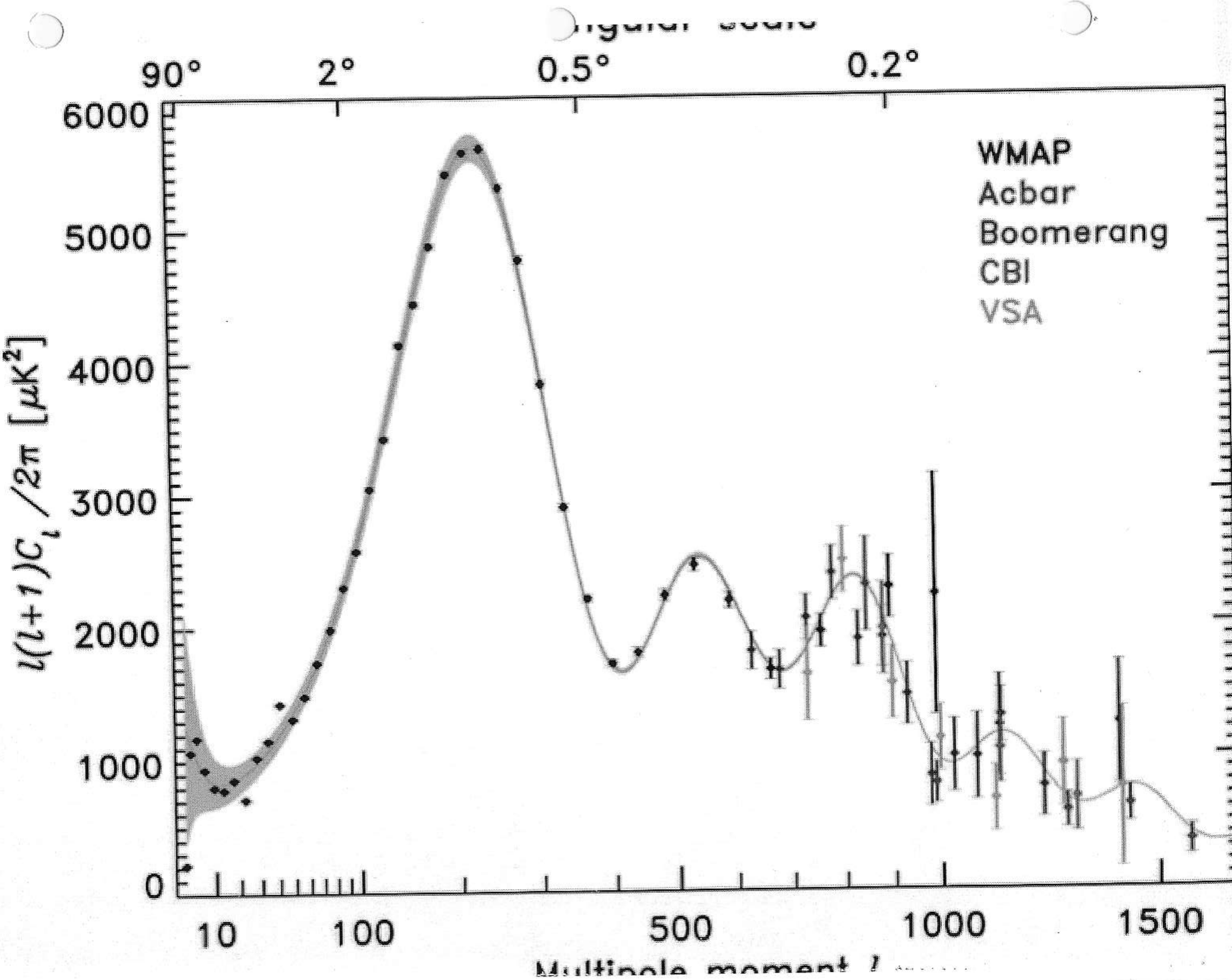

Angular scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ [μK^2]
WMAP
Acbar
Boomerang
CBI
VSA
10
100
500
1000
1500
Multipole moment l

Cosmic Microwave Background: Anisotropies

Anisotropies in the specific intensity are directly related to temperature fluctuations at this wavelengths $\Rightarrow$ detector measures the "brightness temperature".

Fluctuations of the temperature are expanded in a series of spherical harmonics $Y_{\ell m}(\theta;\varphi)$:

$$\frac{\Delta T}{T}(\theta;\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta;\varphi)$$

We have $\boxed{\theta \simeq \frac{180°}{\ell}}$ and $\langle a^*_{\ell' m'} a_{\ell m} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}$

The angular power spectrum is $\underline{C_\ell = \langle |a_{\ell m}|^2 \rangle}$.

Cosmic variance:

$$C_\ell(\text{obs}) = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 = \langle |a_{\ell m}|^2 \rangle$$

Poisson noise for small ℓ: $\frac{\Delta C_\ell}{C_\ell} \simeq \frac{1}{\sqrt{2\ell+1}}$

for $\ell \sim 200$ $\frac{\Delta C_\ell}{C_\ell} \sim 5\%$ only from cosmic variance.

The error can be reduced by averaging over $\ell = \ell \pm \Delta\ell$.

over a few neighboring l : $\bar{C}_l = \frac{1}{2\Delta l+1} \sum_{l'=l-\Delta l}^{l+\Delta l} C_{l'}$

Let's consider a few first l values:

$(T \sim 10^5 \Delta T)$ $l=0$ (monopole) is just the average temperature

$\left(\frac{\Delta T}{T} \simeq 10^{-3}\right)$ $l=1$ (dipole) is generally attributed to the motion of the detector with respect to the CMB. $\frac{\Delta T}{T} \simeq 10^{-3}$

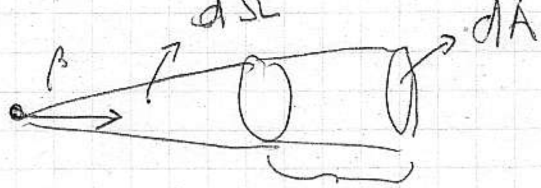
Not a simple doppler effect: doppler effect increases energy of photons by ΔE $(1+\beta\cos\theta)$ with $\beta = \frac{v}{c} \simeq 10^{-3}$
but $\Delta T \propto \frac{\Delta E}{\Delta \nu}$ and also $\Delta\nu \propto (1+\beta\cos\theta)$ $\Rightarrow$
ΔT does not change. Two other effects:
1) moving observer sweeps up more photons $T' = T(1+\beta\cos\theta)$
2) aberration: solid angle for moving observers decreases by $(1+\beta\cos\theta)^{-2}$ $\Rightarrow$ Intensity increases by
$d\Omega' \propto (1+\beta\cos\theta)^2$.

Net effect : $I'(\nu') = (1+\beta\cos\theta)^3 I(\nu)$ $\Rightarrow$ $I(\nu) \propto \frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}$

$\nu' = (1+\beta\cos\theta)\,\nu$ $\Rightarrow$ $T(\theta) = T_0(1+\beta\cos\theta)$

$v_{earth} = 390 \pm 30$ km/s $\Rightarrow$ $v_{LG} = 600$ km/s in the direction of Hydra-Centaurus ($l = 268°$ $b = 27°$)

Dipole:



$$d\Omega' = \frac{d\Omega}{(1+\beta\cos\theta)^2}$$

$$c\,dt + \beta\cos\theta \cdot c\,dt$$
$$= c\,dt\,(1+\beta\cos\theta)$$

$$I' = \frac{\langle n_\nu \rangle\, h\nu'\, c\,dt\,(1+\beta\cos\theta)\cdot \cancel{dA}}{\cancel{dt'}\, d\nu'\, \cancel{dA'}\, d\Omega'}$$

$$dt \sim dt'$$
$$dA \sim dA'$$

$$I' = \frac{\langle n_\nu \rangle\; h\nu' \cdot c}{d\nu'}\;\frac{(1+\beta\cos\theta)^3}{d\Omega} = \frac{\langle n_\nu \rangle\, h\nu\, c}{d\nu} \cdot \frac{(1+\beta\cos\theta)^3}{d\Omega}$$

$$\begin{cases} \nu' = \nu\,(1+\beta\cos\theta) \\ d\nu' = d\nu\,(1+\beta\cos\theta) \end{cases}$$

$$\langle n_\nu \rangle = \frac{2\nu^2\, d\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$I' = I\,(1+\beta\cos\theta)^3 = \frac{2h\nu^3(1+\beta\cos\theta)^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$I' = \frac{2h\nu'^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu'}{kT(1+\beta\cos\theta)}} - 1}$$

$$T' = T\,(1+\beta\cos\theta)$$

c.v.d.

$\ell = 2$ (quadrupole): first mode of cosmological interest. Contamination from the galaxy.

$\ell \geq 2$ **Sachs-Wolfe effect:**

a) 

$$\frac{\Delta T}{T} \simeq \frac{\delta\varphi}{c^2}$$

effect of grav redshift

b) Time dilation: $\Delta t \to \delta a$

$(T \propto a^{-1})$, $a \propto t^{\frac{2}{3}}$, $\frac{\Delta t}{t} \propto \frac{\delta\varphi}{c^2}$ GR

$$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta t}{t} = -\frac{2}{3}\frac{\delta\varphi}{c^2}$$

Total S-W: $\frac{\Delta T}{T} = \frac{1}{3}\frac{\delta\varphi}{c^2} = \frac{1}{3}\frac{\delta\rho}{\rho}\left(\frac{\lambda}{\underbrace{ct}_{R_H}}\right)^2$

Integrated S-W effect:

$$\frac{\Delta T}{T} = 2\int \frac{\Delta\dot{\varphi}}{c^2}\,dt$$

$= 0$ if $\Omega_0 = 0$

$\neq 0$ if $\Omega_0 \neq 0$

$\neq 0$ including non-linear collapse.

The power of S-W fluctuations is given by:

$$C_\ell = 4\pi^2 \int_0^\infty k^2 dk \, \frac{j_\ell^2(k\tau)}{k^4} P(k)$$

conformal time

$$P(k) = k^n T^2(k)$$

↳ transfer function

At small ℓ ($\ell < 200$) $P(k) \sim k^n$ because $k < k_{eq}$ (thus $T(k) = 1$)

$n \simeq 1$

$$C_\ell = 4\pi^2 \int_0^\infty \frac{dk}{k} \, j_\ell^2(k\tau) = \text{const}[\tau] = \frac{1}{2\ell(\ell+1)}$$

$\Rightarrow$ $C_\ell \, \ell(\ell+1) = \text{const}\,[\ell]$

Small ℓ are used to constrain the normalization of the power spectrum (σ_8) and the spectral index n.

The main problem ~~is con~~ for determining cosmological parameters is COSMIC VARIANCE.

The first acustic peak:

Is the peak with largest power: the scale correspond to the horizon at time of recombination $\Rightarrow$

can be used as a rod to measure geometry of the universe (Ω_k)

In the homework we found (assuming $\Omega_\Lambda = 0$):

$$\theta_{rec} = \left(\frac{\Omega_m}{1+z_{rec}}\right)^{\frac{1}{2}} \simeq 0.87^\circ \left(\frac{\Omega_m}{0.3}\right)^{\frac{1}{2}}$$

There are 2 sets of peaks:

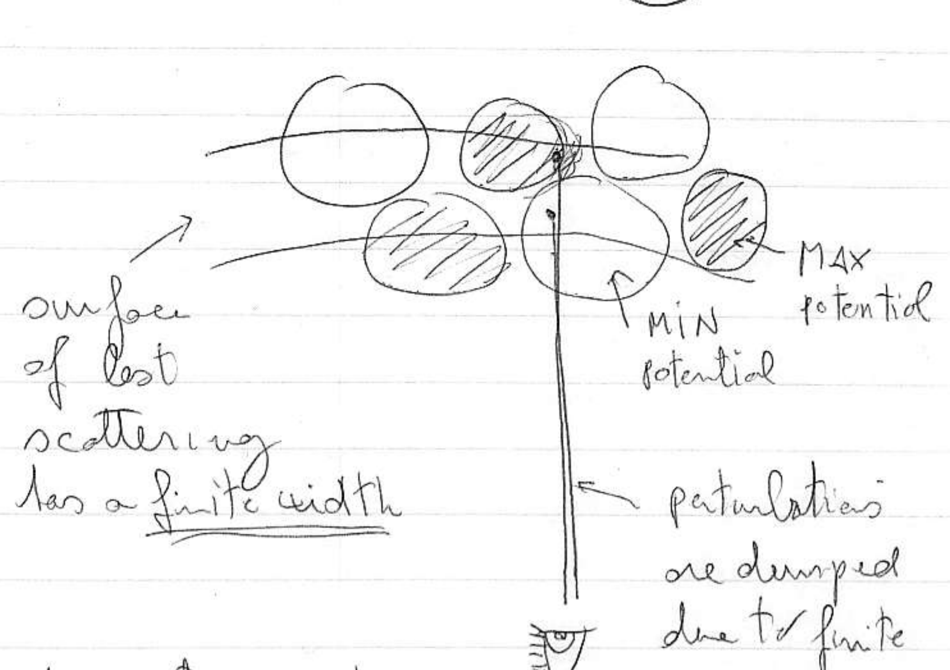
1) Due to continuation of S-W effect (the strongest). Amplitude of peaks is independent of $\Omega_b h^2$.

2) Due to doppler effect

The amplitude of the doppler peaks is increasing with $\Omega_b h^2$ (because photons are coupled more strongly to gas)

3) On small scales peaks amplitude decreases due to Silk-damping.

Summary:



1) Dipole: due to motion with respect to CMB

2) Quadrupole: contaminated by galactic but has cosmological importance

3) Small-ℓ: $\rightarrow$ Sachs-Wolf effect $\Rightarrow C_\ell \ell(\ell+1) = \text{const}$
$\downarrow$
Integrated Sachs-Wolf (only for $\Omega_0 \neq 1$ or non-linear effects). Is a time dependent effect. Gives extra power at small ℓ.

4) Acustic peaks consist of 2 sets:
- odd peaks are a continuation of the S-W effect, arise from potential fluctuations.
- even peaks arise from velocity fluctuations (Doppler effect). The ratio of amplitude of odd to even peaks depend on $\Omega_b h^2$.
The location of peaks depends on Ω_0.

↗ damping increases with decreasing Ω_b
because thickness of last scattering surface increases!

5) On small scales, $\ell > \ell_D \simeq 900 \left(\frac{\Omega_b h^2}{0.025}\right)^{\frac{1}{2}}$ fluctuations are damped (Silk damping) $\propto \exp\left[-\left(\frac{\ell}{\ell_D}\right)^{1.5}\right]$

6) Gravity waves: contribute with $C_\ell \ell(\ell+1) = \text{const}$ at small ℓ ($\ell < 100$).

7) Reionization: perturbations are damped by $e^{-\tau_e}$ where τ_e is the optical depth to Tompson scattering of the IGM.

Cosmic Confusion:

free parameters: $\Omega_m, \Omega_\Lambda; \Omega_k; \Omega_b; h; n_s;$

Ω_k ↑ curvature; n_s ↑ spectral index scalar perturbations

$n_T; \sigma_8; \tau_e; C_2^S / C_2^T$

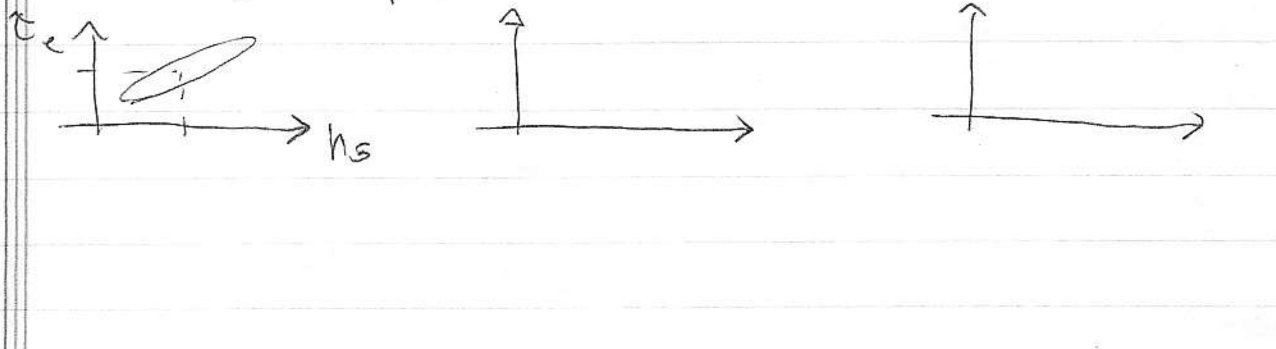
σ_8: normalisation of power spectrum

τ_e: (reionization)

C_2^S / C_2^T: amplitude of tensor part

n_T ↑ spectral index tensor perturbations

Most of these parameters are degenerate with each other: The C_ℓ are the same for different sets of cosmological parameters.



(240A)

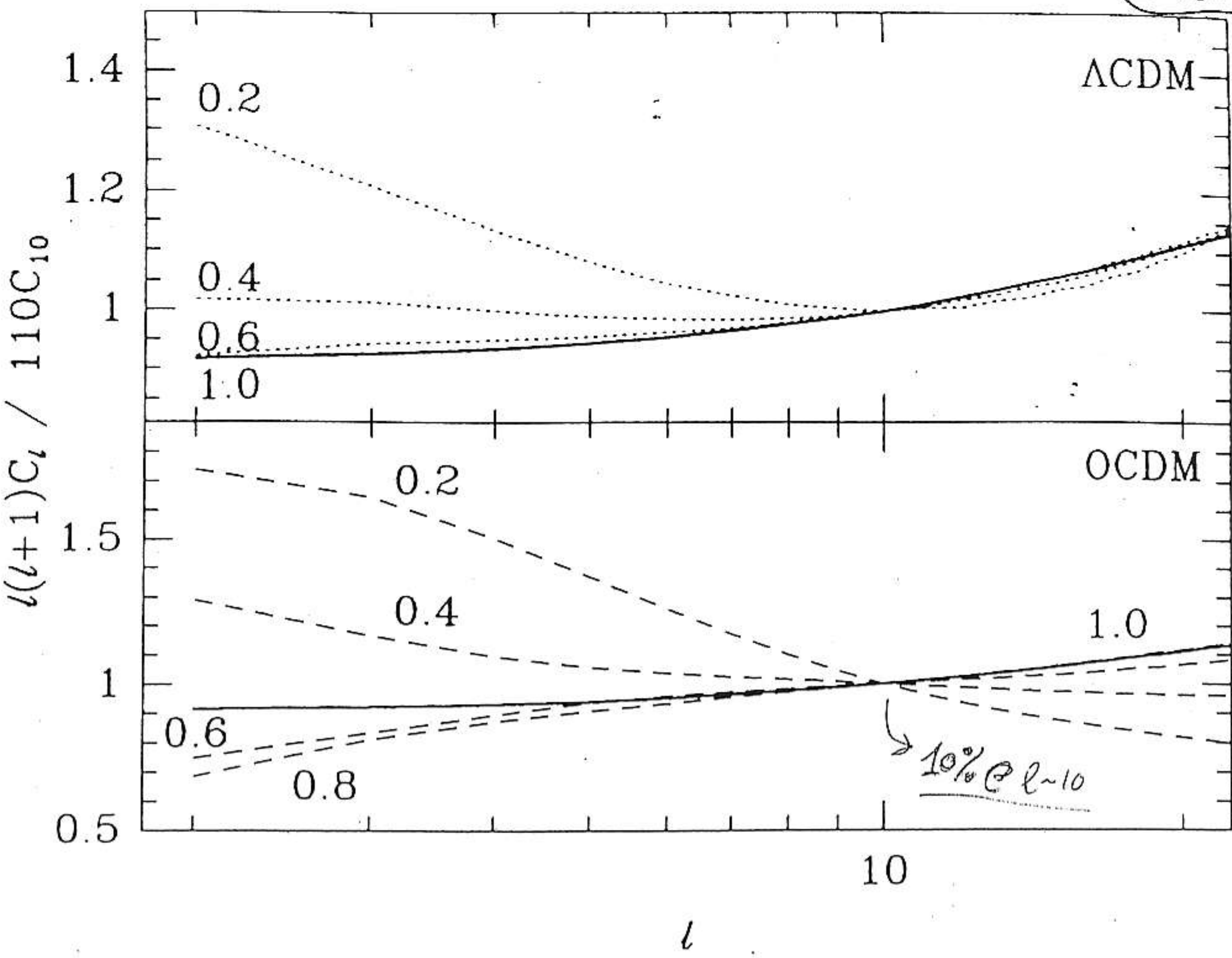


FIG. 7.—Shapes of the angular power spectra plotted for various CDM models. The upper panel shows spatially flat models with $\Omega_0 + \Omega_\Lambda = 1$, and the lower panel shows models with $\Omega_\Lambda = 0$. All models have $n = 1$ and are labeled with the appropriate value of Ω_0.

from Bunn & White 1997, ApJ, 480, 6

$h''' = 0$ $\quad \left.\begin{matrix} RD \\ MD \end{matrix}\right\} h \sim \eta^2$

$\Omega_0 = 1$

$\Omega_0 < 1 \longrightarrow \Omega_0 = 0.3 + \Omega_k = 0.7 \rightarrow \left.\begin{matrix} RD \\ CD \end{matrix}\right\} h \not\propto \eta^2 \quad h''' \neq 0$

$\Omega_0 = 0.3 + \Omega_\Lambda = 0.7 \rightarrow \left.\begin{matrix} RD \\ VD \end{matrix}\right\} h \not\propto \eta \quad h''' \neq 0$

$\Delta = -\frac{2}{k^2} \underbrace{h''}_{\text{cnst}} e^{ik\mu(\eta_R - \eta)} + \underbrace{\Delta_{ISW}}_{\text{integrated SW}} \rightarrow$ $\Delta_{ISW} \neq 0$ at low redshift.

$\Delta_{ISW} = 0$ if matter dominated.

255A

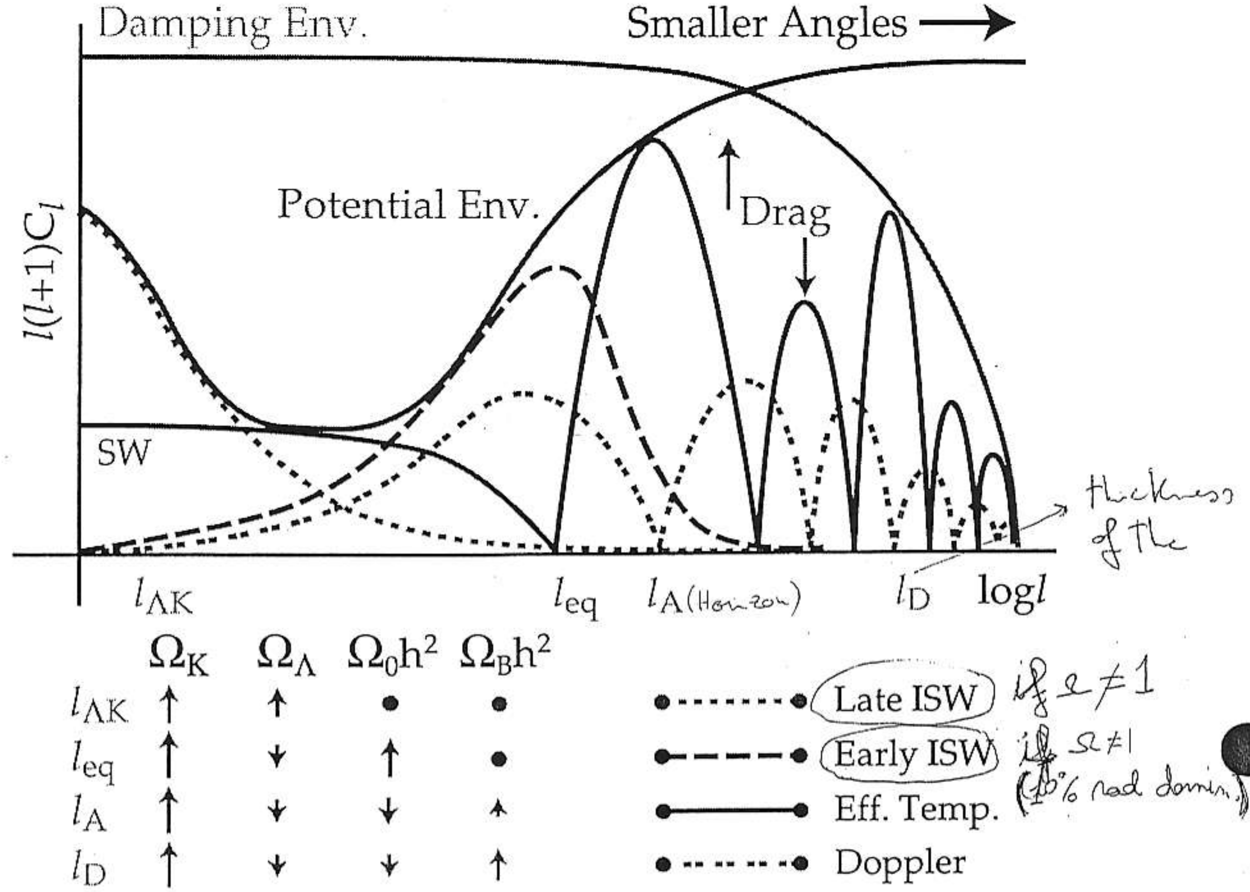


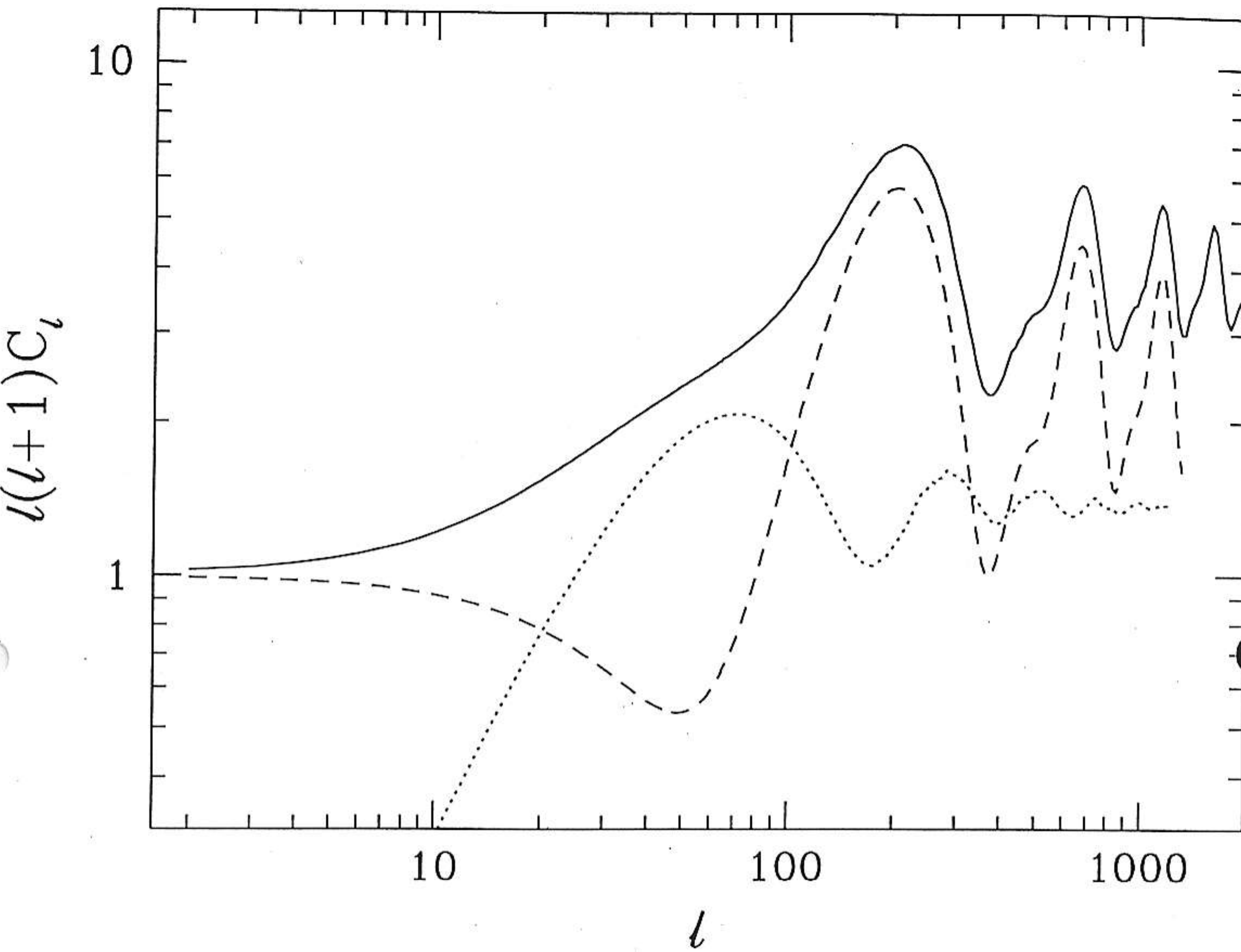
Figure B2. Anisotropy spectrum: power in anisotropies $\ell(\ell+1)C_\ell$ per logarithmic interval in $\ell \sim \theta^{-1}$. The decomposition into various physical effects allows one to extract four fundamental scales in the spectrum: $\ell_{\Lambda K}$ and $\ell_{\rm eq}$ which enclose the Sachs-Wolfe (SW) plateau in the potential envelope, ℓ_A the acoustic spacing, and ℓ_D the characteristic scale of the diffusion damping envelope. Here a scale-invariant adiabatic case is shown for illustration purposes. These scales may be combined to infer the four fundamental cosmological parameters $\Omega_K (\equiv 1 - \Omega_\Lambda - \Omega_0)$, Ω_Λ, $\Omega_0 h$ and $\Omega_B h^2$. Baryon drag enhances all compressional (here odd) maxima of the acoustic oscillation, and can probe the spectrum of fluctuations at last scattering and/or $\Omega_B h^2$.

Box 2: Power Spectrum

The scale invariant adiabatic model illustrates how the anisotropy spectrum encodes cosmological information (see Fig. B2). It is conventionally denoted $\ell(\ell+1)C_\ell$ and represents the power per logarithmic interval in temperature fluctuations on angular scales $\ell \sim \theta^{-1}$. Highly accurate numerical results for the spectrum in this model have long been available[41,42,43] with only moderate improvements to match the increasing precision of experiments[44,45] (see also Fig. 4). Here we present a more schematic description that better illuminates the physical content and also may more easily be adapted to alternate models.

Hu, Sugiayma & Silk, astro-ph/9604166

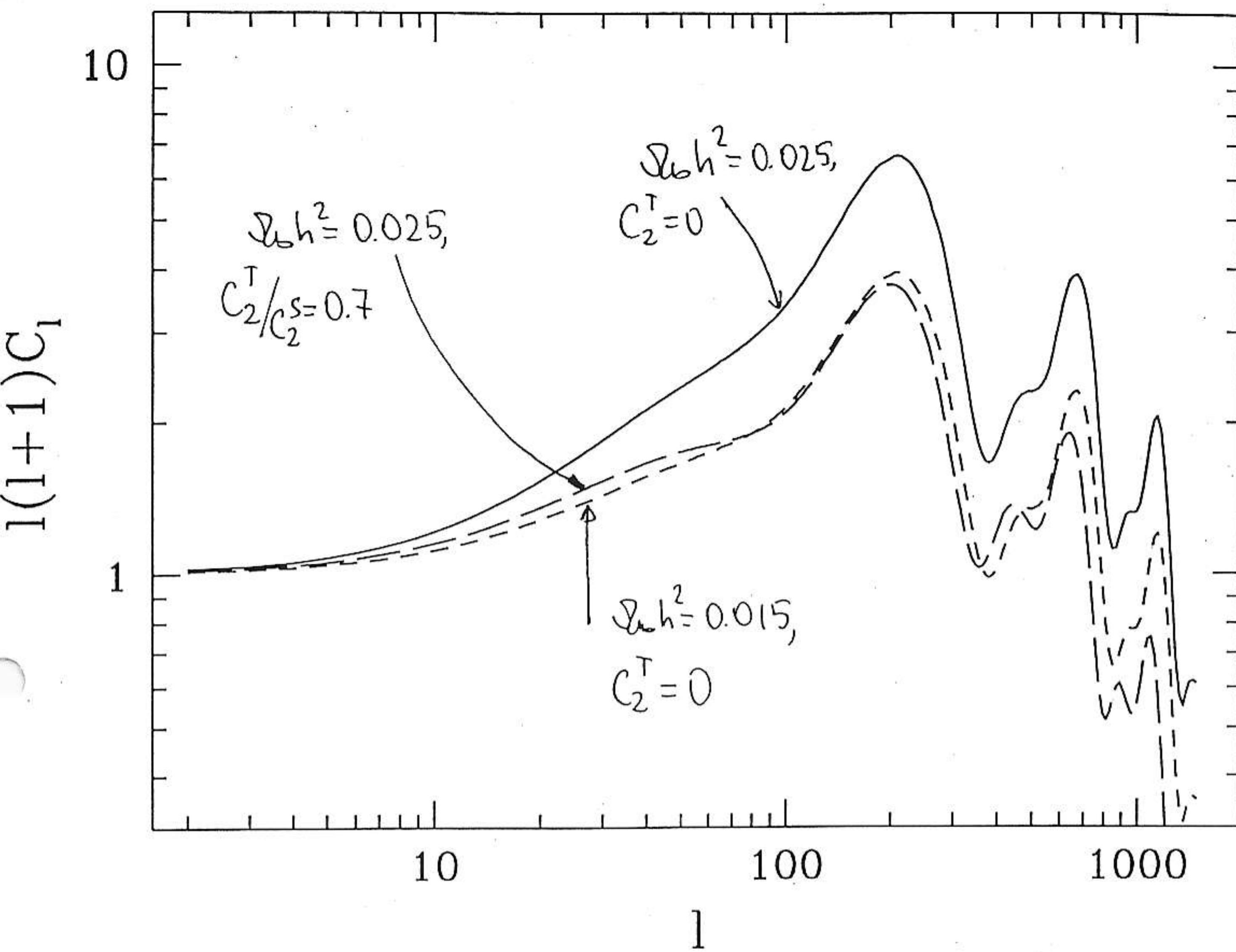
(248C)



$\Omega_b h^2 = 0.025$ case

dashed line: only term with cos is retained in [247.1]

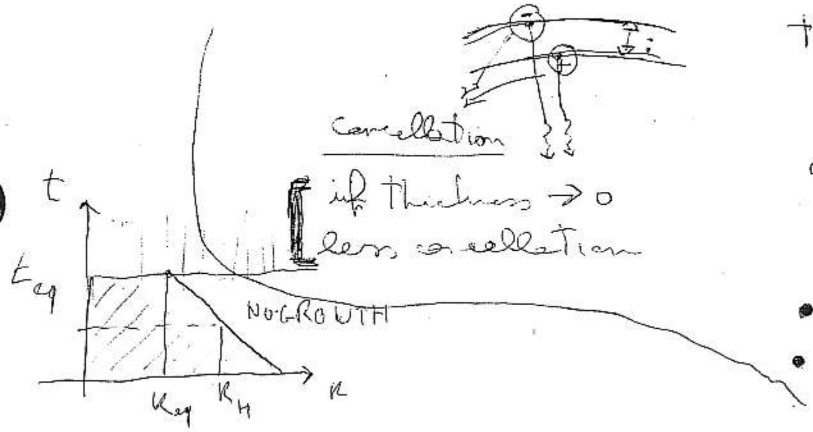
dotted line: only the term with sin is retained.



Example of Cosmic Confusion:

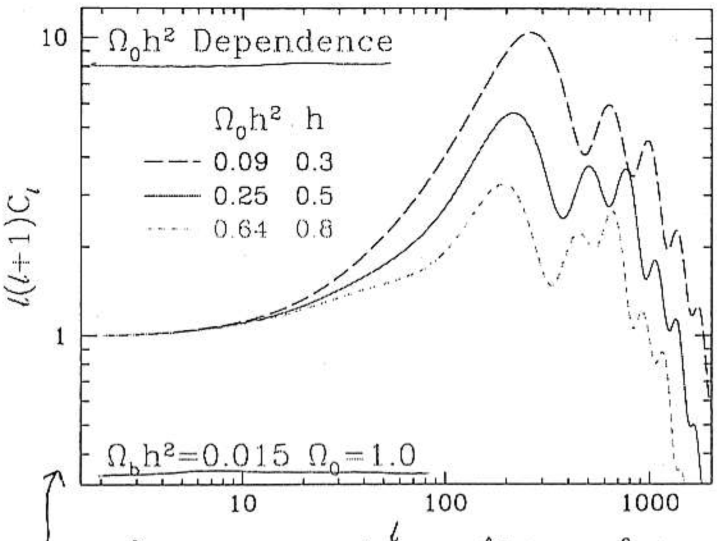
$\Omega_b h^2 = 0.025$ + 70% gravity waves looks very similar to $\Omega_b h^2 = 0.015$ + no gravity waves, when they are normalized to the same C_2 value (the same COBE normalization) thickness of the surface scattering

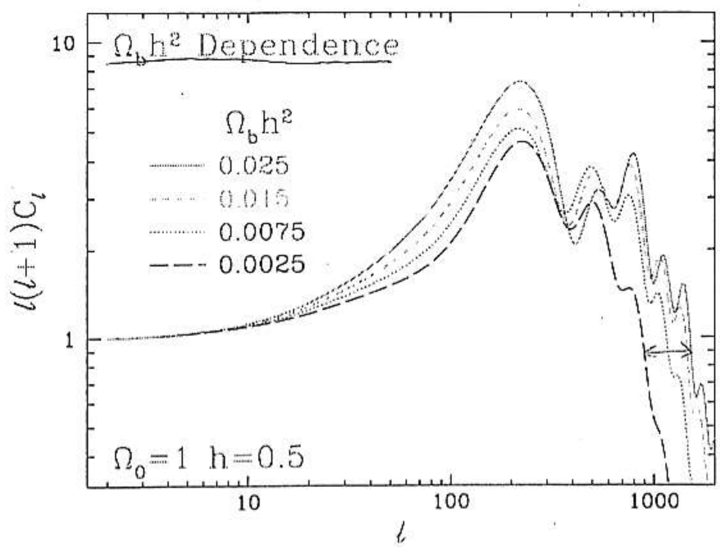
(255B)



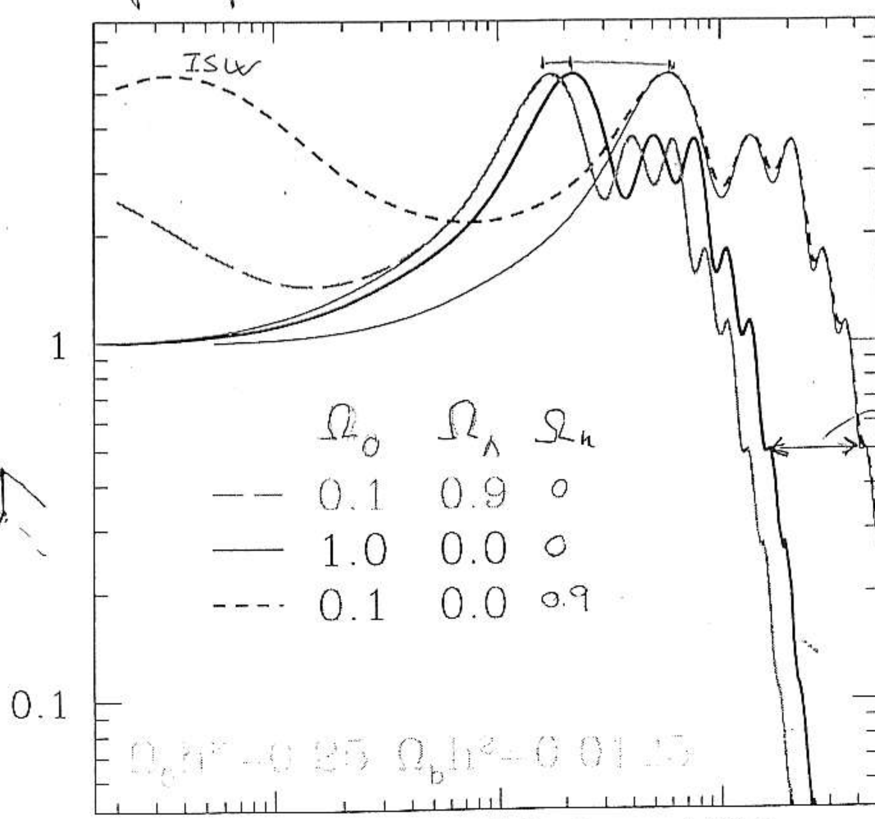
coupling $\propto \frac{1}{c\sigma_T n_e t}$

- more barions ⇒ peaks get higher.
- if Ω_b is higher the coupling is better ....

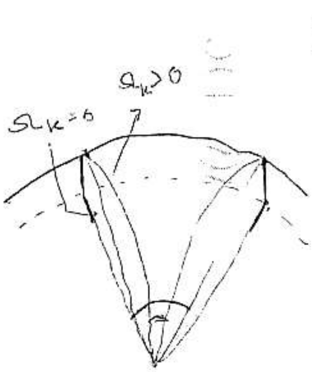




if matter dominated earlier
had more time to form potential wells.



Silk damping shift of the same amount.

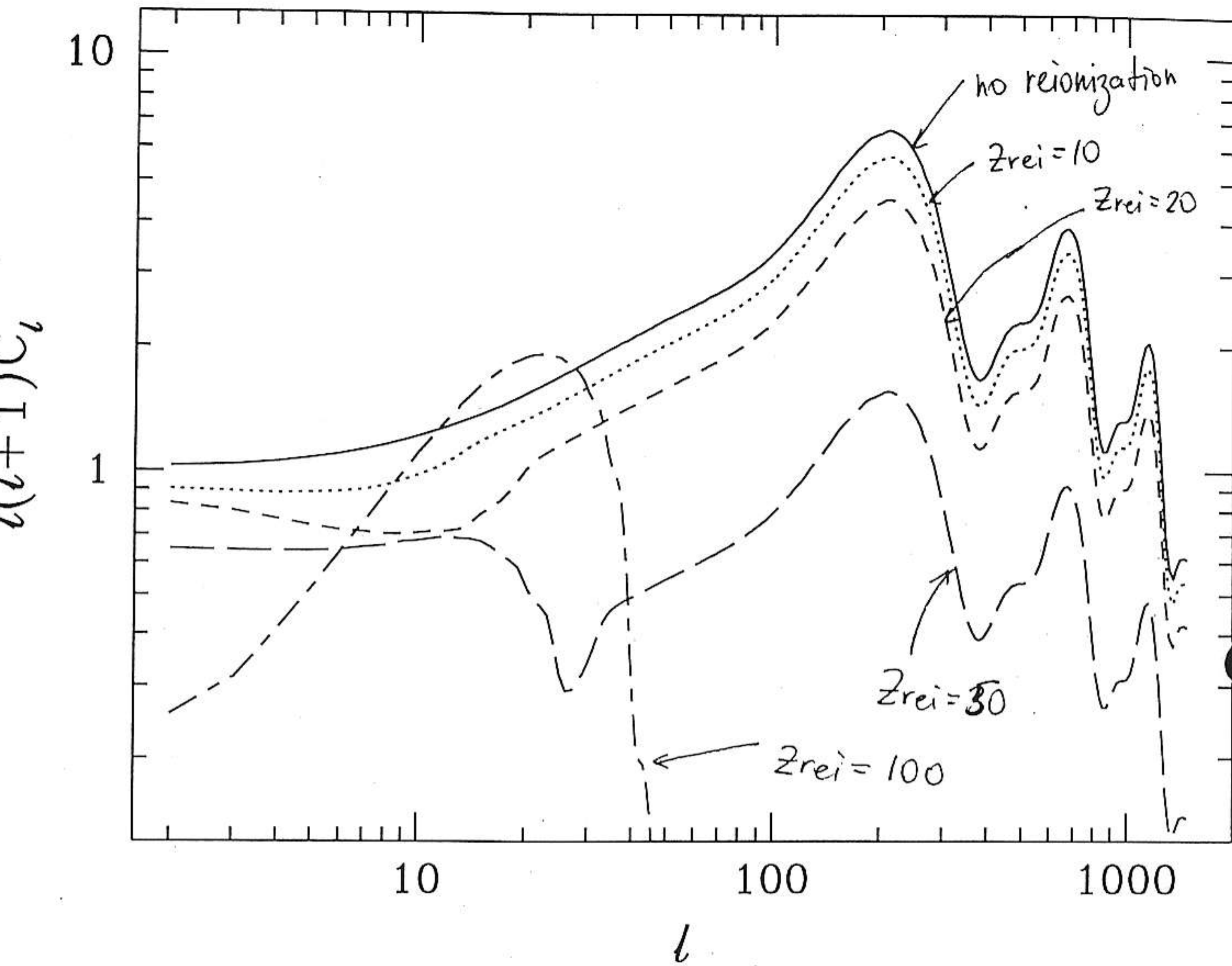


$\Omega_\Lambda \neq 0$ age of the Universe changes.

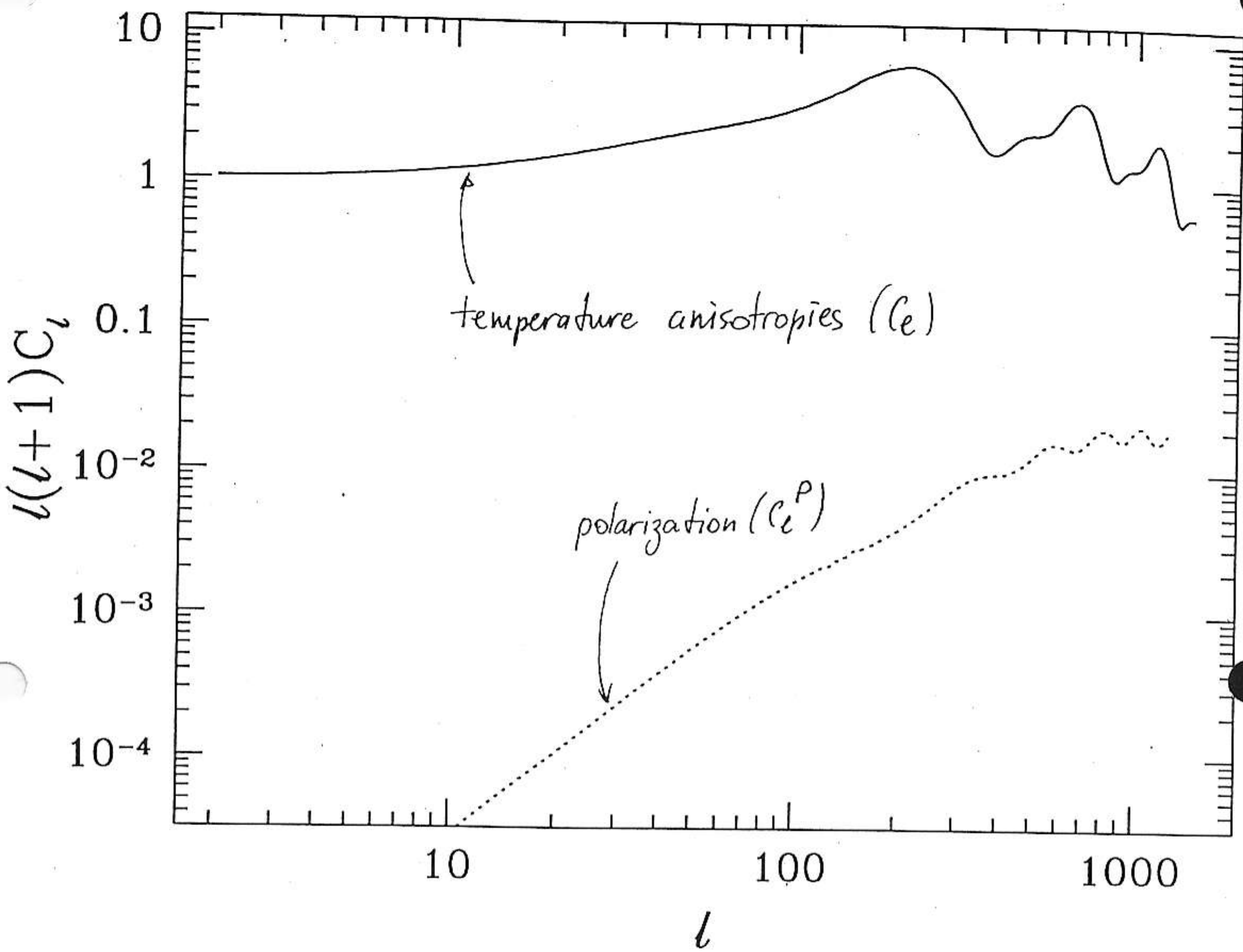
$\rho \propto \frac{\Omega_0}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$

from Wayne Hu's homepage, http://www.sns.ias.edu/~whu

282A

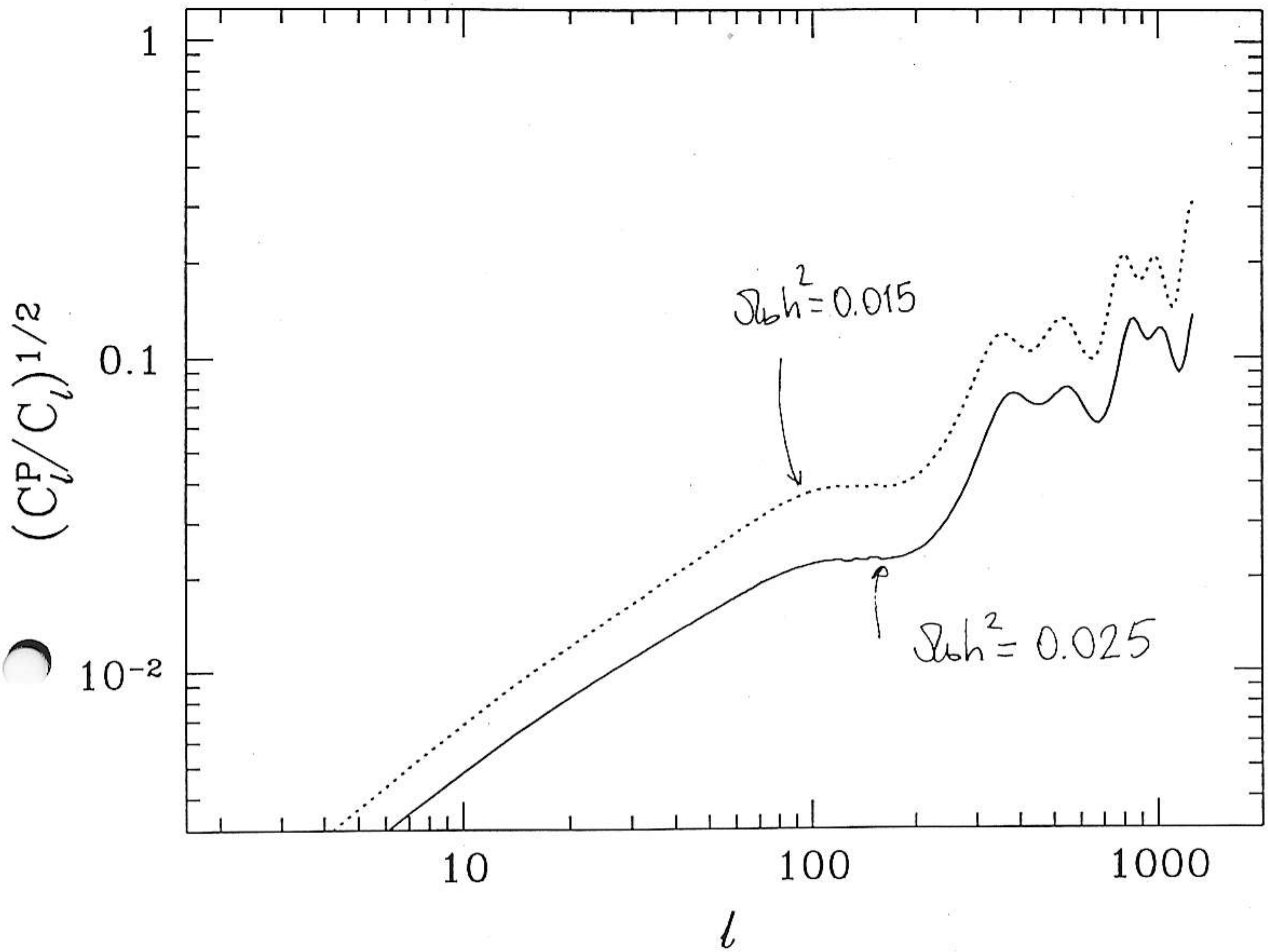


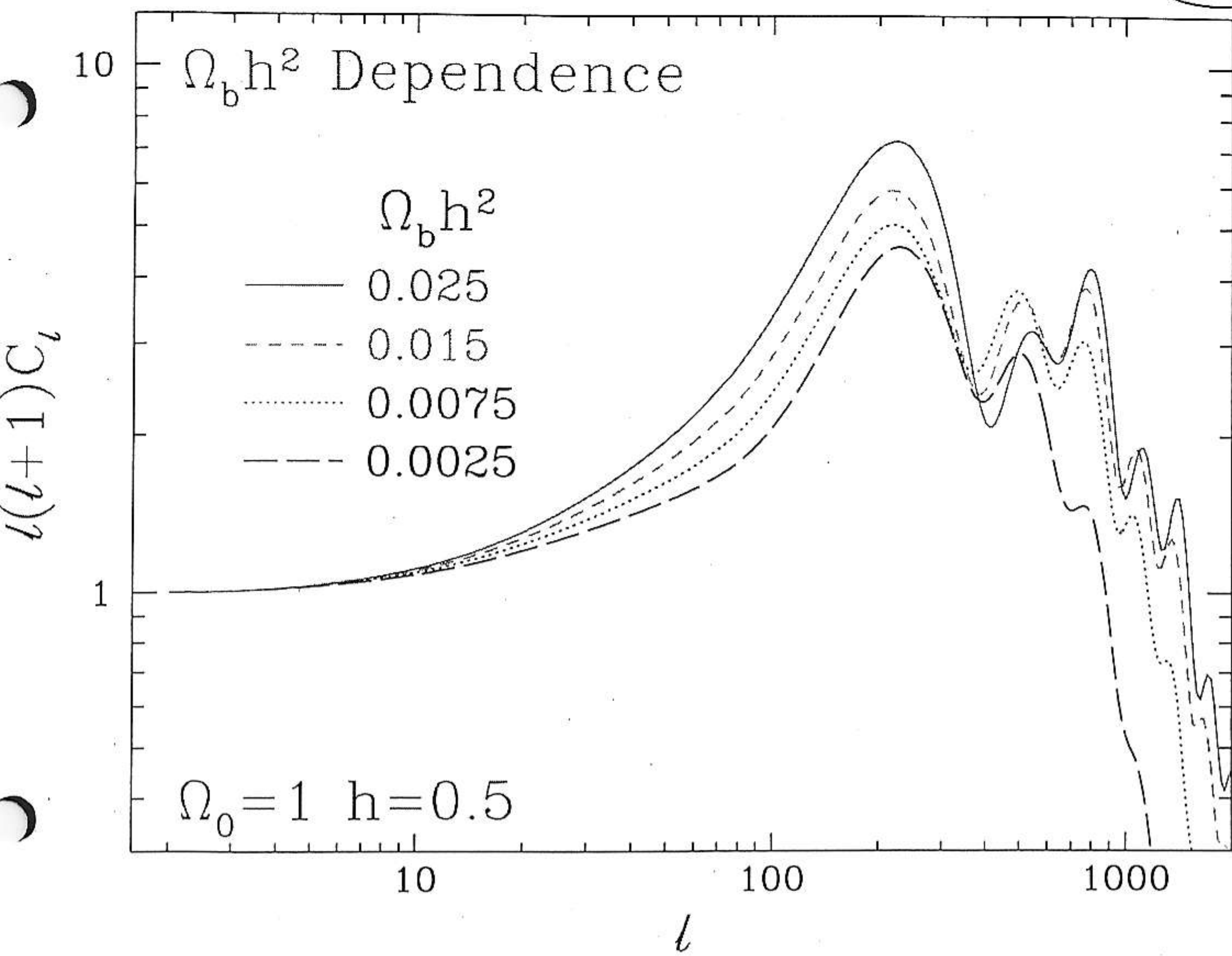
All for $\delta\eta_i/\eta_i = 0.1$



$\Omega_b h^2 = 0.025$ case

(262B)

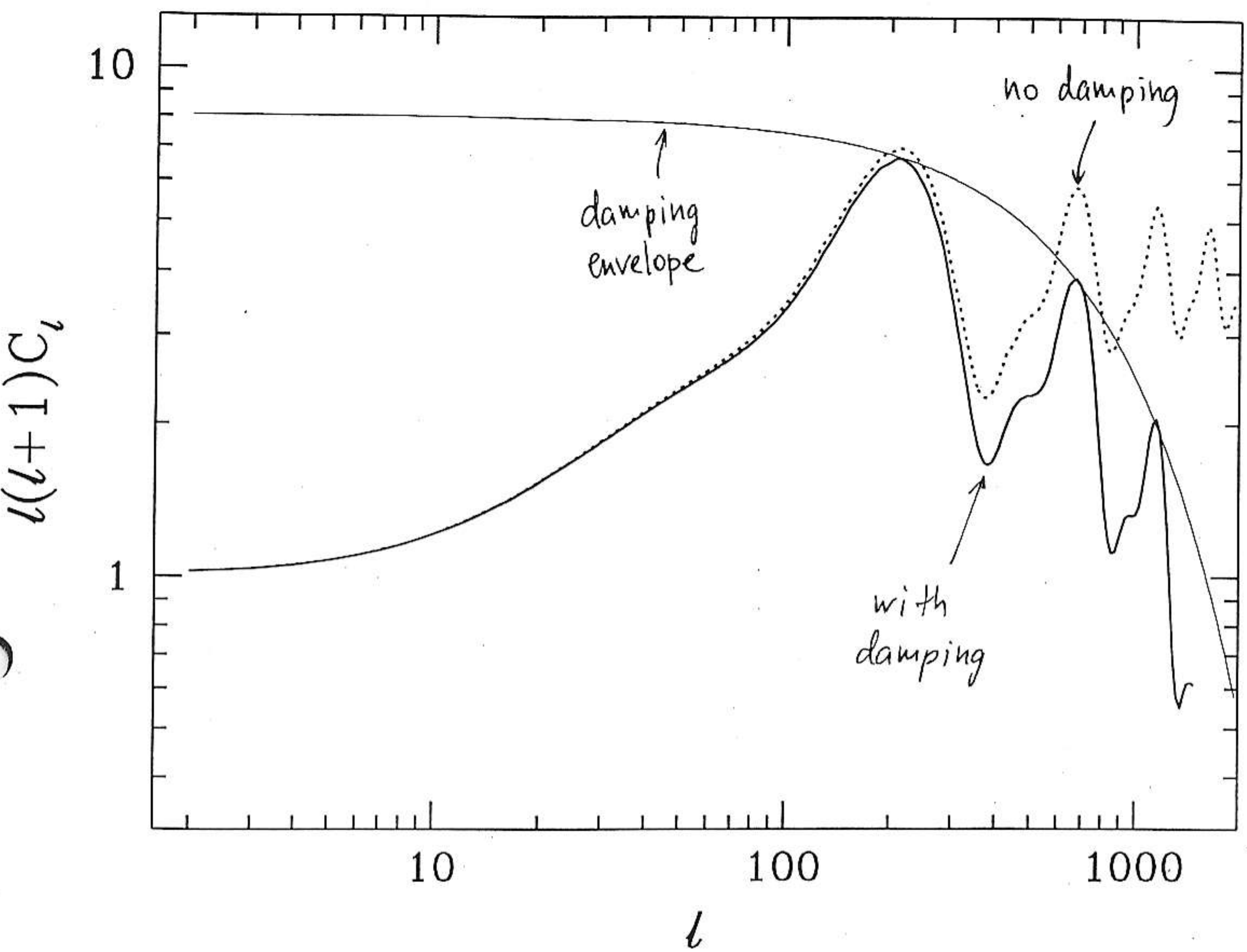




from Wayne Hu homepage:

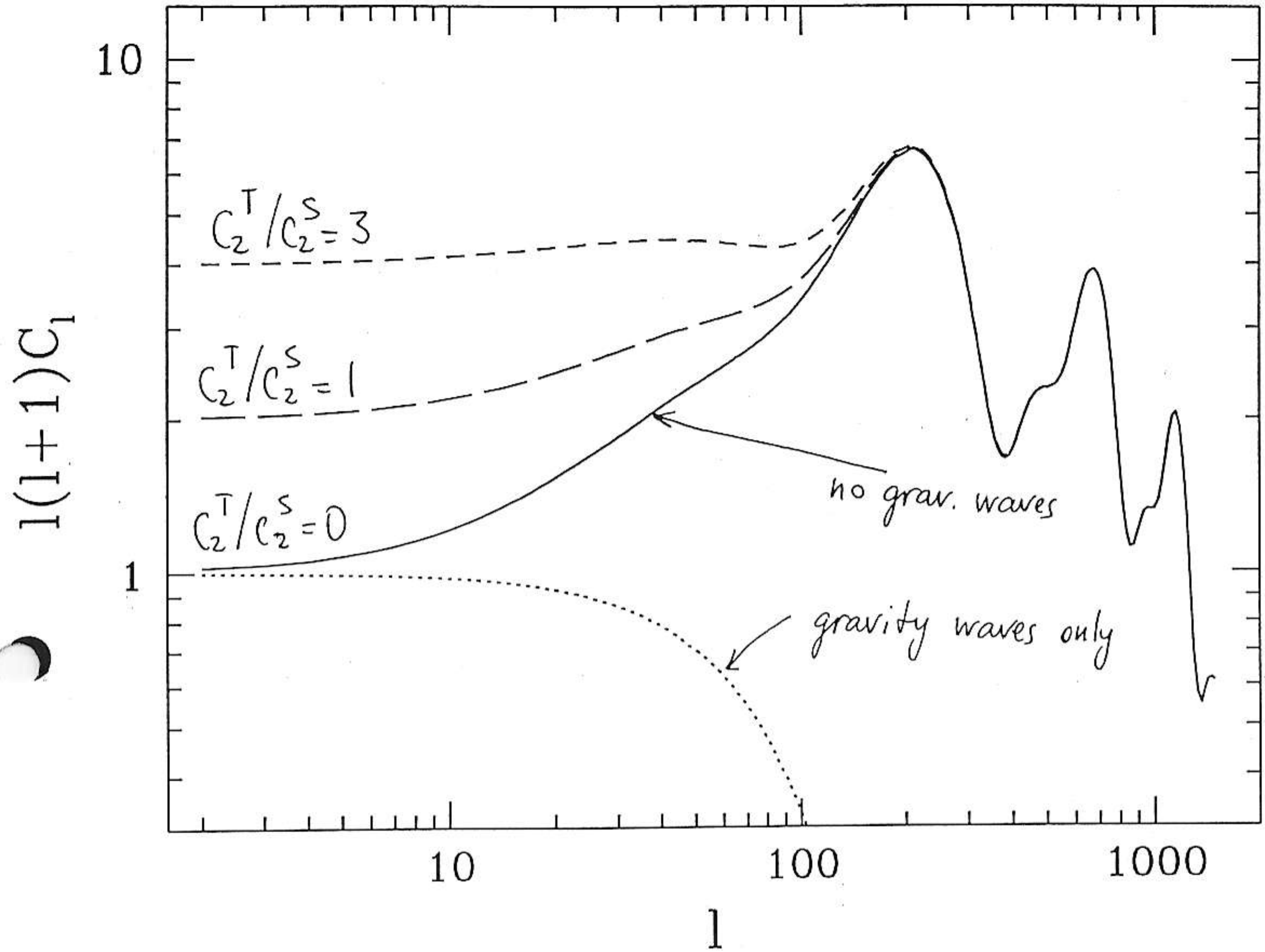
http://www.sns.ias.edu/~whu

254A



$J_\ell(x)$ $\quad x \sim \pi \cdot \ell$

$\ell_D \sim \frac{x_D}{\pi} \sim 10^3$



$\Omega_b h^2 = 0.025$ case with and without gravity waves.

T = tensor
S = scalar