ASTR615 Fall 2015 Problem Set #3

Due Oct 21, 2015

Topics for this problem set include round-off error and linear algebra.

1. As an example of an unstable algorithm, consider integer powers of the "Golden Mean" $\phi = (\sqrt{5}-1)/2$. It can be shown that $\phi^{n+1} = \phi^{n-1} - \phi^n$, i.e. successively higher powers of ϕ can be computed from a single subtraction rather than a more expensive multiply. Write a single-precision program to compute a table consisting of the columns n, ϕ^n computed from the recursion relation, and ϕ^n computed directly (i.e. $\phi^n = \phi\phi^{n-1}$), for n ranging from 1 to 20. Is the round-off error random? What happens in double precision?

2. Write a program to compute the instantaneous spin period of a rigid body made up of identical, discrete, point particles. Use the fact that the angular momentum is

$$\mathbf{L} = \sum_i m_i(\mathbf{r}_i \times \mathbf{v}_i) = \mathbf{I}\boldsymbol{\omega}, \tag{1}$$

where m_i is the mass of particle i, $\mathbf{r}_i$ and $\mathbf{v}_i$ are its position and velocity vectors with respect to the centre of mass, $\boldsymbol{\omega}$ is the spin vector, and $\mathbf{I}$ is the inertia tensor

$$\mathbf{I} = \sum_i m_i(r_i^2\mathbf{1} - \mathbf{r}_i\mathbf{r}_i),$$

where $\mathbf{1}$ is the unit matrix.[1] Write a program to solve Eq. (1) for $\boldsymbol{\omega}$ (feel free to use the routines in *Numerical Recipes).* The spin period is then $2\pi/|\boldsymbol{\omega}|$.

   (a) Test your code by reading the data file

   `http://www.astro.umd.edu/~ricotti/NEWWEB/teaching/ASTR415/ps2.dat`

   which is in the format x y z v_x v_y v_z (i.e. 6 values to a line separated by white space). The units are mks (SI). What is the spin period in hours?

   (b) Make a graphical representation of the body using your favorite graphing package. If you use 2-D projections, be sure to include enough viewing angles to get a complete picture.

[1] For continuous bodies the summations are replaced by volume integrations and the particle masses become a mass density. In the present case the m_i's can be omitted entirely since the particles are identical. The expression $\mathbf{r}_i\mathbf{r}_i$ is dyadic product of the 3 dimensional vectors $\mathbf{r}_i$, producing a 3x3 tensor $\mathbf{T}$ with elements $T_{lm} = r_l r_m$, where $l, m = 1, 2, 3$ span over the x, y, z components of the vector $\mathbf{r}_i$.