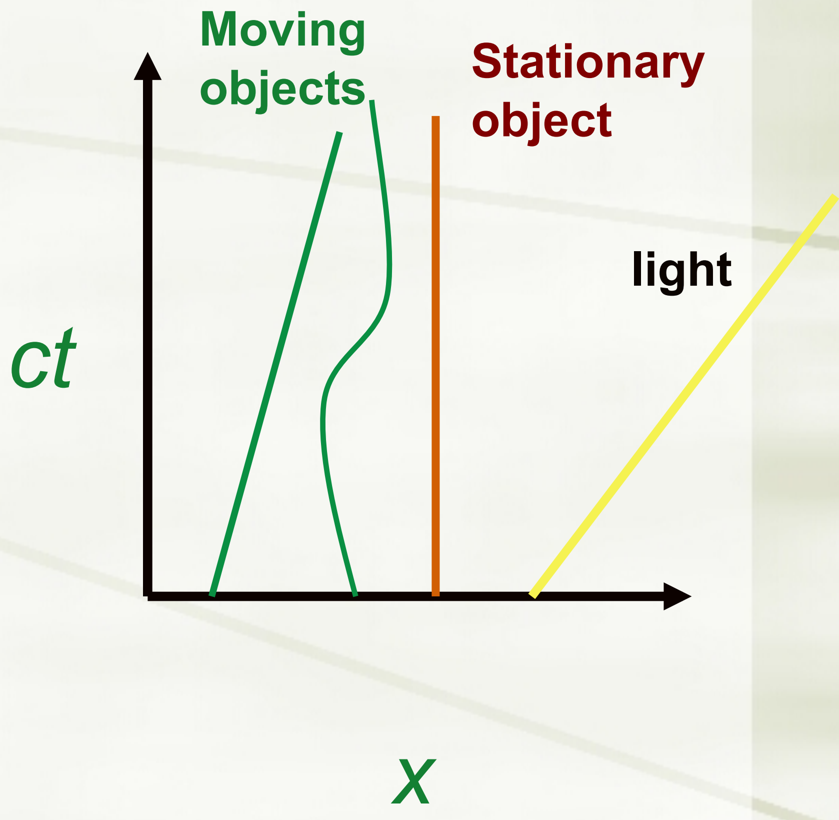
Causality

- Can causality be proved?
  - No, it is an axiom of physics
- What if causality doesn't hold?
  - Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - This would be a true "end of science."
- So we will *insist* on causality as we continue to explore relativity.

Space-time diagrams

- Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- This is simplest to do for events that take place along a line in space (one-dimensional space)
  - Plot as a 2D graph
  - use two coordinates: *x* and *ct*
- Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image): *x*, *y* and *ct*
- Can also be generalized to events taking place in 3D space using a 4D graph, but this is difficult to visualize



World lines of events

Distances in time and space

- Two events A and B separated by distance Δs in space (x, y, z):

  $$\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$$

  (Thanks, Pythagoras!)

  where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

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- Two events A and B separated by distance Δs in time (t):

  $$\Delta s = [(c\Delta t)^2]^{1/2}$$

  where $\Delta t = t_A - t_B$, and we've multiplied by c to make the units of Δs come out as a distance
- Two events A and B separated in x and t:

  $$\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$$

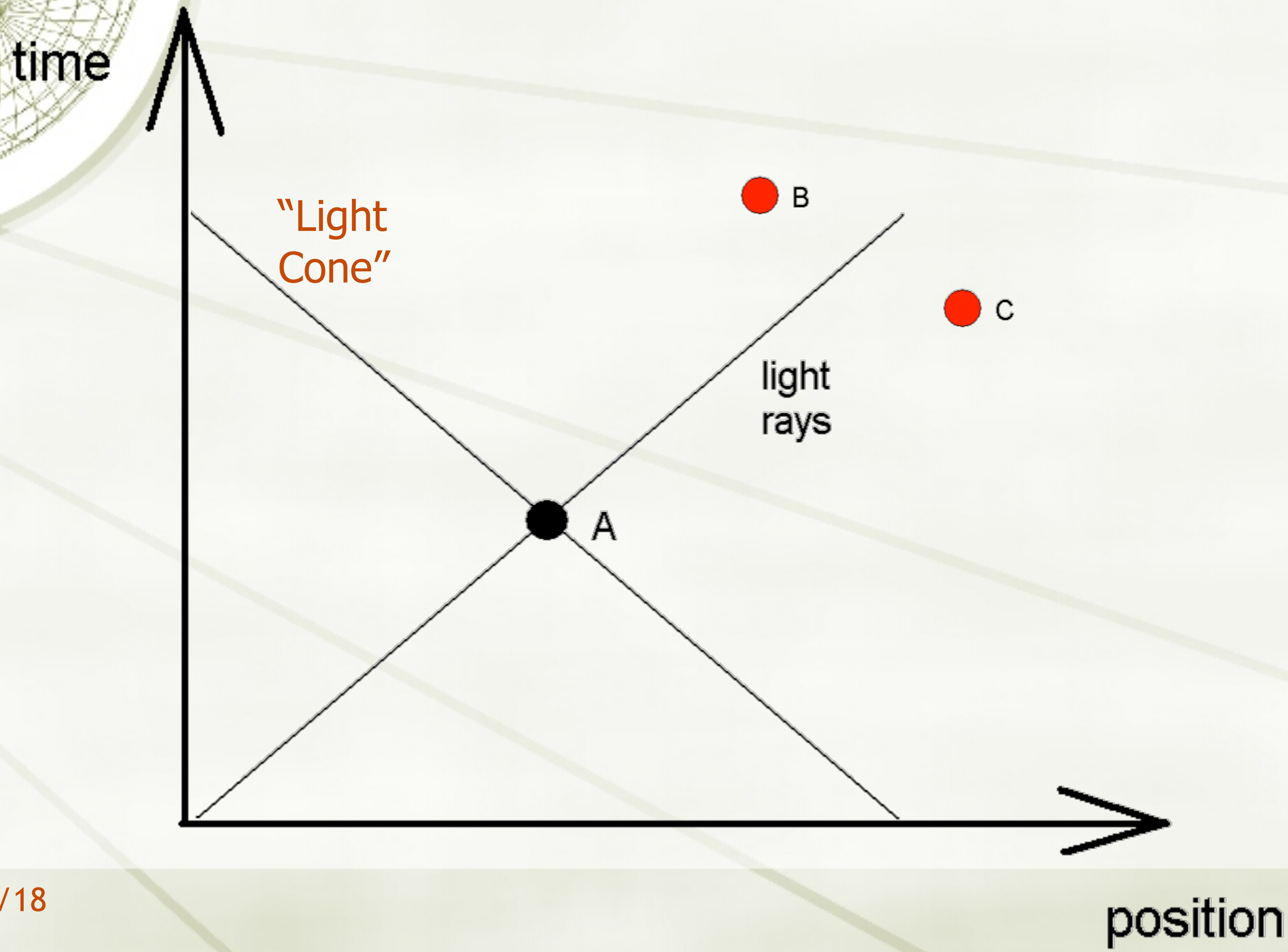
Space-time intervals

- Two events A and B in space-time are separated by an **invariant interval**, given by

$$\Delta s=[(c\Delta t)^2 -(\Delta x)^2 -(\Delta y)^2 -(\Delta z)^2]^{1/2}$$

where $\Delta t = t_A - t_B$, $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$,
- The formula is analogous to Pythagorean equation, but modified to account for the difference between space (x) and time (ct)
- The invariant space-time interval is an important quantity because it is independent of the frame in which it is measured; *all* observers agree on it!
  - This is true even though the Δt, Δx , *etc individually* are different for different observers (due to time dilation, space contraction)
  - The invariant interval is equal in value to (proper time of event) $\times$ c
- Space-time interval is zero for any two points on light ray world line
- Proper time between two events connected by a curved world line is computed by adding up results for small straight intervals along curve
  - Even if two curved world lines start and end at the same place, they may result in different proper time intervals

Light cone for event "A"

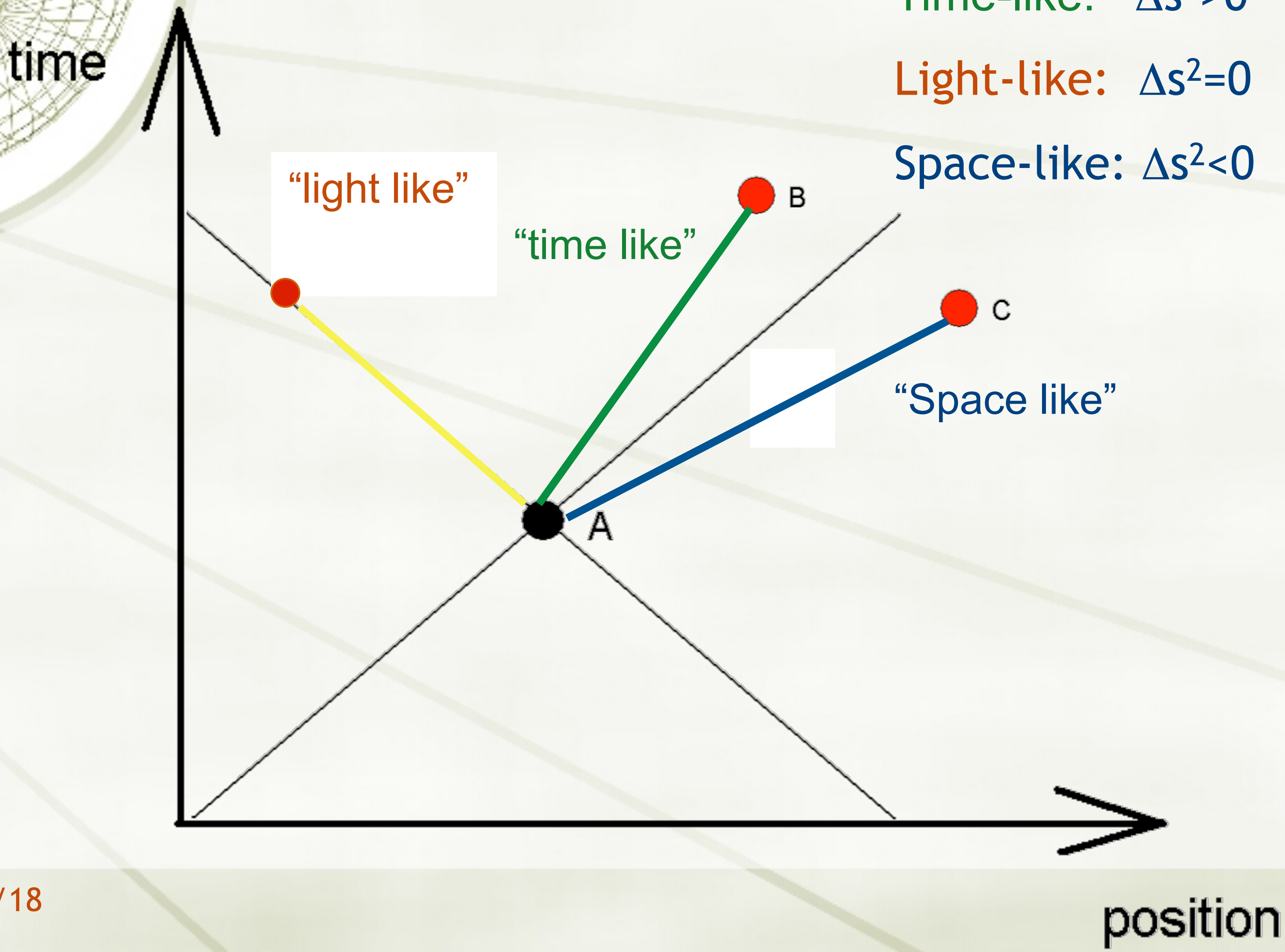


Different kinds of space-time intervals

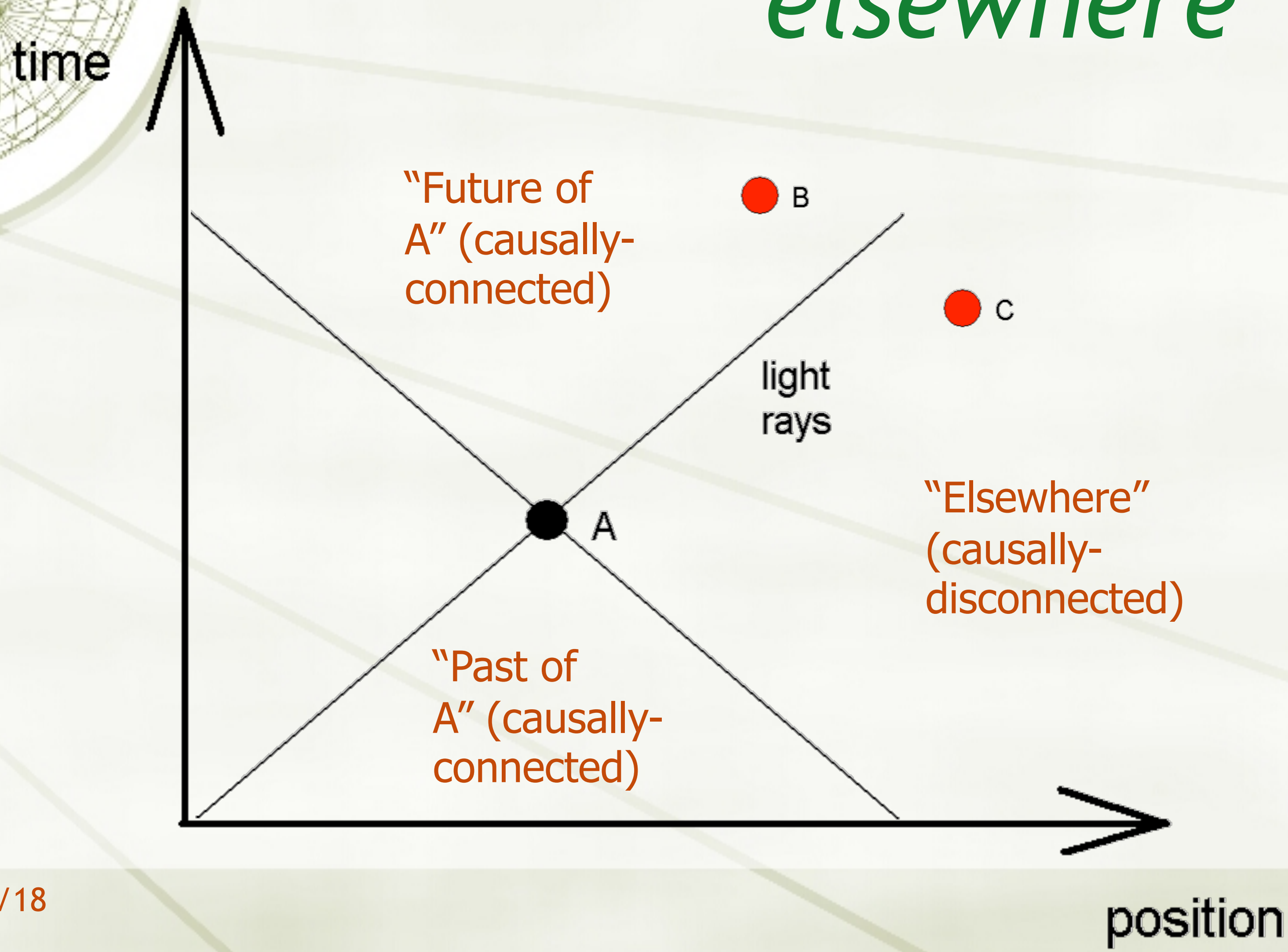
Time-like: $\Delta s^2 > 0$

Light-like: $\Delta s^2 = 0$

Space-like: $\Delta s^2 < 0$

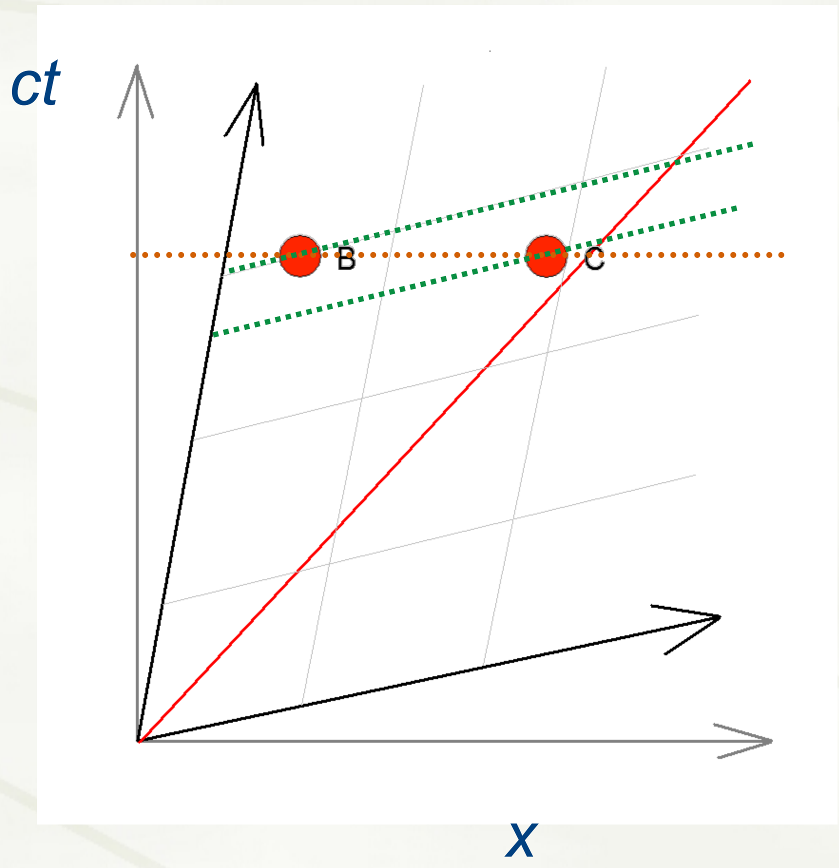


Past, future and “elsewhere”.



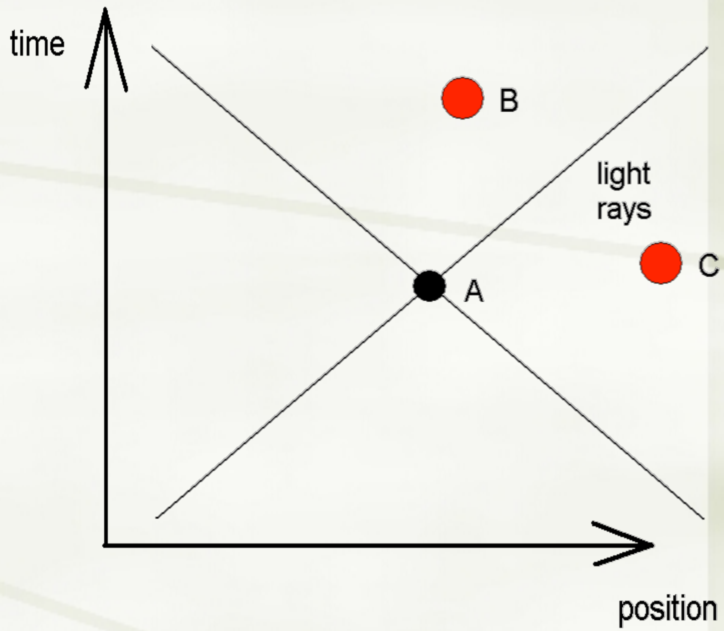
Spacetime diagrams in different frames

- Changing from one reference frame to another…
  - Affects time coordinate (time dilation)
  - Affects space coordinate (length contraction)
  - Leads to a distortion of the space-time diagram as shown in figure.
- Events that are simultaneous in one frame are not simultaneous in another frame



Causality

- Events A and B…
  - Cannot change order of **A** and **B** by changing frames of reference.
  - **A** can also communicate information to **B** by sending a signal at, or less than, the speed of light.
  - This means that **A** and **B** are causally-connected.
- Events A and C…
  - Can change the order of **A** and **C** by changing frame of reference.
  - If there were any communication between **A** and **C**, it would have to happen at a speed faster than the speed of light.
- If idea of **cause and effect** is to have any meaning, we must conclude that no communication can occur at a speed faster than the speed of light.



The twin paradox

- Suppose Andy (A) and Betty (B) are twins.
- Andy stays on Earth, while Betty leaves Earth, travels (at a large fraction of the speed of light) to visit her aunt on a planet orbiting Alpha Centauri, and returns
- When Betty gets home, she finds Andy is greatly aged compared with herself.
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- Is this correct?

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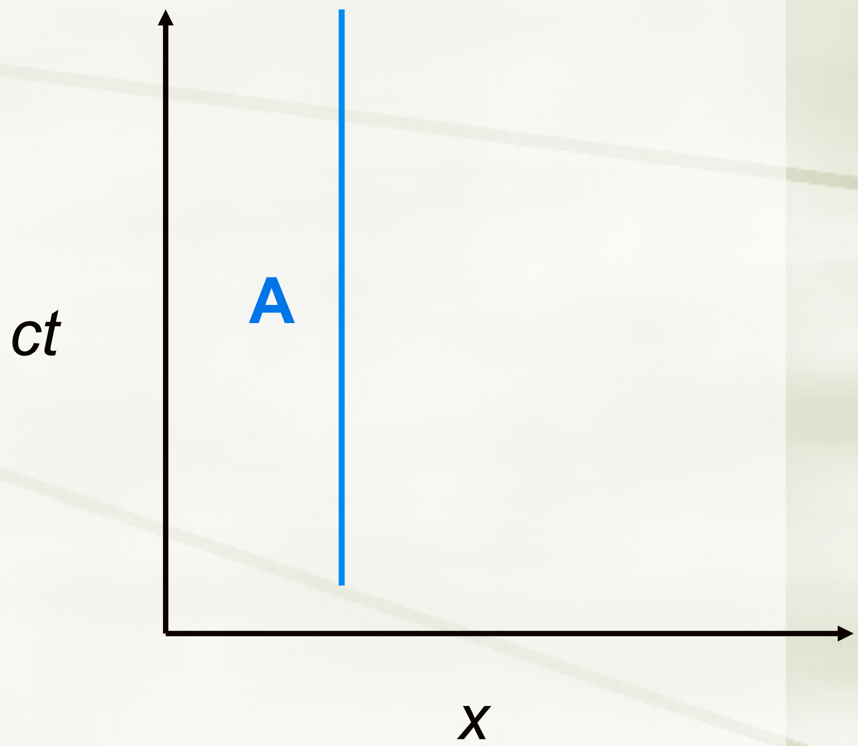
- Andy attributes this to the time dilation he observes for Betty’s clock during her journey
- Is this correct?
- What about reciprocity? Doesn’t Betty observe Andy’s clock as dilated, from her point of view? Wouldn’t that mean she would find him much older, when she returns?
- Who’s really older?? What’s going on???

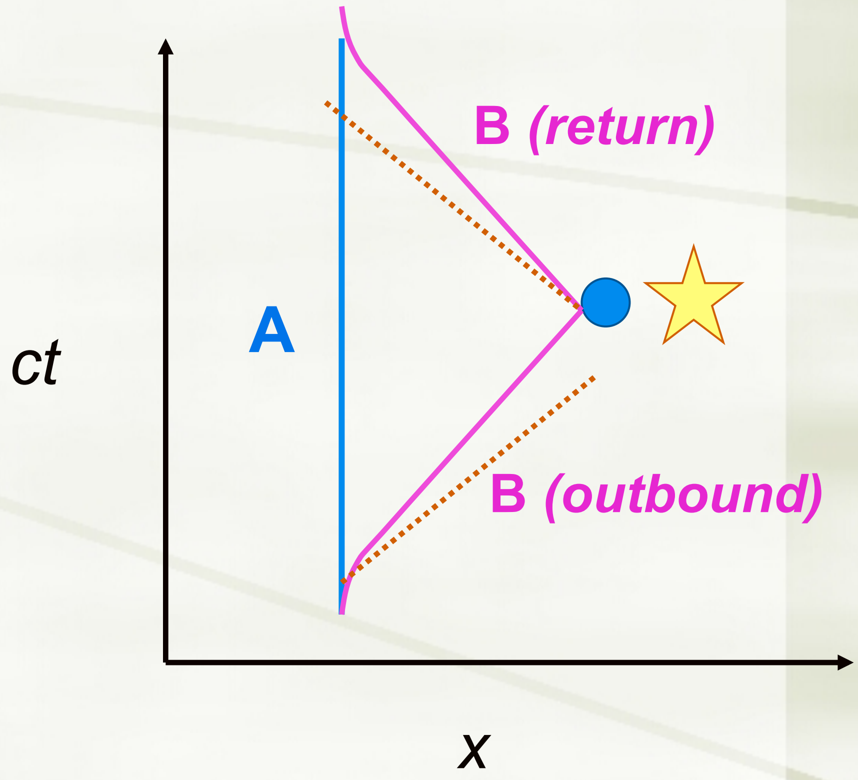
Andy's point of view

- Andy's world line, in his own frame, is a straight line



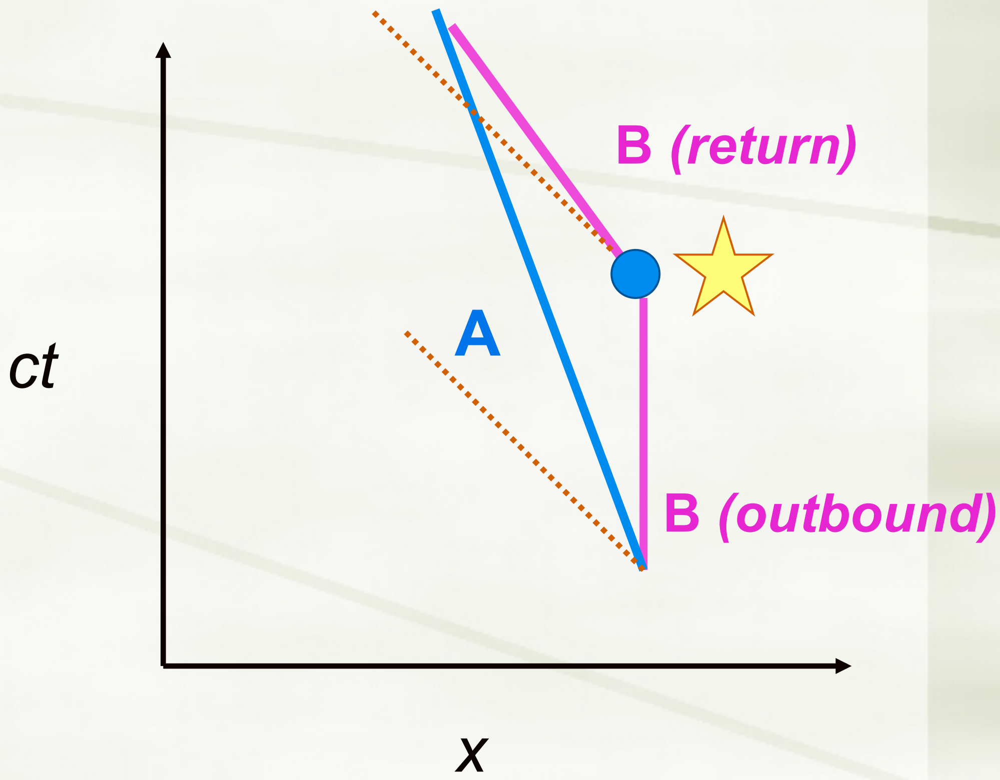
Andy's point of view

- Andy's world line, in his own frame, is a straight line
- Betty's journey has world line with two segments, one for outbound (towards larger x) and one for return (towards smaller x)
- Both of Betty's segments are at angles <45° to vertical, because she travels at v<c
- If Andy is older by Δt years when Betty returns, he expects that due to time dilation she will have aged by $\Delta t/\gamma$ years
- Since $1/\gamma = (1-v^2/c^2)^{1/2} < 1$, Betty will be younger than Andy, and the faster Betty travels, the more age difference there will be



Betty's point of view

- Consider frame moving with Betty's outbound velocity
- Andy on Earth will have straight world line moving towards smaller *x*
- Betty's return journey world line is not the same as her outbound world line, which instead points toward smaller *x*
- Both Andy's world line and Betty's return world line are at angles < 45 ° to vertical (inside of the light cone)
- Betty's return world line is closer to light cone than Andy's world line



- For frame moving with Betty's return velocity, the situation is similar

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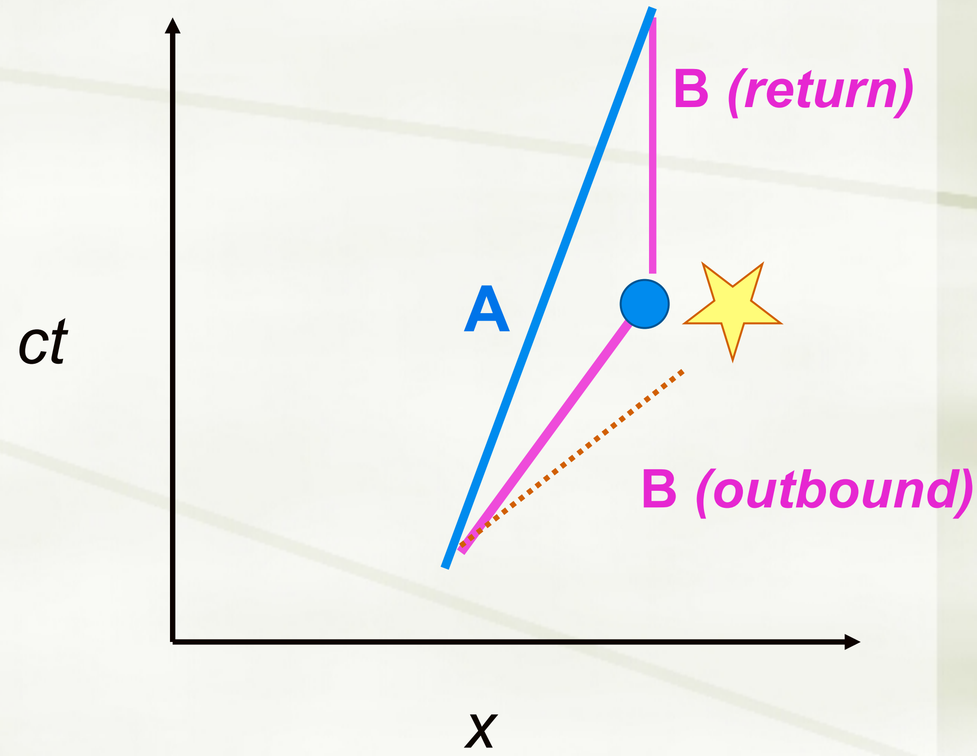
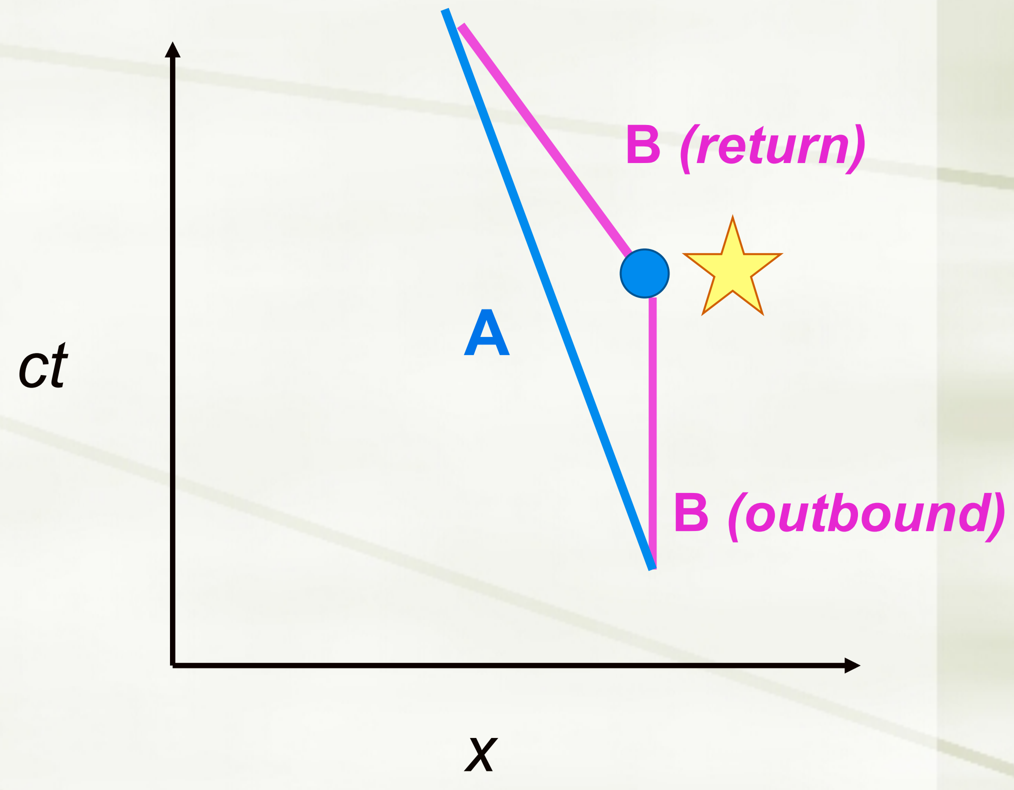
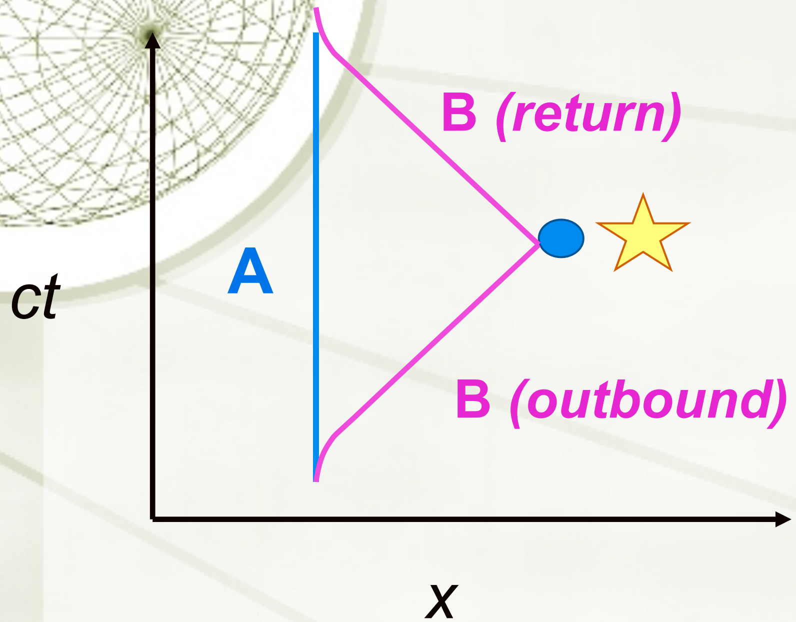


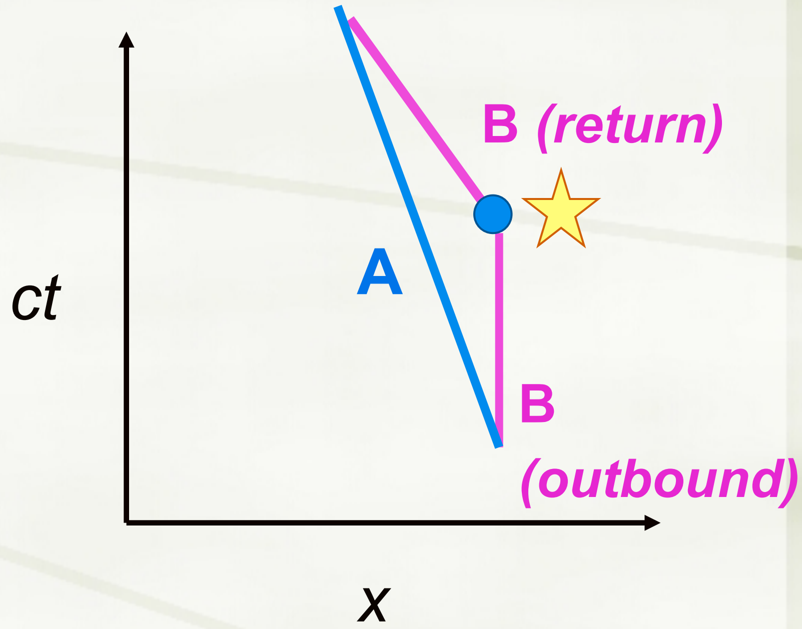
Solution of the paradox

- *From any perspective*, Andy's world line has a single segment
- *From any perspective*, Betty's world line has two different segments
- There is no *single* inertial frame for Betty's trip, so reciprocity of time dilation with Andy cannot apply for whole journey
- Betty's proper time is truly shorter -- she is younger than Andy when she returns



Different kinds of world lines

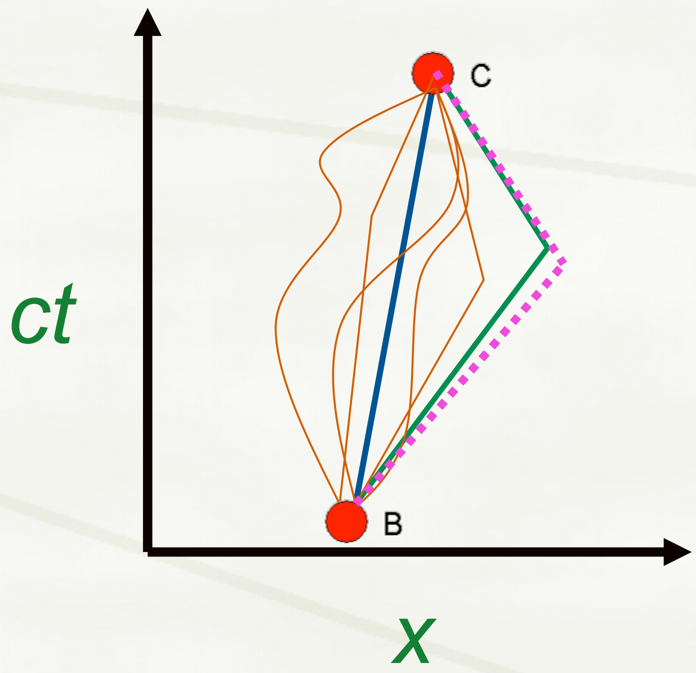




- Regardless of frame, Betty's world line does not connect start and end points with a straight line, while Andy's does
- This is because Betty's journey involves **accelerations**, while Andy's does not
- Acceleration from one velocity to another produces a curved world line – here a quick change, so a kink in the world line

More on invariant intervals

- Considering all possible world lines joining two points in a space-time diagram, the one with the **longest proper time** (=invariant interval) is always the **straight world line that connects the two points**
- The **light-like world lines** (involving reflection) have the **shortest proper time** -- zero!
- Massive bodies can minimize their proper time between events by following a world line *near* a light-like world line



Next time…

- Special Relativity III & General Relativity I
  - Einstein's formula for energy
  - Equivalence of mass and energy
  - Mass turning into energy
  - Energy turning into mass
  - Redshifting of light
  - Need for General Relativity