ASTR415 Spring 2009 TERM PROJECT

Due May 7th, 2009

YOUR FIRST COSMOLOGICAL SIMULATION OF A BLACK HOLE!
(also known as secondary infall problem)

Write a report and prepare a class presentation on the results of your numerical simulations of dark matter accretion around a black hole at rest in an expanding Universe. This problem is quite difficult for an undergraduate level course: do not worry too much if your results do not agree with the expected solution!

In order to solve this problem you need to generalize the integration package you wrote for PS5 so that it can solve the N-body problem in an expanding Universe assuming spherical geometry and fixed time steps. The following ODEs should be integrated in order to solve the problem (see also my handwritten notes):

$$\frac{dx}{dt} = (1+z)v_p \tag{1}$$

$$\frac{dv_p}{dt} = -H(t)v_p - (z+1)^2\frac{G\Delta M}{x^2}. \tag{2}$$

This equations describe the motion of spherical shells of dark matter around the black hole in comoving coordinates: $v_p(x)$ is the peculiar velocity of a shell at comiving distance x from the black hole and $\Delta M(< x) = M(< x)(t) - M(< x)(t = t_i)$ is the difference between the mass within x at time t and the original mass at time $t = t_i$ when the Universe was homogeneous (constant density). To convert the distances in physical units r use the formula: $r = x/(1+z)$, where the redshift z depends on time as $z+1 = (t/t_0)^{-2/3}$ and where $t_0 = 13$ Gyr is the age of the Universe. The Hubble parameter in a flat matter dominated Universe is: $H = H_0(1+z)^{3/2} = H_0(t/t_0)^{-1}$, where $H_0 = 70$ km/s.

Alternatively you can integrate the equation as a function of redshift z rather than time t (it may be easier). Using the relationship $dz/dt = -H(1+z)$ you get the following equation:

$$\frac{dx}{dz} = -\frac{v_p}{H(z)} \tag{3}$$

$$\frac{dv_p}{dz} = \frac{v_p}{1+z} + \frac{z+1}{H(z)}\frac{G\Delta M}{x^2}. \tag{4}$$

Assume initial conditions as follows: start the simulation at redshift $z_i = 3000$ and assume that the dark matter has constant density $\rho = 3.8 \times 10^{-29}$ g/cm^3. Remember to place a black hole of mass $M_{BH} = 1$ M$_\odot$ at $x = 0$. If I forgot to provide enough explanations to solve the problem feel free to ask me. Have fun!

Hints: The dark matter should create a spherical halo around the black hole with mass $M(< r) \propto r^{0.75}$ *and have a total mass within the halo radius that increases with decreasing redshift as* $M \propto 1/(1+z)$.