Clusters of Galaxies Overview

- Probes of the history of structure formation
  Dynamical timescales are not much shorter than the age of the universe

• Studies of their evolution, temperature and luminosity function can place strong constraints on all theories of large scale structure
  - and determine precise values for many of the cosmological parameters

Provide a record of nucleosynthesis in the universe- as opposed to galaxies, clusters probably retain all the enriched material created in them
  •Measurement of the elemental abundances and their evolution provide f fundamental data for the origin of the elements
  •The distribution of the elements in the clusters reveals how the metals were removed from stellar systems into the IGM

Clusters should be "fair" samples of the universe"
  •Studies of their mass and their baryon fraction reveal the "gross" properties of the universe as a whole
  •Much of the entropy of the gas is produced by processes other than shocks-
  - a major source of energy in the universe ?
  - a indication of the importance of non-gravitational processes in structure formation ?

Todays Material

- How do we know that clusters are massive
  - Virial theorem
  - Lensing
  - -X-ray Hydrostatic equilibrium (but first we will discuss x-ray spectra) Equation of hydrostatic equilibrium (*)
- What do x-ray spectra of clusters look like

se Kaiser sec 26.2-26.4

*Hydrostatic equilibrium

$$\nabla \mathbf{P} = -\rho_g \nabla \phi(r)$$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter) **P** is gas pressure and ρ_g is the gas density ($\nabla f = (\partial f/x_1, \partial f/x_2 \ldots \partial f/x_n$)

How to Start....Using Galaxy Dynamics

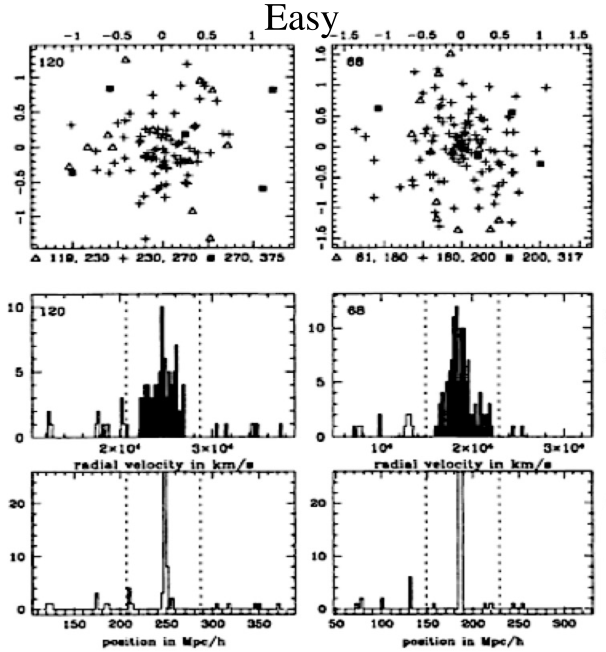
Basic procedure

- first identify the cluster via some sort of signal (e.g. an overdensity of galaxies)
- deduce cluster membership
  - this is not easy and the inclusion of even small fractions of interloper galaxies that are not gravitationally bound to the cluster can lead to a strong mass bias)
- estimate a cluster mass using galaxy positions and velocities as input

    Viral theorem

    Jeans eq

Easy

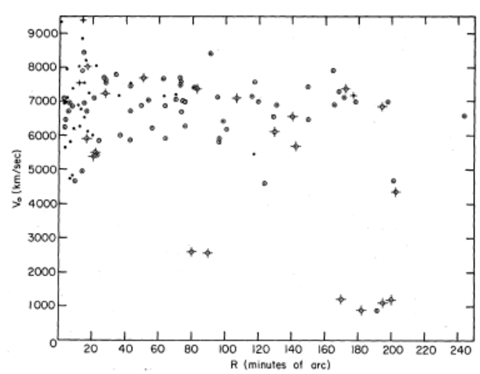


2 clusters

bottom panel is 'true' distance of clusters, middle is velocity histogram, top is position of galaxies (+ for galaxies in cluster)
van Haarlem,Frenk and White 1997

The First Detailed Analysis

- Rood et al used the King (1969) analytic models of potentials (developed for globular clusters) and the velocity data and surface density of galaxies to infer a very high <u>mass to light ratio</u> of ~230.
- Since "no" stellar system had M/L>12 dark matter was necessary



Rood 1972- velocity vs position of galaxies in Coma

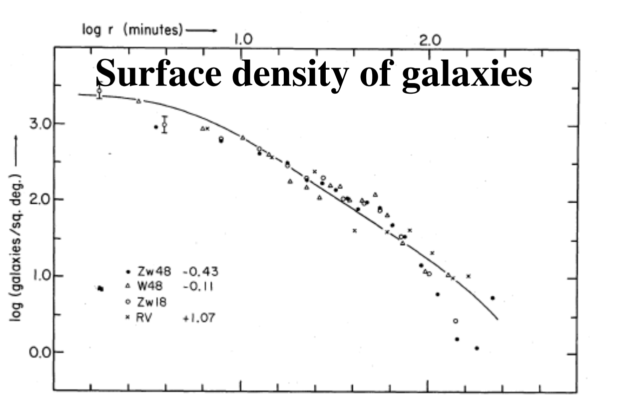


FIG. 5.—Surface densities, corrected for backgrounds given in table 2. For this fitting, logarithms of

Paper is worth reading ApJ 175,627

Virial Theorem (Longair 3.5.1; see also Kaiser sec 26.3)

- The virial theorem states that, for a stable, self-gravitating, spherical distribution of equal mass objects (stars, galaxies, etc), the total kinetic energy of the objects is equal to -1/2 times the total gravitational potential energy.

$2<T>=-<U_{TOT}>$

T is the time average of the Kinetic energy and U is the time overage of the potential energy In other words, the potential energy must equal 1/2 the kinetic energy.

- Consider a system of **N** particles with mass m and velocity v.
- kinetic energy of the total system is K.E. (system) = $1/2\ m\ N\ v^2 = 1/2\ M_{tot} v^2$

$U \sim 1/2 GN^2m^2/R_{tot} = 1/2 GM^2_{tot}/R_{tot}$ (eq 3.13 Longair dimensional analysis)

If the orbits are random KE=1/2U (virial theorem)

$M_{tot} \sim 2Rv^2_{tot}/G$

No assumptions been made about the velocity distributions of the particles.

The virial theorem applies to all cases provided the system is in dynamical equilibrium- "virialized"

For more detailed derivation see Longair eqs 3.4-3.16 also http://www.sjsu.edu/faculty/watkins/virialth.htm

Virial Theorem Actual Use (Kaiser 26.4.2)

- Photometric observations provide the surface brightness Σ_{light} of a cluster. Measurements of the velocity dispersion σ^2_v together with the virial theorem give $\sigma^2_v \sim U/M \sim GM/R \sim G\Sigma_{mass} R$

  Σ_{mass} is the projected **mass** density.

At a distance **D** the mass to light ratio (M/L) can be estimated as $M/L = \Sigma_{mass}/\Sigma_{light} = \sigma^2_v / GD\Theta\Sigma_{light}$; Σ_{light} is the surface brightness- a direct measurable

Notice all the terms are observable ! D= cluster distance, G=gravitational constant **Θ** is the angular size of the cluster. (see Kaiser eq 26.13)

Compare M/L with what is expected for stellar systems (e.g. galaxies)

- The virial theorem is exact, *but requires that the light traces the mass- it will fail if the dark matter has a different profile from the luminous particles.*

Mass Estimates

- While the virial theorem is fine it depends on knowing the time averaged orbits, the distribution of particles etc etc-
- If the system is spherically symmetric, a suitably weighted mean separation R_{cl} can be estimated from the observed surface distribution of stars or galaxies and so the gravitational potential energy can be written $|U| = GM^2/R_{cl}$.
- The mass of the system is using $T = ½ |U|$
- $M = 3\sigma^2 R_{cl}/G$. (The & White 1986); R_{cl} depends on the density distribution
- In useful units this gives $M=3R_G\sigma^2_v/G= 7.0\times 10^{12}R_G(\sigma_v)^2M_\odot$

where R_G is in units of Mpc and σ_v is the velocity dispersion in units of 100km/sec

Would like better techniques

- Gravitational lensing
- Use of spatially resolved x-ray spectra

- The viral mass estimator is given by

$$M(<r) = \frac{3\pi N \sum_i v_{z,i}(<r)^2}{2G\sum_{i\neq j}\frac{1}{R_{ij}}}$$

- where $v_{z,i}$ is the galaxy line-of-sight velocity and R_{ij} is the projected distance between two galaxies.

Mass Estimate

- As pointed out by Longair The application of the theorem to galaxies and clusters is not straightforward.
- only radial velocities can be measured from the Doppler shifts of the spectral lines, not the 'true' velocity dispersion.
- Assumptions need to be made about the spatial and velocity distributions of stars in
- the galaxy or the galaxies in a cluster e.g :that the galaxies have the same spatial and velocity distribution as dark matter particles, and that all galaxies have the same mass
- If the velocity distribution is isotropic, the velocity

dispersion is the same in the two perpendicular directions as along the line of sight and so $<v^2> = 3<v_r>^2$ where v_r is the radial velocity which is measureable.

If the velocity dispersion is independent of the masses of the stars or galaxies, the total kinetic energy is $T = 3/2M <v_r>^2$ (3.18)

- If the system is spherically symmetric, a suitably

weighted mean separation R_{cl} can be estimated from the observed surface distribution of stars or galaxies and so the gravitational potential energy is

$|U|= GM^2/R_{cl}$ and thus using the viral theorem $M=3<v_r>^2 R_{cl}/G$ 3.18 (Longair)

Mass Determination

- for a perfectly spherical system one can write the **Jeans equation for a spherical system** where ν is the density of a tracer - this is used a lot to derive the mass of elliptical galaxies
- Anisotropy parameter $\beta(r) = 1 - \sigma(r)_\theta^{2}/\sigma(r)^2_r$

$$GM(\leq r) = -r\sigma_r^2\{[d\ln\nu/d\ln r + d\ln\sigma_r^2/d\ln r_r] + 2\beta(r)\}$$

$\sigma^2_\phi = \sigma^2_\theta$ Spherical symmetry.

$\sigma^2_r << \sigma^2_\theta$ Nearly circular
$\sigma^2_r >> \sigma^2_\theta$ Nearly radial

•Notice the nasty terms

All of these variables are 3-D; we observe projected quantities !

Both rotation and random motions (σ-dispersion) can important.

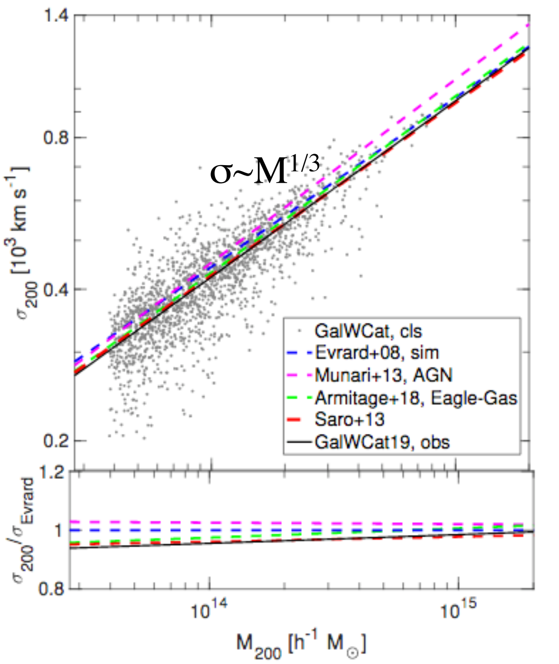
Standard practice is to make an apriori assumption about form of potential, density distribution or velocity field.

https://ned.ipac.caltech.edu/level5/Sept03/Merritt/Merritt2.html

see arXiv:1907.05061v2 for a recent application of this technique

Using Velocity Dispersion

- When virial equilibrium is satisfied, the specific thermal energy of dark matter in a halo of mass M and radius R will scale with its potential energy,GM/R and $\sigma^2 \sim M^{2/3}$ using virial mass estimator
- Using this estimator and modern redshift surveys can produce a large catalog of masses under the assumption that the potential has a certain form



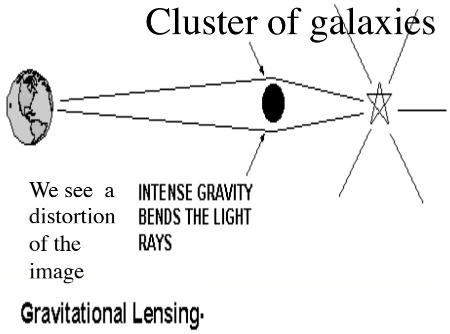
Mass vs velocity dispersion
GalWeight 1907.05061.pdf

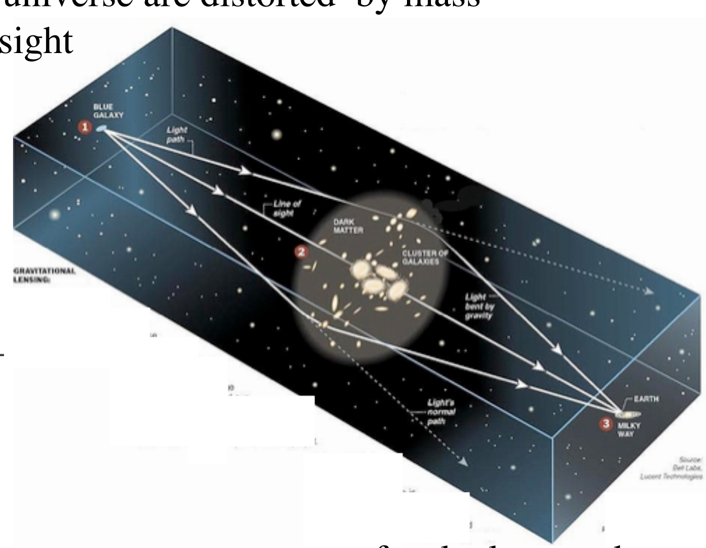
Light Can Be Bent by Gravity- Read sec 4.7 Longair

gravitational lensing. Light rays propagating to us through the inhomogeneous universe are distorted by mass distributed along the line of sight

The 'more' mass- the more the light is bent

Cluster of galaxies





faculty.lsmsa.edu

Amount and type of distortion is related to amount and distribution of mass in gravitational lens

Zwicky Again

- Gravitational lensing probes directly the total mass distribution, independent of the distribution of baryonic matter

Zwicky 1937

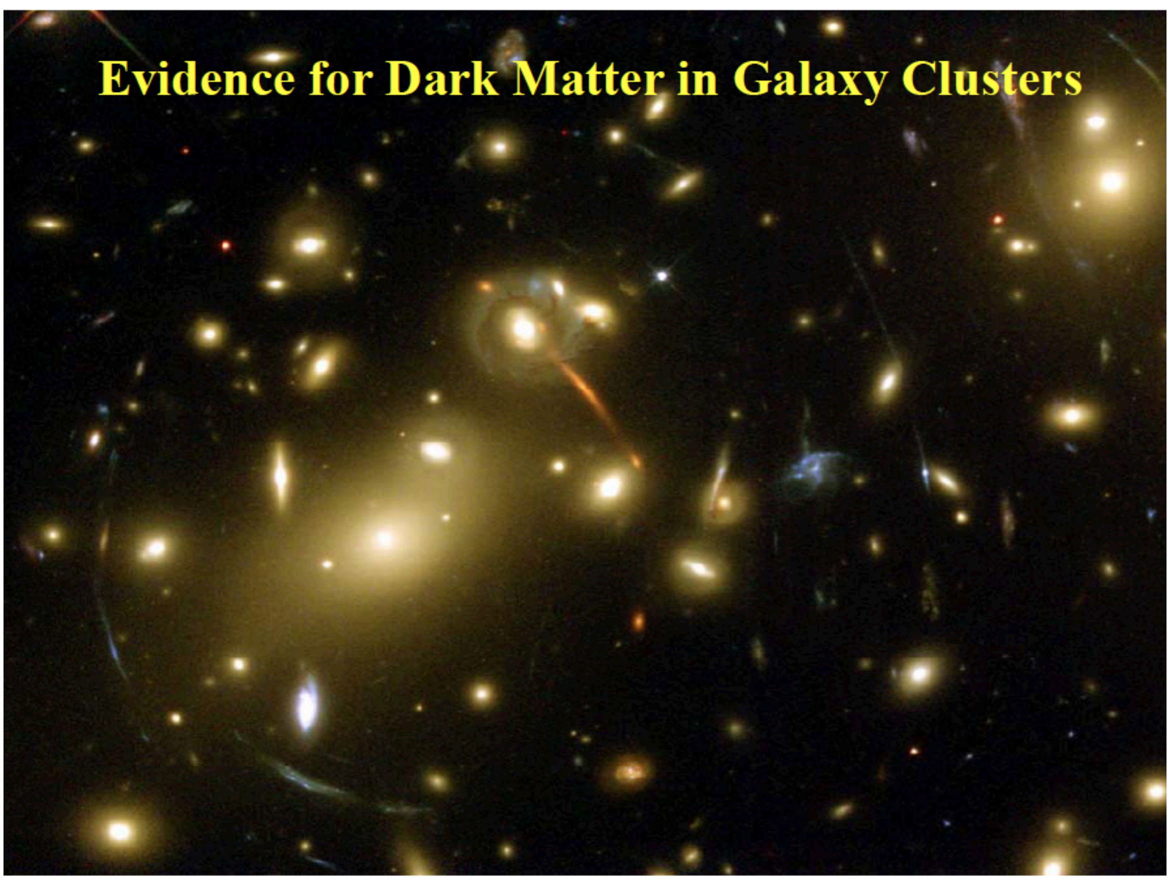
Nebulae as Gravitational Lenses

The discovery of images of nebulae which are formed through the gravitational fields of nearby nebulae would be of considerable interest for a number of reasons.

(1) It would furnish an additional test for the general theory of relativity.

(2) It would enable us to see nebulae at distances greater than those ordinarily reached by even the greatest telescopes. Any such *extension* of the known parts of the universe promises to throw very welcome new light on a number of cosmological problems.

(3) The problem of determining nebular masses at present has arrived at a stalemate. The mass of an average nebula until recently was thought to be of the order of $M_N = 10^9 M_\odot$, where $M_\odot$ is the mass of the sun. This estimate is based on certain deductions drawn from data on the intrinsic brightness of nebulae as well as their spectrographic rotations. Some time ago, however, I showed[2] that a straightforward application of the virial theorem to the great cluster of nebulae in Coma leads to an average nebular mass four hundred times greater than the one mentioned, that is, $M_N' = 4 \times 10^{11} M_\odot$. This result has recently been verified by an investigation of the Virgo cluster.[3] Observations on the deflection of light around nebulae may provide the most direct determination of nebular masses and clear up the above-mentioned discrepancy.



Basics of Gravitational Lensing Sec 4.7 Longair

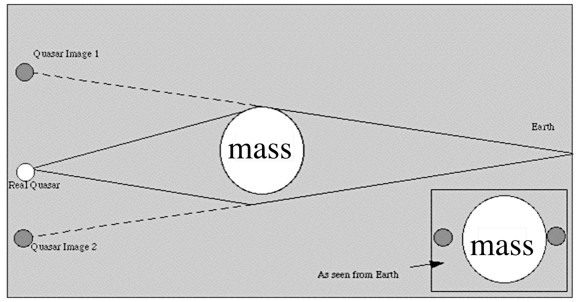
- See Lectures on Gravitational Lensing by

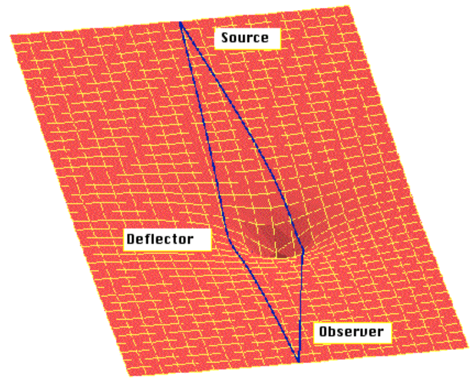
Ramesh Narayan Matthias Bartelmann or
  http://www.pgss.mcs.cmu.edu/1997/Volume16/physics/GL/GL-II.html

For a detailed discussion of the problem

- Rich centrally condensed clusters can produce **giant arcs** when a background galaxy happens to be aligned with one of the cluster caustics*. Can have multiple images
- Every cluster produces **weakly distorted images of large numbers of background galaxies.**
  - These images are called arclets and the phenomenon is referred to as weak lensing.
- The deflection of a light ray that passes a point mass M at *impact parameter b* is eq 4.28

  $\Theta_{def} = 4GM/c^2b$; $R_{Sch} = 2GM/c^2$





The Angle we Measure

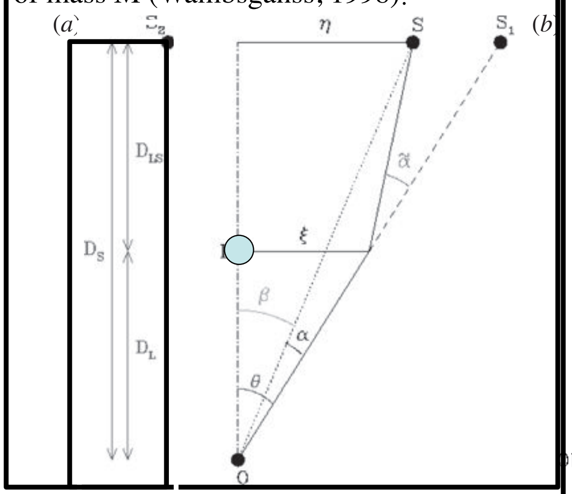
- We have to divide the angle by the distance ratio so that
- e.g the angle we measure is twice the Schwarzschild radius divided by the impact parameter and scaled by the distance ratio D_{LS}/D_S

Gravitational lensing has two major advantages:

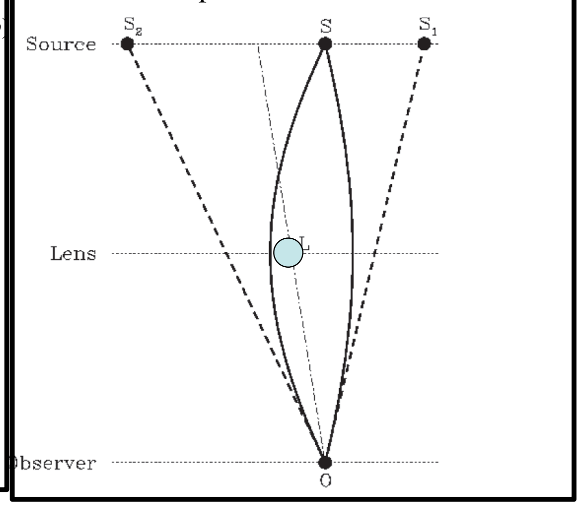
- its foundation in the theory of gravity is straightforward, and it is sensitive to matter (and energy) inhomogeneities regardless of their internal physical state.
- Under the assumptions that gravitational lenses are weak, and are much smaller than cosmological length scales, the effects of gravitational lensing are entirely captured by a two-dimensional effective lensing potential. (Matthias Bartelmann & Matteo Maturi 1612.06535.pdf)

- See https://ned.ipac.caltech.edu/level5/March04/Kochanek2/Kochanek_contents.html for a complete course on lensing !

the deflection of light by a deflector, or lens, of mass M (Wambsganss, 1998).



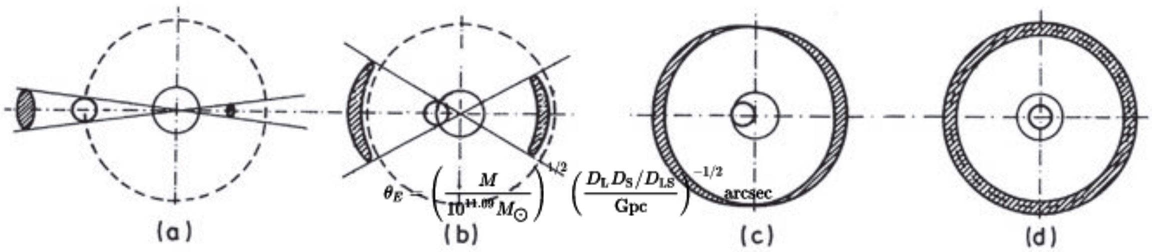
the two light paths from the source to the observer for a point mass



D_S= distance to source
D_{LS} distance from source to lens
D_L distance from us to lens (L)

Longair Figure 4.11

Arcs- Strong Lensing



- Changes in the appearance of a compact background source as it passes behind a point mass. The dashed circles correspond to the Einstein radius (really an angle). When the lens and the background source are precisely aligned, an Einstein ring is formed with radius equal to the Einstein radius θ_E .

Lensing

- Einstein radius is the scale of lensing
- For a point mass it is
- $\theta_E = ((4GM/c^2)(D_{ds}/D_d D_s))^{1/2}$
- or in more useful units
- $\theta_E = (0.9'')M_{11}^{\ 1/2}\ D_{Gpc}^{\ -1/2}$
- Lens eq

$\beta\ = \theta-(D_{ds}/D_d D_s)4GM/\theta c^2$.

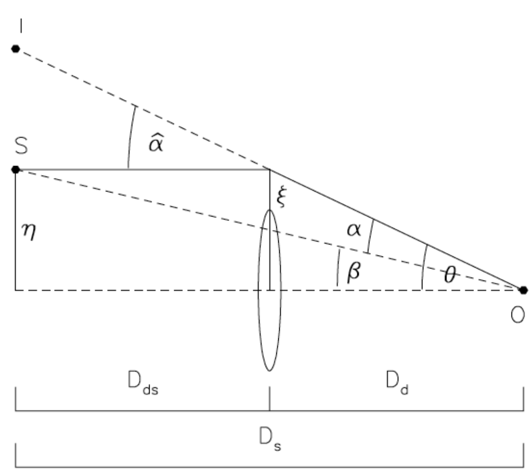
or

$\beta\ = \theta-\theta^2_E/\theta$

β 2 solutions

Any source is imaged twice by a point mass lens

Gravitational light deflection preserves surface brightness because of the Liouville theorm



- The optical properties of a lumpy universe are similar to that of a block of glass of inhomogeneous density where the refractive index is $n(r) = (1- 2\phi(r)/c^2)$ with *$\phi(r)$ the Newtonian gravitational potential*. In an over-dense region, ϕ is negative, so n is slightly greater than unity. In this picture we think of space as being flat, but that the speed of light is slightly retarded in the over-dense region.

Lensing

- assume –

matter inhomogeneities which cause the lensing are local perturbations.

- Light paths propagating from the source past the lens 3 regimes
- 1) light travels from the source to a point close to the lens through unperturbed spacetime.
- 2) near the lens, light is deflected.
- 3) light again travels through unperturbed spacetime.

- For a single point lens at the origin there will be two images

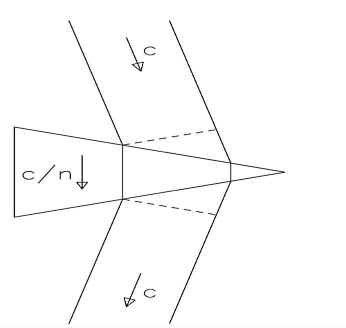
The effect of spacetime curvature on the light paths can be expressed in terms of an effective index of refraction, *n*, (e.g. Schneider et al. 1992)

$\boldsymbol{n} = 1 -(2/c^2)\ \phi(r)$;$\phi(r)$ the Newtonian gravitational potential

As in normal optics, for refractive index $\boldsymbol{n} > 1$ light travels slower than in free vacuum.

effective speed of a ray of light in a gravitational field is

$v = c/n \sim c-(2/c)\phi$



Ways of Thinking About Lensing (Kaiser sec 33.5)

- This deflection is just twice what Newtonian theory would give for the deflection of a test particle moving at v = c where we can imagine the radiation to be test particles being pulled by a gravitational acceleration.

- another way to look at this using wave-optics; the inhomogeneity of the mass distribution causes space-time to become curved. The space in an over-dense region is positively curved.

  Light rays propagating through the over-density have to go a slightly greater distance than they would in the absence of the density perturbation.

  Consequently the wave-fronts get retarded slightly in passing through the over-density and this results in focusing of rays.

- The deflection of light by a point mass *M* due to the bending of space-time amounts to precisely twice that predicted by a Newtonian calculation,
- $\alpha = [4GM/bc^2]$, (4.28)

b is the 'impact parameter' ~ the distance of closest approach of the light ray to the deflector.

As an example, we now evaluate the deflection angle of a point mass M (cf. Fig. 3). The Newtonian potential of the lens is

$$\Phi(b,z) = -\frac{GM}{(b^2+z^2)^{1/2}} , \tag{5}$$

where b is the impact parameter of the unperturbed light ray, and z indicates distance along the unperturbed light ray from the point of closest approach. We therefore have

$$\vec{\nabla}_\perp \Phi(b,z) = \frac{GM\,\vec{b}}{(b^2+z^2)^{3/2}} , \tag{6}$$

where $\vec{b}$ is orthogonal to the unperturbed ray and points toward the point mass. Equation (6) then yields the deflection angle

$$\hat{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi \, dz = \frac{4GM}{c^2 b} . \tag{7}$$

Note that the Schwarzschild radius of a point mass is

$$R_S = \frac{2GM}{c^2} , \tag{8}$$

so that the deflection angle is simply twice the inverse of the impact parameter in units of the Schwarzschild radius. As an example, the Schwarzschild radius of the Sun is 2.95 km, and the solar radius is 6.96×10^5 km. A light ray grazing the limb of the Sun is therefore deflected by an angle $(5.9/7.0) \times 10^{-5}$ radians $= 1.''7$.

Narayan and Bartellman 1996

Einstein radius is the scale of lensing (see derivation in Longair pg 118)

- For a point mass it is

$\theta_E = [(4GM/c^2)(D_{LS}/D_L D_S)]$ (4.30)-

- or in more useful units

$\theta_E = (0.9'')M_{11}^{1/2} D_{Gpc}^{-1/2}$ (4.32)

$\theta_E \sim 1.6\ (M_{15}/M_\odot)^{1/2} / D^{-1/2}_{Gpc}$ arcmin . D in units of Gpc

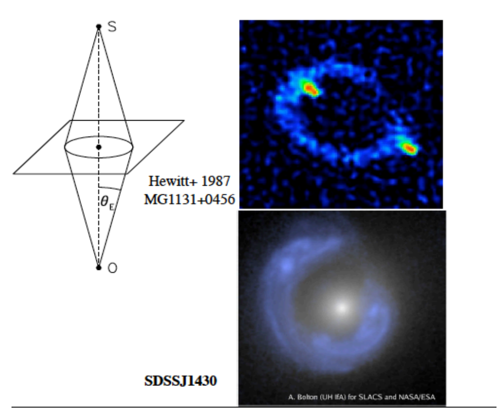
where $D = (D_S D_L / D_{LS})$.

- The gravitational deflection of the light rays is $\alpha = 4\pi\sigma^2/c^2$.
- For a singular isothermal sphere, the gravitational deflection is independent of the distance at which the light rays pass by the lens

Einstein Ring

When lens, source, and observer lie on the same line get Einstein ring

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$



Condition for formation of lensed image (Σ is surface mass density)

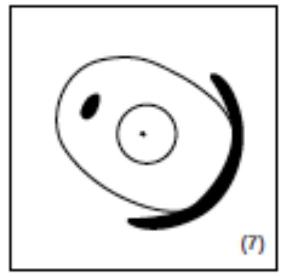
$\Sigma_{cr} > [c^2/4\pi G] D_{LS}/D_S D_L \sim 0.35\,\mathrm{g\ cm^{-2}}/[D]$ D in Gpc;

see 4.35-4.39

Cluster Lensing

- Clusters can be more or less well modeled by isothermal spheres which has a density distribution of $\rho \sim 1/(b^2+r^2)$ (b being a core radius)
- This gives a lensing solution with the Einstein radius
- $\theta_E = 28.8\sigma_{1000}^2 \, D_{LS}/D_s$ arcsec
- σ is in units of 1000km/sec as is appropriate for clusters-
- a robust expression for estimating the masses of clusters of galaxies

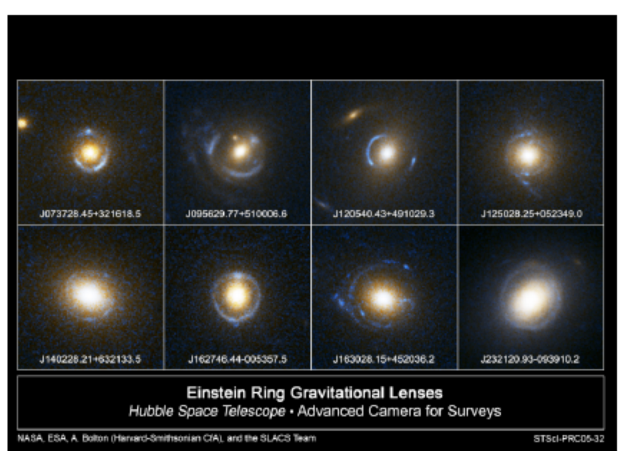
Strong lensing of background sources only occurs if they lie within the Einstein angle θ_E of the axis of the lens- e.g. the giant arcs



Lensing

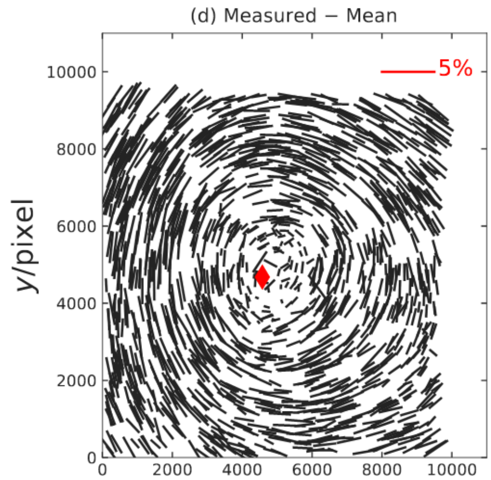
- This is exact for a single isothermal sphere model of the mass of the lensing object and the Einstein radius is
- $\theta_E \approx 4.4\times10^{14} M_\odot (r_t/30")^2 \, (D_L D_s / D_{LS})$ D in units of Gpc
- where D_s is the distance to the source, D_{LS} is the distance to the source as seen from the lens
- The Einstein ring happens when the background source is exactly aligned with the foreground mass

Good for estimating the masses of clusters of galaxies (Fort and Mellier, 1994).see https://ned.ipac.caltech.edu/level5/March14/Weinberg/Weinberg6.html- Clusters of galaxies as cosmological probes and weak lensings

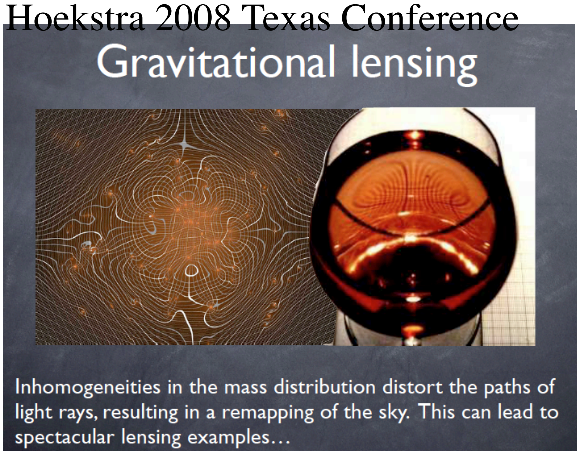


Large Scale Mass Distribution

- Cluster shear measurements Dietrich et al 2019



Hoekstra 2008 Texas Conference

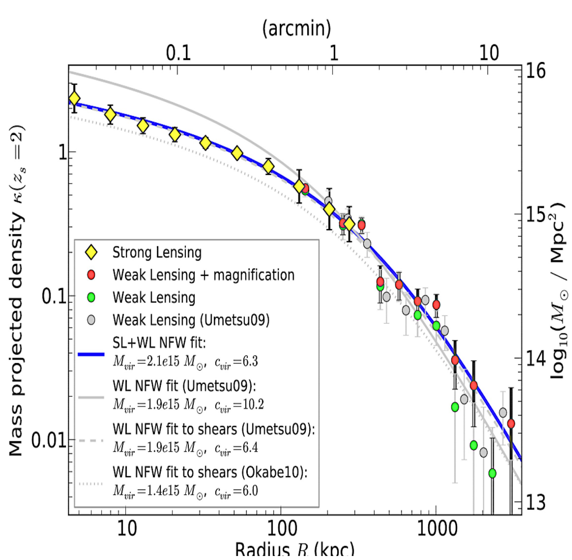


Recent Results

- Detailed lensing studies of >70 clusters have now been done (e.g. CLASH project 2018ApJ...860..104U Umetsu, K et al Canadian Cluster Comparison Project Hoekstra et al 2015MNRAS.449..685, Subaru data Niikura et al 1504.01413.pdf)
- Can combine strong and weak lensing

Results,in general, are consistent with NFW density profile

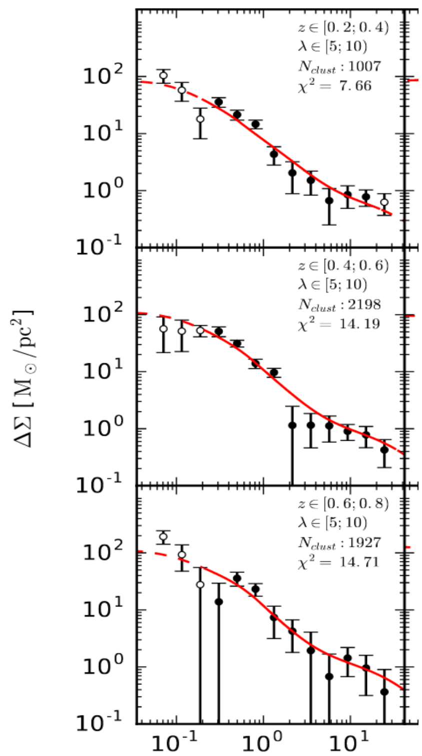
$\rho_{NFW}(r)=\rho_c/(r/r_s)(1+r/r_s)^2$



Recent Results

- The DES has produced a very large number of cluster weak lensing signals.(8,000 clusters) to z~0.8
- While each one is very noisy they can be stacked in mass, redshift to derive statistical results (Melchior et al 1610.06890.pdf) on cluster surface mass profiles.

- DES= Dark energy survey (https://www.darkenergysurvey.org/



R(Mpc)

NFW Profile

- Stacked mass profile of 50 clusters compared to a theoretical model of the potential of a cluster (NFW profile Niikura et al 2016)

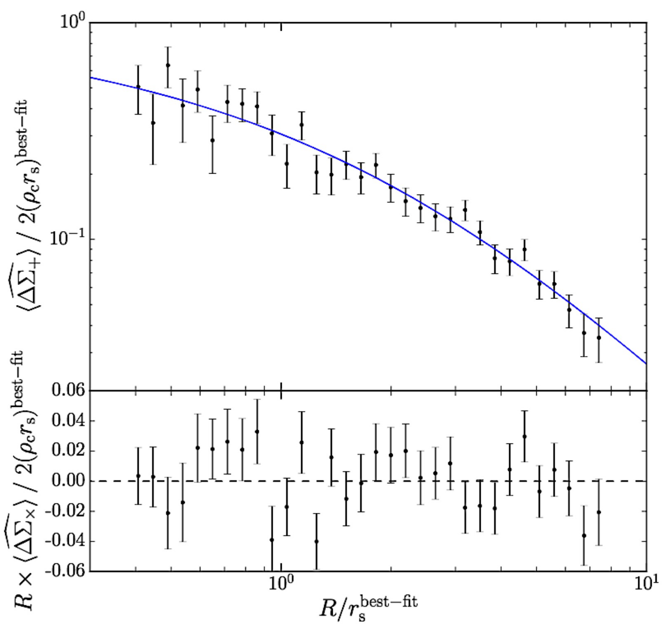
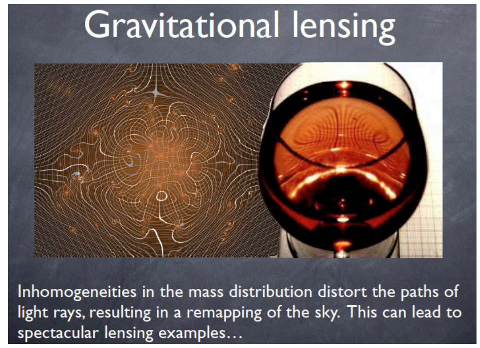
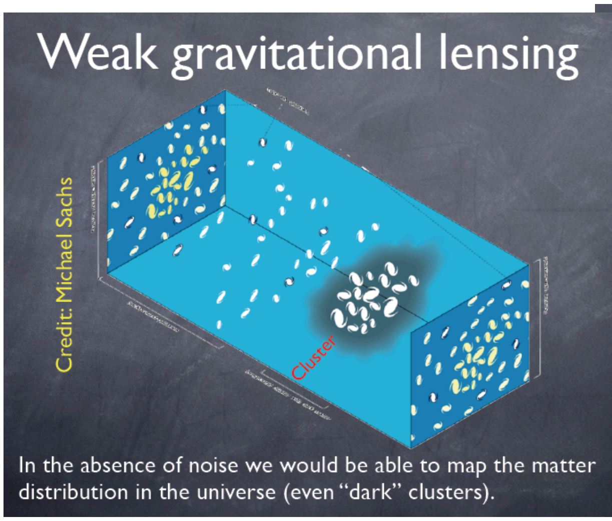


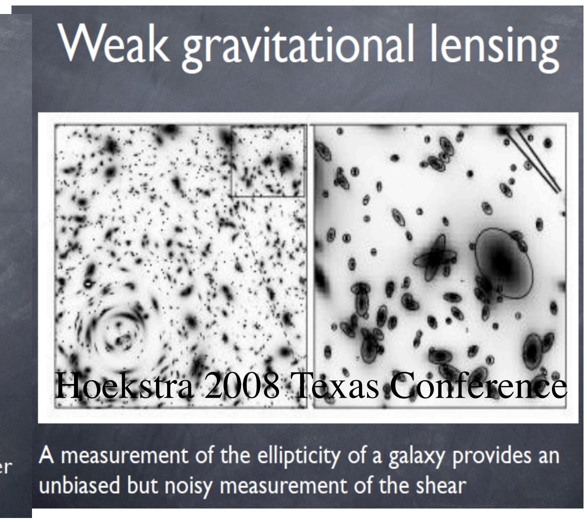
Figure 6. *Upper panel:* The stacked distortion profile measured from 50

Weak Lensing

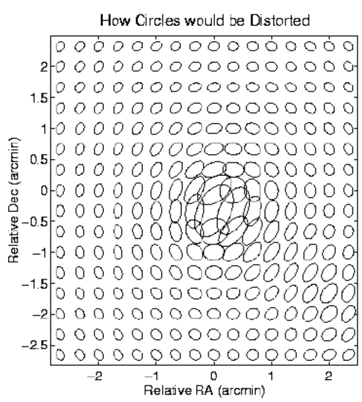
- Look for the distortion of the shape of the background objects

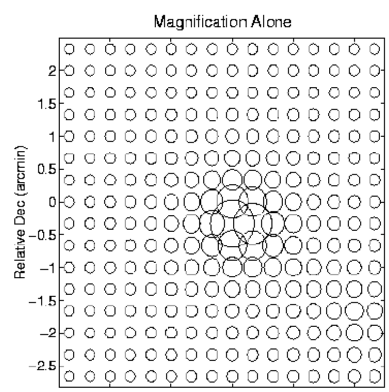


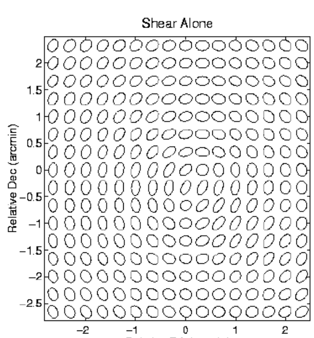


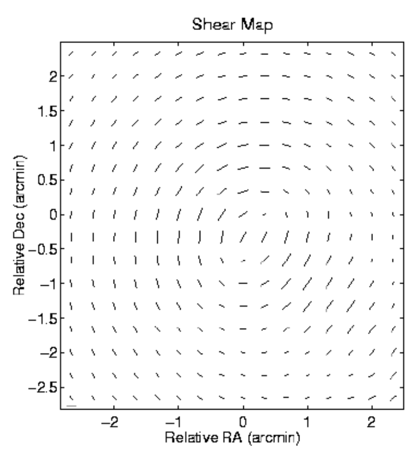


What Does the Lensing Signal look like ? From George F. Smoot

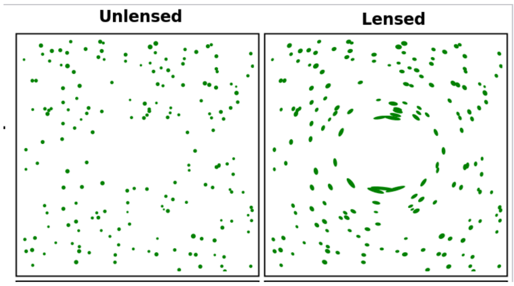


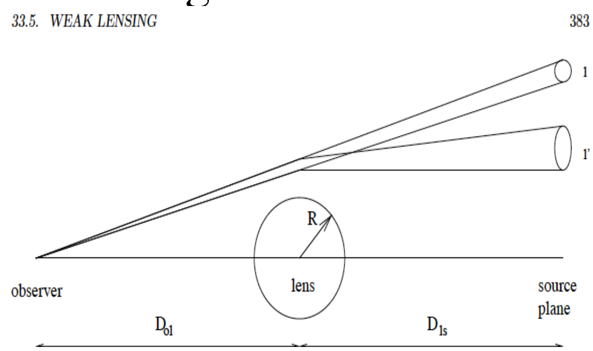






Weak Lensing





- Causes a circular image to appear elliptical (the image is 'sheared')-the strength of the effect scales as the product of the density and the size of the object; it is proportional to the surface density of the lens.
- the fractional stretching of the image, also known as the 'image-shear', is $\gamma = l'/(l - 1)$
- It is statistical in nature since the distortion for a given object is rather small (See Schenider presentation on the web page)

Weak Lensing

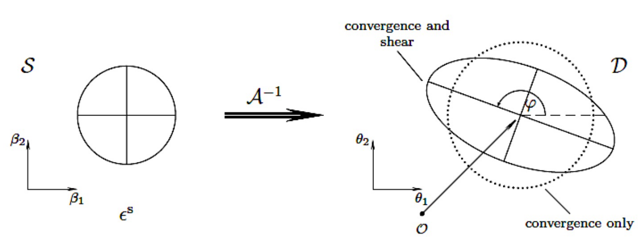
Weak lensing directly measures the projected mass

in detail it measures the ratio,***k***, of the surface density of the cluster to the critical surface density $\Sigma_{crit}=(c^2/4\pi G)(D_S/D_L D_s)$

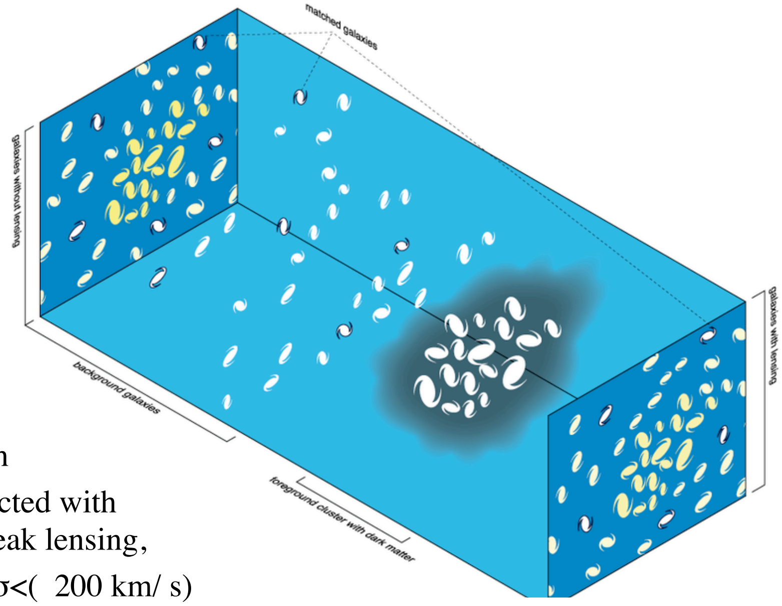
$$\kappa = \frac{1}{2}\nabla^2\phi$$

However we cannot measure the potential directly ! We can measure the distortion in the images of background galaxies- the distortion is caused by lensing

- the amplitude of distortion (called shear) is related to the potential



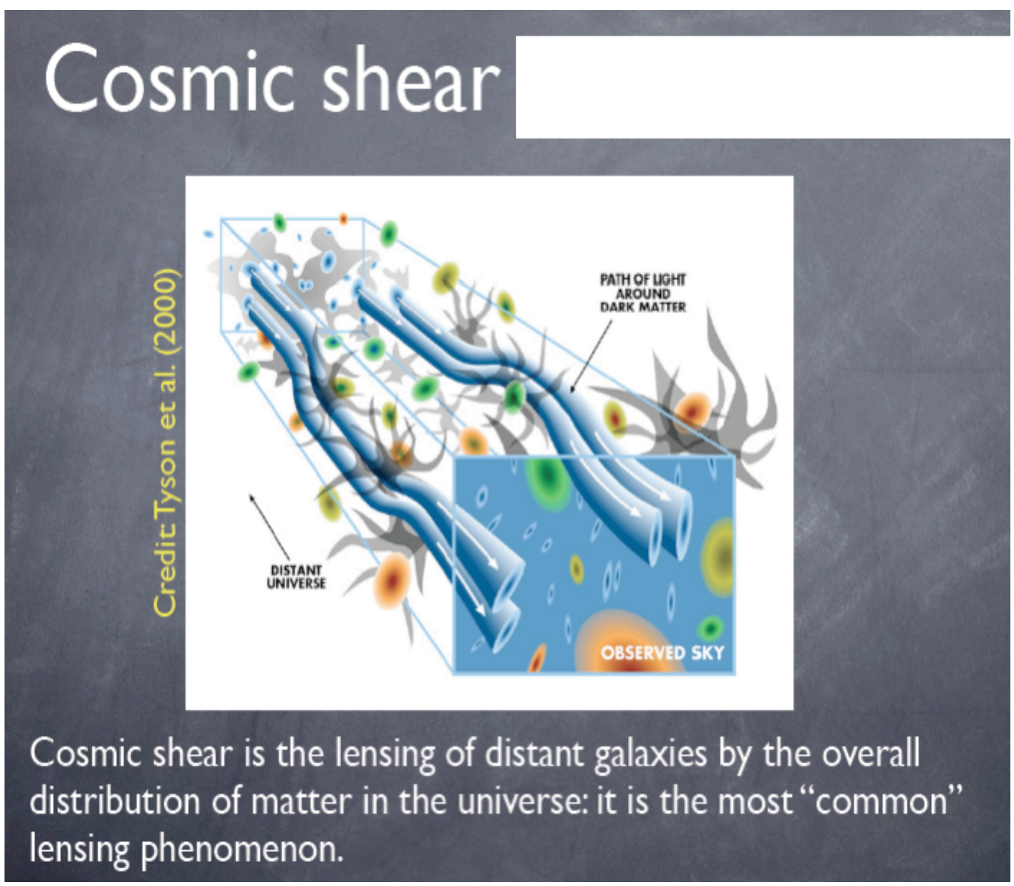
Weak Lensing



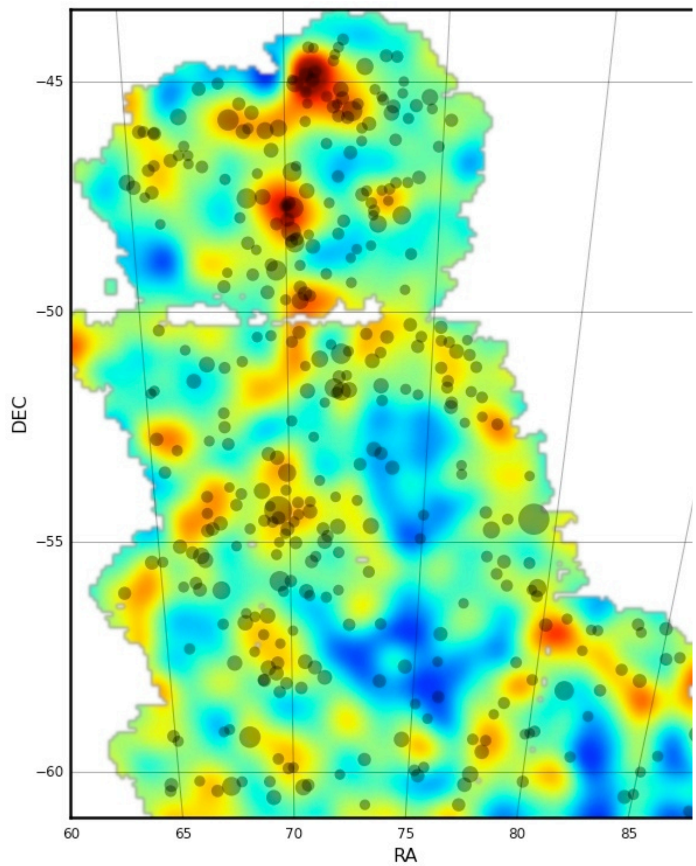
- clusters of galaxies with

$\sigma >$(600 km/ s can be detected with sufficiently large S/N by weak lensing,

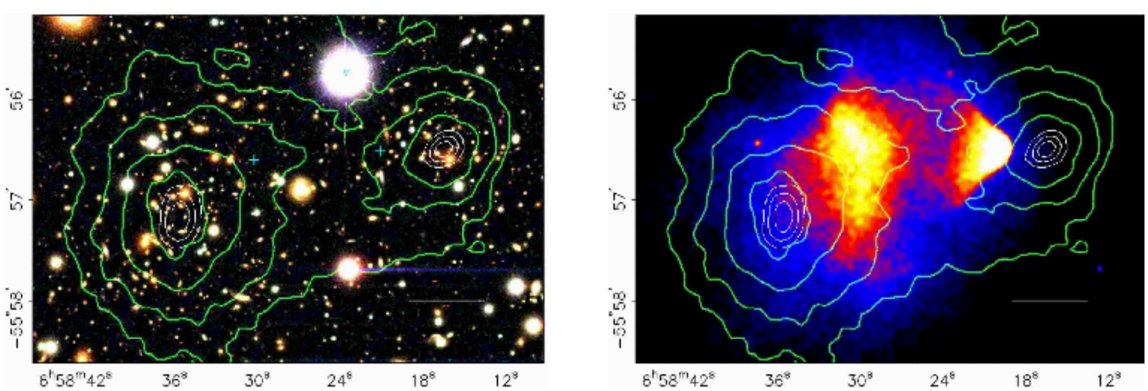
- but individual galaxies $\sigma <$(200 km/ s) are too weak as lenses to be detected



- The detailed distribution of dark matter traced across a large area of sky by **weak lensing**: yellow and red represent relatively dense regions of dark matter and the black circles represent galaxy clusters (Chang et al 2015)



Lensing and Dark Matter



- Left panel (Clowe et al 2004) optical imaging and mass contours from lensing
- Right panel is the x-ray image with the lensing contours showing that the dominant baryonic component (hot gas) and the total mass (dominated by dark matter) were not in the same place

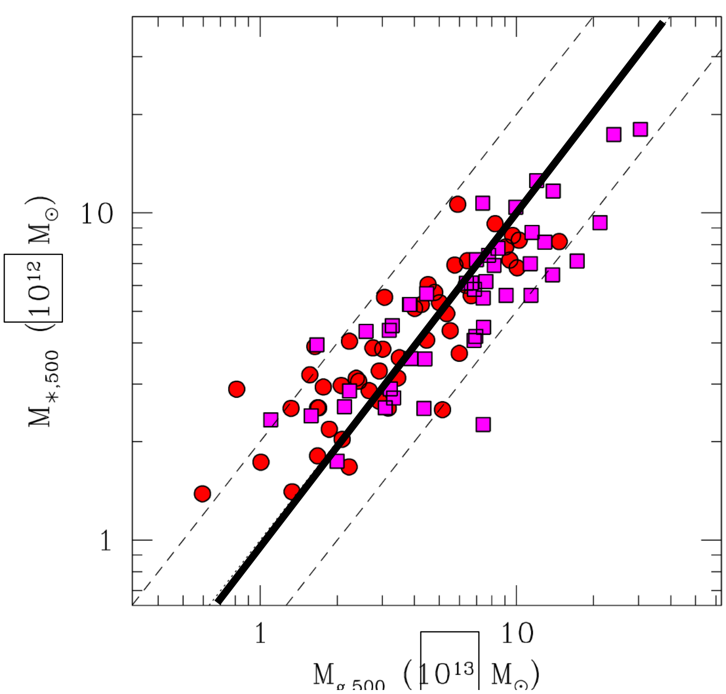
END OF LENSING....

X-rays from Clusters of Galaxies

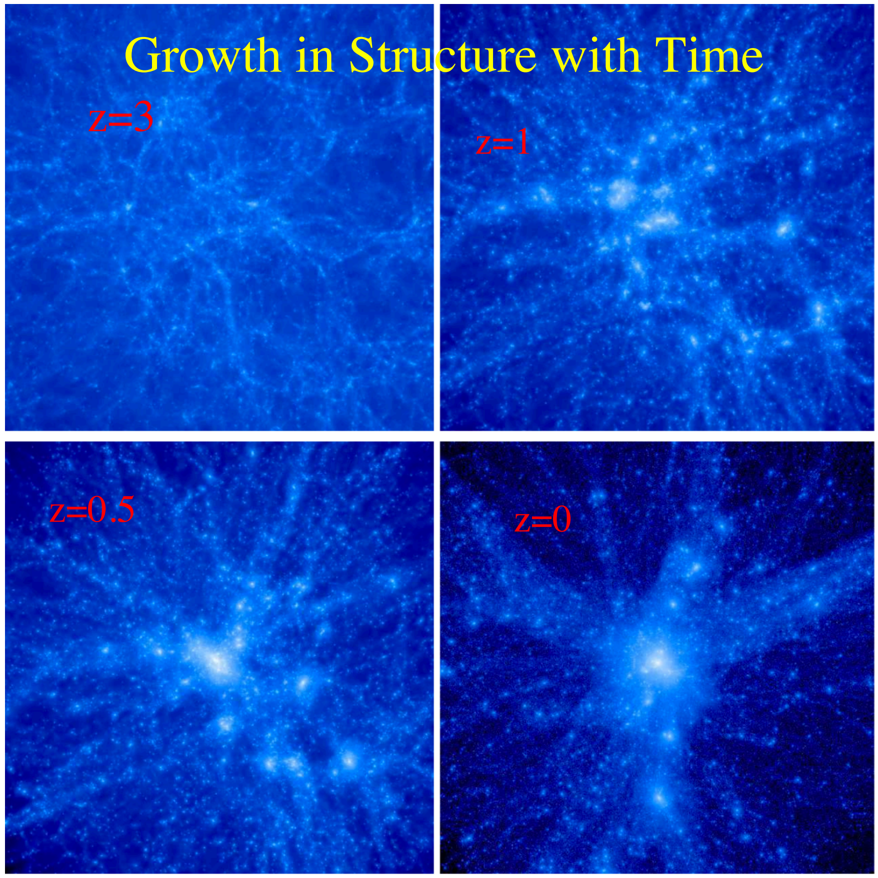
- The baryons *thermalize* to > 10^6 K making clusters strong X-ray sources- the potential energy of infall is converted into kinetic energy of the gas.
- Most of the baryons in a cluster are in the X-ray emitting plasma - only 10-20% are in the galaxies.
- Clusters of galaxies are self-gravitating accumulations of dark matter which have trapped hot plasma (intracluster medium - ICM) and galaxies: (the galaxies are the least massive constituent)

Mass in Gas vs Stars

Mass in gas is ~10x that in stars in low (z<0.1 red) and high (z~0.5) purple clusters
(Kravtsov and Borgani 2012)



Simulation is centered on what will become a massive cluster



What we try to measure with X-ray Spectra

- From the x-ray spectrum of the gas we can measure a mean temperature, a redshift, and abundances of the most common elements (heavier than He).
- With good S/N we can determine whether the spectrum is consistent with a single temperature or is a sum of emission from plasma at different temperatures.
- Using symmetry assumptions the X-ray surface brightness can be converted to a measure of the ICM density.

What we try to measure II

If we can measure the temperature and density at different positions in the cluster then assuming the plasma is in hydrostatic equilibrium we can derive the **gravitational potential and hence the amount and distribution of the dark matter.**

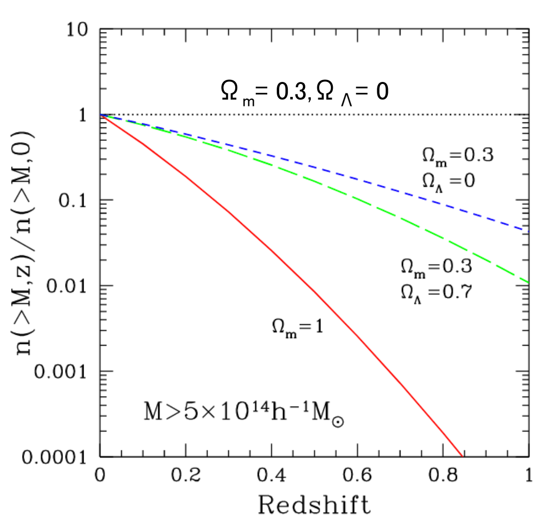
There are two other ways to get the gravitational potential :

- The galaxies act as test particles moving in the potential so their velocities and positional distribution provides a measure of total mass (Viral theorem)
- The gravitational potential acts as a lens on light from background galaxies (previous slides).

Why do we care ?

Cosmological simulations predict distributions of masses and how it evolves over cosmic time.

If we want to use X-ray selected samples of clusters of galaxies to measure cosmological parameters (main science goal of eRosita on SRG) then we must be able to relate the observables (X-ray luminosity and temperature) to the theoretical masses.

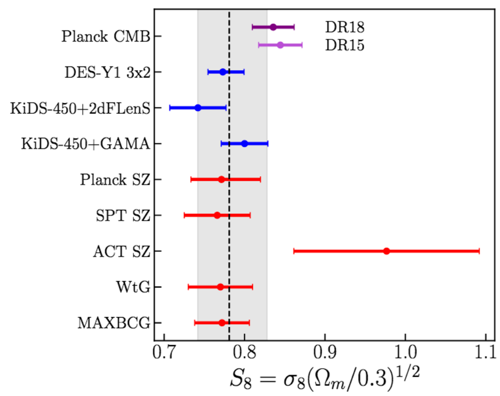


Effect of change of cosmological parameters are on number of clusters as a function of redshift

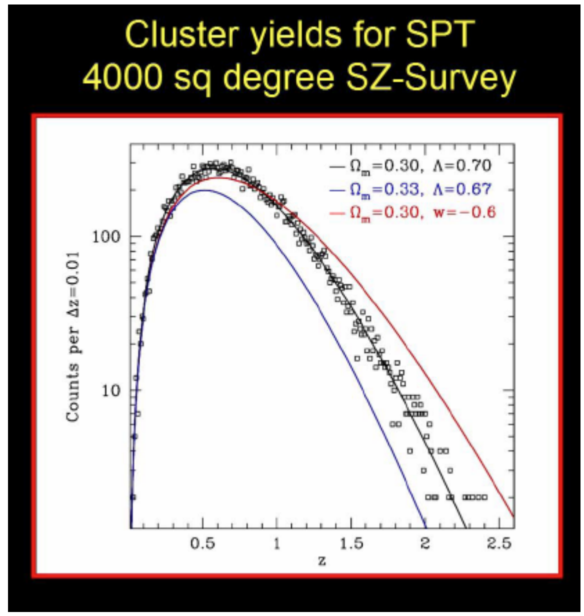
Sensitivity of Cluster Numbers vs Cosmology

The number of clusters per unit mass
per unit volume is very sensitive to the cosmological parameters

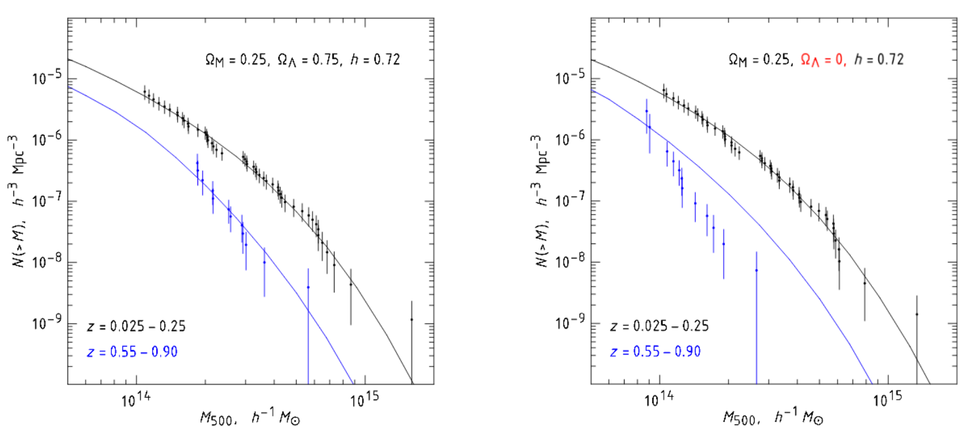
clusters- grey band



Costanzi et al 2018



What Do the Data Show??



eRosita will increase sample size by ~50 and redshift range