Synchrotron Radiation: II. Spectrum

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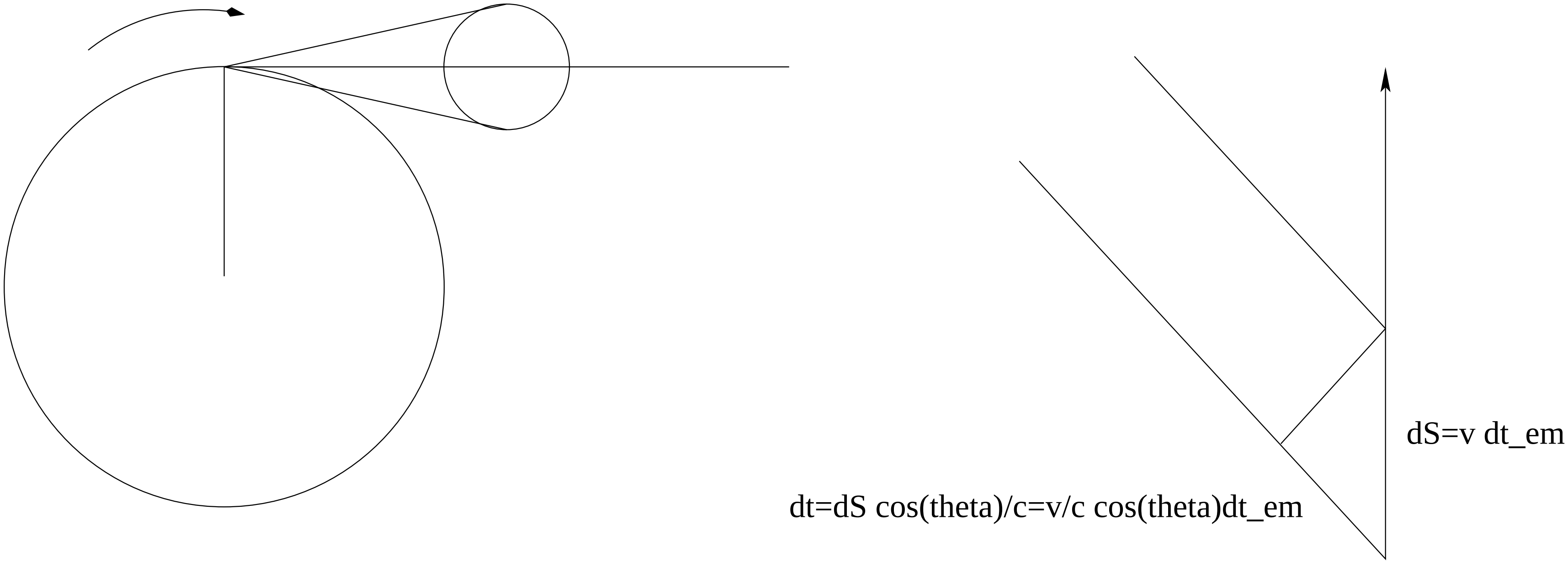
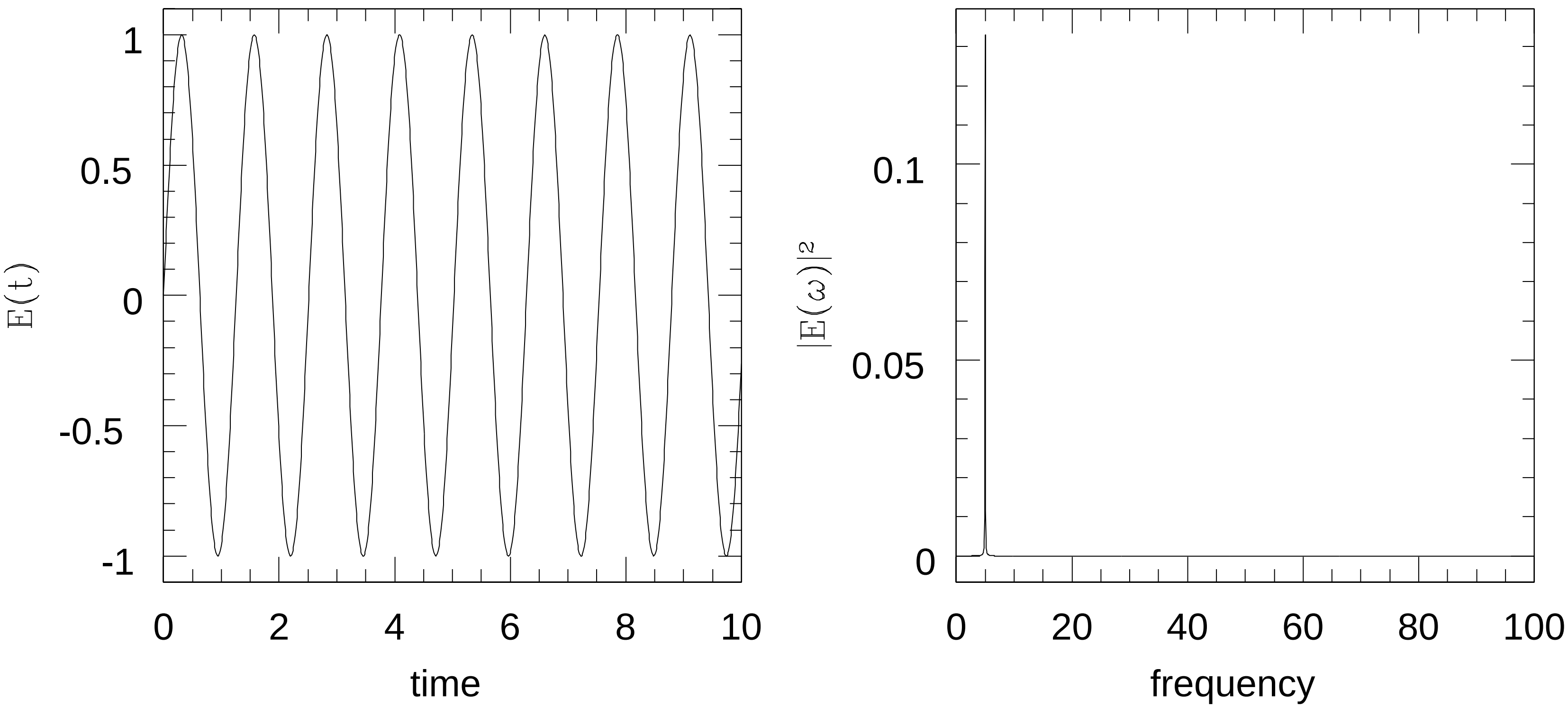

dS=v dt_em
dt=dS cos(theta)/c=v/c cos(theta)dt_em

Spectrum. I. Mono-energetic CRs

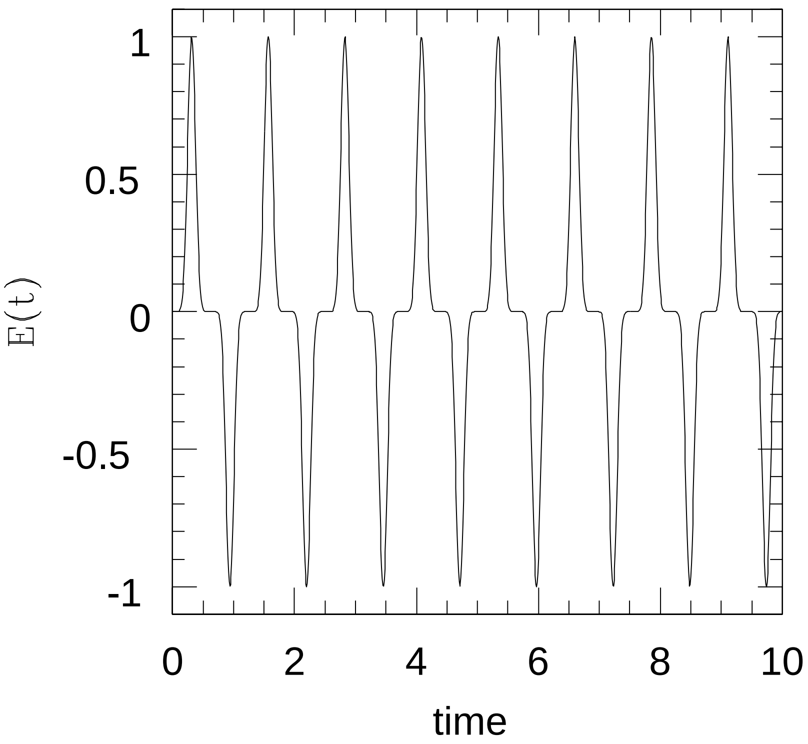
Effect of beaming:

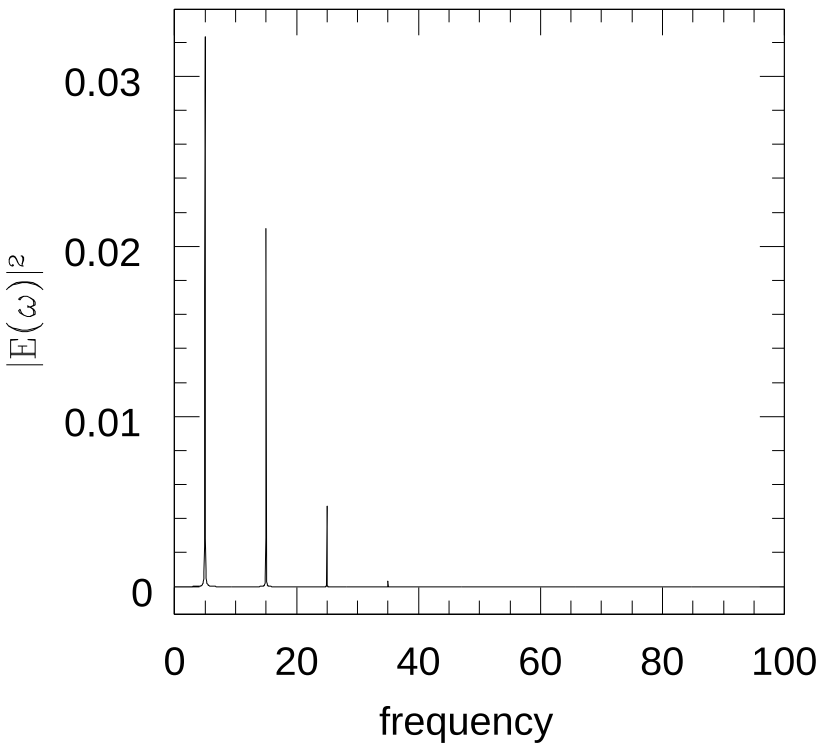
- Opening angle $\delta\theta = 2/\gamma$
- $dt_{em} = (\delta\theta/\omega_B) \sim 1/\gamma\omega_B \sim \omega_L$
- Thus the frequency of the pulse is shorter by a factor of γ

Form cyclotron to synchrotron

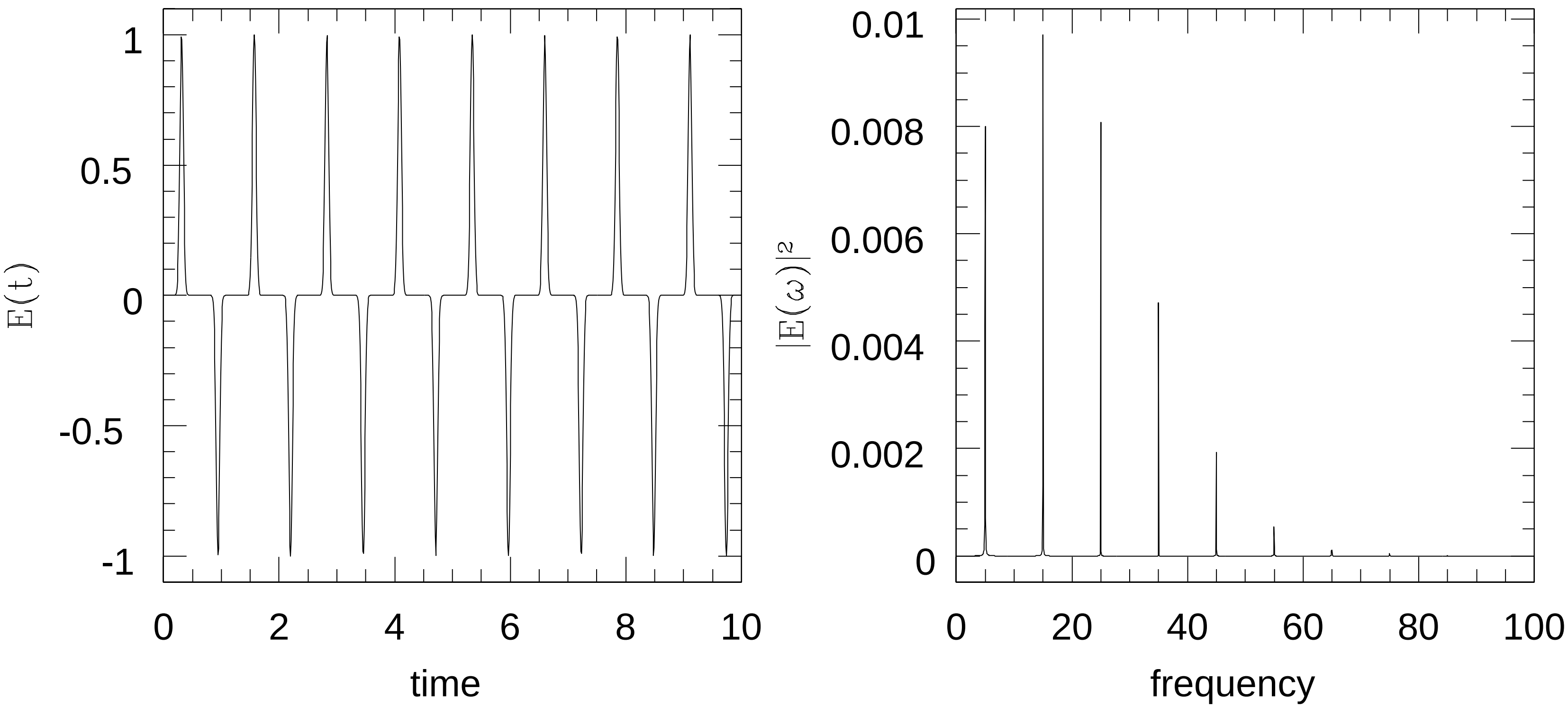


Form cyclotron to synchrotron





Form cyclotron to synchrotron



Relativistic effects make the frequency higher by another factor γ^2

- In the observer frame of reference:

$$\delta t_{rec} = \delta t_{em}(1 - \beta \cos\theta)$$

where $\theta \sim \delta\theta$ is the viewing angle

- Thus,

$$\begin{aligned} dt_{rec} &= dt_{em}(1 - \beta + \beta\delta\theta^2/2) = \\ &= dt_{em}(1/2\gamma^2 + 1/2\gamma^2) = dt_{em}/\gamma^2 \end{aligned}$$

- Similarly can derive superluminal motions proposed by M.J. Rees: $v_{obs} \sim ds\delta\theta/dt_{rec} \sim v_{em}\gamma$, that can be $> c$.
- Including the pitch angle: $\omega_c = 3/2\gamma^3\omega_B \sin\alpha = 1.5\gamma^2\omega_L \sin\alpha$ where $\omega_L = \frac{qB_0}{mc}$ Larmor freq.

Spectrum. I. Mono-energetic CRs

Details of the spectral shape are not important as we will see later.

$$F(x) \propto \begin{cases} \propto x^{1/3} \text{ for } x \ll 1 \\ \propto x^{1/2} \exp\left(-x\right) \text{ for } x \gg 1 \end{cases}$$

- where $x = \omega/\omega_c$. Maximum of $F(x)$ at $x = 0.29$
- What is the normalization of $P(\omega)$?

$$\frac{dP}{d\omega} \sim \frac{P}{\omega_c} \sim \frac{(2/3)\gamma^2\beta^2 sin^2\alpha(q^4/m^2c^3)B_0^2}{(3/2)\gamma^2(qB_0/mc)\sin\alpha} = \frac{4}{9}\frac{B_0 q^3 sin\alpha}{mc^2}$$

- Indeed consedering the correct normalization of $F(x)$ we have,

$$\frac{dP(\omega)}{d\omega} = \frac{\sqrt{3}}{2\pi}\frac{B_0 q^3 \sin\alpha}{mc^2}F(x)$$

Spectrum. II. Power-law distribution of CRs

$$n_\gamma d\gamma = n_0 \gamma^{-p} d\gamma$$

with $p \sim 2.5$.

$$P \propto \int_1^\infty P(\gamma) n_\gamma d\gamma$$

where $P(\gamma) \propto \gamma^2$. Assume for simplicity $F(x) = \delta(x-1)$. Set $\nu' = \nu_c = \gamma^2 \nu_L$, $d\nu' = 2\gamma d\gamma \nu_L$,

$$P(\nu) \propto \int \delta(\nu - \nu') \left(\frac{\nu\prime}{\nu_L} \right)^{(1-p)/2} \frac{d\nu'}{2\nu_L} \propto \nu^{-(p-1)/2}$$

Thus, Synchrotron is characterized by a power law spectrum with slope $-(p-1)/2 \sim -0.7$. The flux now depends on the combination of n_0 and B_0. Need more info to measure the magnetic field!

Synchrotron self-absorption

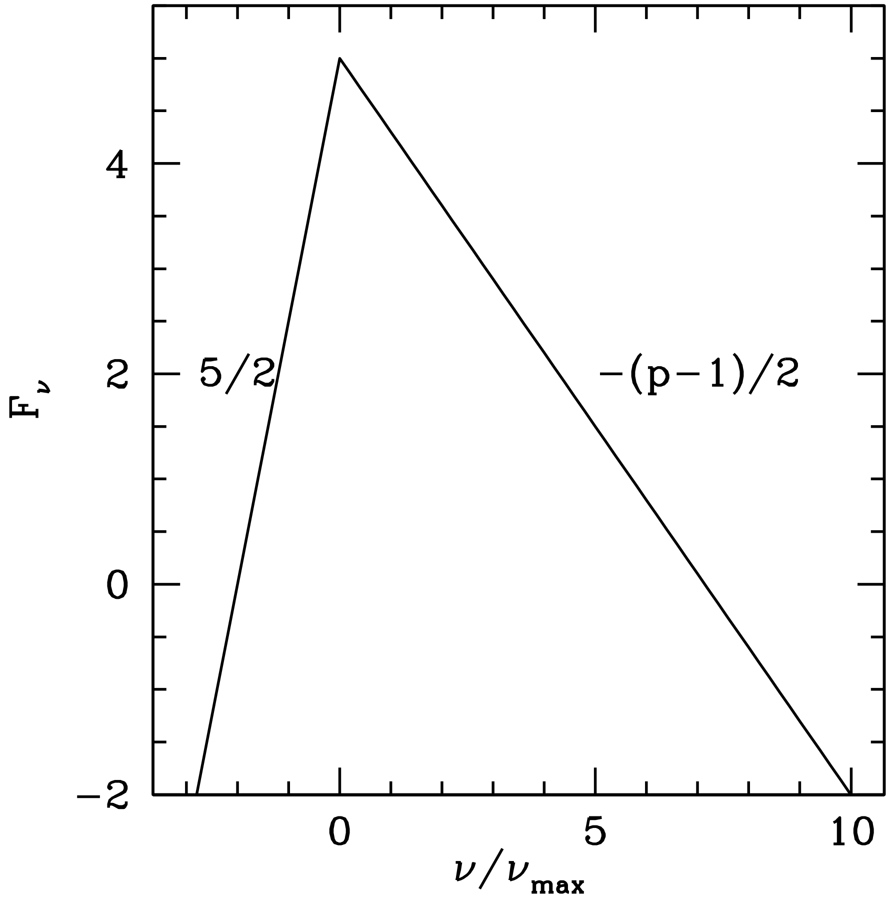
- We have seen the synchrotron emission mechanism: what about absorption? and stimulated emission?
- We can have both: absorption is important at low frequencies. Why?
- For synchrontron the source function is $S_\nu \propto B_0^{-1/2}\nu^{5/2}$.
- Here is the qualitative derivation. For BB:

$$S_\nu = \frac{2\nu^2}{c^2}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right) \rightarrow \frac{2\nu^2}{c^2}KT$$

- kT is energy of thermally excited harmonic oscillator. Replace kT with appropriate energy. $\epsilon = \gamma m_e c^2 = m e_c^2(\nu/\nu_L)^{1/2}$. Thus, $S_\nu \propto B_0^{-1/2}\nu^{5/2}$.
- What is the flux in the optically thick regime?

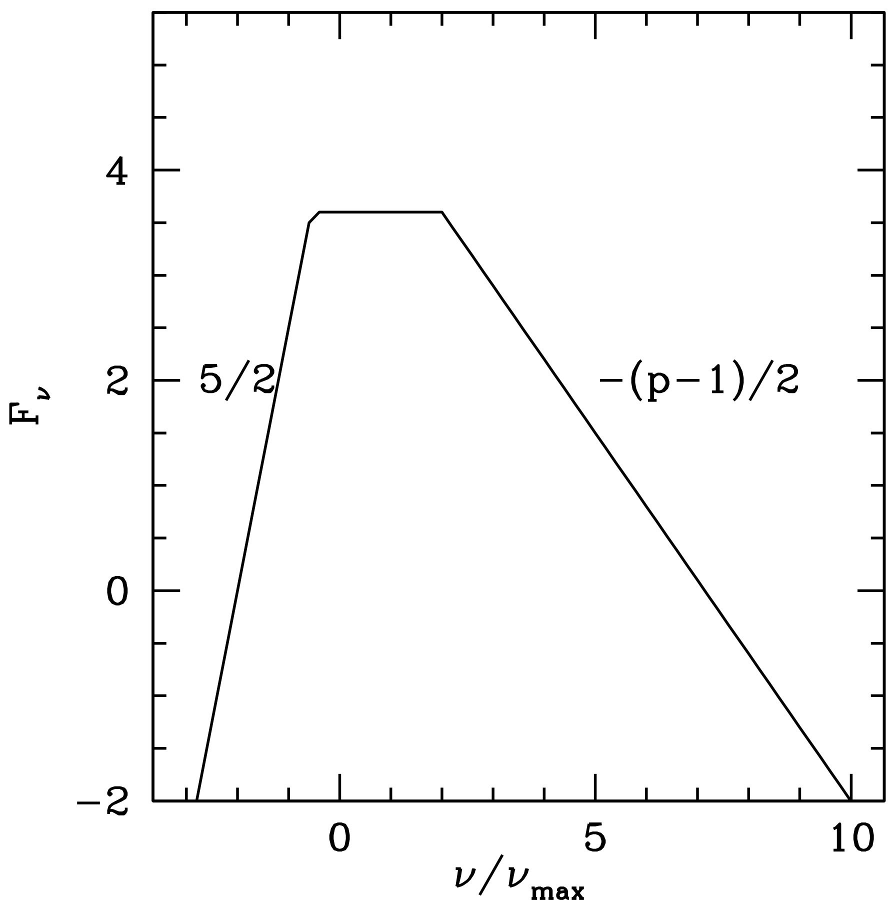
First of all we have $I_\nu = S_\nu$ in optically thick regime.

$$F_\nu^{max} = \pi I_\nu (R_{source}/dist)^2 \propto B_0^{-1/2} \nu_{max}^{5/2} (R_{source}/dist)^2$$



Can break the degeneracy $n_0 B_0$ and measure magnetic field.

- In addition there seems to be a maximum flux of synchrotron radiation from compact radio sources. Why?



Inverse Compton losses

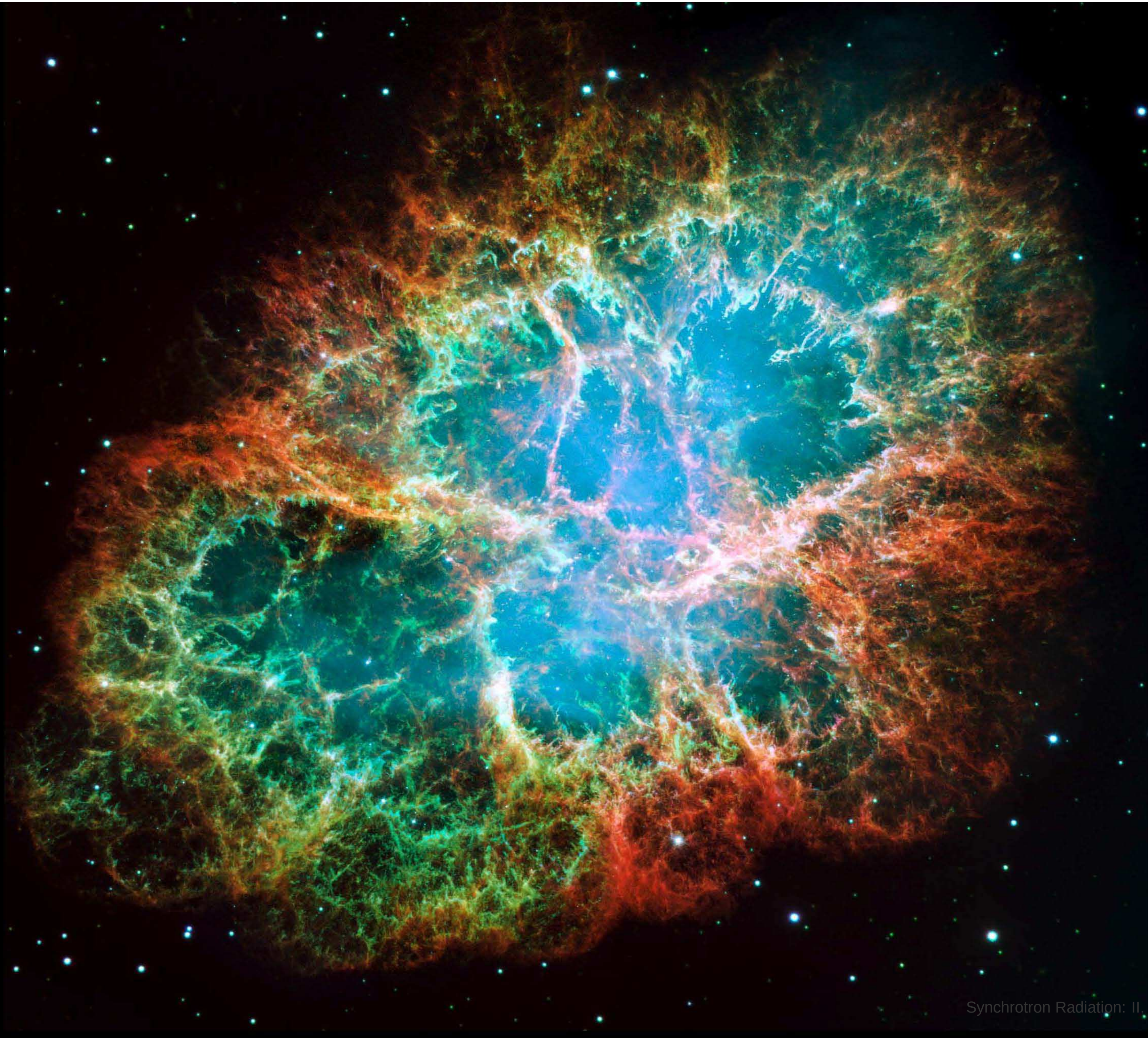
- $T_b = \frac{c^2 I_\nu}{2 k_B \nu^2}$ brightness temperature

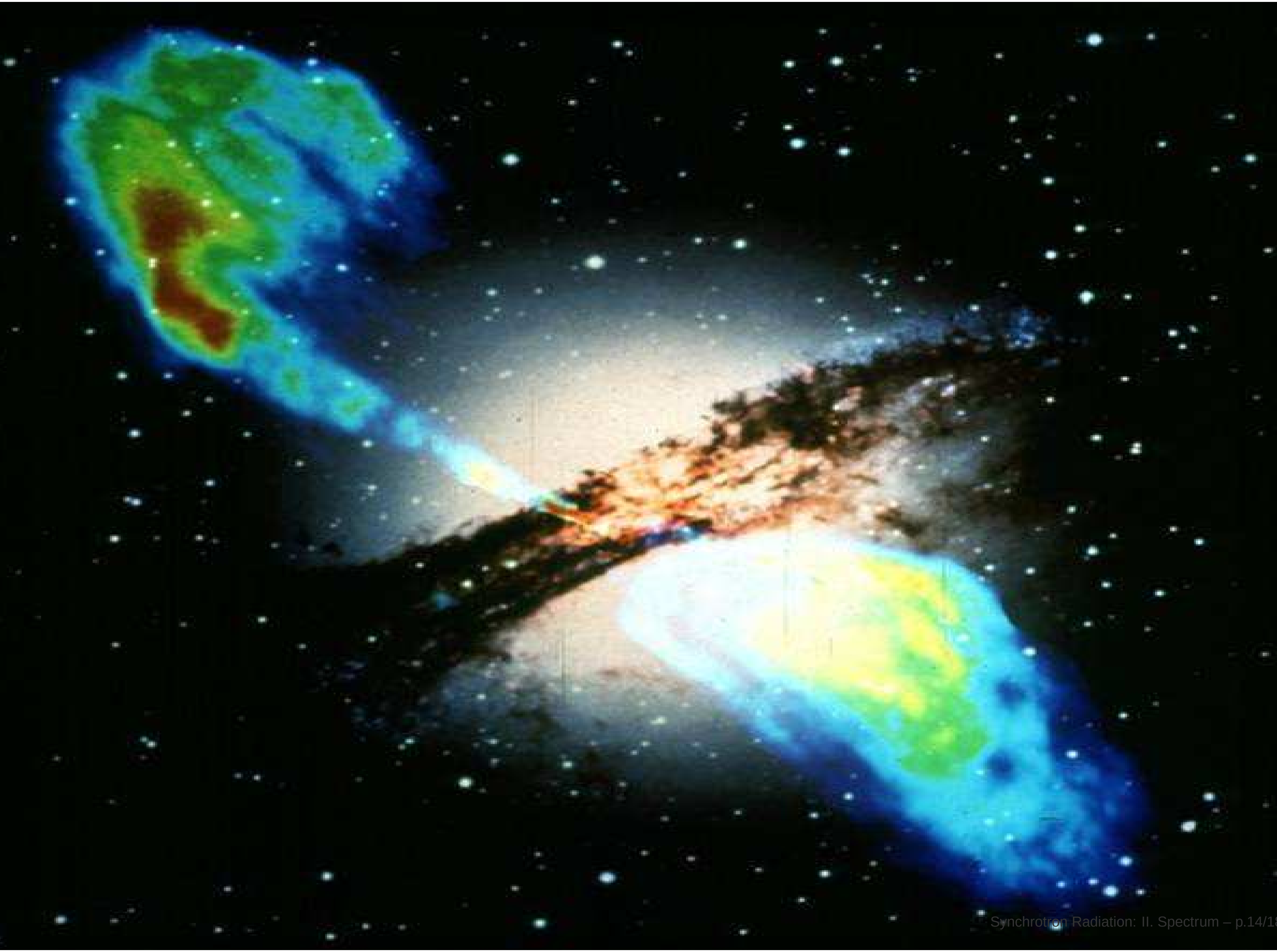
In 1969 Kellermann and Pauling-Tohth noted that in compact radio sources $T_b(max) < 10^{12}$ K (clearly this is non-thermal emission)

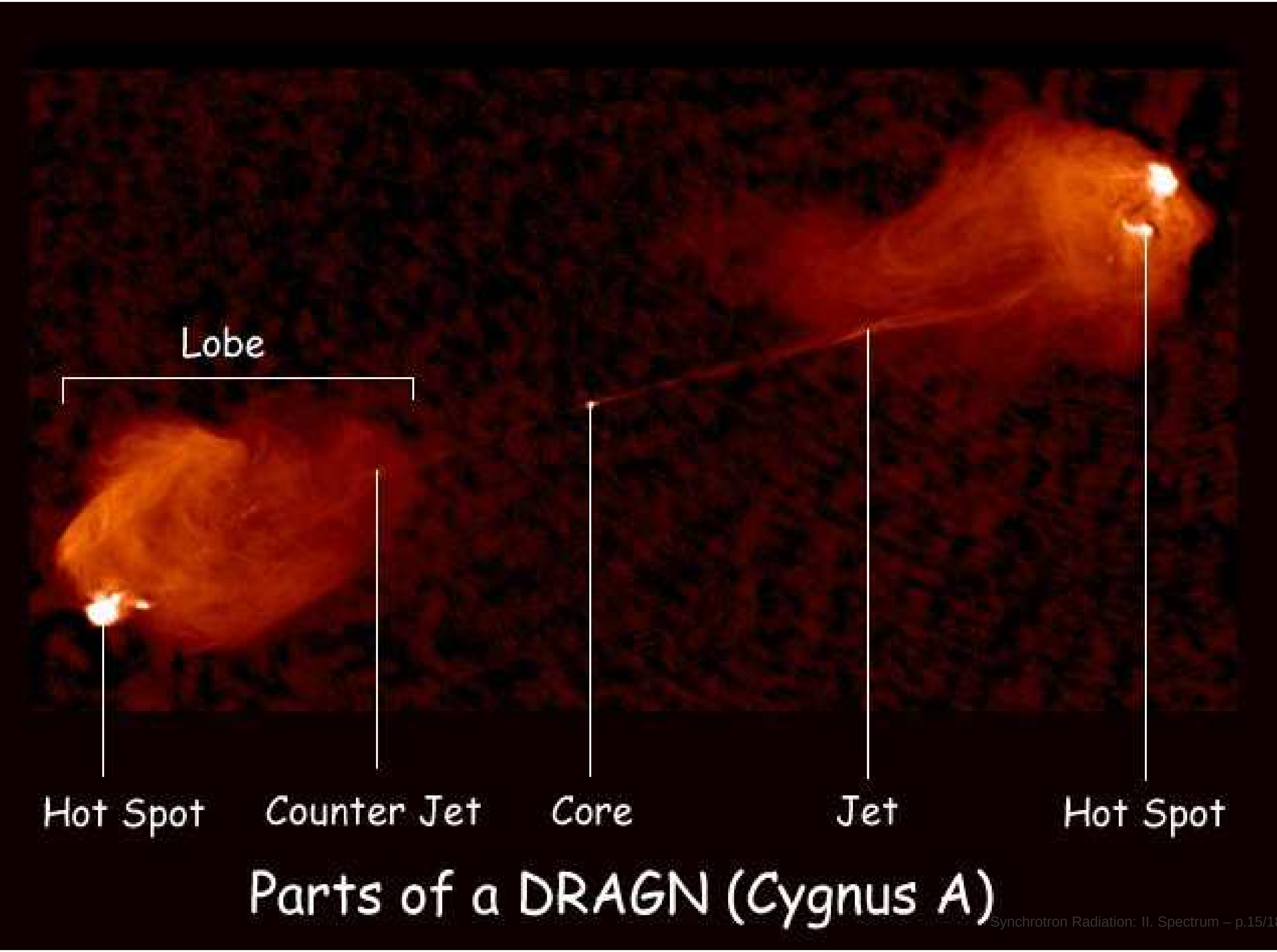
- How can this observation be explained?
- Recall that $\frac{L_c}{L_s} = \frac{U_{ph}}{U_B}$ where $U_{ph} \propto F_\nu \nu$
- For $B_0 = 10^{-3}$ Gauss, $\gamma = 10^3 \rightarrow \gamma^2 \omega_L \sim 10^9$ Hz
- Compton scattering with ultra-rel electrons of GHz photons $\rightarrow \gamma^2 \nu \sim 10^{15}$ Hz (optical wavelengths)

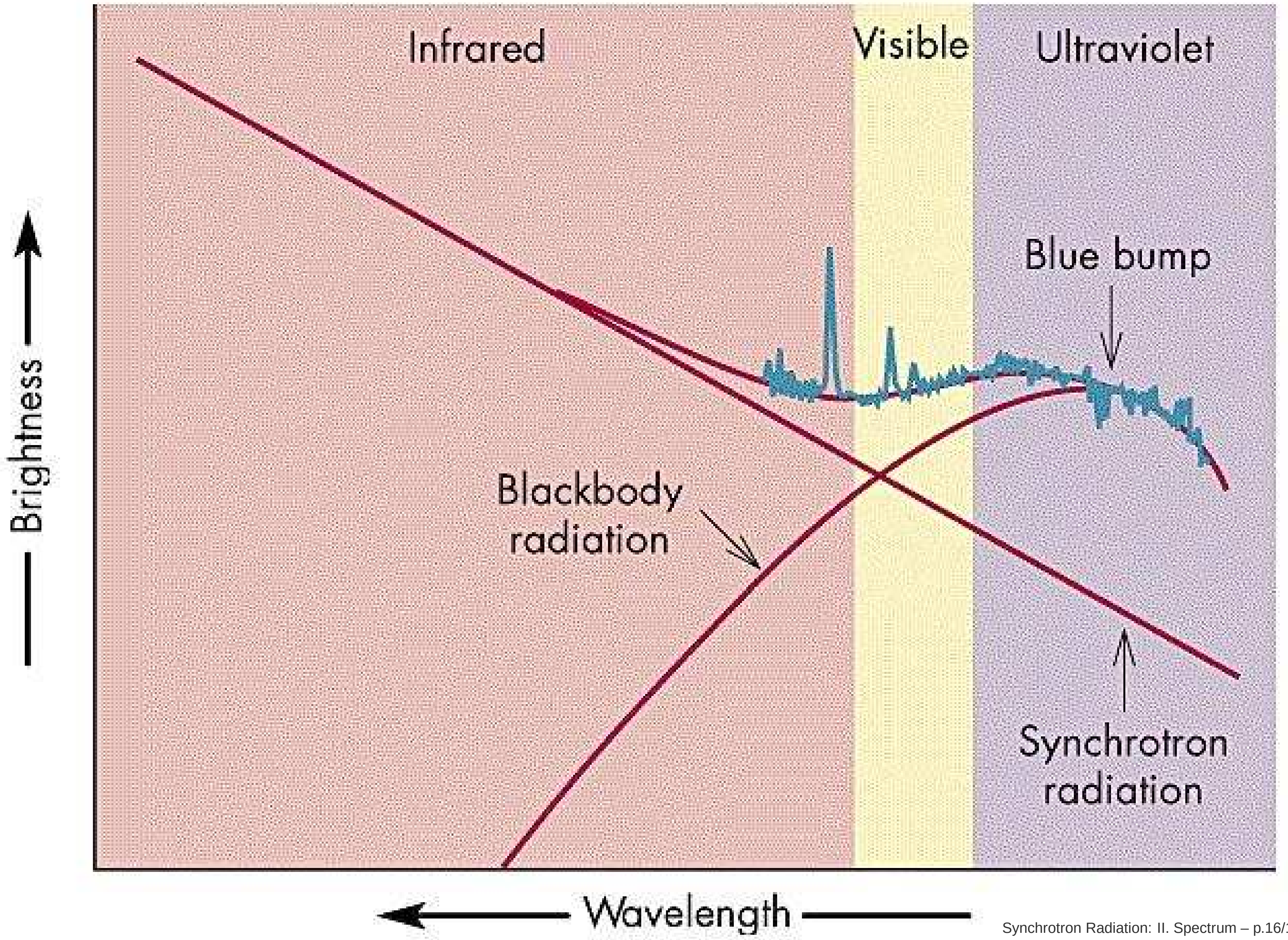
Astrophisical sources of synchrotron radiation

- Pulsars
- SN remnants (for example, the Crab nebula)
- Gamma ray bursts
- Radio Galaxies (jets from AGN)






Lobe
Hot Spot
Counter Jet
Core
Jet
Hot Spot
Parts of a DRAGN (Cygnus A)


Infrared
Visible
Ultraviolet
Blue bump
Blackbody radiation
Synchrotron radiation
Brightness
Wavelength

Application to Radio Galaxies

In 1959 Burbidge noticed a problem with the energetic requirements of radio galaxies

- In radio galaxies synchrotron dominates the entire spectrum from 10 meter to mm wavelengths
- Near equipartition of U_{CR} and U_B minimizes the energy requirement to produce the observed luminosity
- $R_{lobes} \sim 30$ kpc, $B_0 \sim 10^{-5}$ Gauss (from peak of spectrum near optically thick synchrotron)
- $E_{tot} = E_{CR} + E_{mag} \sim 2E_{mag} = (4\pi/3)R^3_{lobes}\frac{B_0^2}{8\pi} \sim 10^{58}$ ergs
- This is an enormous amount of energy: about energy generated by 10^7 SN explosions

In addition synchrotron cooling of lobes is extremely short:

$$m_e c^2 \dot{\gamma} = -P_{Sync} = -2\beta^2\gamma^2 c\sigma_T U_B \sin^2\alpha$$

$$t_{cool} = -\frac{\gamma}{\dot{\gamma}} \sim \frac{m_e c}{2\sigma_T U_B \gamma \sin^2\alpha}$$

$$t_{cool} \sim 10^7 \; yrs \; \mathrm{for} \gamma = 10^4 \mathrm{and} B_0 = 10^{-5} \; Gauss$$

Need engine that keeps pumping energetic electrons: SMBH at the center of galaxy.
However, our assumption of $\gamma = const$ in the derivation of the sychrotron radiation is valid because the period of gyration is typically of the order of seconds: $\ll t_{cool}$.