Lecture 4: Newton's Laws & Galilean Relativity

- Newton's profound perspective
- Newton's Laws of Motion… 3 ways
- Newton's Law of Gravitation

Newton's profound perspective

- Newton formulated a **universal** theory of motion and gravity
  - Same laws of physics operate anywhere and anytime in the Universe
  - Tears down the wall that Aristotle built between Earthly laws and Heavenly laws

I : Newton's laws of motion

- **Newton's first law : If a body is not acted upon by any forces, then its velocity remains constant**
- Notes
  - Remember that velocity is a vector quantity (it has direction as well as magnitude)
  - This law sweeps away the idea that "being at rest" is a natural state… this was a major change of thinking (originated with Galileo)!

- **Newton's second law : If a body of mass M is acted upon by a force F, then its acceleration a is given by F=Ma**
- Notes
  - Remember that both F and a are vectors
  - This law defines the "inertial mass" as the degree to which a body resists being accelerated by a force

- **Newton's third law - If a body A exerts a force F on body B, then body B exerts a force -F on body A**
- Notes
  - This is the law of "equal and opposite reaction"
  - We will see later that this law is closely tied to conservation of momentum

Review of Goddard's pioneering work on rockets

- "Professor Goddard does not know the relation between action and reaction and the needs to have something better than a vacuum against, which to react. He seems to lack the basic knowledge ladled out daily in high schools."…

-1921 New York Times editorial

II : Momentum

- **Definition : If an object of mass m is moving with velocity $\underline{V}$, its momentum $\underline{p}$ is given by $\underline{p}=m\underline{V}$**

- **The total momentum $\underline{p}_{tot}$ of a number of objects with masses m_1, m_2, … and velocities $\underline{V}_1$, $\underline{V}_2$, … is just the (vector) sum of the objects' separate momenta**

$$\begin{aligned} \underline{p}_{\text{tot}} &= m_1\underline{V}_1 + m_2\underline{V}_2 + m_3\underline{V}_3 + \ldots \\ &= \sum_{i=1}^{N} m_i\underline{V}_i \end{aligned}$$

- **Conservation of momentum : The total momentum of a system of particles is constant if no external forces act on the system**
- Proof for a two particle system…
  - Consider two particles with masses m_1 and m_2
  - They exert forces on each other, but there is no force being applied to the pair as a whole
  - At some instant in time, they have velocities $\underline{V}_1$ and $\underline{V}_2$
  - So momentum is $\underline{p}=m_1\underline{V}_1+m_2\underline{V}_2$

- Consider some instant in time Δt later… individual velocities will have changed due to forces that particles exerted on each other… let new velocities be $\underline{V}_1$' and $\underline{V}_2$'
- Difference between new and old momentum is

$$\begin{aligned}
\Delta \underline{p} &= \underline{p}_{\text{new}} - \underline{p}_{\text{old}} \\
&= m_1(\underline{V}_1' - \underline{V}_1) + m_2(\underline{V}_2' - \underline{V}_2) \\
&= m_1 \Delta \underline{V}_1 + m_2 \Delta \underline{V}_2 \\
&= \left(m_1 \frac{\Delta \underline{V}_1}{\Delta t} + m_2 \frac{\Delta \underline{V}_2}{\Delta t} \right) \Delta t \\
&= (\underline{F}_1 + \underline{F}_2) \Delta t \\
&= (\underline{F}_1 - \underline{F}_1) \Delta t \\
&= 0
\end{aligned}$$

Newton's third law used here!

- Proof for a general (many particle) system follows very similar lines

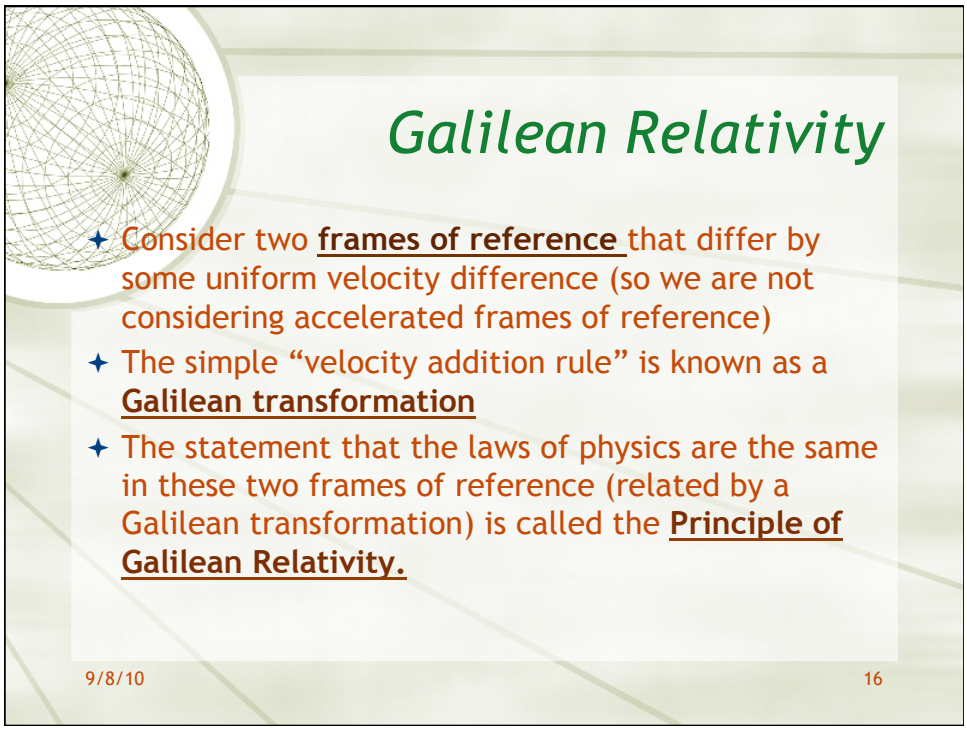
- We now see that Newton's laws can be rephrased entirely in terms of momentum…
  - **Second law**… the rate of change of momentum of a body is equal to the force applied to that body
  - **First law** is special case of the Second law… the momentum of a body is unchanged if there are no forces acting on body
  - **Third law**… the momentum of an isolated system of objects is conserved

III : Symmetries and frames of reference

- The idea of symmetry is very important in modern advanced physics! Let's have a glimpse of symmetry in action…

- Consider…
  - Two equal, connected masses M at rest.
  - At some time, they are suddenly pushed apart by a spring
  - They must fly apart with the same speed in opposite directions (*what else could possibly happen… why would one mass "decide" to move faster*?)

- Now think of same situation, but the two connected masses are initially moving at velocity V. Let's turn this into the above situation by "moving along with the masses at velocity V"
  - Change perspective to bring masses to rest…
  - Do same problem as before…
  - Change back to the original perspective…
  - You have "changed your frame of reference".
  - The "velocity addition" rule is called a **Galilean transformation.**
  - We assume that, after changing our reference frame and using a Galilean transformation, the laws of physics are the same. This is called **Galilean Relativity**.
  - Then find that momentum before = momentum after


Galilean Relativity
Consider two frames of reference that differ by some uniform velocity difference (so we are not considering accelerated frames of reference)
The simple "velocity addition rule" is known as a Galilean transformation
The statement that the laws of physics are the same in these two frames of reference (related by a Galilean transformation) is called the Principle of Galilean Relativity.
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- How do Newton's laws fit into **this** picture?
  - N1 comes directly from Galilean Relativity (there is no difference between a state of rest and a state of motion)
  - N2 and N3 are exactly what's needed to make sure that momentum is conserved and so is related to the symmetry of space
  - So… Newton's laws are related to the symmetry of space and the way that different frames of reference relate to each other.

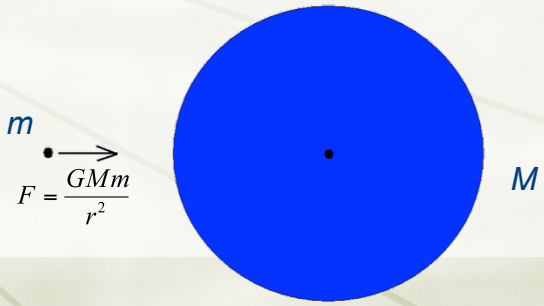
IV: NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's law of Gravitation: A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by

$$F = \frac{Gm_1m_2}{r^2}$$

- "G" is called the Gravitational constant (G = 6.67×10^{-11} N m^2 kg^{-2} in mks units)
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings.

- Newton's Law of Gravitation defines the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass *M* (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass *M* at the sphere's center.



Inertial and gravitiational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a$$ m_I=inertial mass

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2}$$ m_G=gravitational mass

So, acceleration due to gravity is:

$$a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) varies, the rate at which objects fall in a gravitational field will vary…

At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Equivalence of inertial and gravitational mass

- ***Experimentally***, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)
- So, m_I/m_G must be the *same* for all bodies
- And we can choose the constant "G" such that $m_I = m_G$, and $a = GM/r^2$
- This is the **weak equivalence principle**: gravity is equivalent to (indistinguishable from) any other acceleration.