What are the possible energy sources?

- Accretion?
  - Release of gravitational potential energy as matter falls into black hole
  - YES! Thought to be primary power source of all systems just discussed

- Rotational energy of black hole? Spinning (Kerr) BHs see 13.11.2
  - Tapping the rotational energy of a spinning black hole $1/2 I\Omega^2$ can be very large
  - May be important in some settings… but can only be tapped if accretion occurring!

Rotating black holes- remember the extra special nature of accelerated frames

- Roy Kerr (1963)
  - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has "**static surface**" inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: "**frame dragging**".
  - **Ergosphere** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Spinning BH- Longair sec 13.11.2

- A black hole with angular momentum ***J*** has a metric

$ds^2=(1-2GMr/\rho c^2)dr^2-\{(1/c^2)[4GMra\sin^2\theta/\rho c]\,dr d\phi\}+(\rho/\Delta)dr^2+$
$\rho d\theta^2+(r^2+a^2+2GMra^2\sin^2\theta/\rho c^2)\sin^2\theta\, d\phi^2]$-- Longair eq 13.63

r,θ,ϕ– usual polar coordinates

where $a=(J/Mc)$ is the angular momentum per unit mass (dimensions of distance) and
$\Delta=r^2-(2GMr/c^2)+a^2$; $\rho=r^2+a^2\cos^2\theta$

Just like Schwarschild metric it becomes singular but at a radius where
$\Delta=r^2-(2GMr/c^2)+a^2=0$; the larger root is
$r_+=GM/c^2+[(GM/c^2)^2-(J/Mc)^2]^{1/2}$
for J>0 this is smaller than the Schwarschild radius
there is a maximum angular momentum ***$J=GM^2/c$; for this value of J the** <u>horizon</u> **is at***
r_+***=GM/c ; 1/2 of the Schwarzschild radius***

- If the black hole is not rotating (a=J/M=0), the Kerr line element reduces to the Schwarzschild line element

Spin Energy

- All the spin energy of a Black Hole resides outside the horizon and can, in principle, be extracted
- For a maximally spinning BH the most energy that can be extracted is $0.29Mc^2$
- Particles are dragged in angle ϕ

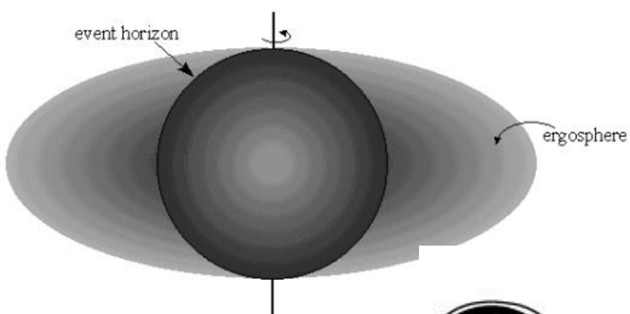
as they fall radially inward, even though no forces act on them

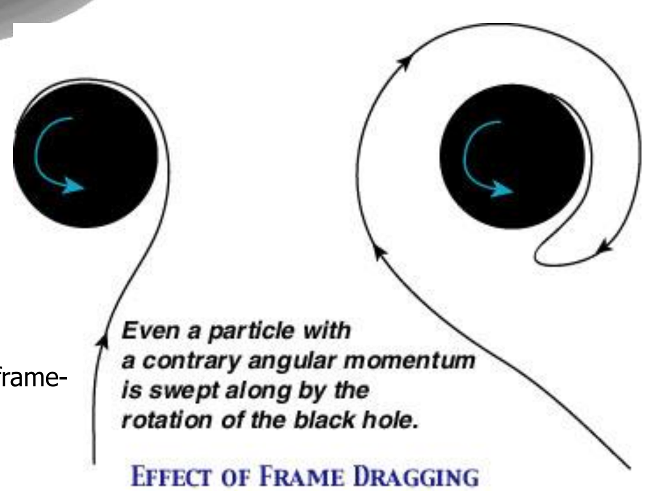
Schwarzschild and Kerr Metric

- for a <u>Schwarzschild</u> BH the innermost stable radius is $3r_G=6GM/c^2$ - there are no **stable** circular orbits at smaller radii
  – the binding energy from this orbit is 0.0572 of the rest mass energy
- For a Kerr the innermost stable radius is at $r_+=GM/c^2$ The spinning black hole drags the the inertial frame-
- The smaller critical radius allows more energy to be released by infalling matter
  – For a Kerr BH 0.423 of the energy can be released.

- There is another 'fiducial' radius in the Kerr solution, that radius within which all light cones point in the direction of rotation, the 'static' radius, r_{static}.
- Between r_{static} and r_+ is a region called the 'ergosphere' within which particles must rotate with the black hole and from energy might be extracted (Penrose process).

Frame dragging by rotating black hole





tested in 2004

http://www.universetoday.com/9984/frame-dragging-confirmed/

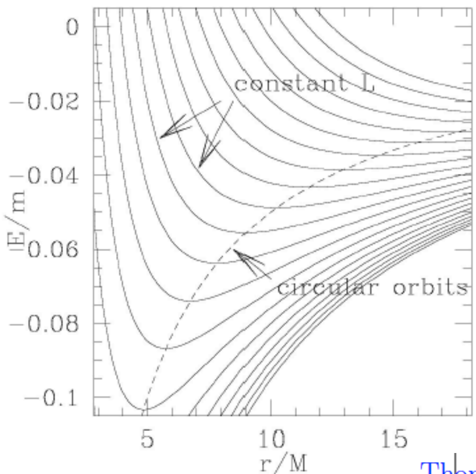
Graphics: University of Winnipeg, Physics Dept.

The innermost stable circular orbit (ISCO)

circular orbit extremizes binding energy E of test mass m at const. angular momentum L

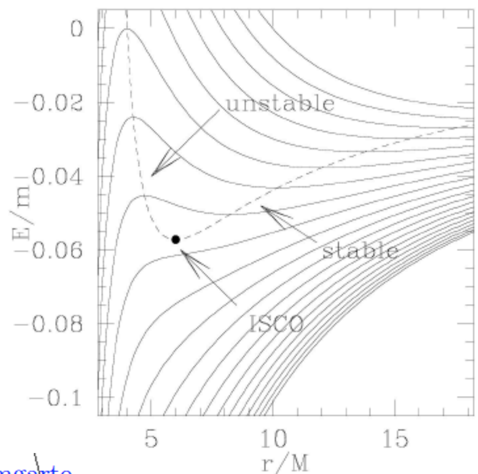
Newtonian point mass

$$\frac{E}{m} = -\frac{M}{r} + \frac{L^2}{2r^2}$$



Schwarzschild black hole

$$\frac{E}{m} = \left(\left(1-\frac{2M}{r}\right)\left(1+\frac{L^2}{r^2}\right)\right)^{1/2} - 1$$

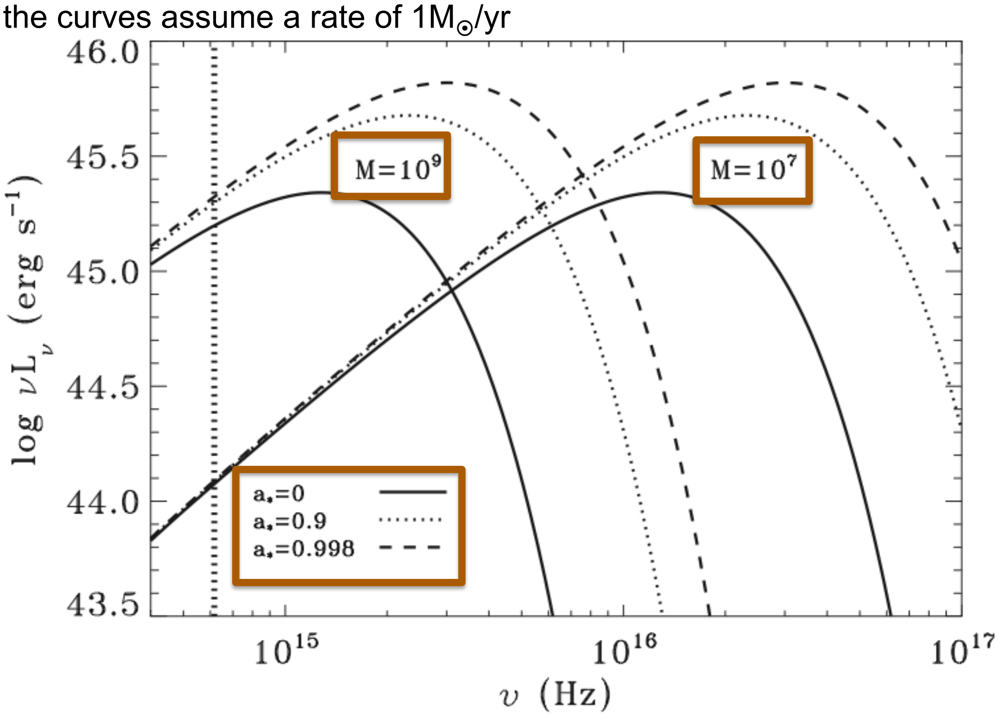


Thomas Baumgarte

Bowdoin College

Effect of BH Mass and Spin on Emitted Spectrum

all the curves assume a rate of $1 M_\odot$/yr



What can come **out** of black hole?

...more than you might think!

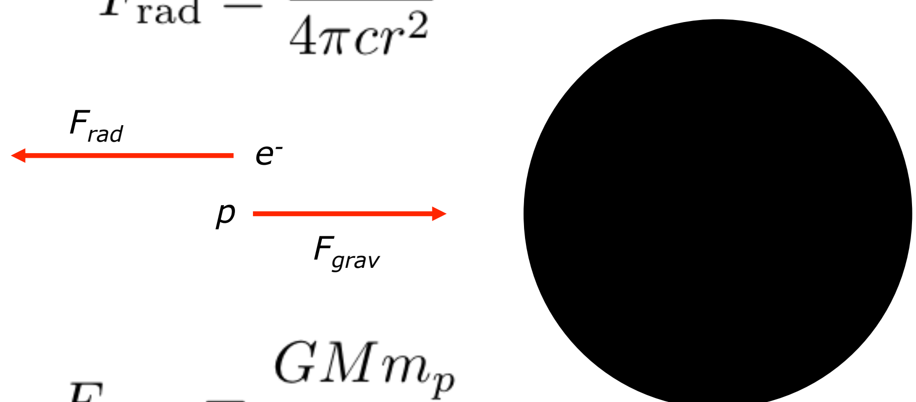
- Magnetic fields threading ergosphere can attach to and drag surrounding matter, reducing the black hole's spin and energy
- "Hawking Radiation"(pg 438): black hole slowly evaporates due to quantum mechanics effects
  - Particle/antiparticle pair is created near BH
  - One particle falls into horizon; the other escapes
  - *Energy to create particles comes from gravity outside horizon*

$$t_{evap} = 10^{10}\,yrs \times \left(\frac{M}{10^{12}\,kg}\right)^3$$

  - Solar-mass black hole would take 10^{65} years to evaporate!
  - Mini-black holes that could evaporate are not known to exist now, but possibly existed in early Universe

How luminous can an accreting black hole be?- this is the same Eddington limit as we discussed for neutron stars

$$F_{\rm rad} = \frac{L\sigma_T}{4\pi c r^2}$$



$$F_{\rm grav} = \frac{GMm_p}{r^2}$$

- The accreting matter is pushed away if

$$F_{\mathrm{rad}} > F_{\mathrm{grav}}$$

- This is the Eddington limit (L_{Edd}). Acts as effective upper limit to the luminosity of accretion powered object. Numerically:

$$L > \frac{4\pi G m_p c}{\sigma_T} M$$

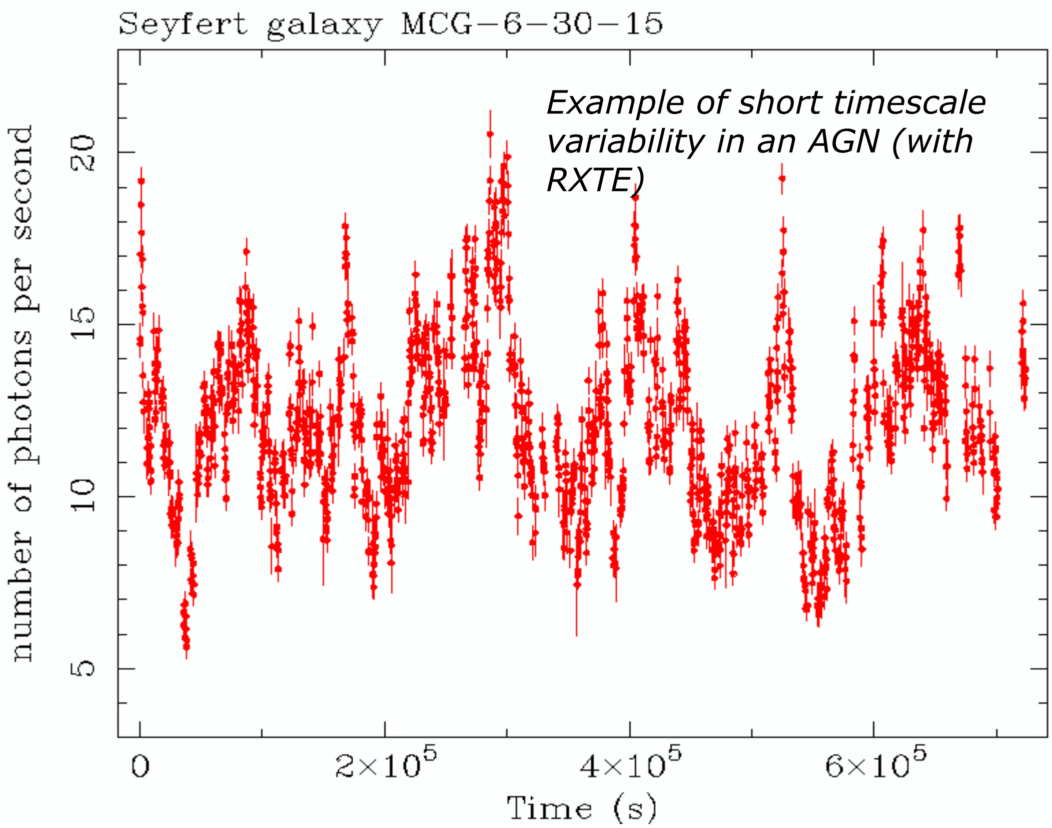
$$L_{\mathrm{Edd}} \approx 1.3 \times 10^{31} \left(\frac{M}{M_\odot} \right) \mathrm{W}$$

General properties of emission from black hole systems

- Emission usually variable on wide variety of timescales
  - Galactic black hole binaries : millisecond and up
  - AGN : minutes and up
  - Most rapid variability approaches light-crossing timescale limit $R \sim c\tau$

- Significant emission over **very** broad spectral range (radio to hard X-ray or gamma-rays)-NS and WDs tend to have 'thermal' like spectra (<u>relatively</u> narrow in wavelength)

- Lack of a signature of a surface - not a pulsar, no boundary layer emission (no x-ray bursts), no 'after glow' from cooling

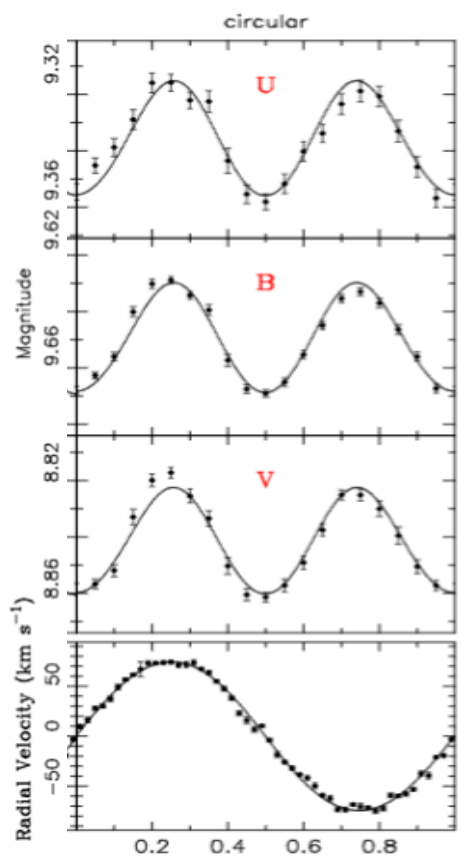


Evidence for black holes

- Galactic black hole candidates – the same sort of dynamical evidence we have for neutron stars! ~20 known
- Black hole mass from orbit of companion star- Cyg X-1 first galactic black hole discovered
  - Period 5.6 days
  - K = V sin i = 75km/s
  - Analysis of orbit shows that

$$f = \frac{K^3 P}{2\pi G} = \frac{M_1^3 (\sin i)^3}{(M_1 + M_2)^2}$$

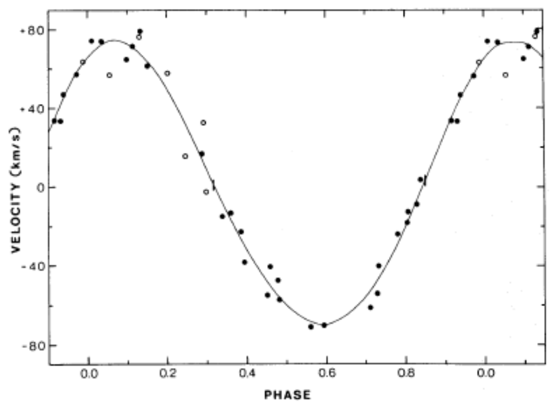
  - "Mass function" f can be measured…
  - M_{BH}>f



Cyg X-1
Orosz 2011

Discovery of black holes

- First evidence for an object which 'must' be a black hole came from discovery of the X-ray source Cygnus X-1
  - Binary star system… black hole in orbit around a massive O-star; period =5.6 days - not eclipsing
  - Mass of x-ray emitting object 7-13 $M_{\odot}$- too high for a NS. Object emits lots of x-rays, little optical light.
  - X-rays produced due to accretion of stellar wind from O-star
  - 2kpc away



Velocity curve of the stellar companion
It is a massive O star

$f(M) = P_{orb}v^2/2\pi G = M_1 \sin^3 i/(1 + q)^2$.
$q=M_2/M_1$; v^2 is the maximum measured velocity
the value of the mass function is the absolute minimum mass of the compact star

Evidence for black holes- Longair 19.3.2

- For Supermassive Black Holes

Dynamics of 'Test particles'

Orbits of gas disks around mass compact objects at the centers of other galaxies- best case is NGC4258 (water maser orbits)

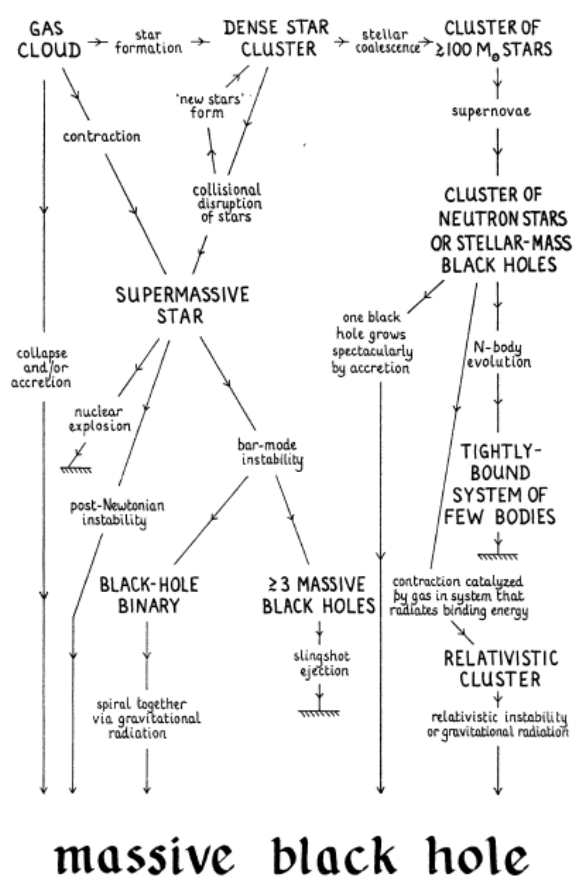
Stellar orbits around a compact mass at the center of our own Galaxy (most solid case for any black hole)

Of course what these data give is the mass inside a given radius. If the mass density is higher than (?) it must be a black hole

- Emission from the region of 'strong gravity'

Extreme gravitational redshifting of emission lines in the X-ray spectrum of some accreting black holes

In a dense region all roads lead to a black Hole (Ress 1984 ARAA)



Some Scales (Rees 1984)

A central mass M has a gravitational radius

$$r_g = \frac{GM}{c^2} = 1.5 \times 10^{13}\, M_8 \text{ cm}, \qquad 1.$$

where M_8 is the mass in units of $10^8\, M_\odot$. The characteristic minimum time scale for variability is

$$r_g/c \simeq 500\, M_8 \text{ s}. \qquad 2.$$

A characteristic luminosity is the "Eddington limit," at which radiation pressure on free electrons balances gravity:

$$L_E = \frac{4\pi G M m_p c}{\sigma_T} \simeq 1.3 \times 10^{46}\, M_8 \text{ erg s}^{-1}. \qquad 3.$$

Related to this is another time scale

$$t_E = \frac{\sigma_T c}{4\pi G m_p} \simeq 4 \times 10^8 \text{ yr}. \qquad 4.$$

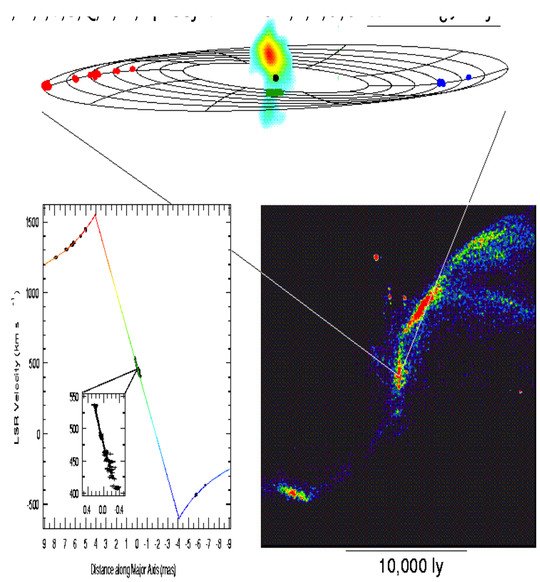
The time scale to grow a black hole if it Were accreting at the Eddington luminosity

The characteristic black body temperature if the Eddington luminosity is emitted at r_g

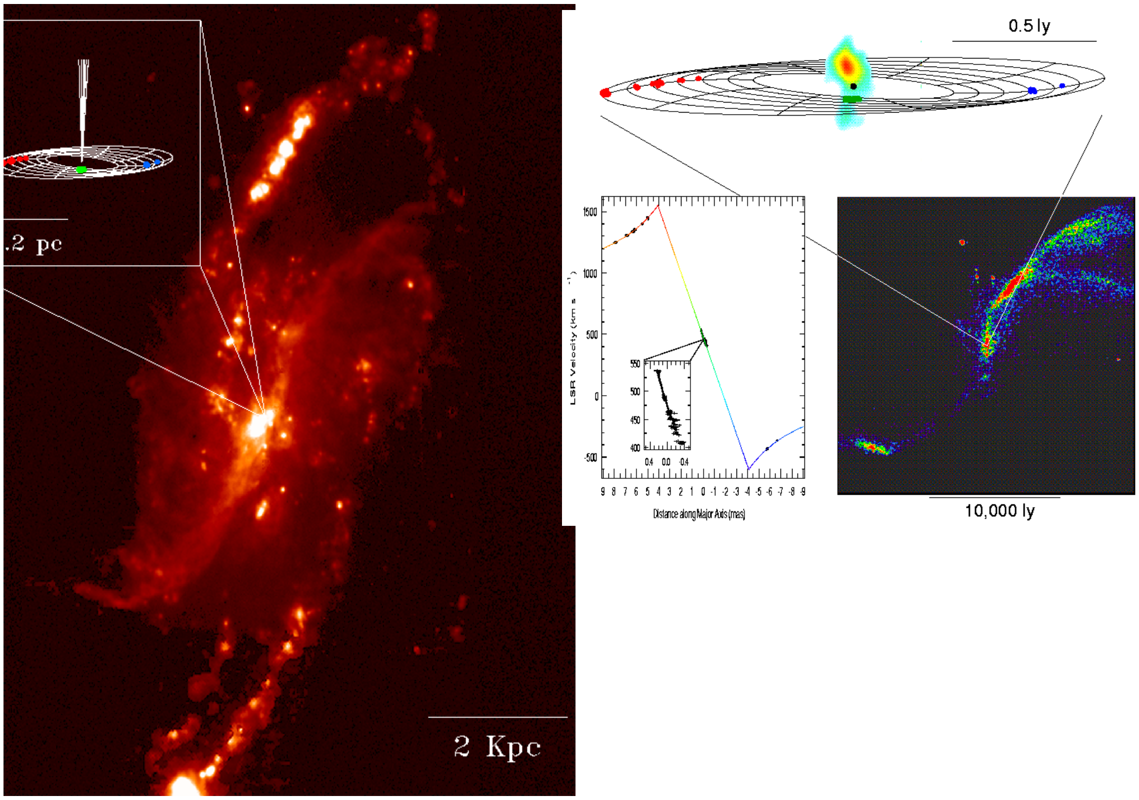
$$T_E \simeq 5 \times 10^5\, M_8^{-1/4}.$$

Examples of Astrophysical Black Holes

- We know that black holes come in 2 size scales
  - 5-20 M_{sun} ; the result of stellar evolution
  - 10^6-10^9 M_{sun} super massive black holes that reside in the centers of most massive galaxies
  - They may also come in another size scale; intermediate mass black holes with $50<M_{sun}<10^3$
- Detailed stellar evolution calculations indicate that for a star with roughly solar metallicity the maximum mass of the remnant black hole is ~20 M_{sun} so what about the recent GW event ?



Miyoshi et al



NGC4258- Rotation of Maser Disk

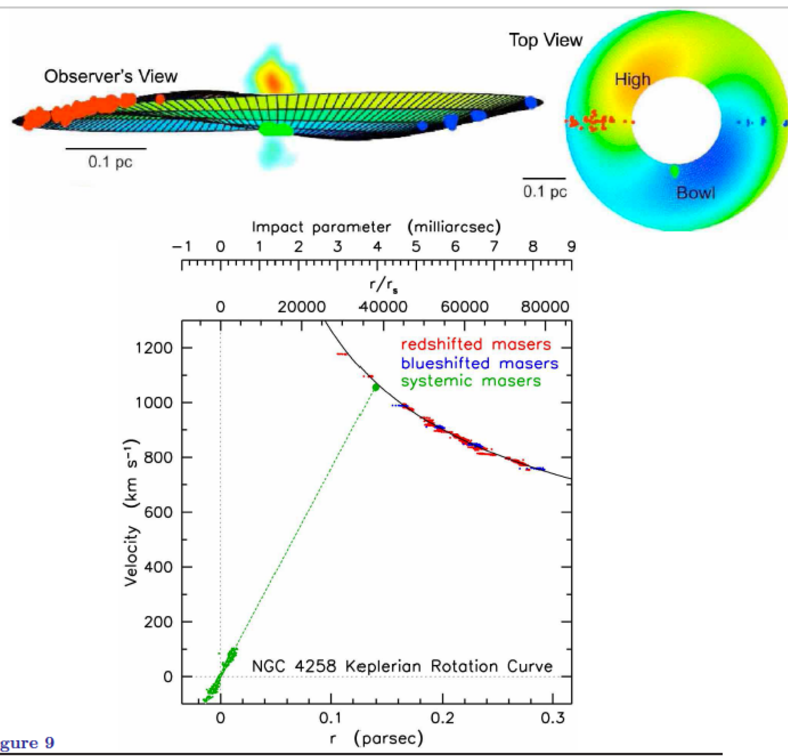
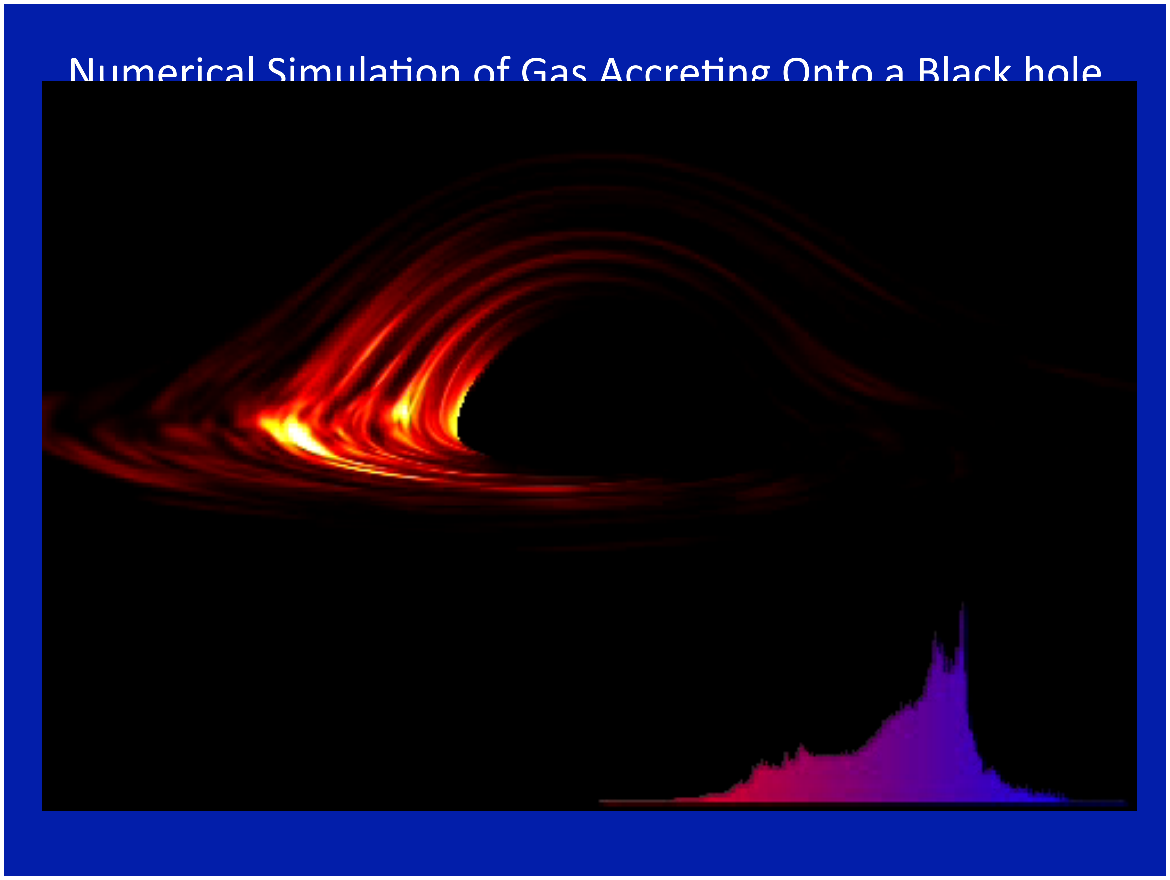
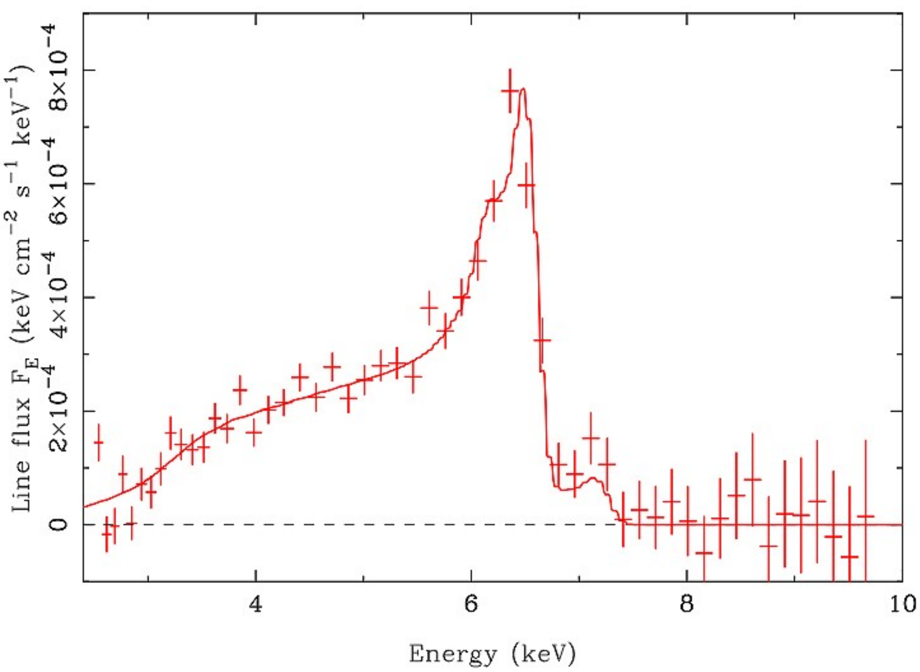


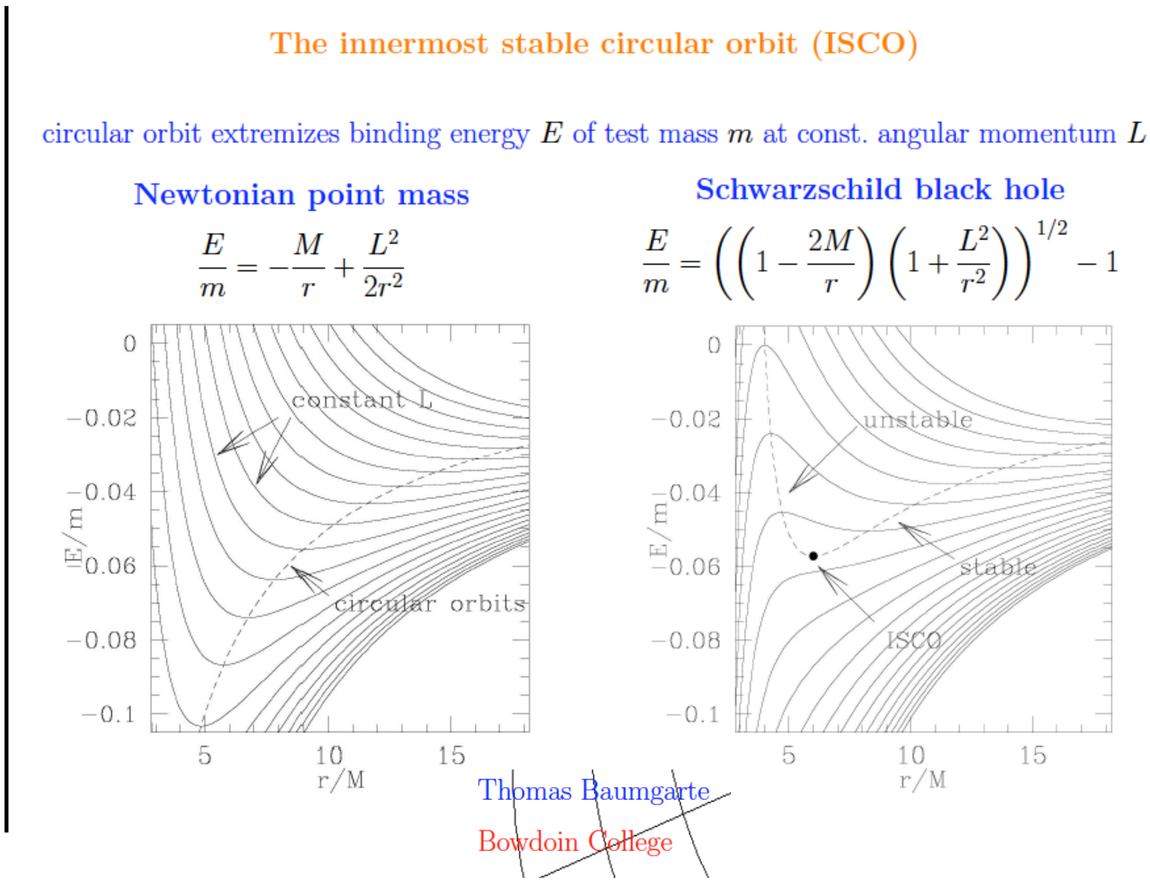
Figure 9

(*top*) Schematic views of the almost-edge-on, warped maser disk of NGC 4258 (from Moran 2008) with warp parameters from Herrnstein et al. (2005) and including the inner contours of the radio jet.





Broad iron line in MCG-6-30-15 (Fabian et al. 2002)



How Hot is a Black Hole??

Lets go back to the accretion disk spectrum- Longair 14.52-14.54

As matter flows inward the radiated luminosity is
L=G$\mathcal{M}$M/ 2R^{2}dR ($\mathcal{M}$=dM/dt)
Dividing by the area – get luminosity per unit area and equate this to the black body emission formula

$$G\mathcal{M}M/8\pi R^3=\sigma_{SB}T^4$$

correct dependence on M,$\mathcal{M}$ and R but wrong normalization

Have to account for: Boundary conditions at the inner edge of the disk

Get T=[3G$\mathcal{M}$M/8πR^3{1-sqrt(R/R$_{in}$}]$^{1/4}$ where R$_{in}$ is the innermost radius

How Hot is a Black Hole??

Lets go back to the accretion disk spectrum- Longair 14.52-14.54

$T(R)=[3G\mathcal{M}M/8\pi R^3]^{1/4}(R/R_{in})]^{-3/4}$

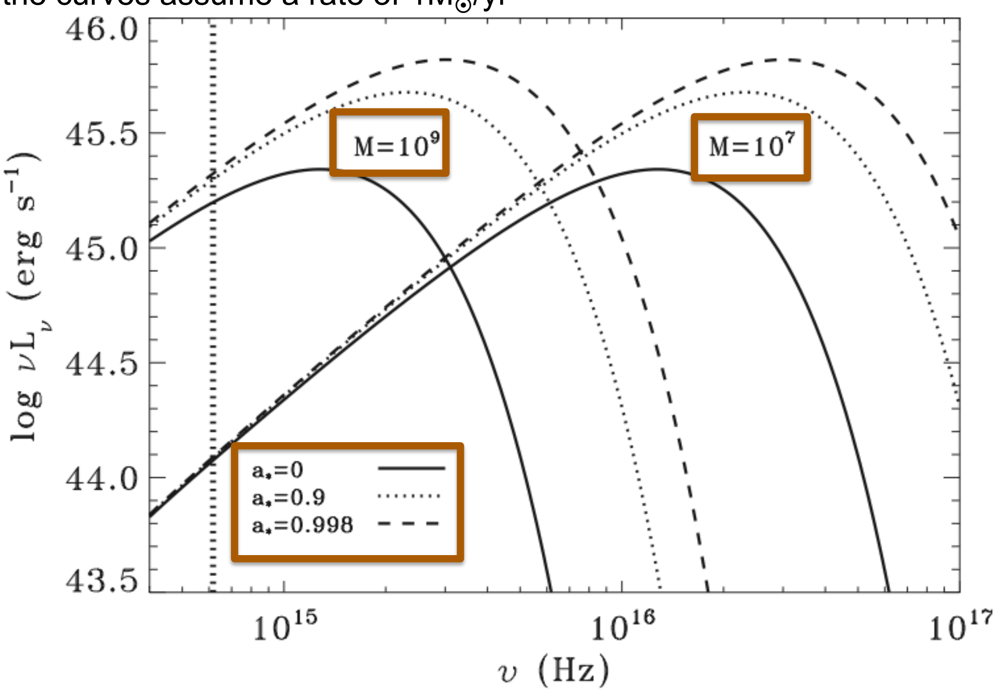
Temperature at fixed number of R_s decreases as $M^{-1/4}$
- **disks around more massive black holes are cooler at a fixed R/R_S and $\mathcal{M}/\mathcal{M}_{Edd}$**

now recasting in terms of the Eddington limit where $\mathcal{M}_{Edd}$ is the mass accretion rate for the Eddington limit and we assume that the conversion efficiency of mass into energy is 10% and M_8 is the BH mass in units of 10^8 M⊙

$$T=6.3\times10^5[\mathcal{M}/\mathcal{M}_{Edd}]^{1/4}[M/M_8]^{-1/4}(R/R_{in})]^{-3/4}k$$

Effect of BH Mass and Spin on Emitted Spectrum

all the curves assume a rate of $1M_\odot$/yr



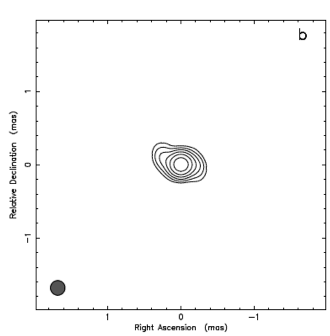
How Hot are Black Holes

For a black body the spectrum peaks at E= 2.8kT

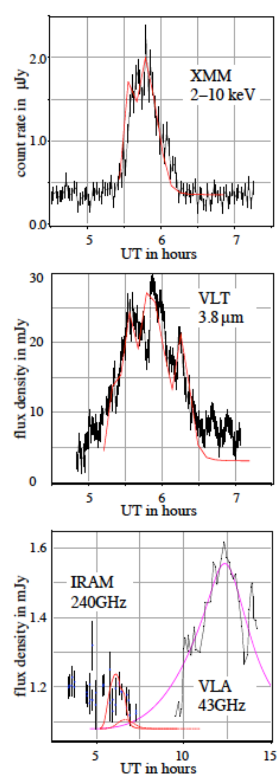
Expect disk emission in AGN accreting close to the Eddington limit to be strong in the ultraviolet - origin of the broad peak in quasar SEDs in the blue and UV

The Center of the Milky Way

- The center of the MW is called Sagitarius A*(SgrA*) from the name of the radio source at the dynamical center of the MW.
- This is also the location of a weak, time variable x-ray (log L_x~34- 100x less than a typical xray binary) and IR source
- The radio source is very small (<0.0005"<$50R_s$ for $M=4x10^6M_\odot$ BH at d=8kpc)
- At SgrA* 1"=0.04pc=1.2x10^{17} cm ,0.5mas=6AU



Radio image of SgrA*



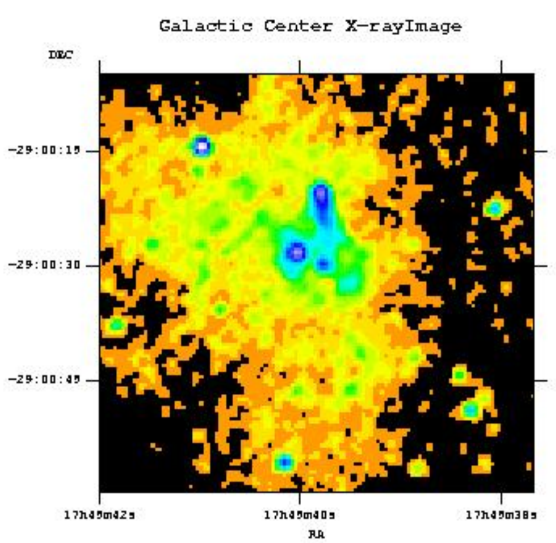
Radio, near IR and Radi light curves

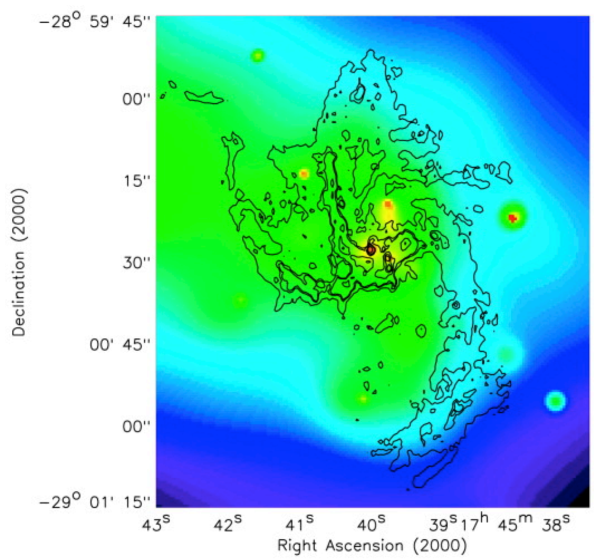
Some Problems with Sgr A*

- There is lots of gas for accretion in the galactic center from the ISM and stellar winds
- Yet the observed luminosity is very low ($L/L_{Edd} \sim 10^{-10}$)
- What happens to the accretion energy- where does the mass and energy go

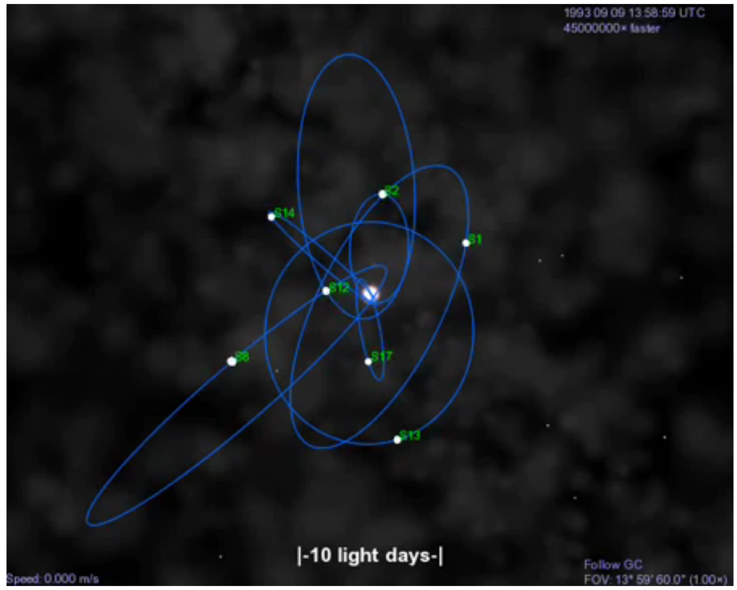
- Sgr A* is similar to >95% of all massive galaxies- they have big black holes, but low luminosities

Radio and X-ray Image of MW Center

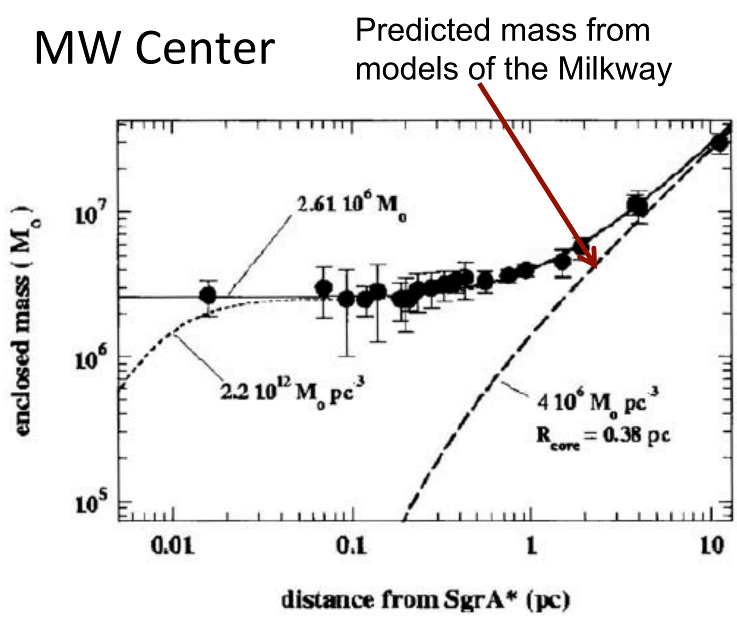




Motion of Stars Around the Center of the Milkyway- see
http://www.youtube.com/watch?v=ZDxFjq-scvU
http://www.mpe.mpg.de/ir/GC/



MW Center



- Two teams led by R. Genzel and A. Ghez have measured the 3-D velocities of individual stars in the galactic center
- This allows a determination of the mass within given radii
- The inferred density of the central region is $>10^{12} M_\odot/pc^3$

•As shown by Genzel et al the stability of alternatives to a black hole (dark clusters composed of white dwarfs, neutron stars, stellar black holes or sub-stellar entities) shows that a dark cluster of mass $2.6 \times 10^6\ M_{sun}$, and density

$20 M_{sun} pc^{-3}$ or greater can not be stable for more than about 10 million years

Velocity Distribution of Stars Near the Center of the MW

A Supermassive Black Hole in the Milky Way 29

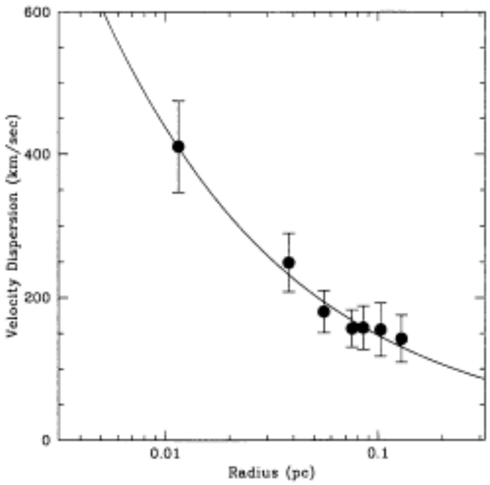


Figure 7. The projected stellar velocity dispersion as a function of projected distance from Sgr A* is consistent with Keplerian motion, which implies that the gravitational field is dominated by mass within 0.1 pc.

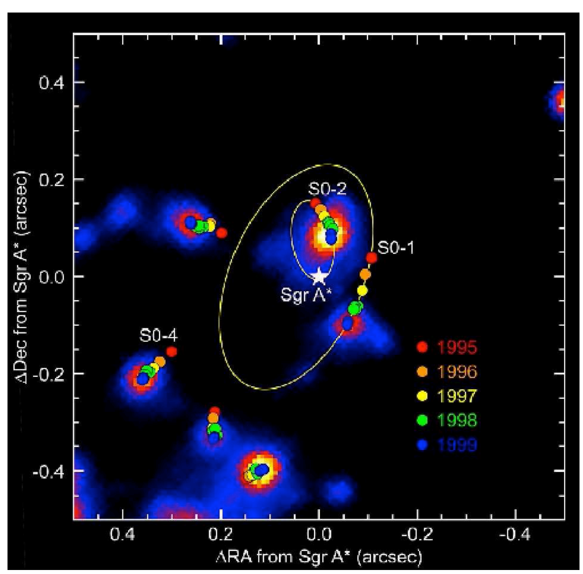
Ghez et al 1998

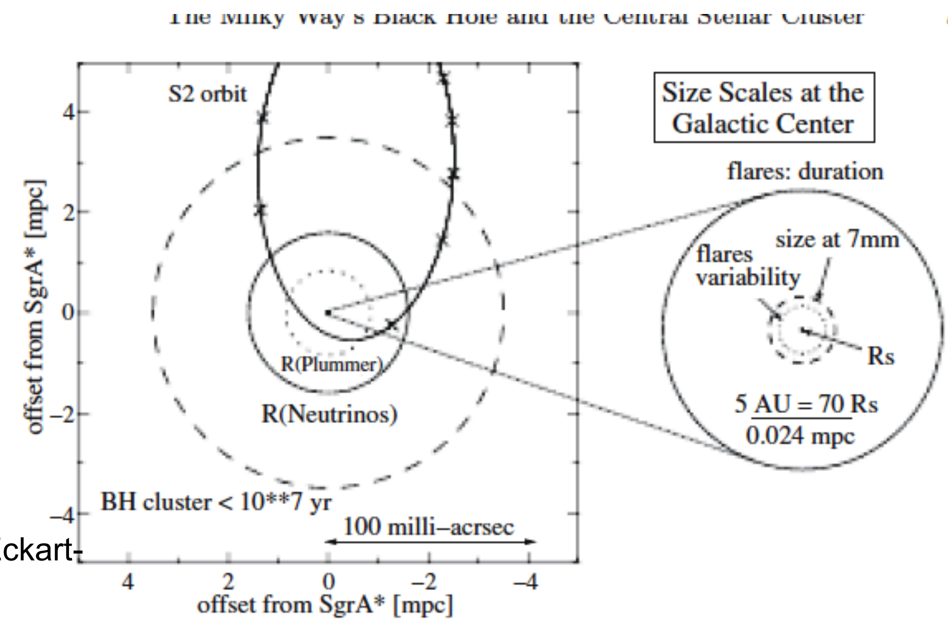
- While stars are moving very fast near the center (Sgr A*) the upper limit on its velocity is 15 km/sec

If there is equipartition of momentum between the stars and SgrA* then one expects

- $M_{SgrA*} > 1000M_{\odot}(M_*/10M_{\odot})(v_*/1500km/sec(v_{sgrA*}/15km/sec)^{-1}$

Where we have assumed that the star stars we see have a mass $10M_{\odot}$ and a velocity of 1500 km/sec

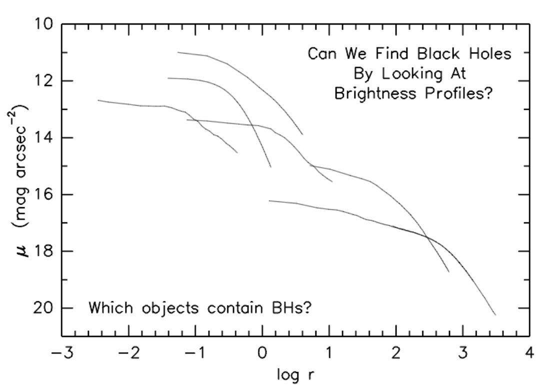




Eckart

What About Other Supermassive Black Holes

- At the centers of galaxies- so much more distant than galactic stellar mass black holes
- First idea: look for a 'cusp' of stars caused by the presence of the black hole- doesn't work, nature produces a large variety of stellar density profiles… need dynamical data
- Dynamical data: use the collisionless Boltzman eq (conceptionally identical to the use of gas temperature to measure mass, but stars have orbits while gas is isotropic)
-



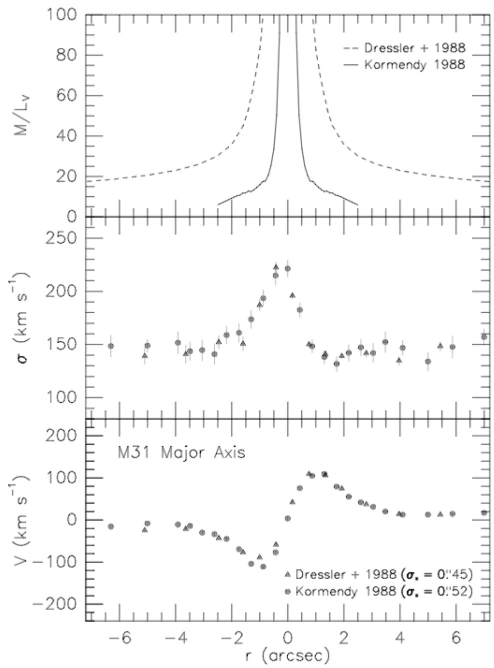
Kormendy and Richstone (2003)

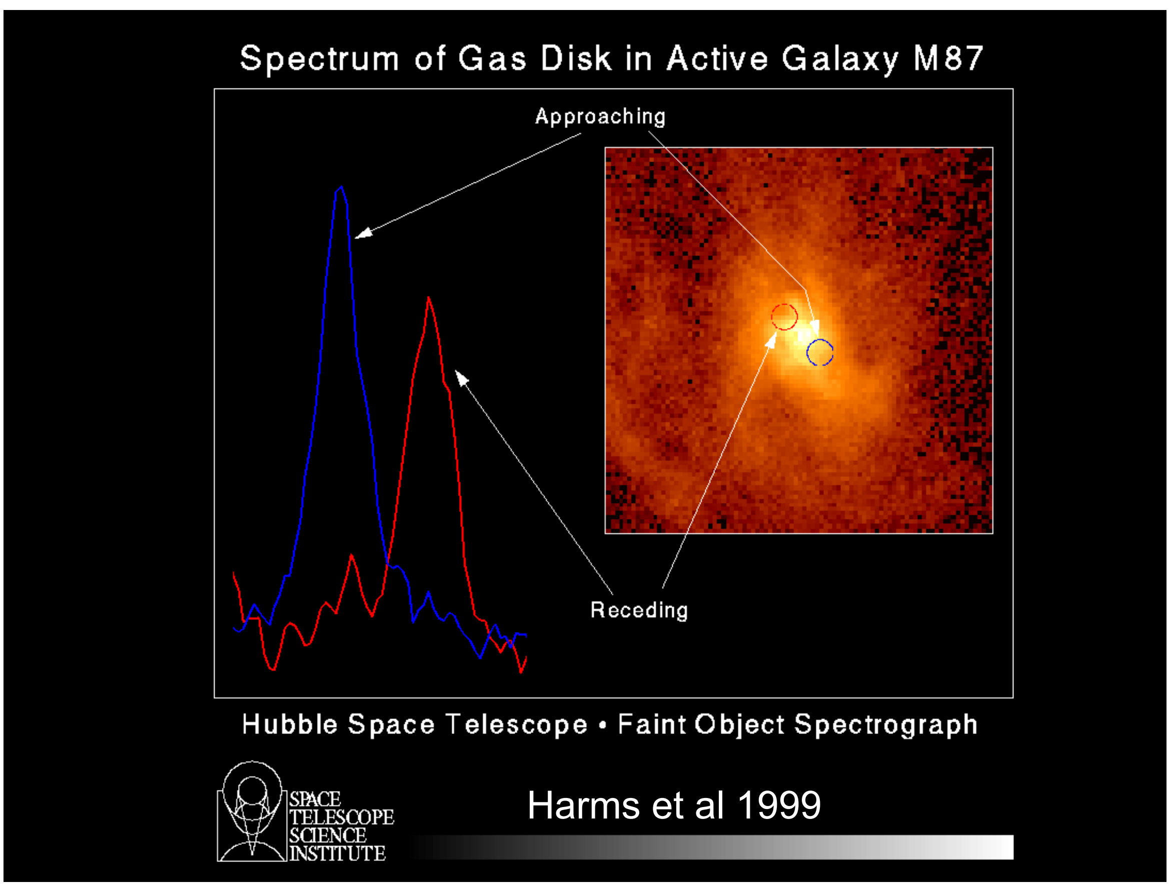
Longair 19.3. eq 19.8 and extensive discussion

$$M(r) = \frac{V^2 r}{G} + \frac{\sigma_r^2 r}{G}\left[-\frac{d\ln\nu}{d\ln r} - \frac{d\ln\sigma_r^2}{d\ln r} - \left(1-\frac{\sigma_\theta^2}{\sigma_r^2}\right) - \left(1-\frac{\sigma_\phi^2}{\sigma_r^2}\right)\right]$$

Example of data for the nearest galaxy M31

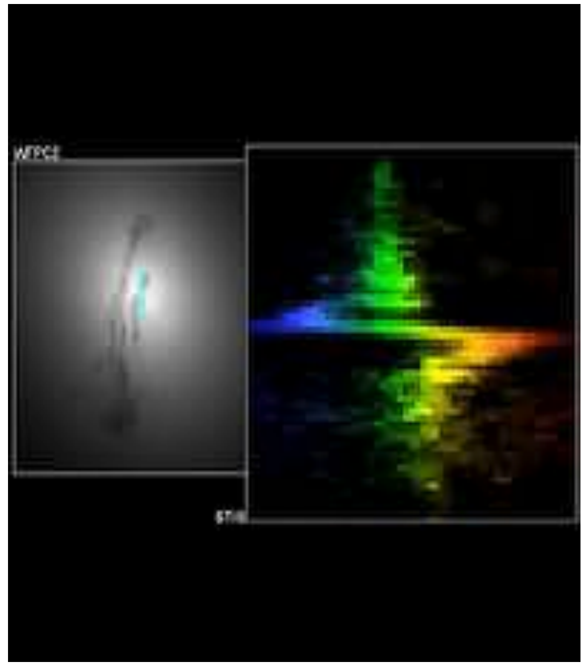
- Notice the nasty terms
- V_r is the rotation velocity σ_r $\sigma_{\theta,}$ σ_ϕ are the 3-D components of the velocity dispersion ν is the density of stars
- All of these variables are 3-D; we observe projected quantities !
- The analysis is done by generating a set of stellar orbits and then minimizing
- Rotation and random motions (dispersion) are both important.
- Effects of seeing (from the ground) are important- Hubble data





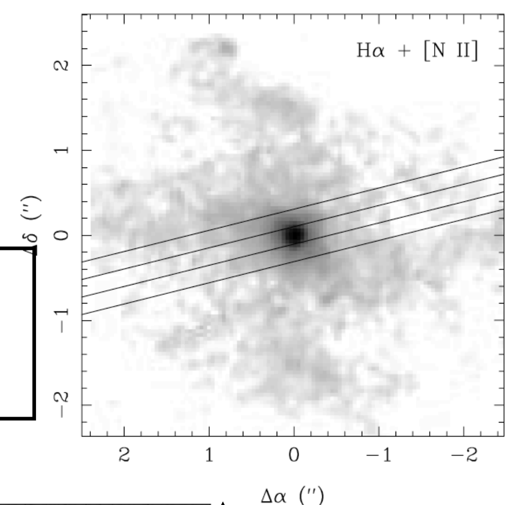
How to Measure the Mass of a SuperMassive Black hole

- Image of central regions and Velocity of gas near the center of M84 a nearby galaxy (Bower et al 1998) -
- The color scale maps the range of velocity along the slit, with blue and red color representing velocities (with respect to systemic) that are blueshifted and redshifted, respectively.
- The dispersion axis (horizontal) covers a velocity interval of 1445 km s-1, while the spatial axis (vertical) covers the central 3 arcsec;.



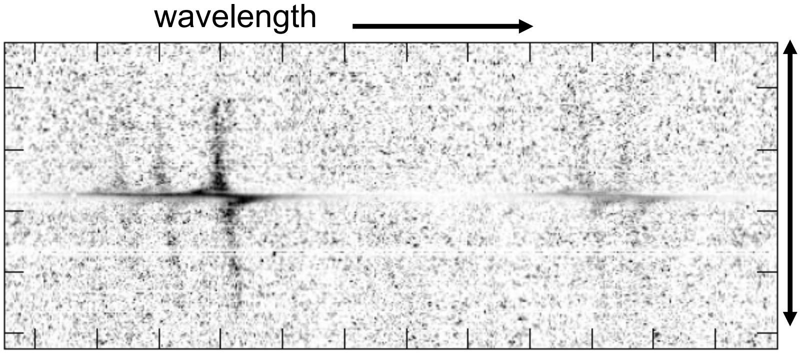
Measurement of Kinematics o Gas

- Image of optical emission line emitting gas around the central region of the nearby giant galaxy M84



HST STIS Observations of the Nuclear Ionized Gas in the Elliptical Galaxy M84
G. A. Bower, R. F. Green, D.

wavelength



Position on sky along slit

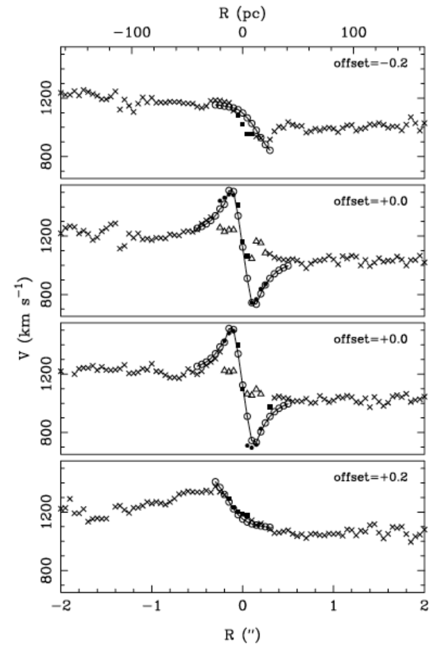
Analysis of Spectral Data for M84

- Mass of central object $1.5 \times 10^9\ M_{sun}$

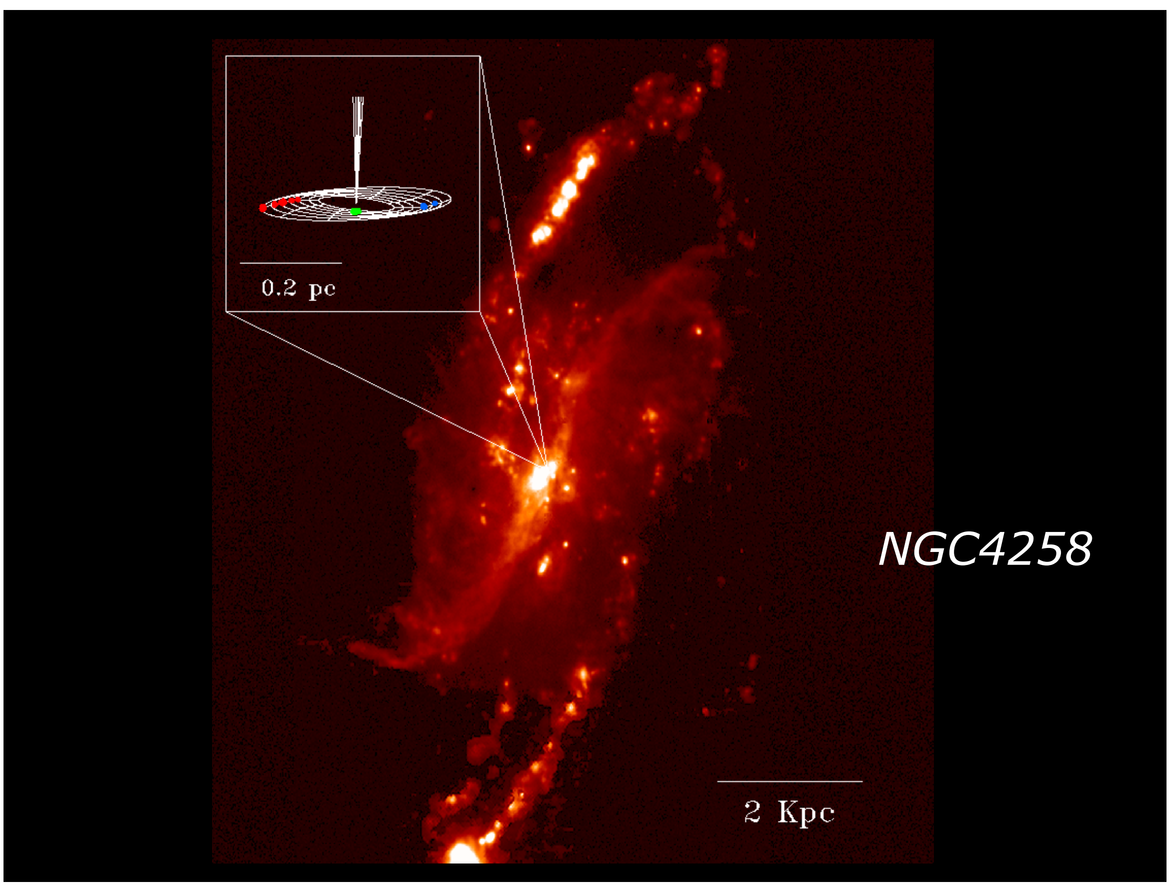
TABLE 1
KEPLERIAN DISK MODEL PARAMETERS

Parameter	Best Fit	Uncertainty Range
Black hole mass ($M_\odot$) ……………	1.5×10^9	$(0.9–2.6) \times 10^9$
Disk inclination (deg) ……………	80	75–85[a]
Disk P.A. (deg) ……………………	83	80–85
Gas systemic velocity (km s^{-1}) ……	1125	1100–1150
Intensity law ………………………	$I(r) \propto r^{-1}$	…
$I(r)$ inner radius (pc) ……………	1	0.3–3
$V(r)$ inner radius (pc) ……………	0.03	0.01–0.1
PSF σ (arcsec) ……………………	0.05	0.04–0.06

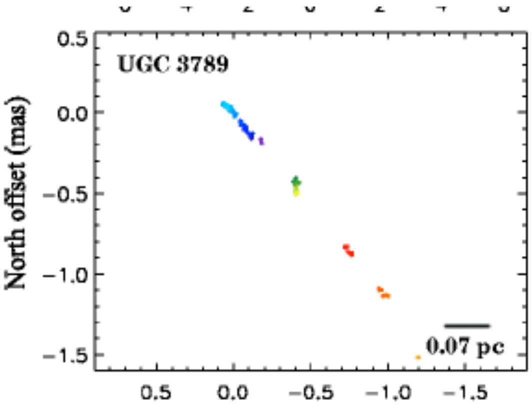
[a] Lower mass requires lower inclination.

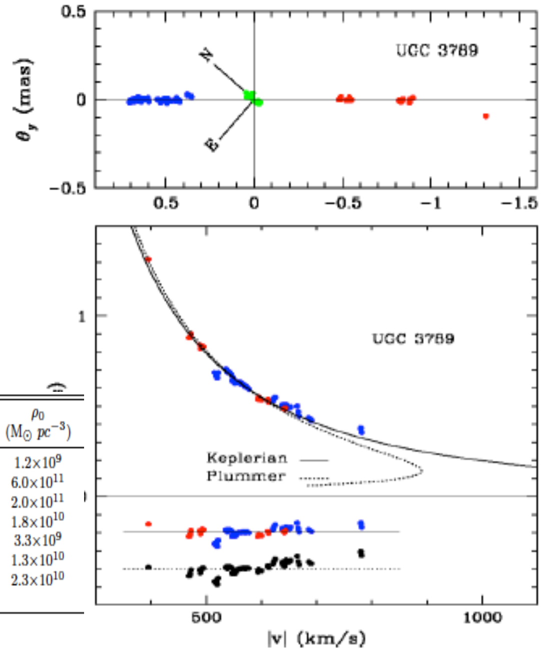


Velocity of gas vs distance from center of emission along 3 parallel lines



Other Masers

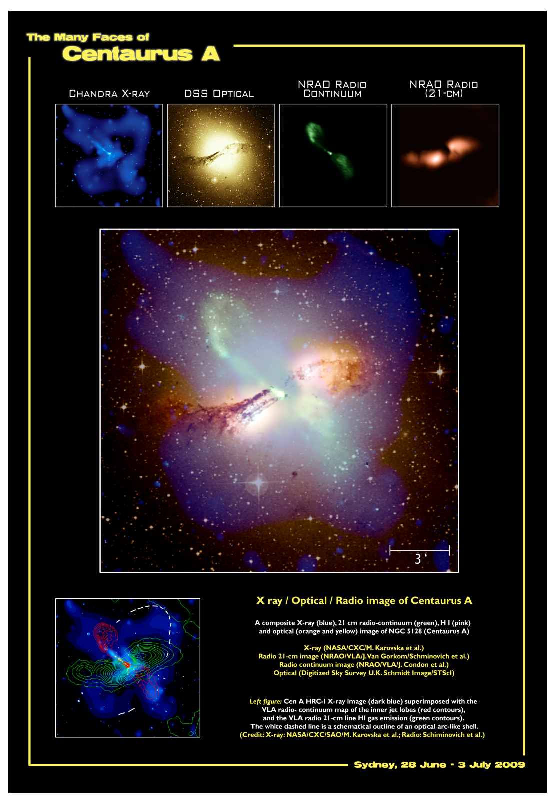




Name	Dist. (Mpc)	BH mass (10^7 $M_\odot$)	Disk Size (pc)	P.A. (°)	Incl. (°)	r_c (pc)	ρ_0 ($M_\odot$ pc^{-3})
NGC 1194	52	6.4±0.3	0.53-1.30	157	85	0.250	1.2×10^9
NGC 2273	26	0.76±0.04	0.028-0.085	153	84	0.015	6.0×10^{11}
UGC 3789	50	1.12±0.05	0.09-0.32	41	> 88	0.024	2.0×10^{11}
NGC 2960	71	1.14±0.05	0.13-0.36	−131	89	0.055	1.8×10^{10}
NGC 4388	19	0.84±0.02	0.24-0.29	107	–	0.090	3.3×10^9
NGC 6264	136	2.84±0.04	0.23-0.78	−85	90	0.083	1.3×10^{10}
NGC 6323	105	0.93±0.01	0.13-0.30	10	89	0.046	2.3×10^{10}

Kuo et al 2010

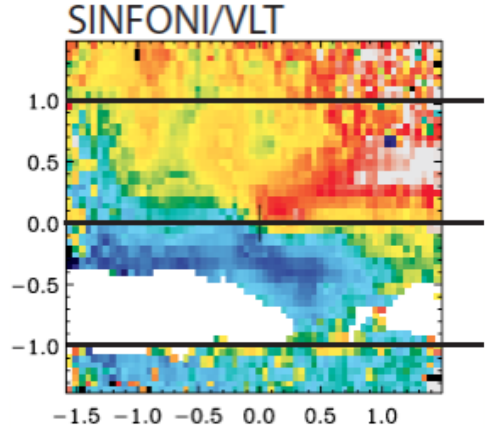
Centaurus-A The Nearest AGN

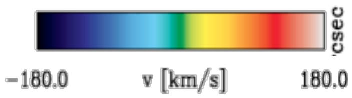


Centaurus -A

- 2 dimensional velocity maps for gas and stars allow assumptions to be checked (Neumayer et al,Cappelari et al)

Gas Velocities





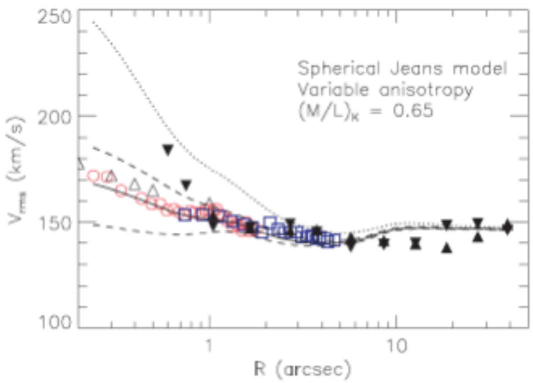
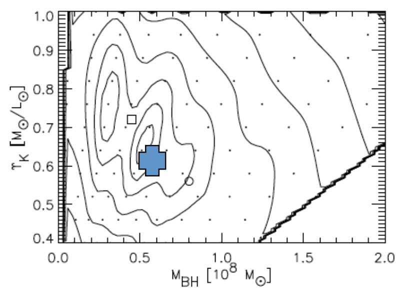


Figure 4: Spherical anisotropic Jeans model

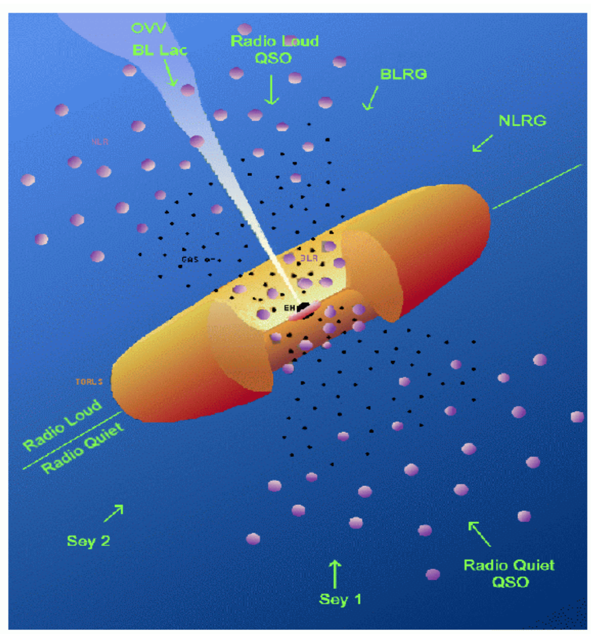
$M_{\rm BH} = (5.5 \pm 3.0) \times 10^7 M_\odot$.



Constraints from stars compared to those from Gas Velocities

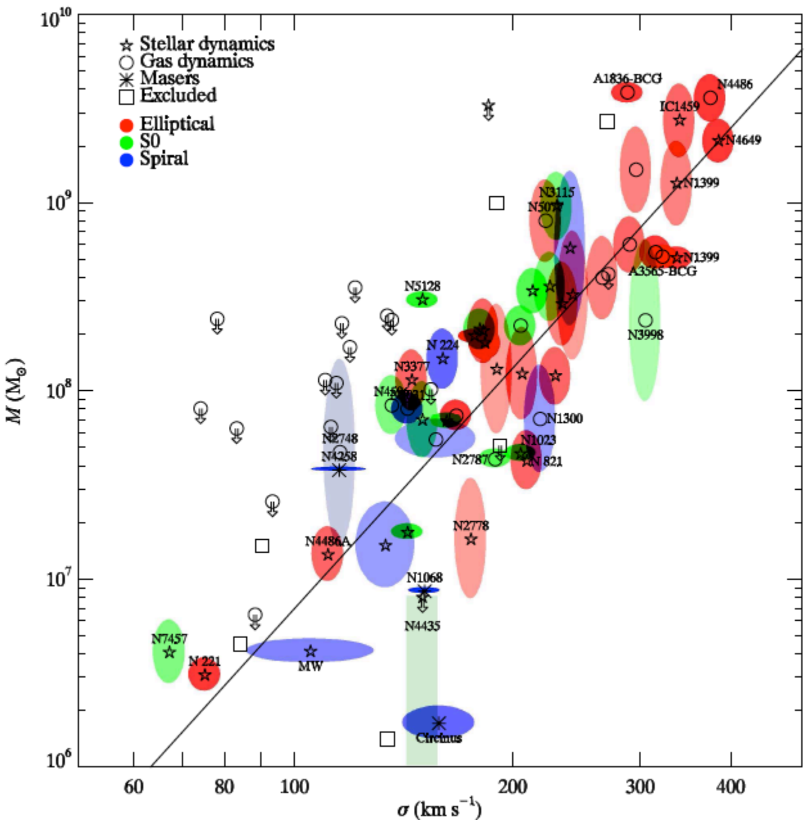
AGN- Alias Active Galactic Nuclei

- AGN are 'radiating' supermassive black holes-
  - They go by a large number of names (Seyert I, Seyfert II, radio galaxies, quasars, Blazars etc etc)
  - The names convey the observational aspects of the objects in the first wavelength band in which they were studied and thus do carry some information
- See http://nedwww.ipac.caltech.edu/level5/Cambridge/Cambridge_contents.html for an overview



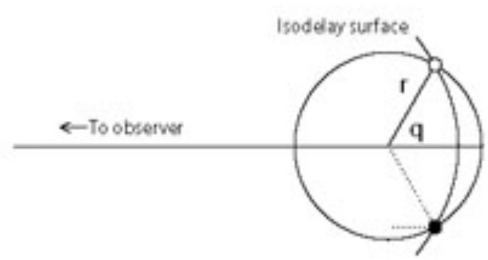
Urry and Padovani 195

- All the Nearby Galaxies with Dynamical Masses for their Central Black Holes (Gultekin 2009)
- There seems to be a scaling of the mass of the black hole with the velocity dispersion of the stars in the bulge of the galaxy
- $M_{BH} \sim 10^{-3}\, M_{bulge}$
- Galaxies know about their BH and vice versa

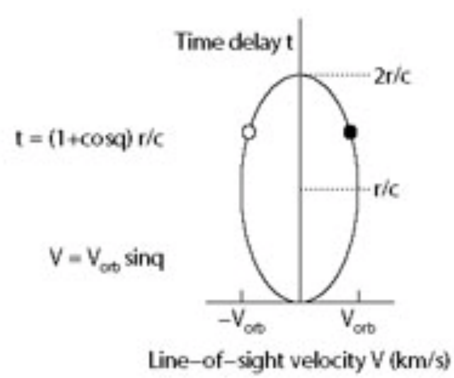


What About AGN in General??

- We believe that the incredible luminosity of AGN comes from accretion onto a black hole
- However the 'glare' of the black hole makes measuring the dynamics of stars and gas near the black hole very difficult



- Technique: reverberation mapping (Peterson 2003)
  - The basic idea is that there exists gas which is moderately close to to the Black Hole (the so-called broad line region) whose ionization is controlled by the radiation from the black hole
  - Thus when the central source varies the gas will respond, with a timescale related to how far away it is

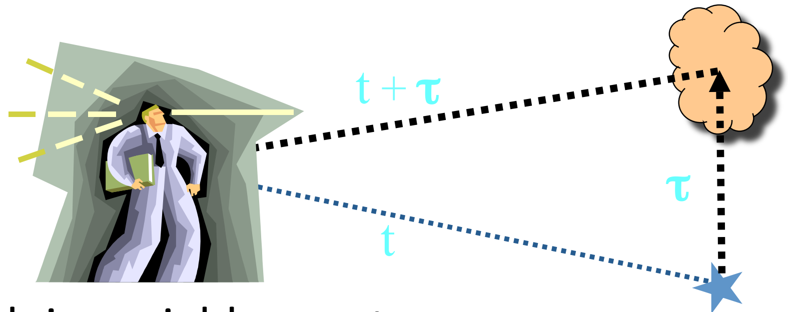


Virial Mass Estimates- Longair 20.5

$M_{BH} = f\ v^2 R_{BLR}/G$

Reverberation Mapping:

- $R_{BLR} = c\,\tau$



- v_{BLR}

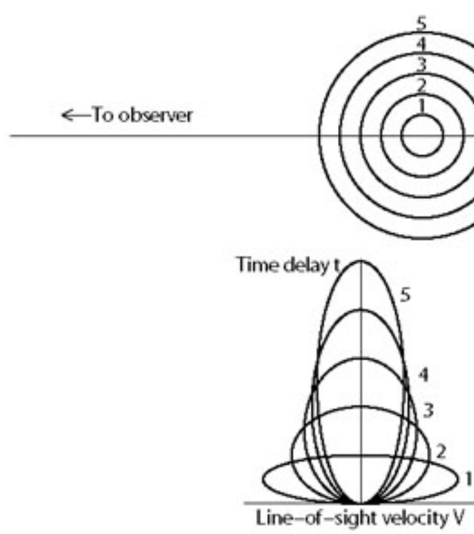
Line width in variable spectrum

The Geometry

- Points (r, θ) in the source map into line-of-sight velocity/time-delay(τ) space (V, τ) according to $V = -V_{orb}\sin(\theta)$, where V_{orb} is the orbital speed, and $\tau = (1 + \cos(\theta))r / c$.
- The idea is that the broad line clouds exist in 'quasi-Keplerian' orbits (do not have to be circular) and respond to the variations in the central source. Lower ionization lines are further away from the central source.
- So

$M_{BH}=frV^2/G$

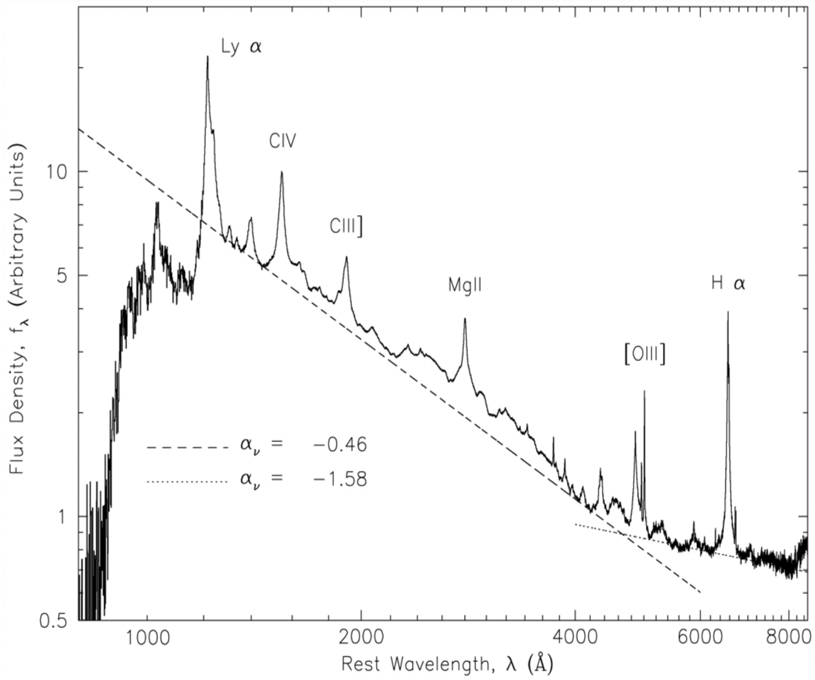
f is a parameter related to geometry- and the orbits of the gas clouds



- AGN (type I) optical and UV spectra consist of a 'feature less continuum' with strong 'broad' lines superimposed

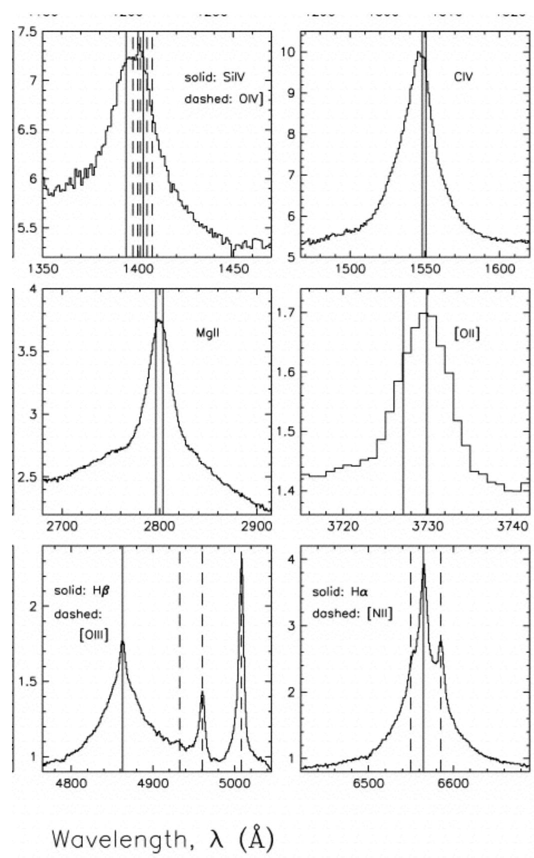
- Typical velocity widths (s, the Gaussian dispersion) are ~2000-5000km/sec

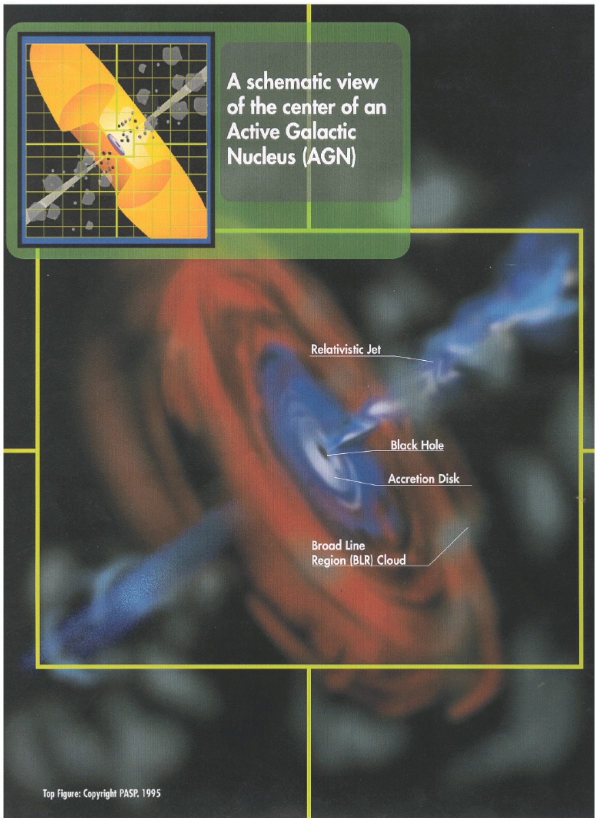
- The broad range of ionization is due to the 'photoionzation' of the gas- the gas is not in collisional equilibrium
- At short wavelengths the continuum is thought to be due to the accretion disk



Van den Berk et al 2001

- A selection of emission lines ranging from high ionization CIV to low ionization Mg II





Source	Distance from central source
X-Ray Fe Kα	3-10 R_S
Broad-Line Region	600 R_S
Megamasers	4×10^4 R_S
Gas Dynamics	8×10^5 R_S
Stellar Dynamics	10^6 R_S

A Quick Guide to Photoionized Plasmas

- Fundamental idea photon interacts with ion and electron is ejected and ion charge increased by 1
- $X^{+q} + h\nu \rightarrow X^{+(q+1)} + e^-$
- Ionization of the plasma is determined by the balance betw photionization and recombinatic
- Photoionization rate is proportional to the number of ionizing photons x number of ionsxthe cross section for interaction and the recombination rate to the number of ions x number of electrons x atomic physics rates

- Steady state ionization determined not by temperature, but by balance between photo-ionization (~F_E spectrum) and recombination (n_e): $n_q \int F_E \sigma^{PI}(E)dE = n_{q+1} n_e \alpha(T_e)$
- $\boxed{\text{Ionization } n_{q+1}/n_q \propto F/n_e \propto L/n_e r^2 \equiv \xi}$

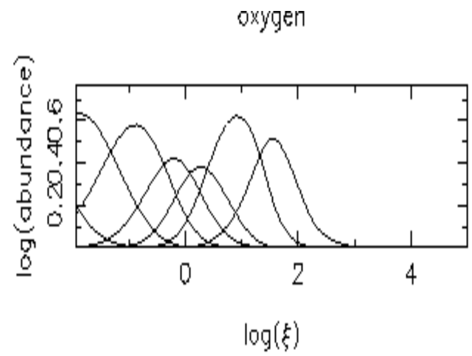
ξ is the ionization parameter (also sometimes called U)

In Other Words

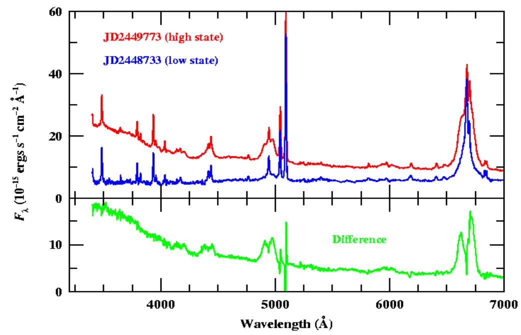
- For each ion:
  - Ionization = recombination
  - ~photon flux ~electron density
- For the gas as a whole
  - Heating = cooling
  - ~photon flux ~electron density

- => All results depend on the ratio photon flux/gas density or "ionization parameter"

- Higher ionization parameters produce more highly ionized lines (higher flux or lower density)



Neutral <----------->fully stripped

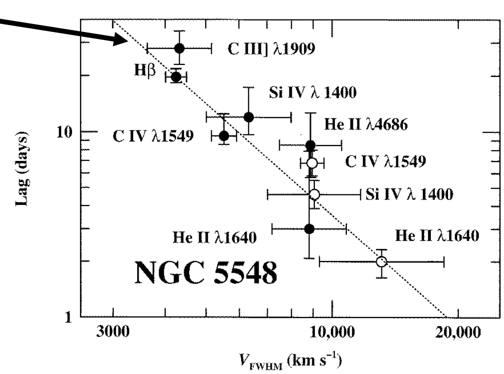


Peterson (1999)

What is Observed??

- The higher ionization lines have a larger width (rotational speed) and respond faster (closer to BH)
- Line is consistent with idea of photoionization, density $\sim r^{-2}$ and Keplerian motions dominate the line shapes ($v \sim r^{-1/2}$)

- Such data exist for ~40 sources
- At present M_{BH} can be estimated to within a factor of a few: $M \propto FWHM^2 L^{0.5}$



Dotted line corresponds to a mass of $6.8 \times 10^7 M_\odot$
Peterson and Wandel 1999