Lecture 8: Special Relativity II

- Simultaneity and causality
- Space-time diagrams
- Reciprocity and the twins paradox

Please continue reading Ch. 7 of text
Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
Suppose sensors on the walls record when the light rays hit the walls.

Since speed of light is constant, light rays will hit opposite walls at precisely the same time. Call these events A and B.
Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
Suppose sensors on the walls record when the light rays hit the walls.

Since speed of light is constant, light rays will hit opposite walls at precisely the same time. Call these events A and B.
Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground. For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.

But astronaut in spacecraft thinks events are simultaneous. Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (Relativity of simultaneity).
Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.

For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.

But astronaut in spacecraft thinks events are simultaneous.

Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (Relativity of simultaneity).
Change frames again!

What about perception of a 3\textsuperscript{rd} observer who is moving faster than spacecraft?

3\textsuperscript{rd} observer sees event B before event A.

So, order in which events happen can depend on the frame of reference.
The laser gun experiment

Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.

- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit before the gun fires?
This is a question of causality.
The events described are causally-connected (i.e. one event can, and does, affect the other event).
In fact, it is not possible to change the order of these events by changing frames, according to Special Relativity theory.
This is true provided that

- The laser blast does not travel faster than the speed of light
- We do not change to a frame of reference that is going faster than the speed of light

To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, anything that transmits any information.
Some things move faster than light!

- But they transmit no information
- E.g., light spot on a distant screen
Some things move faster than light

- But they transmit no information
- E.g., light spot on a distant screen

$V_{\text{spot}} > C$
Causality

Can causality be proved?
- No, it is an axiom of physics

What if causality doesn’t hold?
- Then the Universe returns to being random, unconnected events that can’t be understood or predicted.
- This would be a true “end of science.”

So we will insist on causality as we continue to explore relativity.
Space-time diagrams

Because space and time are “mixed up” in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.

This is simplest to do for events that take place along a line in space (one-dimensional space)
  ✤ Plot as a 2D graph
  ✤ use two coordinates: $x$ and $ct$
  ✤ Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image): $x$, $y$ and $ct$
  ✤ Can also be generalized to events taking place in 3D space using a 4D graph, but this is difficult to visualize

World lines of events
Distances in time and space

Two events A and B separated by distance $\Delta s$ in space $(x, y, z)$:

$$\Delta s = \left[ (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \right]^{1/2}$$

(Thanks, Pythagoras!)

where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$
Distances in time and space

Two events A and B separated by distance $\Delta s$ in space ($x, y, z$):

$$\Delta s = \left[ (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \right]^{1/2}$$

(Thanks, Pythagoras!)

where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

Two events A and B separated by distance $\Delta s$ in time ($t$):

$$\Delta s = \left[ (c\Delta t)^2 \right]^{1/2}$$

where $\Delta t = t_A - t_B$, and we’ve multiplied by c to make the units of $\Delta s$ come out as a distance

Two events A and B separated in $x$ and $t$:

$$\Delta s = \left[ (c\Delta t)^2 - (\Delta x)^2 \right]^{1/2}$$
Space-time intervals

- Two events A and B in space-time are separated by an invariant interval, given by
  \[ \Delta s = [(c \Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2]^{1/2} \]
  where \( \Delta t = t_A - t_B \), \( \Delta x = x_A - x_B \), \( \Delta y = y_A - y_B \), \( \Delta z = z_A - z_B \).
- The formula is analogous to Pythagorean equation, but modified to account for the difference between space (x) and time (ct).
- The invariant space-time interval is an important quantity because it is independent of the frame in which it is measured; all observers agree on it!
  - This is true even though the \( \Delta t \), \( \Delta x \), etc. individually are different for different observers (due to time dilation, space contraction).
  - The invariant interval is equal in value to (proper time of event) \( \times \) c.
- Space-time interval is zero for any two points on light ray world line.
- Proper time between two events connected by a curved world line is computed by adding up results for small straight intervals along curve.
  - Even if two curved world lines start and end at the same place, they may result in different proper time intervals.
Light cone for event “A”

```
... time
```

```
... position
```

```
Light Cone
```

```
light rays
```

- A
- B
- C
Different kinds of space-time intervals

- **Time-like**: $\Delta s^2 > 0$
- **Light-like**: $\Delta s^2 = 0$
- **Space-like**: $\Delta s^2 < 0$

- "Time-like"
- "Light like"
- "Space like"
Past, future and “elsewhere”.

“Future of A” (causally-connected)

“Past of A” (causally-connected)

“Elsewhere” (causally-disconnected)
Spacetime diagrams in different frames

- Changing from one reference frame to another...
  - Affects time coordinate (time dilation)
  - Affects space coordinate (length contraction)
  - Leads to a distortion of the space-time diagram as shown in figure.

- Events that are simultaneous in one frame are **not** simultaneous in another frame.
Events A and B...
- Cannot change order of A and B by changing frames of reference.
- A can also communicate information to B by sending a signal at, or less than, the speed of light.
- This means that A and B are causally-connected.

Events A and C...
- Can change the order of A and C by changing frame of reference.
- If there were any communication between A and C, it would have to happen at a speed faster than the speed of light.
- If idea of cause and effect is to have any meaning, we must conclude that no communication can occur at a speed faster than the speed of light.
The twin paradox

- Suppose Andy (A) and Betty (B) are twins.
- Andy stays on Earth, while Betty leaves Earth, travels (at a large fraction of the speed of light) to visit her aunt on a planet orbiting Alpha Centauri, and returns.
- When Betty gets home, she finds Andy is greatly aged compared with herself.
- Andy attributes this to the time dilation he observes for Betty’s clock during her journey.
The twin paradox

- Suppose Andy (A) and Betty (B) are twins.
- Andy stays on Earth, while Betty leaves Earth, travels (at a large fraction of the speed of light) to visit her aunt on a planet orbiting Alpha Centauri, and returns.
- When Betty gets home, she finds Andy is greatly aged compared with herself.
- Andy attributes this to the time dilation he observes for Betty’s clock during her journey.
- Is this correct?
The twin paradox

- Suppose Andy (A) and Betty (B) are twins.
- Andy stays on Earth, while Betty leaves Earth, travels (at a large fraction of the speed of light) to visit her aunt on a planet orbiting Alpha Centauri, and returns.
- When Betty gets home, she finds Andy is greatly aged compared with herself.
- Andy attributes this to the time dilation he observes for Betty’s clock during her journey.
- Is this correct?
- What about reciprocity? Doesn’t Betty observe Andy’s clock as dilated, from her point of view? Wouldn’t that mean she would find him much older, when she returns?
- Who’s really older?? What’s going on???
Andy’s point of view

- Andy’s world line, in his own frame, is a straight line.
Andy’s point of view

- Andy’s world line, in his own frame, is a straight line
- Betty’s journey has world line with two segments, one for outbound (towards larger x) and one for return (towards smaller x)
- Both of Betty’s segments are at angles $<45^\circ$ to vertical, because she travels at $v<c$
- If Andy is older by $\Delta t$ years when Betty returns, he expects that due to time dilation she will have aged by $\Delta t/\gamma$ years
- Since $1/\gamma = (1-v^2/c^2)^{1/2} < 1$, Betty will be younger than Andy, and the faster Betty travels, the more age difference there will be
Betty’s point of view

- Consider frame moving with Betty’s outbound velocity
- Andy on Earth will have straight world line moving towards smaller $x$
- Betty’s return journey world line is not the same as her outbound world line, which instead points toward smaller $x$
- Both Andy’s world line and Betty’s return world line are at angles $< 45 \, ^\circ$ to vertical (inside of the light cone)
- Betty’s return world line is closer to light cone than Andy’s world line

For frame moving with Betty’s return velocity, the situation is similar
Betty’s point of view

- Consider frame moving with Betty’s return velocity
- Andy on Earth will have straight world line moving towards larger x
- Betty’s return journey world line is not the same as her outbound world line, which instead pointings toward larger x
- Both Andy’s world line and Betty’s return world line are at angles $< 45^\circ$ to vertical (inside of the light cone)
- Betty’s outbound world line is closer to light cone than Andy’s world line
Solution of the paradox

- From any perspective, Andy’s world line has a single segment
- From any perspective, Betty’s world line has two different segments
- There is no single inertial frame for Betty’s trip, so reciprocity of time dilation with Andy cannot apply for whole journey
- Betty’s proper time is truly shorter -- she is younger than Andy when she returns
Different kinds of world lines

- Regardless of frame, Betty’s world line does not connect start and end points with a straight line, while Andy’s does.
- This is because Betty’s journey involves accelerations, while Andy’s does not.
- Acceleration from one velocity to another produces a curved world line - here a quick change, so a kink in the world line.
More on invariant intervals

- Considering all possible world lines joining two points in a space-time diagram, the one with the **longest proper time** (= invariant interval) is always the **straight world line that connects the two points**

- The **light-like world lines** (involving reflection) have the **shortest proper time** -- zero!

- Massive bodies can minimize their proper time between events by following a world line *near* a light-like world line

\[ ct \]

\[ X \]
Next time...

- Special Relativity III & General Relativity I
- Einstein’s formula for energy
- Equivalence of mass and energy
- Mass turning into energy
- Energy turning into mass
- Redshifting of light
- Need for General Relativity