Primordial Nucleosynthesis

- Summary of reactions:

1) $T \gtrsim T_{dec,\nu_e} \simeq 10^{10}\,K$ n and p in th. eq:

$$\begin{cases} n+\nu_e \rightleftarrows p+e^- \\ n+e^+ \rightleftarrows p+\bar{\nu}_e \end{cases}$$

$$\frac{n_n}{n_p}\left(\frac{1}{1+\frac{n_n}{n_p}}\right)$$

$$X_n = \frac{n_n}{n_n+n_p} = \frac{n_n}{n_{TOT}} \simeq 0.17 \qquad \frac{n_n}{n_p} \simeq \exp\left(-\frac{Q}{kT}\right) = \exp\left(-\frac{1.5\cdot 10^{10}\,K}{T}\right)$$

@ freeze-out

2) $n+p \rightleftarrows D+\gamma$

$$X_d \begin{cases} T \gtrsim 10^{10}\,K & X_d \simeq 0 \\ T \simeq 10^{9}\,K & X_d \neq 0 \end{cases} \qquad X_d \to 0.5 \text{ for } T \lesssim 10^9\,K$$

3) But D cannot accumulate because:

$$\begin{cases} D+D \Rightarrow {}^3He + n \\ D+D \Rightarrow T + p \end{cases} \qquad {}^3He + D \Rightarrow {}^4He + p \quad \text{not in equilib.}$$

→ are very fast!

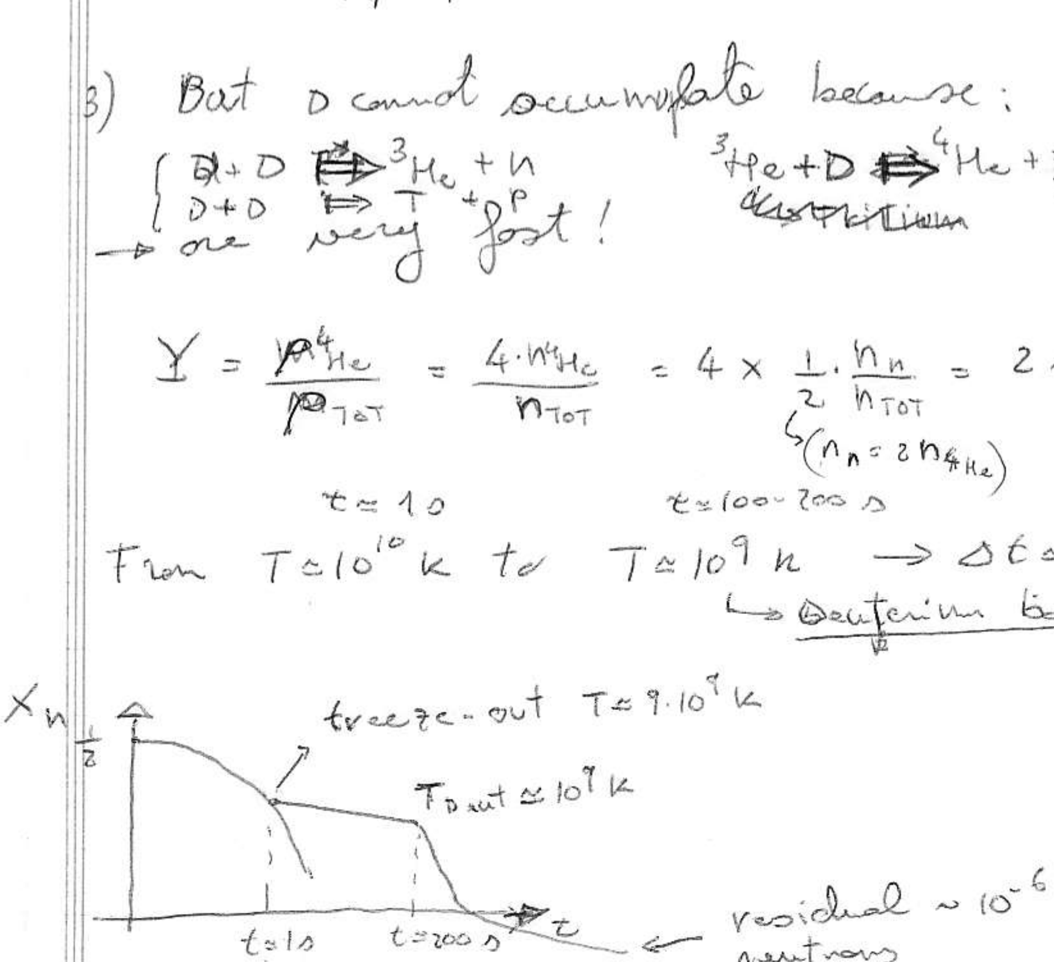
$$Y = \frac{\rho_{^4He}}{\rho_{TOT}} = \frac{4\cdot n_{^4He}}{n_{TOT}} = 4\times\frac{1}{2}\cdot\frac{n_n}{n_{TOT}} = 2\times X_n \simeq 2\cdot 0.17\cdot\overbrace{\exp\left(-\frac{\Delta t}{\tau_n}\right)}^{0.12}$$

$(n_n = 2n_{^4He})$ $\simeq 0.25$ τ_n: 900 s

$t \simeq 1\,s$ $t \simeq 100-200\,s$

From $T \simeq 10^{10}\,K$ to $T \simeq 10^{9}\,K$ $\longrightarrow \Delta t \simeq 100-200\,s$

↳ Deuterium bottle neck



Details:

1) Neutrons:

$\mu_n + \cancel{\mu_\nu} = \mu_p + \cancel{\mu_e}$ $\quad (n+\nu \rightleftharpoons p+e^-)$

$$\frac{n_n}{n_p} = \frac{X_n}{X_p} = e^{\left(-\frac{Q}{kT} + \cancel{\frac{\mu_e - \mu_\nu}{kT}}\right)}$$

where $Q = (m_n - m_p)c^2 = 1.293318$ MeV

$\left(\mu_\nu \simeq \mu_e \simeq 10^{-9} k_B T \qquad \text{we can neglect } \mu_\nu \text{ and } \mu_e\right)$

$$\left.\frac{X_n}{X_p}\right|_{eq} \simeq e^{-\frac{Q}{kT}} = \exp\left(-\frac{1.5\cdot 10^{10}\,K}{T}\right) \simeq 0.17 \text{ at } T \simeq 10^{10} K$$

$\boxed{T_{dec;\nu_e} \simeq 10^{10}\ K}$

$\Rightarrow$ β-decay: $\tau_\beta \simeq 900\,s$ $\quad$ $n \to p + e^- + \bar{\nu}_e$

$$\frac{X_n}{X_p} \simeq 0.17 \exp\left(-\frac{\Delta t}{\tau_\beta}\right) \simeq 0.17 \exp\left(-\frac{\Delta t}{900\,s}\right)$$

2) Neutron freeze-out: $1/t = \Gamma = 6.5\cdot 10^{-6}\, T_9^5$

where $T_9 = \frac{T}{10^9 K}$ $\Longrightarrow$ $t \simeq 1\,s\, \frac{\Theta}{T_9^2}$

where $\Theta = 3.3298\cdot 10^{20}\ \frac{10^{-18}(1+\Delta)}{g^{\frac{1}{2}}}$

At $T_9 < 1$ $e^- - e^+$ annihilation is complete:

$\Delta = 0 \qquad g = g_0 = 3.369 \quad \rightarrow \boxed{\Theta = 181.6}$

For $T_9 > 5$ $e^- - e^+$ annihilation has not started

$\Theta = 0$ $g = \frac{43}{4}$ $\Rightarrow \boxed{\Theta = 101.6}$

neutron freeze out $\Gamma \cdot t = 1 \Rightarrow$

[$T_9 = 9.4$ for $\Theta = 481.6$]

$6.5 \cdot 10^{-6}\, T_9^5 \cdot \frac{101.6}{T_9^2} = 1$ $T_9 = 9.1$ $(t = 1.2$ sec$)$

$T_9 = \left(\frac{1}{6.5 \cdot 10^{-6} \ldots}\right)^{\frac{1}{3}}$

before $e^+ - e^-$ annihilation

$\frac{X_n}{X_p} \simeq e^{-\frac{15}{9.1}} \simeq 0.19$ $\Rightarrow X_n = \frac{0.19}{1+0.19} = 0.16$

Deuterium:

$n + p \leftrightarrows D + \gamma$ $n_i = g_i \left(\frac{m_i kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\mu_i - m_i c^2}{k_B T}\right)$

$i = n; p; d$; $g_n = g_p = 2$ $g_D = 3$; $\mu_n + \mu_p = \mu_D$

$X_p = \frac{n_p}{n_{TOT}}$ $X_n = \frac{n_n}{n_{TOT}}$ $X_D = \frac{n_D}{n_{TOT}}$

$$X_D = \frac{3}{n_{TOT}} \left(\frac{m_D kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\mu_n + \mu_p - \overbrace{(m_n + m_p)c^2 + B_D}^{m_D c^2}}{k_B T}\right)$$

where B_D = binding energy $= (m_p + m_p - m_d)^2 c^2 = 2.225\,MeV =$
$= 2.5 \cdot 10^{10}\,K$

$$X_D = n_{TOT} \left(\frac{m_D}{m_n m_p}\right)^{3/2} \frac{3}{4} \left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} X_n X_p \exp\left(\frac{B_D}{k_B T}\right)$$

$$X_D = X_n X_p \exp\left[-29.33 + \frac{25.82}{T_9} - \frac{3}{2}\ln T_9 + \ln(\Omega_{0b} h^2)\right]$$

Depends weakly on $\Omega_b h^2$:

For $T_9^* \simeq 0.8$ for $\Omega_b h \simeq 0.03$ ($t \sim 200s$) $\rightarrow X_d = X_n X_p$

For $T_9 < T_9^*$ all neutrons captured to form D.

See figure : Y = const $\simeq 0.24$

NOTE

- if η smaller Y decreases because ^{4}He nucleosynth starts later and is more incomplete; D or He increase to compensate the decrease in ^{4}He.

Note: $$\eta = \frac{n_{baryons}(z=0)}{n_\gamma(z=0)} = \frac{1.126\cdot 10^{-5}\,\Omega_b h^2\,cm^{-3}}{411.8\,cm^{-3}}$$

$$\underline{\eta = 2.734\cdot 10^{-8}\cdot\Omega_b h^2}$$

Helium: ($d+d \rightarrow {}^3He + n$ $\quad {}^3He + d \rightarrow {}^4He + p$)

$$\dot{X}_\alpha = \rho_B X_D^2 \left(\frac{{}^3He}{\ldots\ldots} + \frac{\text{Tritium}}{\ldots\ldots} \right)$$

$$\dot{X}_\alpha = 6.5\cdot 10^{-20}\,s^{-1}\; X_n^2 X_p^2 \left(\Omega_b h^2\right)^3 T_9^{17/3}\, e^{\frac{51.64}{T_9}}\, e^{-\frac{4.258}{T_9^{1/2}}}$$

Freeze out point :

$$\dot{X}_\alpha \simeq \frac{X_n}{\tau_n} \implies T_9^* = \frac{51.64}{40.27 + \frac{4.258}{T_9^{1/3}} + \frac{0.205}{T_9^2} + \frac{17}{3}\ln T_9 - 3\ln(\Omega_b h^2)}$$

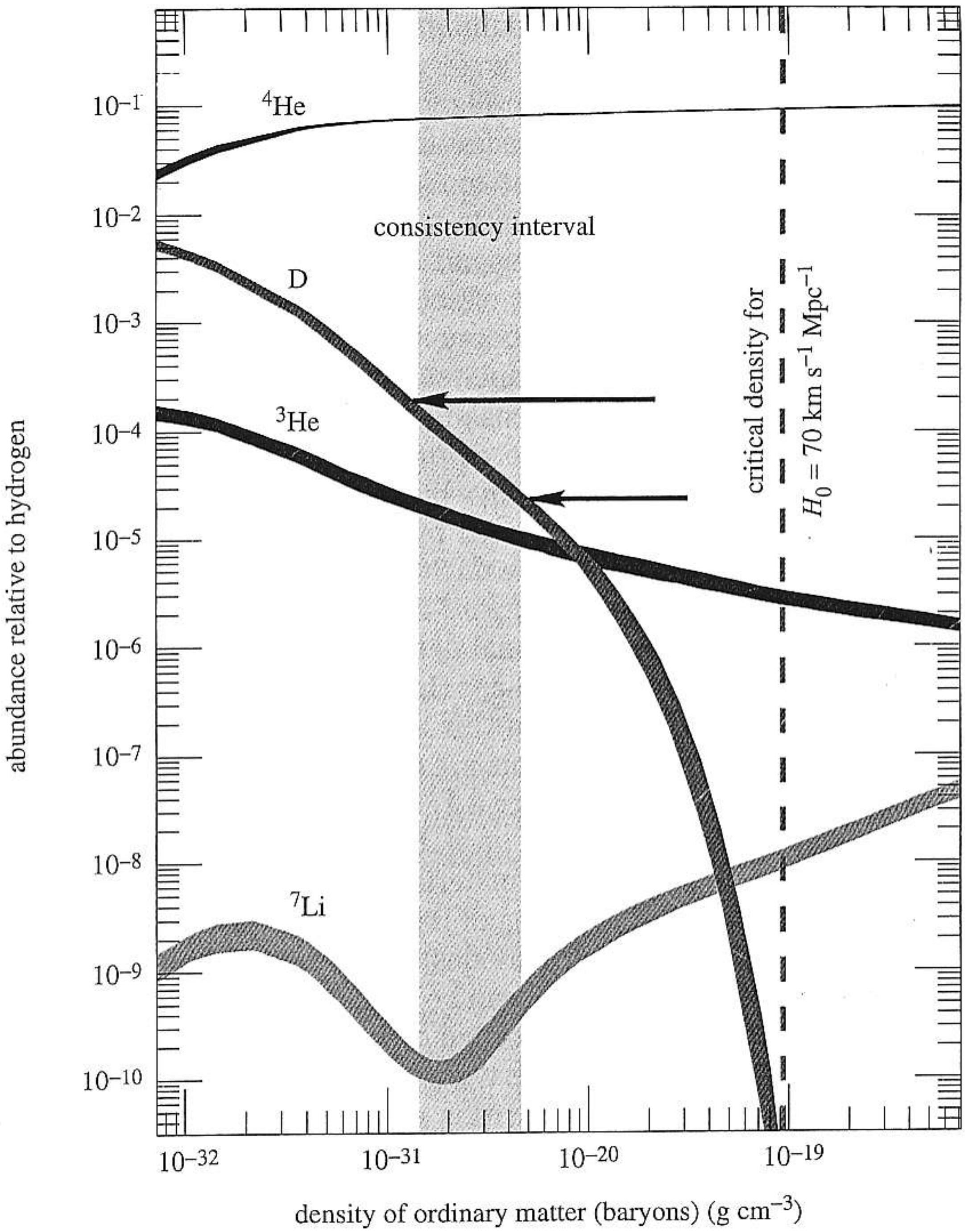


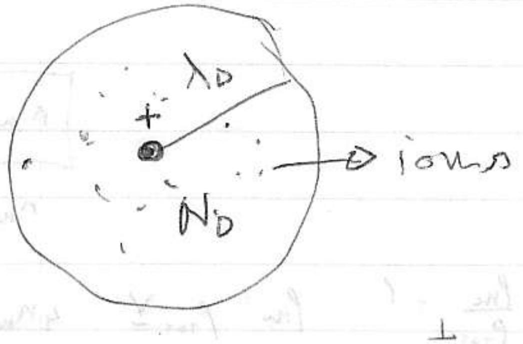
Figure 8.1 Light-element abundance determined by numerical calculations as functions of the matter density, as explained in the text. The arrows mark the possible deuterium abundance. From Schramm and Turner (1996). Picture courtesy of Mike Turner.

8.6.5 Other elements

As far as the abundances of other light elements are concerned one needs to perform a detailed numerical integration of all the rate equations describing the reaction network involved in building up heavier nuclei than ^{4}He. We have no space to discuss the details of these calculations here, but the main results are illustrated in Figure 8.1.

(12) Plasma era, equality, recombination, temperature decoupling

Plasma: energy 2-body interactions << Thermal energy



$N_{ions,D} \gg 1$ plasma.

$N_D = \frac{4\pi}{3} n_e \lambda_D^3 = 1.8 \cdot 10^6 (\Omega_b h^2)^{-\frac{1}{2}}$

$\lambda_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}}$

$\lambda = n_e^{-\frac{1}{3}}$ $\quad \frac{\lambda_D}{\lambda} = 10^2 (\Omega_b h^2)^{-\frac{1}{6}} \gg 1$

$t_{pp} \approx t_{ee} \simeq t_{ep} \simeq t_{e\gamma} \ll t_H$ $\quad \boxed{T_e = T_p = T}$

(screening-effects are negligible)

The Plasma era:

1) Radiative era:

from $e^+ - e^-$ annihilation at $T_e \sim 5 \cdot 10^9 K$ (0.5 MeV)
$\tau \simeq 10$ sec

$$\rho_{\gamma,\nu} = \rho_{0\gamma}^{K_0} \left(\frac{T}{T_{\gamma 0}}\right)^4 = \rho_{0\gamma} K_0 (1+z)^4 \qquad K_0 = (1 + 0.227 N_\nu) = 1.68 \quad (N_\nu = 3)$$

relativistic particles

EQUALITY
~~EQUIVALENCE~~

$$\rho_m = \rho_{0c} \Omega_{0m} (1+z)^3 \qquad \underline{\underline{\rho_m = \rho_{\gamma,\nu}}} \text{ at}$$

$$1 + z_{eq} = \frac{\rho_{0c}\Omega_{0m}}{K_0 \rho_{0\gamma}} = \frac{4.3}{K_0} \times 10^4 \Omega_0 h^2$$

$$K_0 = 1.68 \quad \Omega_0 h^2 \simeq 0.15 \rightarrow 1 + z_{eq} \simeq 3{,}839$$

$$T_{eq} \simeq 10^5 \frac{\Omega_0 h^2}{K_0} K = 8{,}928 K$$

2) ~~Recombination~~ Hydrogen Recombination:

a) He already recombined.

Particles
He; p; e; γ

Ionization fraction $x_e = \frac{n_e}{n_p + n_{H0}} = \frac{n_e}{n_{TOT}}$

is regulated by the Saha formula for a plasma in thermal equilibrium:

$$n_e = n_p \qquad n_{HI} + \overset{n_e}{n_p} \xrightarrow{} (+n_{He}) \rightarrow 0.06\, n_{TOT} \qquad = n_{TOT,H} \qquad n_{HI} = n_{TOT,H} - n_e$$

$$\frac{n_e n_p}{n_H n_{TOT}} = \frac{n_e^2}{(n_{TOT}-n_e)n_{TOT}} = \frac{x_e^2}{1-x_e} = \frac{1}{n_{TOT}}\left(\frac{m_e k T}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{I_H}{k_B T}\right)$$

$I_H = 13.6$ eV = neutral Hydrogen binding energy.

$$\frac{I_H}{k_B} = 1.58 \cdot 10^5 \text{ K}$$

$$\frac{x_e^2}{1-x_e} = \frac{1}{n_{TOT}} \cdot 2.4 \cdot 10^{21} \text{ cm}^{-3}\; T_4^{3/2}\, e^{-\frac{15.8}{T_4}} \qquad \left(T_4 = \frac{T}{10^4 \text{ K}}\right)$$

$$n_{TOT} = \frac{\Omega_b \rho_{crit}}{\mu}(1+z)^3 \qquad \mu = \frac{n_{H} m_p + 4 n_{He} m_p}{n_{TOT}} \approx \overset{1.25}{} m_p$$

$$\begin{cases} n_{e,eq} = 3.2 \cdot 10^{13} \text{ cm}^{-3}\; T_4^{9/4}\, e^{-\frac{7.9}{T_4}} (\Omega_b h^2)^{\frac{1}{2}} \\ n_{H,eq} = 4.2 \cdot 10^5 (\Omega_b h^2) \text{ cm}^{-3}\; T_4^3 \end{cases}$$

Approximate solution → freeze-out: recombination

Ionization balance: $\frac{dn_e a^3}{dt} = \frac{dn_e}{dt} + 3Hn_p = \underbrace{n_H \Gamma}_{\text{coll. ionization}} - \underbrace{R n_e n_p}_{\text{recombination}}$

$n_p = n_e$ neutrality $\qquad \frac{dn_e}{dt} + 3Hn_e = n_H\Gamma - Rn_e^2$

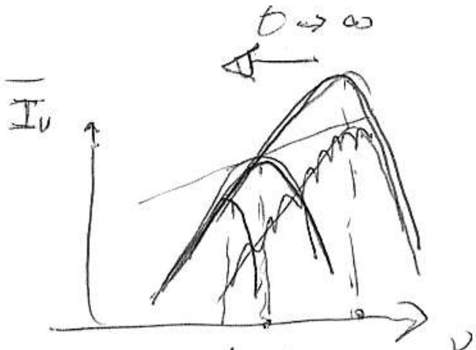
at equilibrium: $n_H \Gamma = R n_{e,eq}^2$

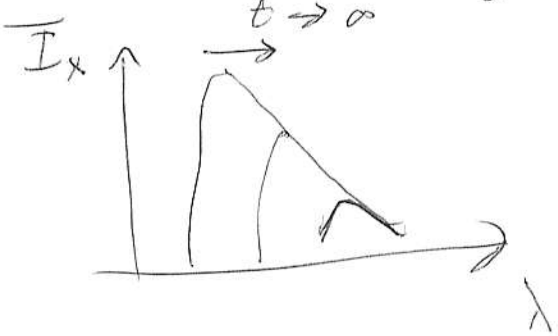
$$\frac{dn_e}{dt} + 3Hn_e = R\left(n_{e,eq}^2 - n_e^2\right)$$

$$\nu = \nu_i \cdot \frac{a_i}{a} \qquad \cancel{\quad} N_\nu = \frac{1}{e^{\frac{h\nu}{kT}}-1} = \frac{1}{e^{\frac{h\nu_i}{kT_i}}-1} \quad \Big| \text{ when } T = T_i \frac{a_i}{a}$$

$$I_\nu(T) = \frac{2h\nu^3}{c} N_\nu = \frac{2h\nu_i^3}{c}\left(\frac{a_i}{a}\right)^3 N_{\nu i} = \left(\frac{a_i}{a}\right)^3 I_{\nu_i}(T_i)$$

$$a^3 I_\nu(T) = a_i^3 I_\nu(T_i) \qquad \boxed{\bar{I}_\nu(T) = \bar{I}_\nu(T_i)}$$





Freeze-out temperature:

$\tau(T_f) \cdot H(T_f) = 1$

$\tau_{rec} \simeq R\, n_{e,eq}$ with $R = 4.3 \cdot 10^{-13}\, T_4^{-0.7}\ \mathrm{cm^3/s}$

$\Rightarrow$ we know $n_{e,eq}$ from Saha equation:

$t_{Rec} \simeq 13.8\ \mathrm{s}^{-1}\, T_4^{1.55}\, e^{-\frac{7.9}{T_4}} (\Omega_b h^2)^{\frac{1}{2}}$

In the matter-dominated era: $H = \frac{2}{3}\frac{1}{t}$

$t = 2.054 \cdot 10^{17}\,\mathrm{s}\, (\Omega_0 h^2)^{-\frac{1}{2}}\, a^{\frac{3}{2}} = 9.3 \cdot 10^{11}\,\mathrm{s}\, (\Omega_0 h^2)^{-\frac{1}{2}}\, T_4^{-\frac{3}{2}}$

$\frac{2}{3}\frac{\tau}{t} = 1 \Rightarrow T_{4,f} = \dfrac{7.9}{30.6 + \frac{1}{2}\ln\frac{\Omega_b}{\Omega_0} + 0.05 \ln T_{4,f}}$

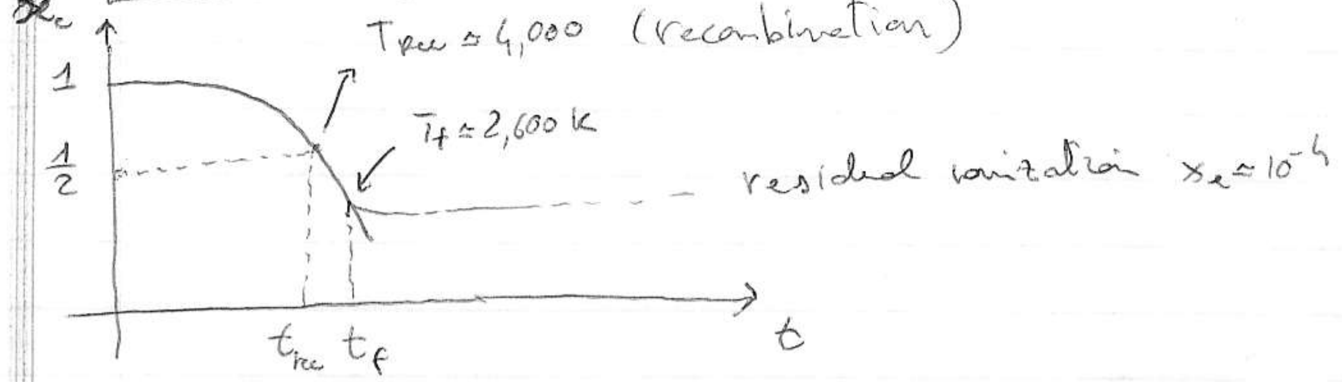
$\Rightarrow x_e(t \to \infty) = \dfrac{n_e(t\to\infty)}{n_{TOT}} = \dfrac{n_{e,eq}(T_f)}{n_{TOT}} = 4 \cdot 10^{-6}\, \dfrac{\Omega_0^{\frac{1}{2}}}{\Omega_b h^2}\, T_{4,f}^{-0.8}$

Plugging in the numbers we get: $\Omega_0 = 0.3\quad \Omega_b h^2 = 0.02$

$T_f = 2{,}600\ \mathrm{K} \Rightarrow z_f = 950\ ;\ x_e \simeq 10^{-4}$

However if we define recombination ~~is defined~~ when

$$\boxed{x_e \simeq x_H = \frac{1}{2}}$$



$1400 \lesssim z_{rec} \lesssim 1600 \qquad \rightarrow z_{rec} \simeq 1500$

$\qquad\qquad\qquad\qquad\qquad\qquad z_f \simeq 1000$

■ Evolution of the CMB spectrum:

$$I(t_i; \nu) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT_i}} - 1} \qquad \text{black-body}$$

the spectrum evolve with redshift : $\boxed{T = T_i \frac{a(t_i)}{a(t)}}$

why?

■ Maintains the same shape:

$\nu \propto a^{-1}$ (redshift) $\qquad \left[\nu = \nu_i \cdot \frac{a_i}{a}\right]$

$$N_\nu = \left(e^{\frac{h\nu}{kT}} - 1\right)^{-1} = \text{const} \qquad T \propto a^{-1}$$

covering

$$\bar{I}(t; \bar{\nu}) = I(t; \nu) \cdot a^3 = \text{const} \qquad \text{Vol} \propto a^3$$

• Note for non-relativistic particles that expand adiabatically similar arguments imply :

$$\boxed{T \propto a^{-2}}$$

non-relativistic particles:
$N_p \propto \exp\left(-\frac{\epsilon_p}{kT}\right) = \text{const}$

$\frac{\epsilon}{kT} \propto \frac{p^2}{kT} \propto \frac{a^{-2}}{T} \Rightarrow T \propto a^{-2}$

number ~~density~~ $\propto \exp\left(-\frac{\epsilon}{kT}\right)$

momentum $p \propto a^{-1}$

(analogy to photons $\epsilon = h\nu = pc$, $p = h\frac{\nu}{c} = \frac{h}{\lambda}$)

• Decoupling :

After recombination $x_e \simeq 1.2\cdot 10^{-5}\,\frac{\Omega_0^{1/2}}{\Omega_b h} \simeq 2\cdot 10^{-4}$

$\Rightarrow$ plasma interacts with CMB photons by Compton-scattering

$$\frac{dT}{dt} \sim \frac{\Delta E}{t_{compt}} \approx -\frac{4}{3} x_e c \sigma_T \frac{U_{ph}}{m_e c^2}\cdot \Delta E \qquad \Delta E = k\Delta T$$

220 Myr

$$t_{compt} = \frac{3}{4} x_e^{-1} \frac{m_e c}{\sigma_T a T^4_{CMB}} = (2.2\cdot 10^{8}\ \text{yr})\left(\frac{x_e}{10^{-4}}\right)^{-1}\left(\frac{100}{1+z}\right)^{+4}$$

$$t_H = \frac{1}{H_0}\frac{1}{\sqrt{\Omega_m}}(1+z)^{-\frac{3}{2}} \simeq 17\ \text{Myr}\left(\frac{h^2\Omega_m}{0.15}\right)^{-\frac{1}{2}}\left(\frac{1+z}{100}\right)^{-\frac{3}{2}}$$

$$t_{compt} = t_H \rightarrow 1+z = 8.8 \cdot \left(\frac{h}{x_e}\right)^{\frac{2}{5}} \Rightarrow$$

$h = 0.7$; $x_e = 2 \cdot 10^{-4}$ $\qquad 1+z_{dec} \cong 200$

[More precisely using $x_e = 1.2 \cdot 10^{-5} \frac{\Omega_0^{\frac{1}{2}}}{\Omega_b h}$

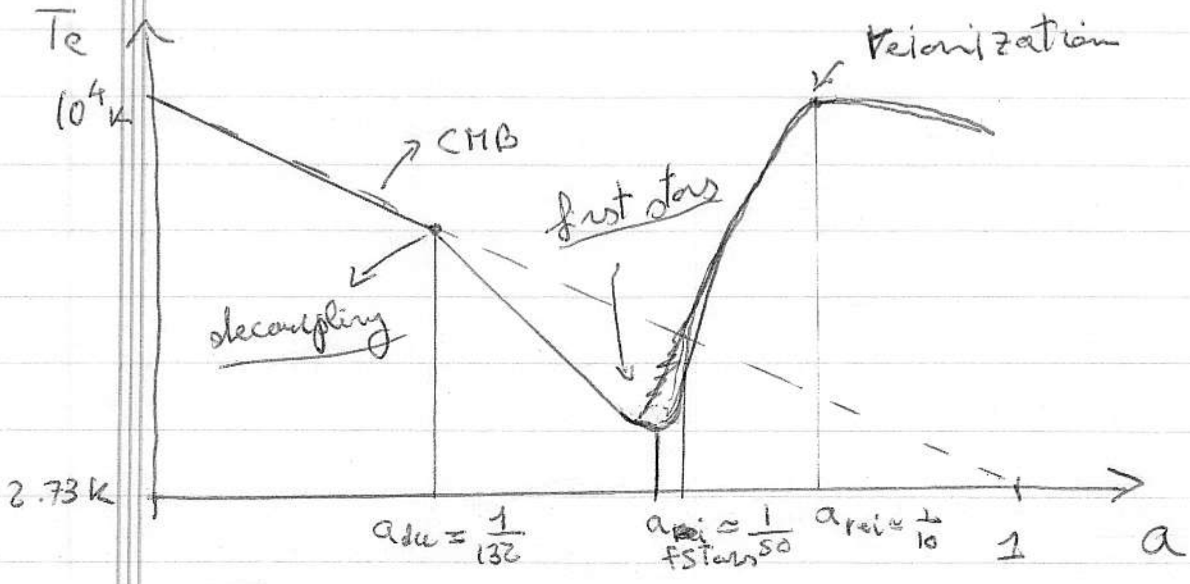
$$1+z_{dec} = 132 \left(\frac{\Omega_b h^2}{0.02}\right)^{\frac{2}{5}}$$]

$$\begin{cases} T_e = T_{CMB} \cong \dfrac{2.73\,K}{a} & z > z_{dec} \\ T_e = \dfrac{2.73\,K}{a^2} \cdot a_{dec} & z < z_{dec} \end{cases}$$

↳ adiabatic expansion

Accurate approximation: (better than 1%)

$$T_e = \frac{2.73\,K}{a} \frac{a_{dec}}{(a^\beta + a_{dec}^\beta)^{\frac{1}{\beta}}} \qquad \beta = 1.72$$



(13) CMB spectrum + distortions

CMB Spectrum distorsions:

Photons interact with matter:

1) Compton scattering $e^- + \gamma \rightarrow e^- + \gamma$
2) Free-Free / Bremsstrahlung $e^- + p \rightarrow e^- + p + \gamma$
3) Ionization / recombination $e^- + p \rightleftarrows H_0 + \gamma$

1) does not create new photons!
2) and 3) create new photons!

μ - distorsion:

Event at $10^4 K < T < 10^7 K$ that inject energy into the plasma: $(\underline{z > z_{eq}})$ $\underline{T_{eq} \sim 9000\,K}$

$\Delta E = \epsilon_{Rad} \cdot \delta \quad \rightarrow \quad \underline{T_e > T}$

Phase I) Compton scattering (does not add photons.)

$\epsilon_R' = \epsilon_R (1+\delta) \qquad n_\gamma' = n_\gamma$

$$I'(t,\nu) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu+\mu}{kT}} - 1}$$

μ = chemical potential

Compton equilibrium spectrum

$$\left[\ \epsilon_R = \frac{4\pi}{c}\int_0^\infty I_\nu d\nu\ \right]$$

$$\epsilon_R' = 2\,\frac{4\pi}{h^3c^3}\int_0^\infty \frac{(h\nu)^3\,d(h\nu)}{e^{\frac{h\nu+\mu}{kT'}}-1} \simeq a_R T'^4\left(1-1.11\frac{\mu}{kT'}\right)$$

$$\left(\text{if } \mu \ll kT'\right)$$

eq. (1) $$n_\gamma' = 2\,\frac{4\pi}{h^3c^3}\int_0^\infty \frac{(h\nu)^2\,(dh\nu)}{e^{\frac{h\nu+\mu}{kT'}}-1} = \frac{30\,\zeta(3)}{\pi^4}\cdot\frac{a_R T'^3}{k_B}\left(1-1.37\frac{\mu}{kT'}\right)$$

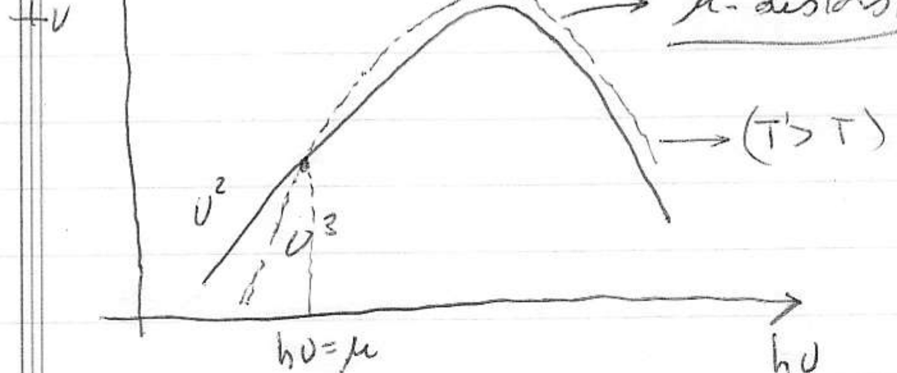
$$\left[T = T'(\dots)^{\frac{1}{3}}\right]$$

since $n_\gamma' = n_\gamma \Rightarrow T^3 = T'^3\left(1-1.37\frac{\mu}{kT'}\right)$

$$T' = T\left(1+0.46\frac{\mu}{kT}\right) \Rightarrow \mathcal{E}_R' = \mathcal{E}_R\left(1+0.72\frac{\mu}{kT}\right)$$

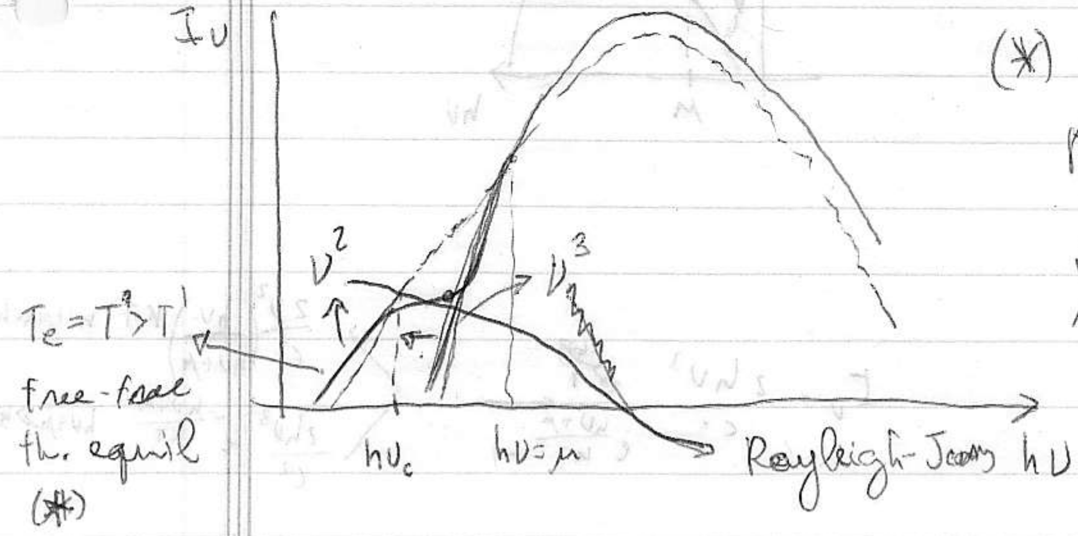
$$\boxed{\delta = \frac{\Delta E}{\epsilon_R} = 0.72\,\frac{\mu}{kT}}$$

also $T_e = T\left(1+\frac{3}{2}\delta\right) = \left(1+1.08\frac{\mu}{kT}\right) > T'$ $\boxed{T_e > T'}$



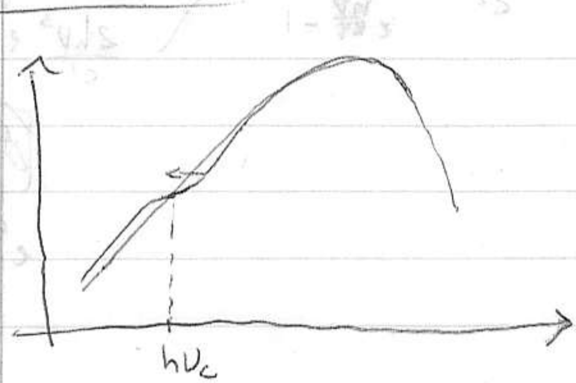
$T_e > T' \rightarrow$

Phase II: Free-Free abs/emission try to establish full thermodynamic eq. [ie, Plank spectrum (with $\mu = 0$)]



(*) constant flux of photons from plasma into radiation
↓
reduction of μ see eq (1)
until $\mu = h\nu_c$

Phase III: back to Planck with $T_f = T(1 + \frac{1}{4}\delta)$



ν_c = frequency at which free-free = compton scattering

$\mu \approx h\nu_c$

Conclusion:

Energy injection after $T = 10^7$ K ($T < 10^7$ K) would be preserved up to now in the form of a μ-distorsion. For $T > 10^7$ K distorsions of the order $\frac{\mu}{kT} < 10^{-4}$ would be erased.

Gas era: late energy injection:

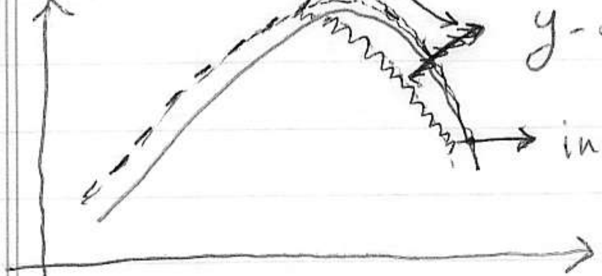
after ~~recombination~~ equality $(T < 10^4\,K)$.

perfect thermodynamic (between e^- and γ) equilibrium cannot be estabilished:

y - parameter:

$$y = \int_0^{t_0} \frac{k\,\Delta T}{m_e c^2} \cdot \sigma_T\, n_e\, c\,dt$$

$$\epsilon_R = \epsilon_R(y=0) \cdot e^{4y}$$



y - distorsion

initial Planck spectrum.

$\Omega_m = 1$ $\qquad t = \frac{2}{3}\frac{1}{H_0}\,a^{\frac{3}{2}}$

$$n_e = x_e \cdot 1.126 \cdot 10^{-5}\ cm^{-3}\ (\Omega_b h^2)\, a^{-3}$$

$$y = 1.027 \cdot 10^{-7} (\Omega_b h) \int_0^1 \underbrace{\left(T_4 - \frac{2.73 \cdot 10^{-4}}{a} \right)}_{\Delta T} x_e \cdot \frac{da}{a^{\frac{5}{2}}}$$

$$T_4 = \frac{T_e}{10^4}$$

Observations of CMB spectrum:

FIRAS on COBE satellite

$T = 2.728 \pm 0.004$ (95% CL)

Spectral distorsions:

$\frac{\mu}{kT_0} = (-1 \pm 4) \cdot 10^{-5}$ $\left|\frac{\mu}{kT}\right| < 9 \cdot 10^{-5}$ 95% CL

$y = (1 \pm 6) \cdot 10^{-6}$ $|y| < 1.5 \cdot 10^{-5}$ 95% CL

NOTE: only distorsions with $\mu < 10^{-4}$ can be smoothed after $e^- - e^+$ annihilation.

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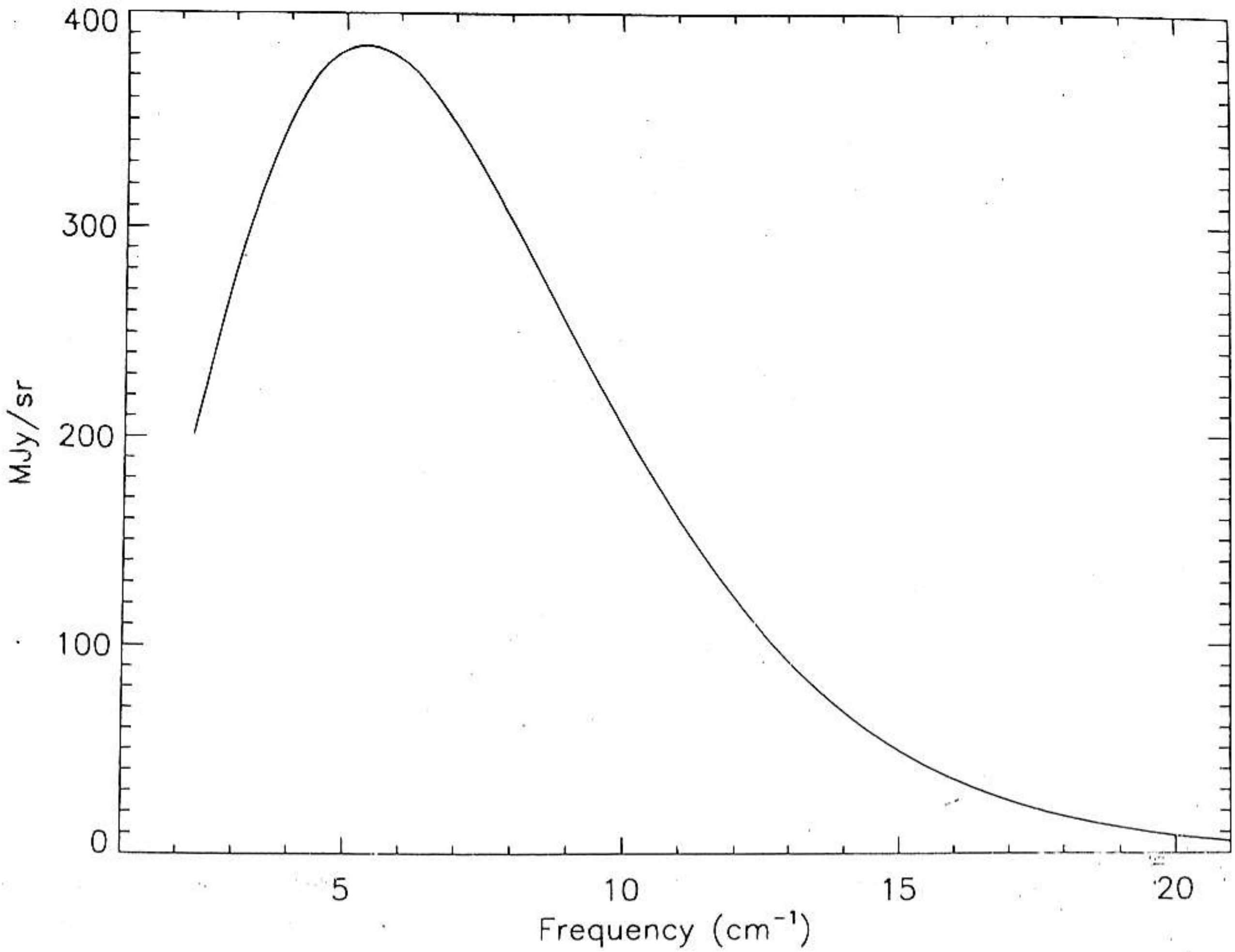

FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

(from Fixsen et al. 1996, ApJ, 473, 576)

Error-bars on this plot ARE SHOWN !

The size of the error-bar on this plot is

$6 \mu m = 6 \cdot 10^{-3}$ MM.

If you blow one tick space on the vertical axis to the size of a room, the error-bar would have the size of a quater!

(152A)

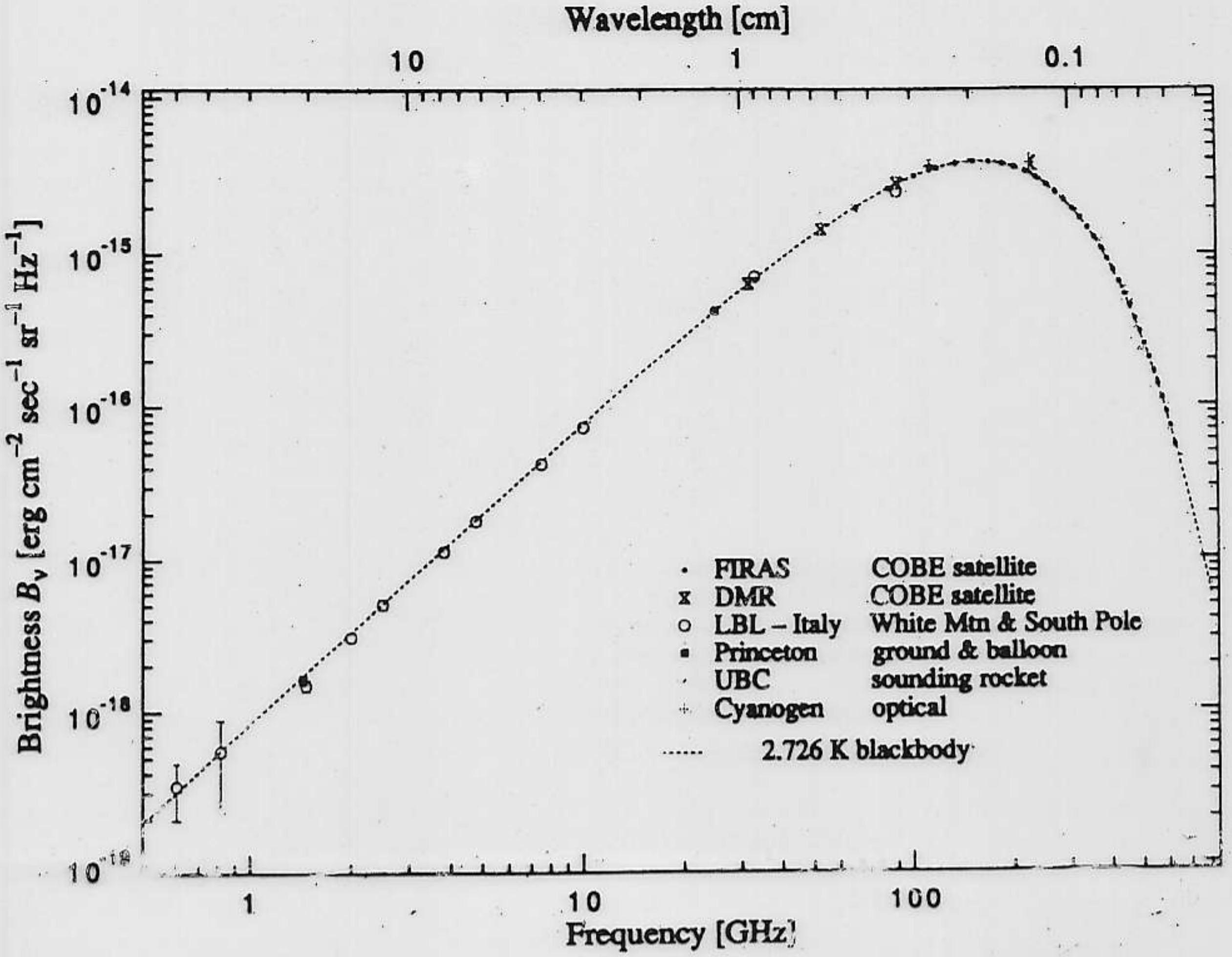

Figure 9.1 The spectrum of the cosmic microwave background as measured by the FIRAS instrument on the COBE satellite. The best–fitting black body spectrum has $T = 2.726 \pm 0.010$ K (95% confidence). Picture courtesy of George Smoot.

(from Coles & Lucchin)

54B

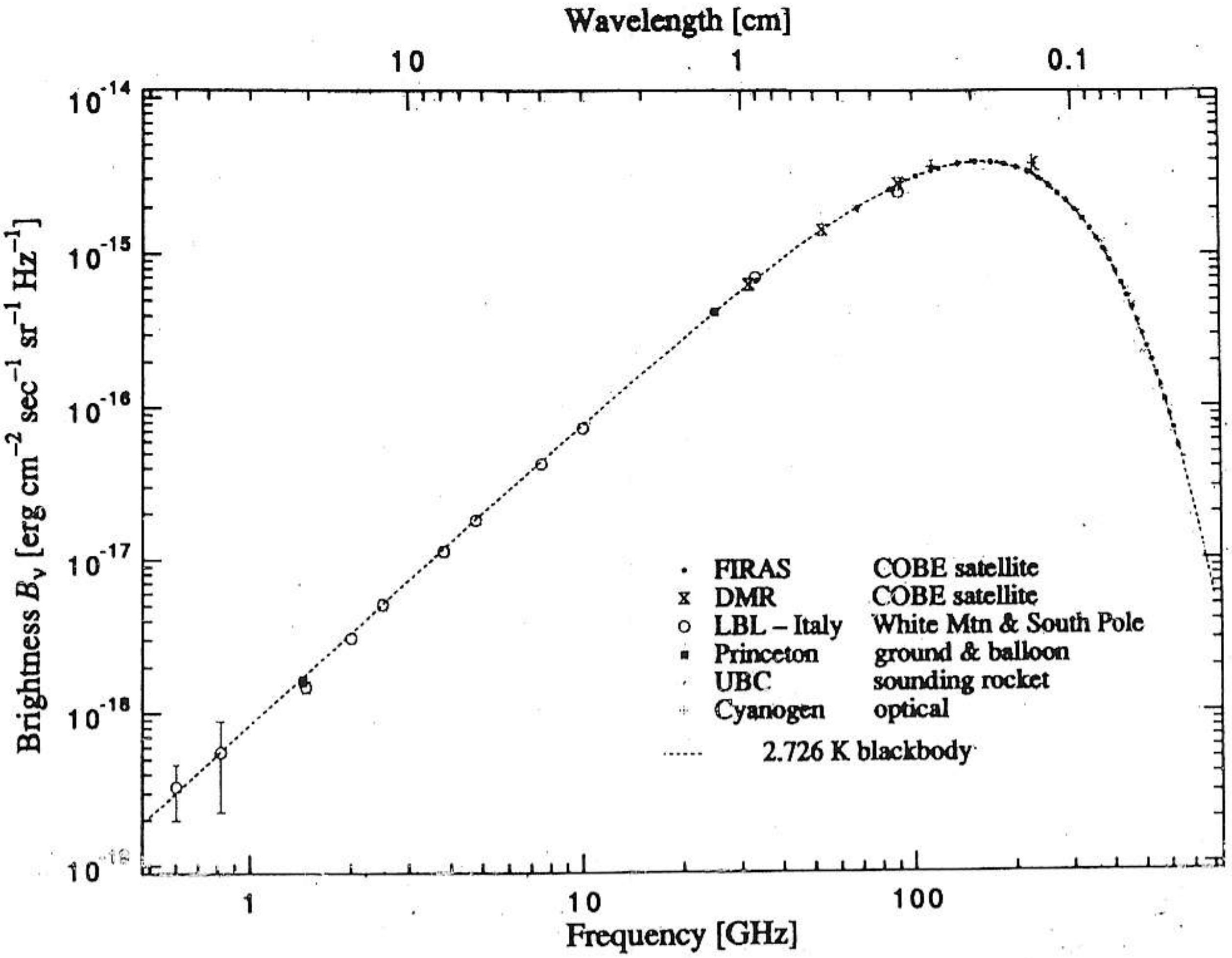

Figure 9.1 The spectrum of the cosmic microwave background as measured by the FIRAS instrument on the COBE satellite. The best-fitting black body spectrum has $T = 2.726 \pm 0.010$ K (95% confidence). Picture courtesy of George Smoot.

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