Hot Big-Bang,
(5) Planck time, phase transitions

Planck Time:

Ch 6.2-6.3

- Singularity and Planck Time: $a \to 0$ $t \to 0$ if

$\ddot{a} < 0$ $\Rightarrow$ $p > -\frac{1}{3}\rho c^2$. Singularity avoidable if $w < -\frac{1}{3}$ $(p = w\rho c^2)$.

- At $t \to 0$ there is not a characteristic time scale other than t_p (constructed with $G, \hbar, c$)

$[G] = 6.68 \cdot 10^{-8}$ erg cm g^{-2} $= [g^{-1} cm^3 s^{-2}]$ $(erg = g\, cm^2 s^{-2})$

$[\hbar] = 1.05 \cdot 10^{-27}$ erg s $= [g\, cm^2 s^{-1}]$

$[c] = 3 \cdot 10^{10}$ cm s^{-1} $[cm\, s^{-1}]$

$$t_p = G^\alpha \hbar^\beta c^\gamma = g^{-\alpha+\beta}\, cm^{3\alpha+2\beta+\gamma}\, s^{-2\alpha-\beta-\gamma}$$

$-2\alpha - \beta - \gamma = 1$

$\alpha = \beta$; $5\alpha = -\gamma$

$-3\alpha + 5\alpha = 1$ $\begin{pmatrix} \alpha = \frac{1}{2} = \beta \\ \gamma = -\frac{5}{2} \end{pmatrix}$

$$t_p = \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}} \simeq 10^{-43}\ s \qquad \text{Planck time}$$

$$\ell_p = c\, t_p = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} \simeq 1.7 \cdot 10^{-33}\ cm$$

$$\rho_p = \frac{1}{G t_p^2} \simeq \frac{c^5}{G^2 \hbar} \simeq 4 \cdot 10^{93}\ g\, cm^{-3}$$

$\left(\cancel{m_p = \rho_p \ell^3 = 2.5 \cdot 10^{-5} g}\right)$

$$n_p = \ell_p^{-3} = \frac{\rho_p}{m_p} = \left(\frac{c^3}{G\hbar}\right)^{3/2} = 10^{98}\ cm^{-3}$$

$$m_p = \rho_p \ell_p^3 = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} = 2.5 \cdot 10^{-5} g \qquad E_p = m_p c^2 = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} = 1.2 \cdot 10^{19}\ GeV$$

$$T_p = \frac{E_p}{k_B} \simeq 1.4 \cdot 10^{32}\ K \qquad \boxed{a_p = \left(\frac{t_p}{2.4 \cdot 10^{19} s}\right)^{\frac{1}{2}} = 5 \cdot 10^{-32}}$$

Alternative derivations:

1) $\Delta E\, \Delta t \sim \hbar$ Heisenberg uncertainty principle

$\Delta E \sim m_p c^2$ $\quad \Delta t \sim t_p$ $\quad m_p \sim \rho_p (c t_p)^3$ $\quad \rho_p \simeq \frac{1}{G t_p^2}$ ...etc

2) $t_{compt} \cdot \Delta E \sim \hbar$ $\quad \Delta E = m_p c^2$

$t_{swartschild} = \frac{2 G m_p}{c^3}$

$\underline{t_{cp}(m_p) = t_{swartz}(m_p)}$ etc...

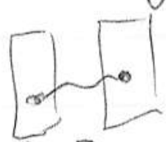
soup of BHs that form and "annihilate"?!

CL

Quantum Cosmology and String theory: (6.4-6.5)

1) Path integral formulation: minimize the action
(Integrate over all possible paths and minimize. The difficulty is to find ~~use~~ the appropriate set of 4-geometries to perform the integration.)
$\boxed{\text{Wave function of the Universe}}$

2) String theory:



M-theory

10-D
compact dimensions

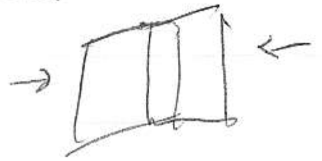
Spectrum of strings determines masses but also constants G, α; etc...

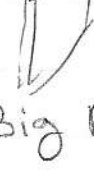
a) Brane world:

Randall-Sundrum → $V(r) = \frac{G M_1 M_2}{r}\left(1 + \frac{1}{r^2 K^2}\right)$
3D brane

$K \sim \frac{1}{\ell_p}$

b) Ekpirotic Universe





Big Bang

Phase transitions and Inflation:

$$T(t) \simeq T_r \cdot \frac{a(t_r)}{a(t)}$$

- Fundamental interactions:

Particles: Fermions: Quarks, Leptons $< \begin{matrix} e^- & \nu_e \\ \mu^- & \nu_\mu \\ \tau^- & \nu_\tau \end{matrix}$
(factional spin) 6 quarks (u, d, s, c, b, t)

1 ~~Weak~~ force: (all particles, mainly leptons) bosons: $W^+ W^-$, Z_0 (short range force)
(QED) 80 GeV 90 GeV

$E_{EW} \simeq 100$ GeV

2 Strong force: hadrons $<$ baryons $p, \bar{p}; n, \bar{n}$; mesons $\pi^+, \pi^-; \pi^0$
(QCD)

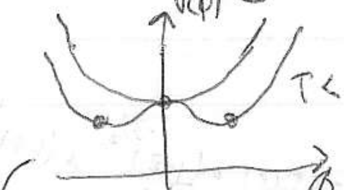
gluons

3 GUT $E_{GUT} \simeq 10^{15}$ GeV
weak + strong

4 · Gravity: Supersimmetry SUSY; string theory ??
$E_{Pl} = 10^{19}$ GeV

5 SUSY; STRING THEORY; QUANTUM COSMOLOGY?

Cosmological phase transitions:



symmetry breaking at a phase transition.

↳ Scalar field (eg/ Higgs field)

1) $T_P \sim 10^{19}$ GeV $< T < T_{GUT} \simeq 10^{15}$ GeV
baryons ≡ anti-baryons ; GUT; $t \sim 10^{-37}$ s

2) $T_{GUT} \simeq 10^{15}$ GeV
• magnetic monopoles (topological defects); baryosynthesis (violation of baryon number conservation): leptons ⟷ baryons

3) $T_{GUT} \leq T < T_{EW} = 100$ GeV
• superheavy bosons disappear (QCD & QED unification no longer holds)
• leptons, quarks, gluons, 4 vector bosons + antiparticles. ($R_H \sim 1$ cm)

4) $T_{EW} < T < T_{QH} \simeq 200$-$300$ MeV

• leptons acquire mass.
• T_{QH} quark-hadron phase transition (QCD)
($t_{QH} \sim 10^{-5}$ s $R_H \sim 1$ km)

Problems:

1) Magnetic monopoles → Topological defects (↓↓↑ ...) (Inflation)
2) Cosmological constant (why now?; what is it?)
3) " Horizon (Inflation)
4) " Flatness (")
5) Origin of initial perturbations (Inflation)
seed of galaxies

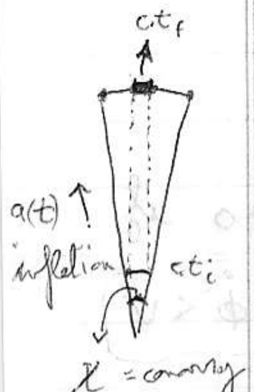
(6) Inflation (I)

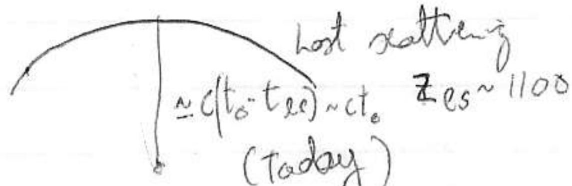
Homogeneity problem:

- Today universe homogeneous on scales $r_H \sim 10$ Gpc

  $\Delta r_{hom} \simeq a_0 \cdot \Delta x_{hom} \sim 10$ Gpc

- At the Planck time: $\Delta r_{hom} = a_{Pl} \cdot \Delta x_{hom} \sim 2 \cdot 10^{-3}$ cm $\sim 10^{30} \cdot l_{Pl}$





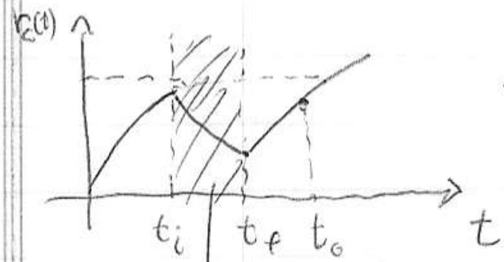
CMB isotropy

$$\begin{cases} r_{Hor}(t=t_0) \simeq 3ct_0 \sim r_H(t_0) \\ r_{Hor,ls}(t=t_{ls}) \; 3ct_0 \cdot a_{ls} \simeq \dfrac{3ct_0}{1+z_{ls}} \end{cases}$$

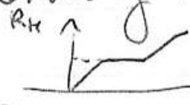
matter dominated

at t_{ls} $\quad r_{Hor,ls} \simeq \dfrac{3ct_0}{1+z_{ls}}$

$$R_H(t_{ls}) \simeq 3ct_0(1+z_{ls})^{-3/2} \simeq r_{Hor,ls}(1+z_{ls})^{-1/2} \simeq \frac{r_{Hor,ls}}{30} << r_{Hor,ls}$$



← Comoving Horizon evolution



(physical)

inflation $\quad \ddot{a} > 0$

$$\left(l_{past}/l_{future} \sim z_{GUT} \exp(-N) << 1 \quad N = H_{inf} \Delta t_{inf} > 70 \right)$$

Flatness problem:

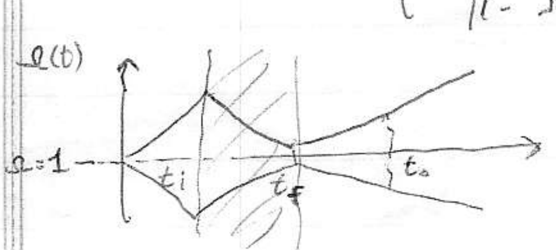
$$\left(\Omega^{-1}(z) - 1\right) = \left(\Omega_0^{-1} - 1\right) \cdot \begin{cases} (1+z)^{-1} & \omega = 0 \\ (1+z)^{-2} & \omega = \frac{1}{3} \\ (1+z)^{2} & \omega = -1 \end{cases} \qquad \left(\propto \rho_0 a_0^2 / \rho a^2\right)$$

$z_{ls} \simeq 10^3$

$z_{GUT} \sim 10^{30}$

$$\begin{cases} |1 - \Omega(z_{ls})| \simeq 10^{-3} \cdot |1 - \Omega_0| \\ |1 - \Omega(z_{GUT})| \simeq 10^{-60} |1 - \Omega_0| \end{cases}$$

$$\left(\begin{array}{c} \Omega \sim 1 + \epsilon \\ |\Omega^{-1} - 1| \simeq |\epsilon| \simeq |\Omega - 1| \end{array} \right)$$



$$\rightarrow \quad |1 - \Omega(t)| \propto \exp(-2H_{inf}\, t)$$
$$(\propto a^{-2})$$

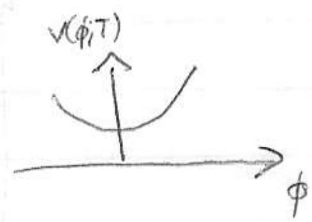
A simple model of Inflation:

Order parameter $\phi \rightarrow$ symmetry breaking

$\downarrow$

scalar field (eg. Higgs field)

$$L_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi;T)$$



$$\begin{cases} \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi;T) \\ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi;T) \end{cases}$$

$(c = \hbar = 1)$ natural units

$\hookrightarrow \simeq F =$ thermodynamic free energy

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G(\rho_\phi + \rho) - \frac{k}{a^2}$$ Friedmann Eq

Eq. of motion for ϕ: $$\underbrace{\frac{d}{dt}\frac{\partial L_\phi a^3}{\partial \dot{\phi}} - \frac{\partial (L_\phi a^3)}{\partial \phi} = 0}$$

$\downarrow$

also from $\left(\frac{\partial \rho}{\partial t} + 3H(\rho_\phi + P_\phi) = 0\right)$ $\dot{\phi}\ddot{\phi} + \frac{\partial V}{\partial \phi}\dot{\phi} + 3H\dot{\phi}^2 = 0$ $\longrightarrow$ $$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

$\frac{\dot{a}}{a} = H$ viscous term; potential

ball moving in a potential with viscosity

Slow-rolling approximation: $\frac{\dot{\phi}}{2} \ll V(\phi) \rightarrow \rho_\phi \simeq$ const

Expansion modes: $\frac{k}{a^2}$ & $\rho \ll \rho_\phi \simeq$ const

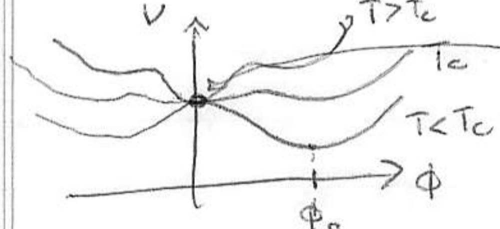
$$\left(\frac{\dot a}{a}\right)^2 = H^2 \simeq \frac{8\pi}{3} G\rho_\phi \simeq \text{const} \to \begin{cases} H^2 \simeq \frac{8\pi G}{3} V(\phi) \\ 3H\dot\phi \simeq -V'(\phi)\end{cases}$$

$$a \propto \exp\left(\frac{t}{\tau}\right) \qquad \text{with } \tau \simeq \left[\frac{3}{8\pi G V(\phi; T_0)}\right]^{\frac{1}{2}} \simeq 10^{-34}\,\text{s}$$

(*) e-foldings: $N = \ln\left(\frac{a_f}{a_i}\right) = -8\pi G \int_{\phi_i}^{\phi_f} \left(\frac{d\ln V}{d\phi}\right)^{-1} d\phi$

- Old inflation (Guth 1981): [Ch. 7.10 (C-L)]

First order phase transition: ΔF finite



supercooling $T<T_c$ $\phi=0$

reheating $T<T_c$ $\phi=\phi_0$

$T \to T_c$ (latent heat)

$F \sim V = F_0 + \alpha\phi^2 + \beta\phi^4 + \dots$

bubble nucleation → bubbles too small do not have time to merge

DOES NOT WORK AS IT IS.

- New inflation (Linde 1982, Albrecht & Steinhardt 1982)

2nd order phase transition; V not flat ← strongly φ coupled

large 3d domains: but does not work well

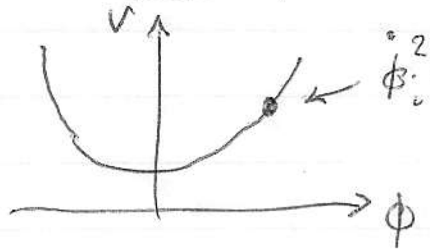
V at $\phi=0$ too flat, φ in thermal eq.

(7) Inflation (2) ; ~~partor~~ Chaotic inflation, perturbations

Chaotic Inflation: (most popular) (Linde 1983)

No phase transition $\downarrow$ ϕ fluctuates spatially

$$\boxed{V = \frac{1}{2} m^2 \phi^2}$$



ϕ at $t = t_i$ uniform on scale $\frac{1}{H(t_i)}$

$$\cancel{\ddot\phi} + 3H\dot\phi = -m^2\phi \quad \xrightarrow[\text{slow-rolling}]{} \quad 3H\dot\phi \simeq -m^2\phi \qquad (1)$$

$\ddot\phi \to 0$

$\dot\phi << m\phi$

$\ddot\phi << m^2\phi$

$$H^2 = \frac{4\pi G}{3}\rho_\phi = \frac{4\pi}{3}\left(\cancel{\frac{1}{2}\dot\phi^2} + m^2\phi^2\right)$$

($G = 1$; $\frac{1}{2}\dot\phi^2 \to 0$)

$$\downarrow$$

$$H = \sqrt{\frac{4\pi}{3}}\, m\phi \qquad (2)$$

$$(1)+(2) \to 3H\dot\phi = 3\sqrt{\frac{4\pi}{3}}\, m\cancel{\phi}\dot\phi = -m^2\cancel{\phi} \qquad \dot\phi = -\frac{m}{\sqrt{12\pi}} = \text{const}$$

$$\phi(t) = \phi_i + \dot\phi(t)\,t = \phi_i - \frac{m}{\sqrt{12\pi}}\cdot t \qquad \tau = \frac{\sqrt{12\pi}}{m}$$

$$\frac{\dot a}{a} = H = \sqrt{\frac{4\pi}{3}}\, m\left(\phi_i - \frac{t}{\tau}\right) \longrightarrow d\ln a = \sqrt{\frac{4\pi}{3}}\, m\left(\phi_i - \frac{t}{\tau}\right)dt$$

$$a(t) \simeq a_i\, e^{2\pi(\phi_i^2 - \phi^2)} \quad \xrightarrow[\phi \to 0]{} \quad \boxed{a_f \simeq a_i\, e^{2\pi\phi_i^2}}$$

details of calculation

$$\ln a\Big|_{a_i}^{a(t)} = \sqrt{\frac{4\pi}{3}}\, m\left(\phi_i t - \frac{t^2}{2\tau}\right) = \sqrt{\frac{4\pi}{3}}\, mt\left(\phi_i - \frac{t}{2\tau}\right) =$$

$$= 4\pi\frac{t}{\tau}\left(\phi_i + \frac{\phi - \phi_i}{2}\right) = 4\pi(\phi_i - \phi)\left(\frac{\phi_i + \phi}{2}\right) = 2\pi(\phi_i^2 - \phi^2)$$

c.v.d.

What is ϕ_i? $\rho_\phi \simeq +V(\phi) \simeq \frac{m^2}{2}\phi_i^2 \simeq \epsilon_{Pl} = 1$

natural units

$\phi_i \simeq \frac{1}{m}$ $m_{Pl} = 10^{-5} g$

If $m \ll 1$ $(m \ll m_{Pl})$ $\rightarrow \phi_i \gg 1 \rightarrow \frac{a_f}{a_i} \ggg 1$

$N = \ln\left(\frac{a_f}{a_i}\right) \simeq 2\pi\phi_i^2 \simeq \frac{1}{m^2}$

NOTE

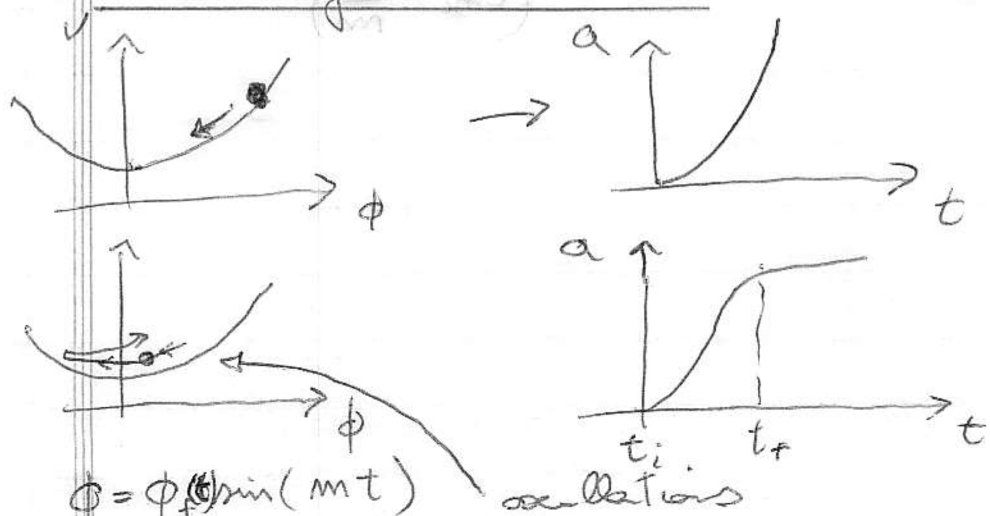
$a \propto e^{Ht}$ $H = \text{cst}$ $\longrightarrow$ de Sitter space-time

$\downarrow$

is static (steady-state universe)

$\left(\frac{\partial g_{ij}}{\partial t} = 0\right.$ for a particular gauge$\left.\right)$

- How inflation ends?



$\phi = \phi_f(t)\sin(mt)$ oscillations

$$\frac{d}{d\phi}\left(\frac{1}{2}m^2\phi^2\right) = m^2\phi$$

$$H^2 = \frac{4\pi}{3} m^2 \phi_f^2 \quad \Rightarrow \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$\boxed{\dot{\phi} = \phi_f(t)\, m \cos(mt) + \dot{\phi}_f \cancel{\sin(mt)}}$$

$$2\dot{\phi}_f\, m \cos mt + \phi_f\left(\cancel{-m^2 \sin mt}\right) + 3H\phi_f\, m \cos mt +$$

$$+ \cancel{m^2 \phi_f \sin mt} = 0$$

$$\dot{\phi}_f = -\frac{3}{2}\underbrace{H}_{\left(\frac{\dot{a}}{a}\right)}\phi_f \qquad \frac{d\phi_f}{\phi_f} \simeq -\frac{3}{2}\frac{da}{a} \qquad \phi_f \propto a^{-\frac{3}{2}}$$

$$\rho_\phi \propto \phi_f^2 \propto a^{-3} \quad \leftarrow \text{matter dominated}$$

$\Downarrow$

How does the universe switches to Rad dominated?

REHEATING $\rightarrow$ couples to other fields during oscillations

Coherent oscillations decay into other particles:
eg. Yukawa coupling to a bosonic particle

$\Rightarrow$ decay produce entropy and re-heat the universe

NOTE: $T_{reheat} < T_{GUT}$ but it has to be large enough to allow baryogenesys ($T_{rh} > T_{nuc}$)

Perturbations from inflation:

$\boxed{CL\ 13.6}$

$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ Slow rolling $\cancel{\ldots}_{3H}$ conditions:

1) $\dot{\phi} \simeq -\frac{V'}{3H}$

2) $\epsilon = \frac{m_p^2}{16\pi}\left(\frac{V'}{V}\right)^2 << 1$ $(V \gg \dot{\phi}^2)$

$H^2 = \frac{8\pi V}{3 m_p^2} \rightarrow a \propto \exp(Ht)$

3) $\eta = \frac{m_p^2 V''}{8\pi V}\left(|\eta| << 1\right)$

Perturbation: $\ddot{\phi}_k + 3H\dot{\phi}_k + \left[\left(\frac{k}{a}\right)^2 + V''\right]\phi_k = 0$

$\ddot{\phi} + 3H\dot{\phi}_k + \left(\frac{k}{a}\right)^2 \phi_k = 0$ 0 as perturbation exit Horizon

damped harmonic oscillator

$\langle|\phi_k|^2\rangle = \frac{H^2}{2k^3}$ (ground state of harmonic oscillator)

↳ similar to Hawking radiation $T_H = \frac{H}{2\pi}$
(de Sitter universe has event Horizon)

If $H \simeq$ constant $\Delta_\phi = P_\phi k^3 \sim H \sim$ constant

$\Delta_H^2(k) \simeq \frac{V^*}{m_p^4 \epsilon^*} \sim$ const $* =$ at Horizon entry

at Horizon entry ↖ $n_H \simeq n_s - 4$ $P_H \sim P k^{-4} \sim \Delta_H k^{-3} \sim k^{-3} \rightarrow \boxed{P \propto k^m}$

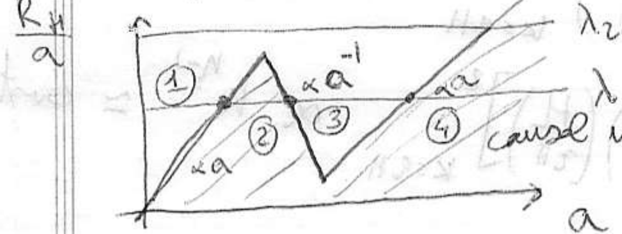
Perturbations from Inflation:

a) Qualitative approach:

$$R_H \simeq \frac{c}{H} \begin{cases} H = \frac{\dot a}{a} \sim \frac{1}{t} & \text{radiation dominated} \quad \underline{R_H \sim ct \propto a^2} \\ H = \text{const} = H_0\sqrt{\Omega_\Lambda} & \text{Inflation } (\Lambda \text{ dominated}) \quad \underline{R_H \sim \text{const}} \\ & (a \propto e^{t}) \end{cases}$$

not 0.7 (inflation Ω_Λ)

no causal interactions



causal interactions perturbations on comoving scale λ

- Quantum fluctuations freeze at exit horizon.

$$\langle|\delta\phi|^2\rangle_{\ln\frac{k}{H}} \simeq \frac{H^2}{2L^3k^3}$$

$$\left(\delta\phi\,(k=H) \simeq \frac{H(\phi)}{2\pi}\right) \quad \left(k = \frac{1}{\ell} \simeq \frac{H}{c} \simeq \frac{1}{r_H}\right)$$

$$P_\phi(k;t_*) = \left(\frac{H}{2\pi}\right)^2\Bigg]_{k=aH}$$

$$\left(\frac{\delta\rho}{\rho}\right)_{\ln\frac{k}{H}} \simeq 2\left(\frac{\delta\phi}{\phi}\right)_{k\sim H} \nearrow \approx 2\cdot P_\phi \qquad (\text{because } \rho_\phi \propto V \propto \phi^2)$$

$$= \frac{2H}{2\pi\phi} = \frac{2}{\sqrt{3\pi}}\, m \qquad \left(H^2 = \frac{4\pi}{3} m^2\phi^2 \rightarrow H = 2\sqrt{\frac{\pi}{3}}\, m\phi\right)$$

$$\left(\frac{\Delta T}{T}\right)_{CMB} \simeq \frac{\delta\rho}{\rho} \approx 10^{-5} \quad \Longrightarrow \quad m \sim 10^{-5} \ll 1$$

and galaxies!

Problem: fine-tuning

SKIP [Read Ch 7.5 Liddle]

Scale invariant spectrum and Grav. waves:

$$P_\phi(k) = \frac{8\pi}{9k^3}\frac{H^2}{\epsilon m_{pl}^2}\Bigg|_{aH=k} = \frac{50\pi^2}{9k^3}\left(\frac{k}{H_0}\right)^{n-1}\delta_H^2\left(\frac{\Omega_m}{D_1(a=1)}\right)^2$$

$$P_h(k) = \frac{8\pi}{k^3}\frac{H^2}{m_{pl}^2}\Bigg|_{aH=k} = A_T\, k^{n_T-3}$$

$$\frac{d\ln P_h}{d\ln k} = n_T - 3 \qquad n_T = 2\frac{d\ln H}{d\ln k}\Bigg|_{aH=k} = -2\frac{k}{H}\frac{aH^2\epsilon}{k^2}\Bigg|_{aH=k} = -2\epsilon$$

$$(\dot H = -aH^2\epsilon)$$

$$\frac{P_\phi}{P_h} \simeq \frac{1}{\epsilon} \simeq \frac{1}{n_T}$$

$$n_s - 1 = \frac{d\ln P_R}{d\ln k} = \frac{d}{d\ln k}\left[\underset{\substack{\downarrow\\ -4\epsilon}}{\ln H^2} - \ln(\epsilon)\right] = -4\epsilon - 2\delta$$

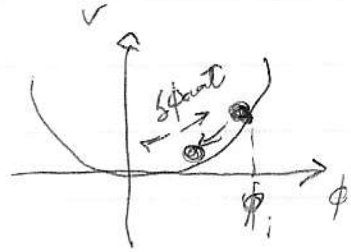
$$\boxed{\begin{aligned}&\frac{\Delta\rho}{\rho} = \frac{24}{5}\sqrt{\frac{\pi}{3}}\, m\,\phi_k^2\Bigg|_{k=H}\\ &k_{com} = \overbrace{\sqrt{\frac{4\pi}{3}}\, m\,\phi_k}^{H_k}\cdot\overbrace{a_f\, e^{-2\pi\phi_k^2}}^{a_k}\end{aligned}} \xrightarrow{\text{invert}} \phi_k(k; a_f)$$

$$k_{com} = a_k H_k$$

$$a_f = a_0\, e^{2\pi\phi_i^2} \qquad (\phi_f = 0) \qquad a_k = a_0\, e^{2\pi(\phi_i^2 - \phi_k^2)}$$

$$a_f = a_k\, e^{2\pi\phi_k^2} \rightarrow a_k = a_f\, e^{-2\pi\phi_k^2}$$

Eternal inflation: (chaotic Inflation)



$\delta\phi_{QUANT} > \delta\phi_{roll}$

$\downarrow$

recover initial conditions

$\downarrow$

new inflationary universe

Fraction of volume that recovers initial conditions is small but exp. expansion $\Rightarrow$ the volume increases with time.

Universe continuously undergoing inflation $\rightarrow$