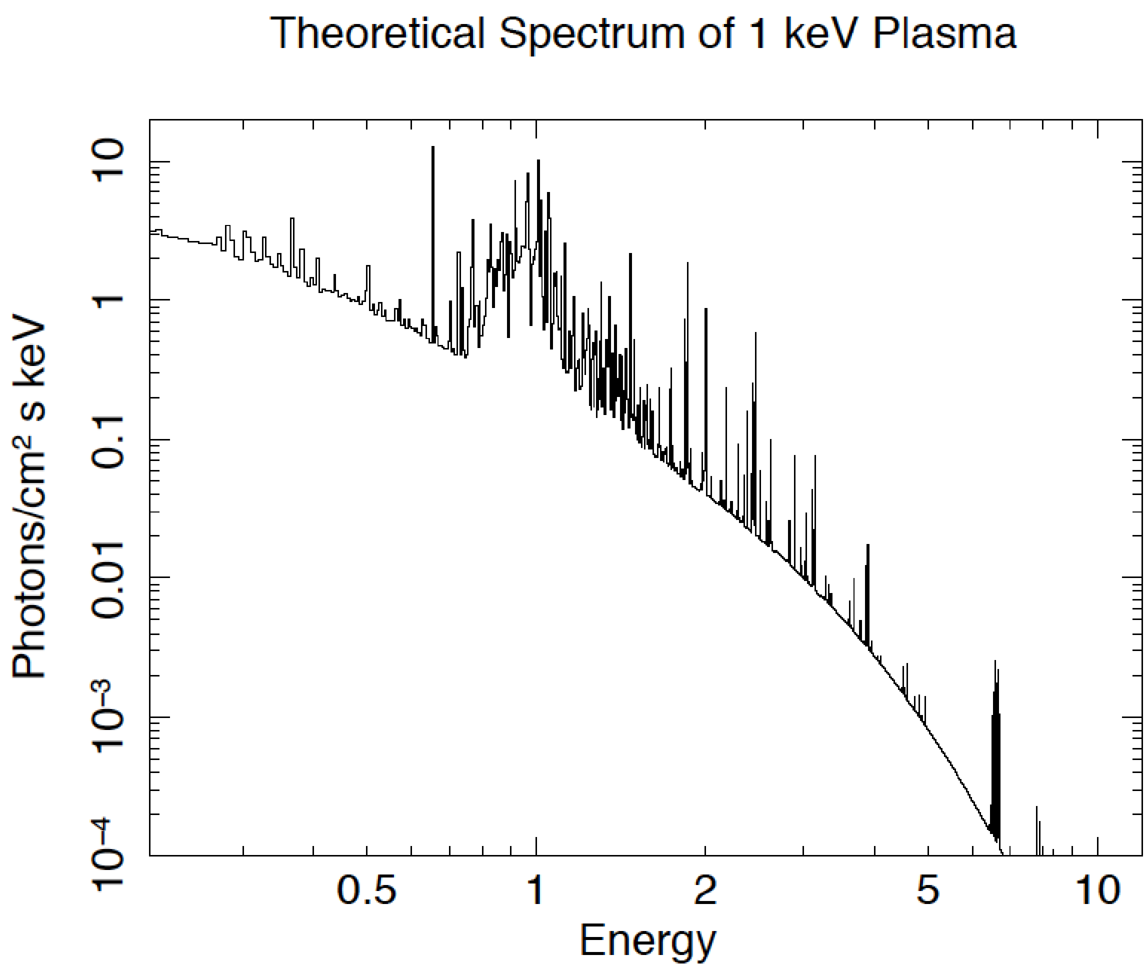
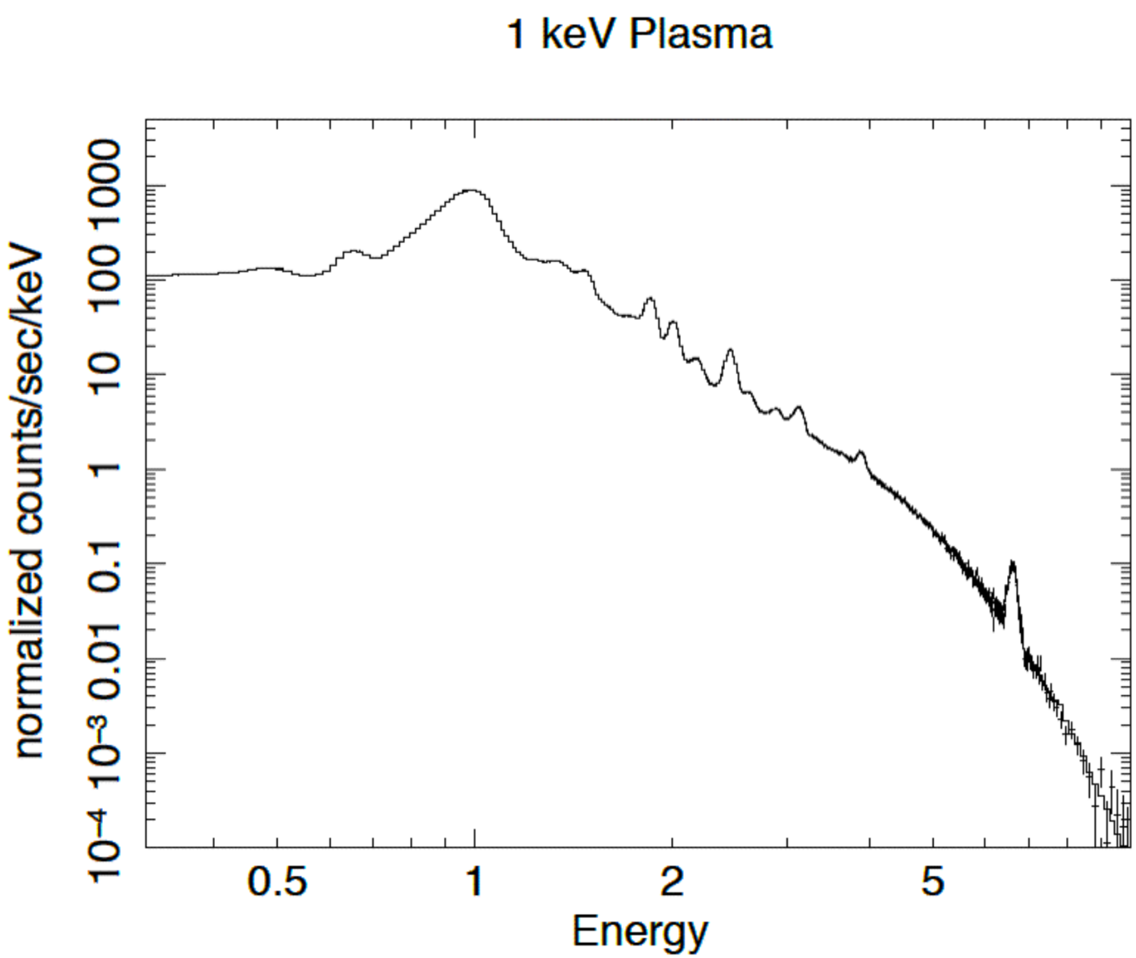
# 1 keV Plasma

- Theoretical model of a collisionally ionized plasma kT=1 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of $10^5$

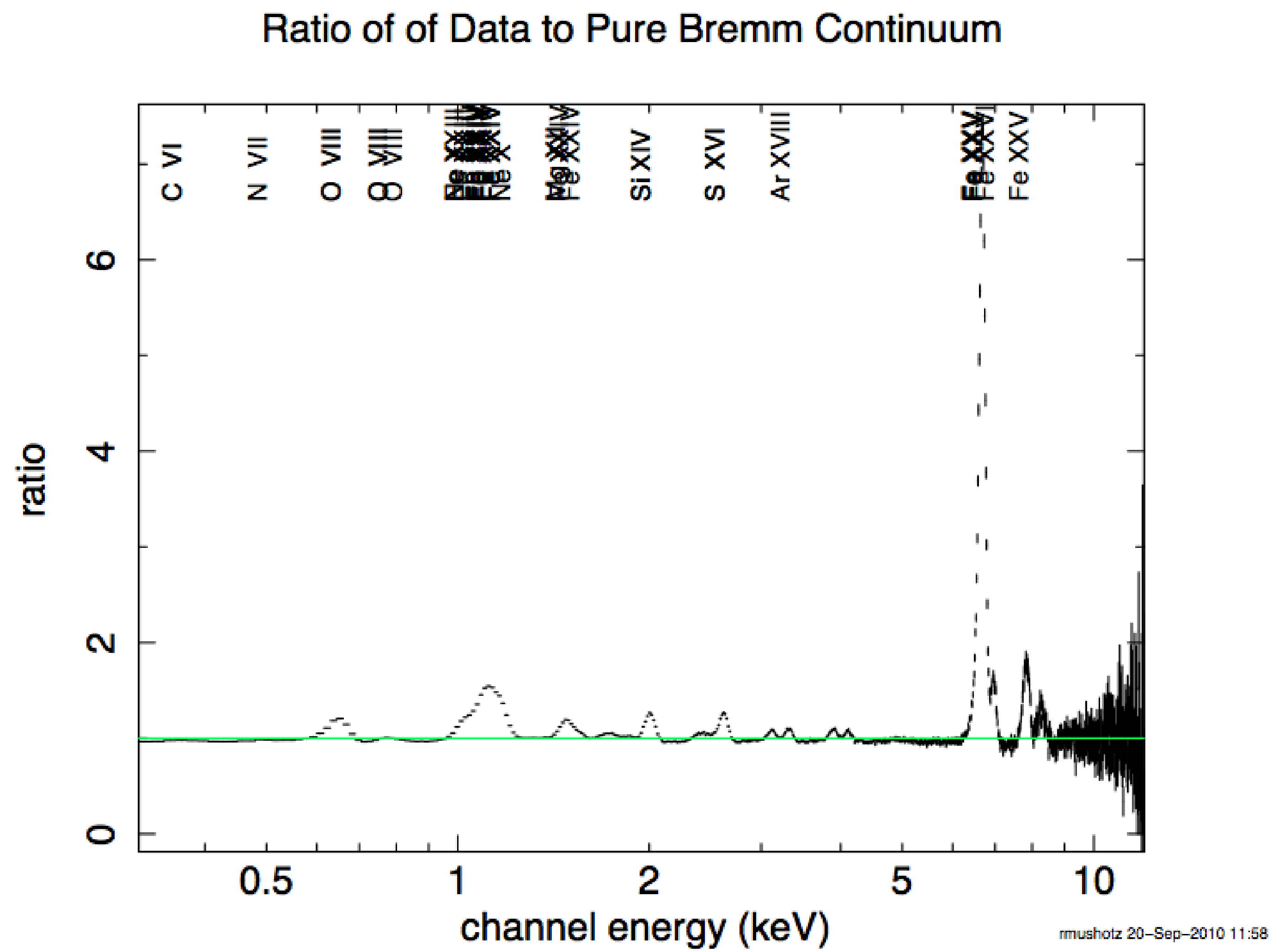


- Observational data for a collisionally ionized plasma kT=1 keV with solar abundances
- Notice the very large blend of lines near 1 keV- L shell lines of Fe
- Notice dynamic range of $10^7$



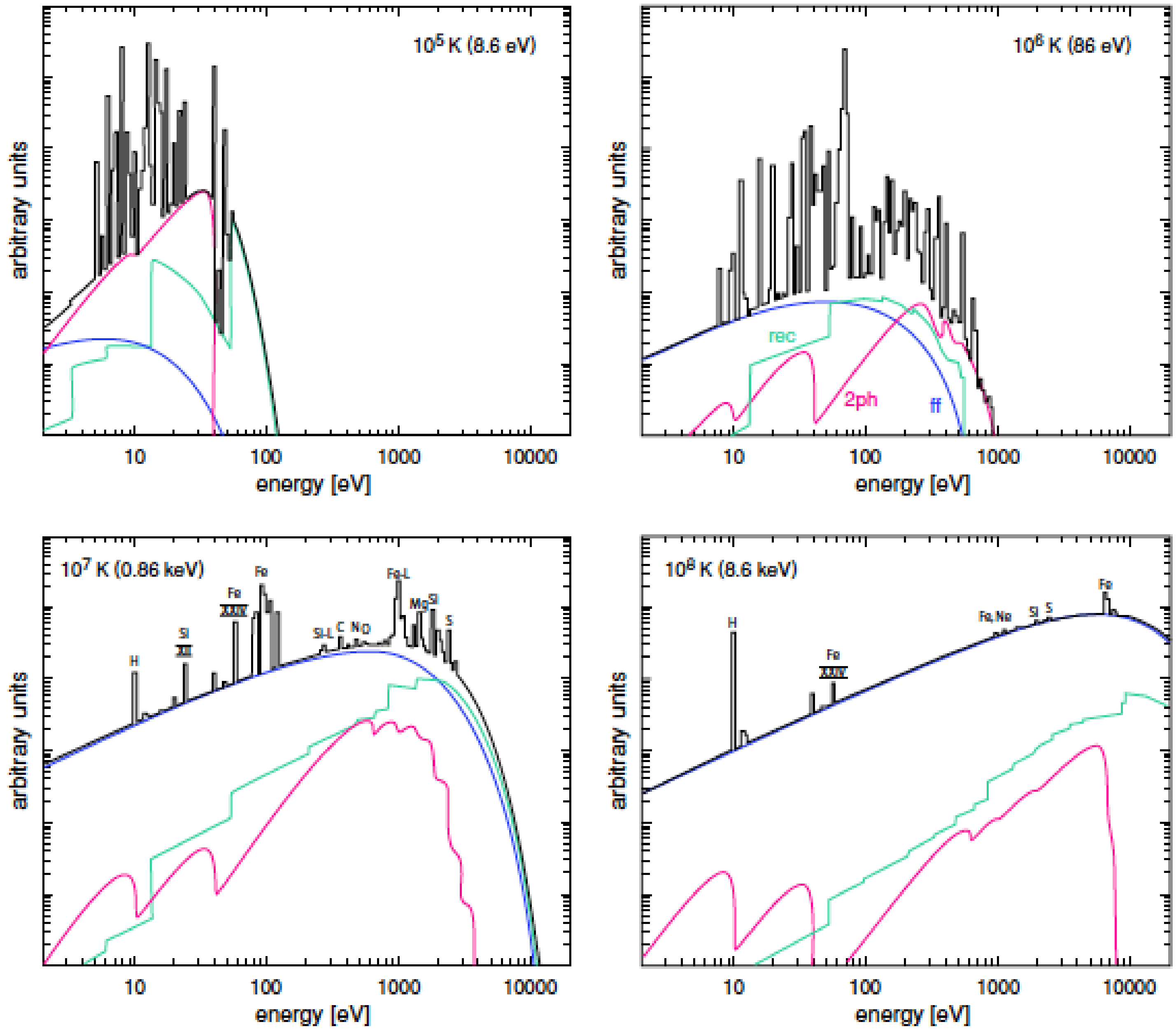
# Collsionally Ionized Equilibrium Plasma

- Ratio of model to a 'pure' H/He plasma
- This plot is designed to show the lines



# Strong Temperature Dependence of Spectra

- Line emission
- Bremms (black)
- Recombination (red)
- 2 photon green



# Relevant Time Scales

- The equilibration timescales between protons and electrons is t(p,e) ~ 2 x $10^8$ yr at an 'average' location
- **In collisional ionization equilibrium population of ions is directly related to temperature**

$$\tau(1,2) = \frac{3m_1\sqrt{2\pi}(kT)^{3/2}}{8\pi\sqrt{m_2}n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}$$

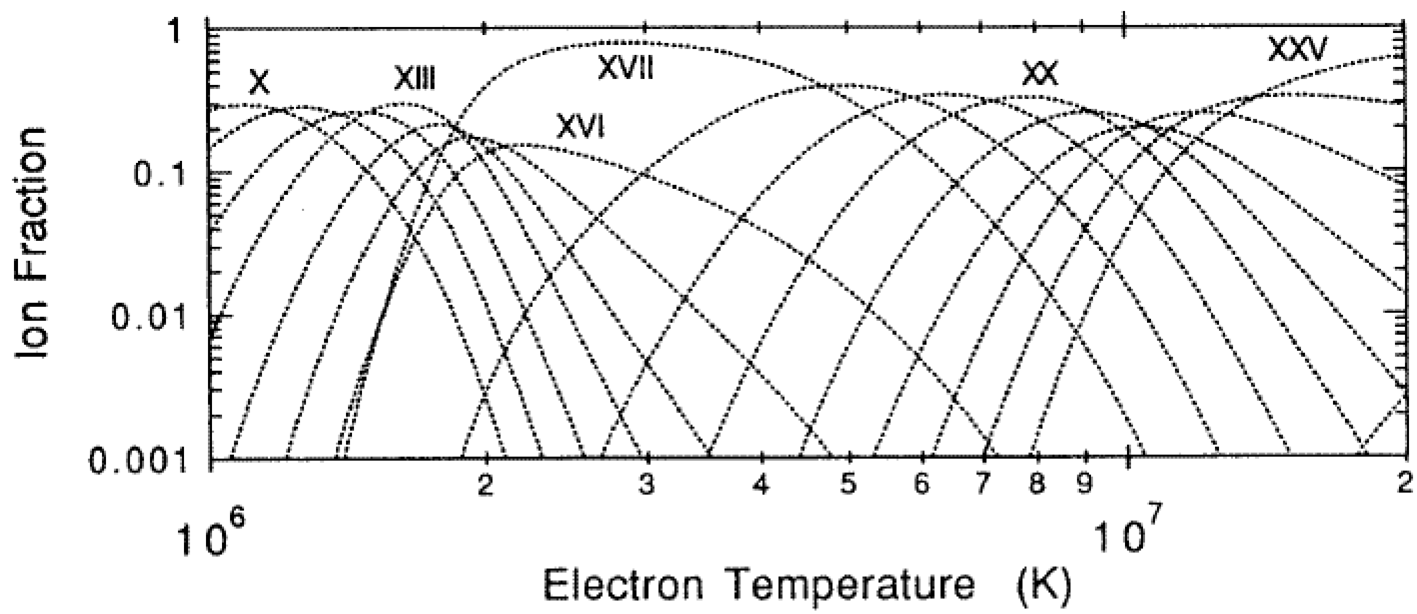
$$\ln\Lambda \equiv \ln(b_{max}/b_{min}) \approx 40$$

$$\tau(e,e) \approx 3\times10^5\left(\frac{T}{10^8\,\mathrm{K}}\right)^{3/2}\left(\frac{n_e}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1}\mathrm{yr}$$

$$\tau(p,p) = \sqrt{m_p/m_e}\,\tau(e,e) \approx 43\tau(e,e)$$

$$\tau(p,e) = (m_p/m_e)\tau(e,e) \approx 1800\tau(e,e)$$

**Ion fraction for Fe vs electron temperature**



# How Did I Know This??

- Why do we think that the emission is thermal bremmstrahlung?
  - X-ray spectra are consistent with model
  - X-ray 'image' is also consistent
  - Derived physical parameters 'make sense'
  - Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)

# Mean Free Path for Collisions/ Energy

- Mean-free-path $\lambda_e$ ~ 20 kpc < 1% of cluster size

$$\lambda_p \approx \lambda_e = \frac{3^{3/2}(kT)^2}{8\sqrt{\pi}\, n_e e^4 \ln\Lambda}$$

$$\approx 23\left(\frac{T}{10^8\ \mathrm{K}}\right)^2\left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1}\mathrm{kpc}$$

At $T>3\mathrm{x}10^7$ K the major form of energy emission is thermal bremmstrahlung continuum

$\varepsilon \sim 3\mathrm{x}10^{-27}\ T^{1/2}\ n^2$ ergs/cm$^3$/sec- how long does it take a parcel of gas to lose its energy?

$\tau \sim nkT/\varepsilon \sim 8.5\mathrm{x}10^{10}$yrs$(n/10^{-3})^{-1}\ T_8^{1/2}$

At lower temperatures line emission is important

# Physical Conditions in the Gas

- the elastic collision times for ions and electrons ) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 x 10^8 yr (T_{gas}/10^8)^{1/2} (D/Mpc)$

- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ (P is the pressure, $\rho_g$ is the gas density, $\gamma=5/3$ is the adiabatic index for a monoatomic ideal gas)
- For an ideal gas with $P=\rho_g kT$ the sound speed depends only on temperature and the mass of the atoms.

# Hydrostatic Equilibrium Kaiser 19.2

- Equation of hydrostatic equil

$$\nabla P = -\rho_g \nabla \phi(r)$$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter)

**P is the gas pressure**

$\rho_g$ **is the gas density**

# Hydrodynamics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \text{ mass conservation (continuity)}$$

$$\rho \frac{Dv}{Dt} + \nabla P + \rho \nabla \phi = 0 \text{ momentum conservation (Euler)}$$

$$\rho T \frac{Ds}{Dt} = H - L \text{ entropy (heating \& cooling)}$$

$$P = \frac{\rho k T}{\mu m_p} \text{ equation of state}$$

Add viscosity, thermal conduction, …

Add magnetic fields (MHD) and cosmic rays

Gravitational potential $\phi$ from DM, gas, galaxies

- density and potential are related by Poisson's equation

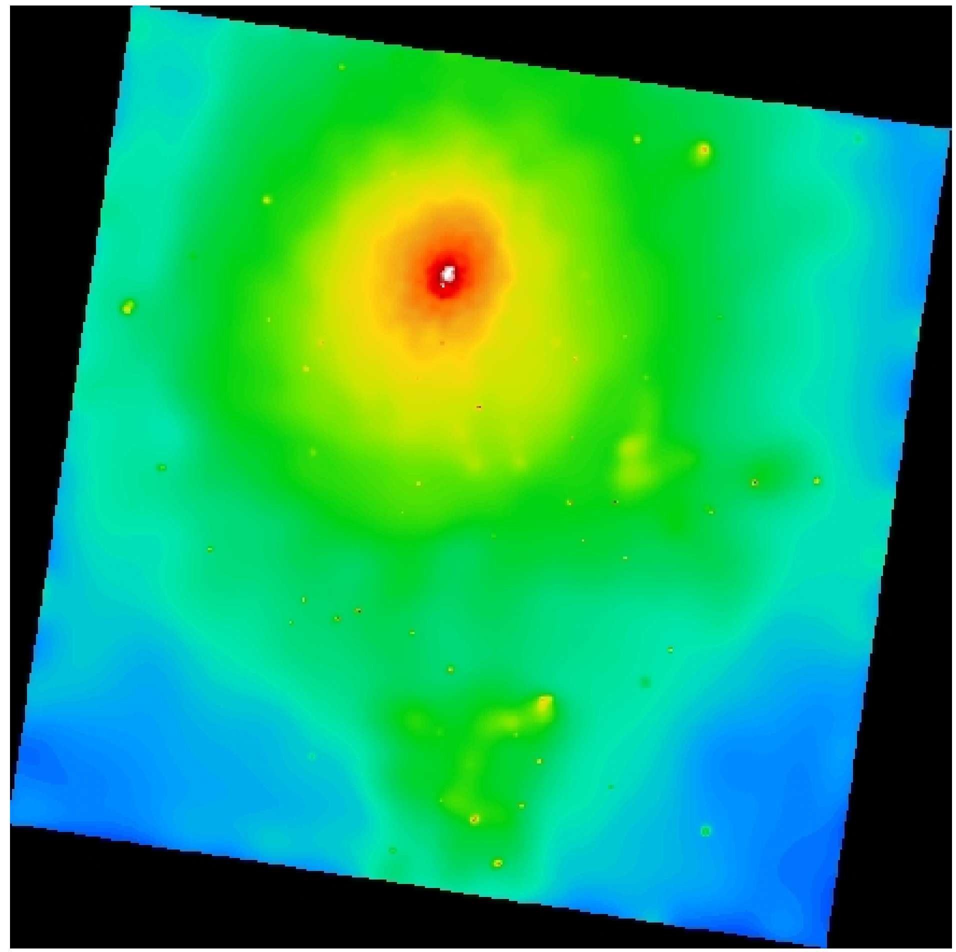
$\nabla^2\phi = 4\pi\rho G$

- and combining this with  the equation of hydrostaic equil

- $\nabla \cdot (1/\rho \nabla P) = -\nabla^2\phi = -4\pi G\rho$

- or, for a spherically symmetric system

$1/r^2\ d/dr\ (r^2/\rho\ dP/dr) = -4\pi\rho G\rho$



# Deriving the Mass from X-ray Spectra

For spherical symmetry this reduces to

$(1/\rho_g)\, dP/dr = -d\phi(r)/dr = GM(r)/r^2$

With a little algebra and the definition of pressure - the total cluster mass can be expressed as

$$M(r)=kT_g(r)/\mu G m_p)\mathbf{r}\ (d\ln T/d\ln r + d\ln \rho_g/d\ln r)$$

k is Boltzmans const, $\mu$ is the mean mass of a particle and $m_H$ is the mass of a hydrogen atom

**Every thing is observable**

The temperature $T_g$ from the spatially resolved spectrum

The density $\rho_g$ from the knowledge that the emission is due to bremmstrahlung

And the scale size, **r**, from the conversion of angles to distance

- De-project X-ray surface brightness profile I(R) to obtain gas density vs. radius, $\rho(r)$

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\varepsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}$$

$$\varepsilon_\nu(r) = -\frac{1}{\pi}\frac{d}{dr^2}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}} = \Lambda_\nu[T(r)]n_e^2(r)$$

- Where $\Lambda$ is the cooling function and $n_e$ is the gas density (subtle difference between gas density and electron density because the gas is not pure hydrogen
- De-project X-ray spectra in annuli T(r)
- Pressure $P = \rho kT/(\mu m_p)$

# X-ray Mass Estimates

- use the equation of hydrostatic equilibirum
-

$$\frac{dP_{\rm gas}}{dr} = \frac{-G\mathfrak{M}_*(r)\rho_{\rm gas}}{r^2} \qquad (3)$$

where $P_{\rm gas}$ is the gas pressure, $\rho_{\rm gas}$ is the density, $G$ is the gravitational constant, and $\mathfrak{M}_*(r)$ is the mass of M87 interior to the radius $r$.

$$P_{\rm gas} = \frac{\rho_{\rm gas} K T_{\rm gas}}{\mu \mathfrak{M}_{\rm H}} \qquad (4)$$

where $\mu$ is the mean molecular weight (taken to be 0.6), and $\mathfrak{M}_{\rm H}$ is the mass of hydrogen atom.

$$\frac{KT_{\rm gas}}{\mu\mathfrak{M}_{\rm H}}\left(\frac{d\rho_{\rm gas}}{\rho_{\rm gas}} + \frac{dT_{\rm gas}}{T_{\rm gas}}\right) = \frac{-G\mathfrak{M}_*(r)}{r^2}dr, \qquad (5)$$

which may be rewritten as:

$$-\frac{KT_{\rm gas}}{G\mu\mathfrak{M}_{\rm H}}\left(\frac{d\log\rho_{\rm gas}}{d\log r} + \frac{d\log T_{\rm gas}}{d\log r}\right) r = \mathfrak{M}_*(r) \qquad (6)$$

**Putting numbers in gives**

$$M(r) = -3.71\times10^{13}\,M_\odot\,T(r)\,r\left(\frac{d\log\rho_g}{d\log r} + \frac{d\log T}{d\log r}\right),$$

T is in units of KeV and r in units of Mpc



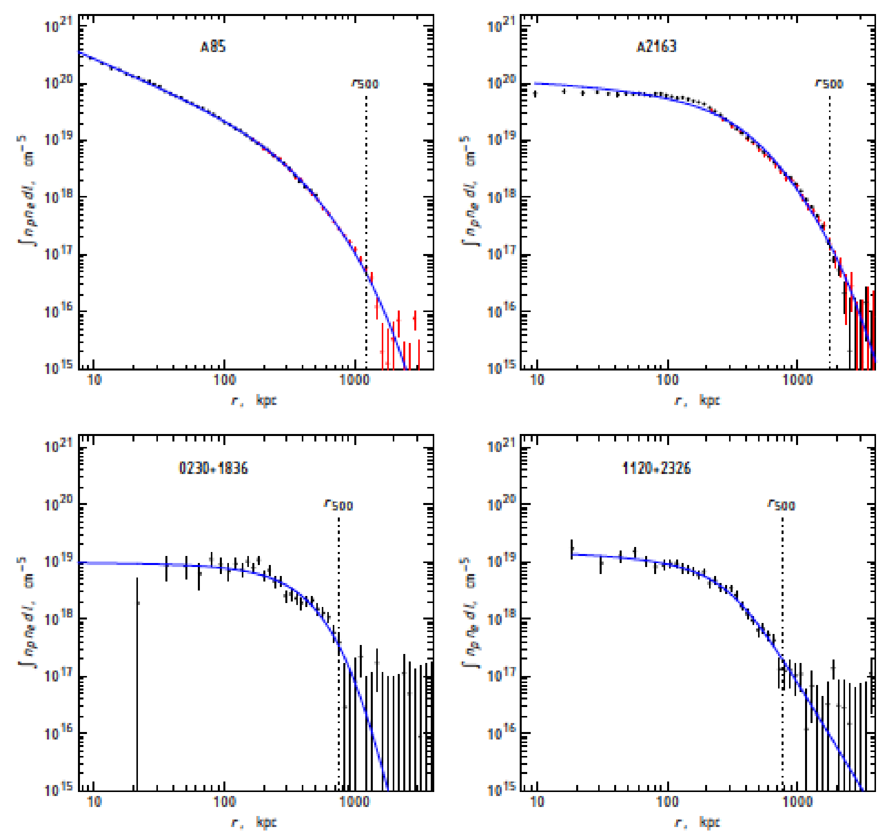
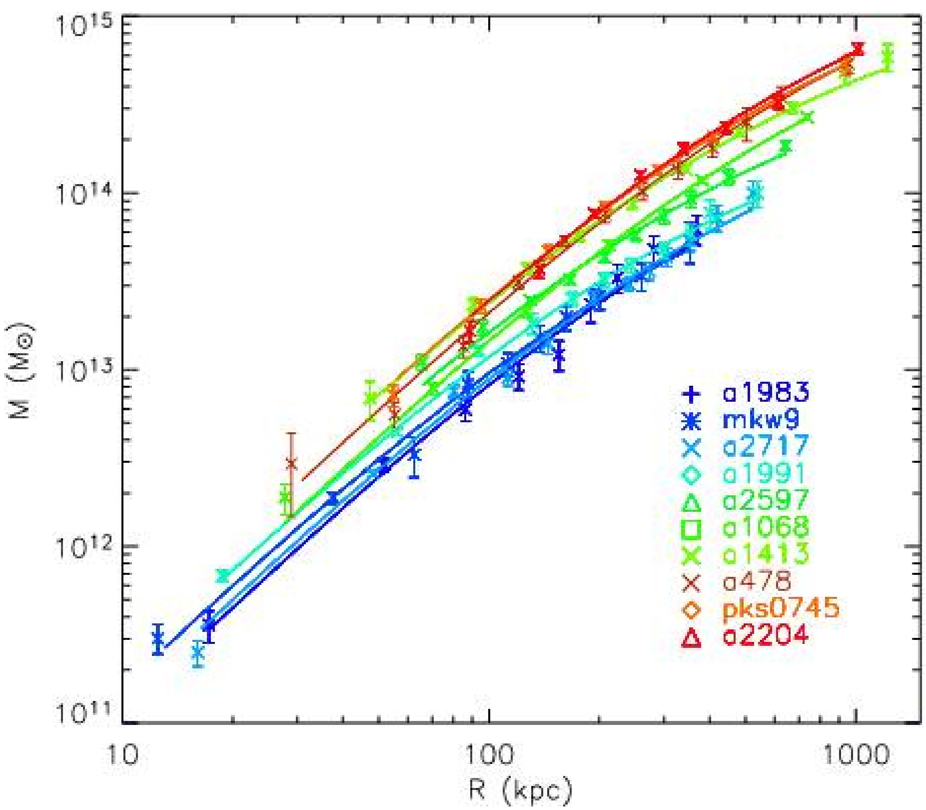


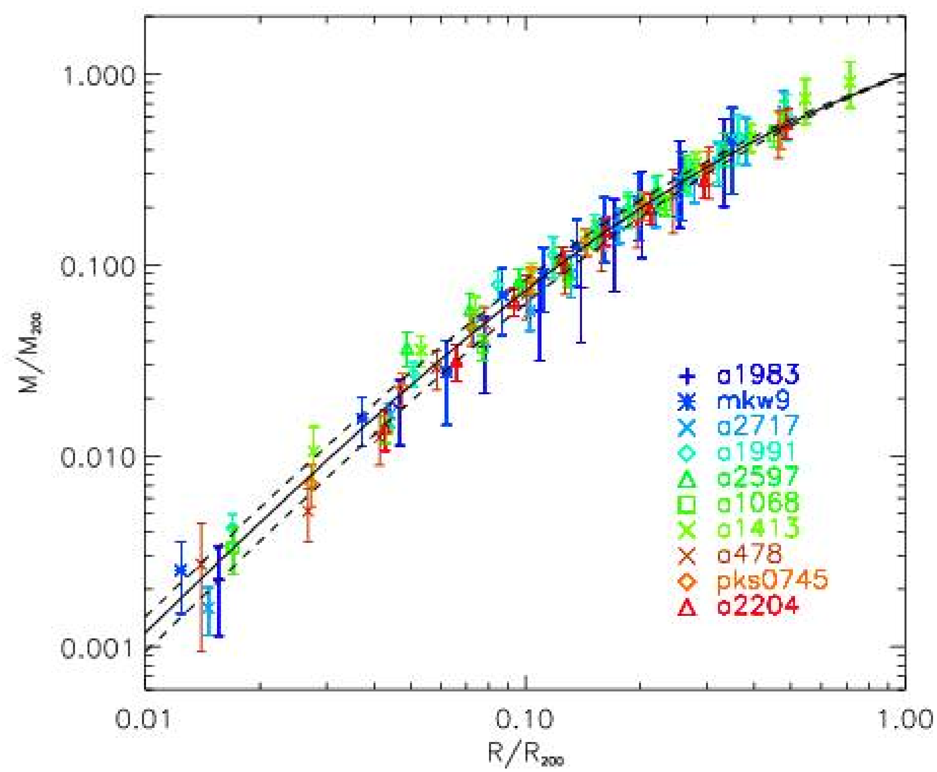

FIG. 5.— Examples of the surface brightness profile modeling for clusters shown in Fig. 3 and 4. The observed X-ray count rates are converted to the projected emission measure integral (see § 3.4 and V06). The black and red data points show the *Chandra* and *ROSAT* measurements, respectively. The best fit models (the projected emission measure integral for the three-dimensional distribution given by eq. 2) are shown by solid lines. The dashed lines indicate the estimated $r_{500}$

# Mass Profiles from Use of Hydrostatic Equilibrium

- Use temperature and density profiles +hydrostatic equilibrium to determine masses




Physical units




Scaled units

- The emission measure along the line of sight at radius r, EM(r), can be deduced from the X-ray surface brightness, $S(\Theta)$:

$EM(r) = 4\pi (1 + z)^4 S(\Theta)/\Lambda(T, z)$ ; $r = dA(z)\,\Theta$

where $\Lambda(T, z)$ is the emissivity in the detector band, taking into account the instrument spectral response,

dA(z) is the angular distance at redshift z.

The emission measure is linked to the gas density $\rho_g$ by:

- $EM(r) = \int_r^\infty \rho^2_g(R)\, R dr/\sqrt{(R^2-r^2)}$
- The shape of the surface brightness profile is thus governed by the form of the gas distribution, whereas its normalization depends also on the cluster overall gas content.

# Density Profile

- a simple model(the β model) fits the surface brightness well
  - $S(r)=S(0)(1/r/a)^2)^{-3\beta+1/2}$ $cts/cm^2/sec/solid$ angle
- Is analytically invertible (inverse Abel transform) to the density profile

  $\rho(r)=\rho(0)(1/r/a)^2)^{-3\beta/2}$

  The conversion function from S(0) to ρ(0) depends on the detector

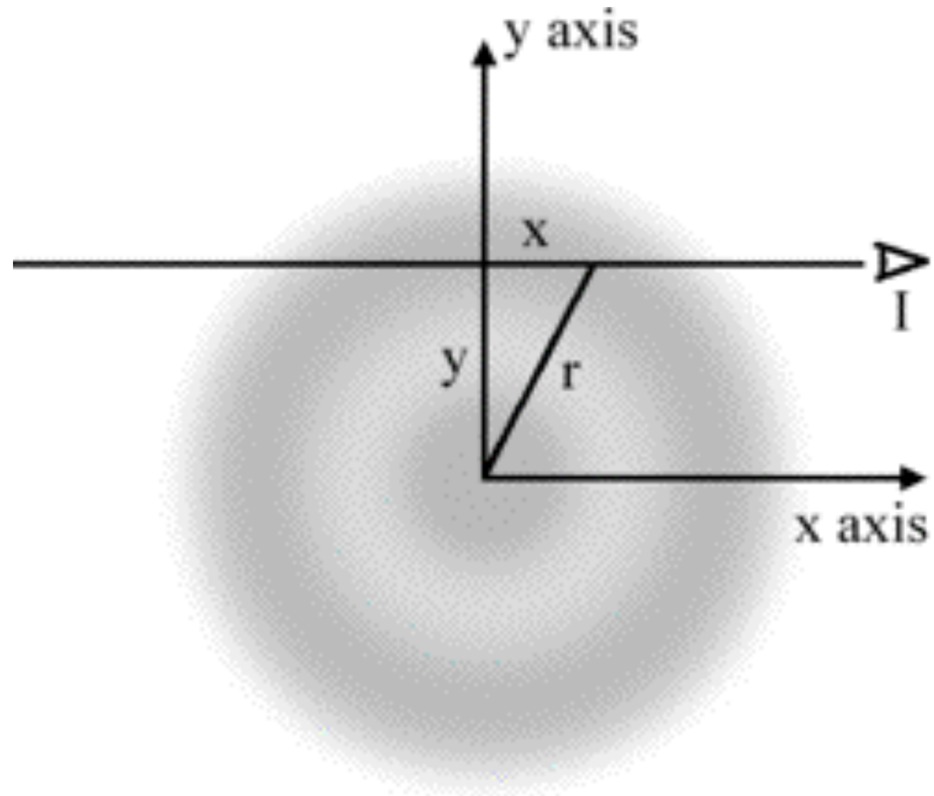
  The quantity 'a' is a scale factor- sometimes called the core radius

  ---------------------------------------------------------------------

- The Abel transform, , is an integral transform used in the analysis of spherically symmetric or axially symmetric functions. The Abel transform of a function f(r) is given by:
- $f(r)=1/p\int_r^{\infty} dF/dy\ dy/\sqrt{(y^{2-}r^2)}$
- In image analysis tthe reverse Abel transform is used to calculate the emission function given a projection (i.e. a scan or a photograph) of that emission function.
- In general the integral is not analytic which makes the

# Abel Transform

A geometrical interpretation of the Abel transform in two dimensions. An observer (I) looks along a line parallel to the x-axis a distance y above the origin. What the observer sees is the projection (i.e. the integral) of the circularly symmetric function f(r) along the line of sight. The function f(r) is represented in gray in this figure. The observer is assumed to be located infinitely far from the origin so that the limits of integration are ±∞



# Discussion in Sarazin sec 5

The gas distributions in clusters can be derived directly from observations of the X-ray surface brightness of the cluster, if the shape of the cluster is known and if the X-ray observations are sufficiently detailed and accurate. This method of analysis also leads to a method for determining cluster masses (Section 5.5.5). The X-ray surface brightness at a photon frequency $\nu$ and at a projected distance $b$ from the center of a spherical cluster is

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\epsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}, \qquad (5.80)$$

where $\epsilon_\nu$ is the X-ray emissivity. This Abel integral can be inverted to give the emissivity as a function of radius,

$$\epsilon_\nu = -\frac{1}{2\pi r}\frac{d}{dr}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}}. \qquad (5.81)$$

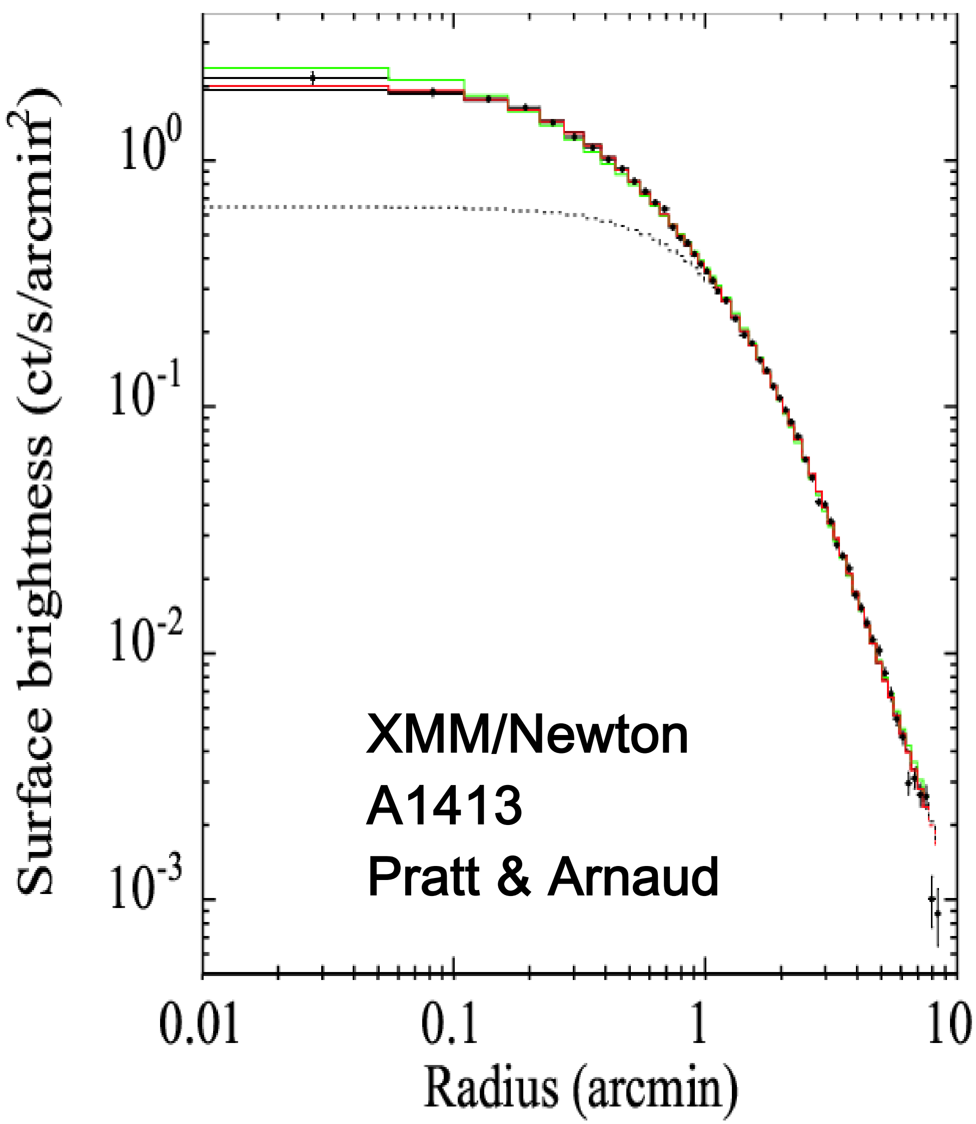
# Surface Brightness Profiles

- It has become customary to use a 'β' model (Cavaliere and Fesco-Fumiano)'
- clusters have <β>~2/3

$$\rho(r) = \frac{\rho_0}{\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{3\beta/2}}$$

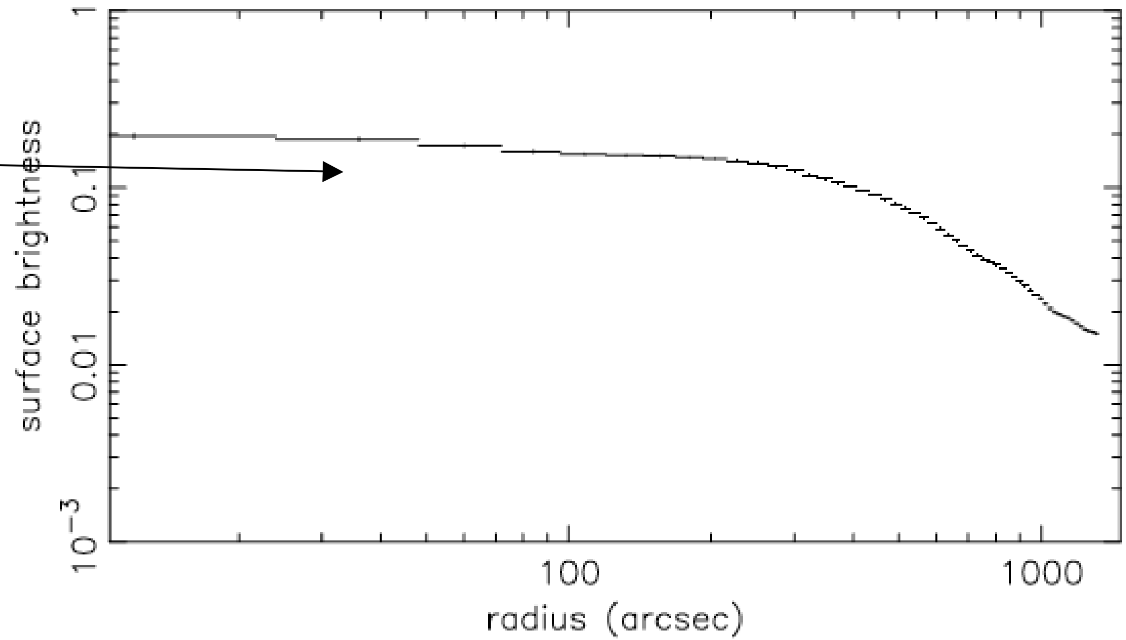
$$\beta \equiv \frac{\mu m_p \sigma_{gal}^2}{kT}$$ but treat as fitting parameter

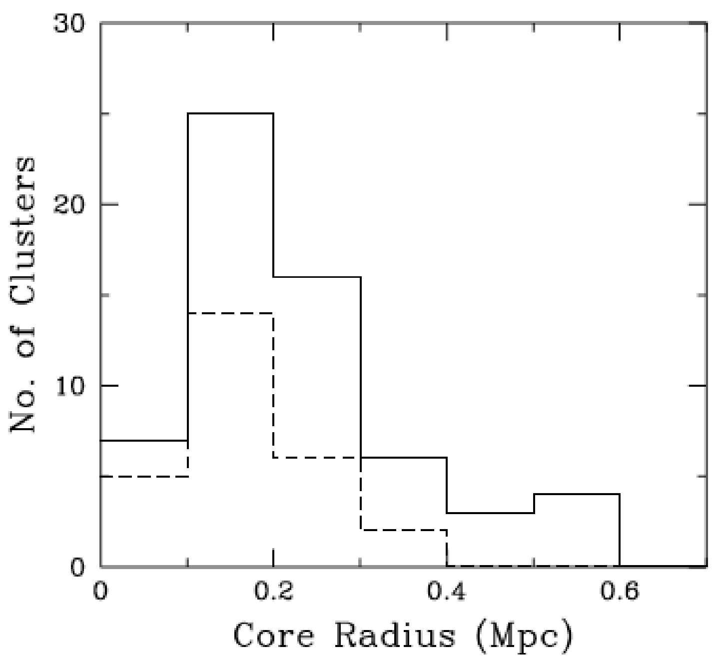
$$I_X(r) \propto \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2}$$

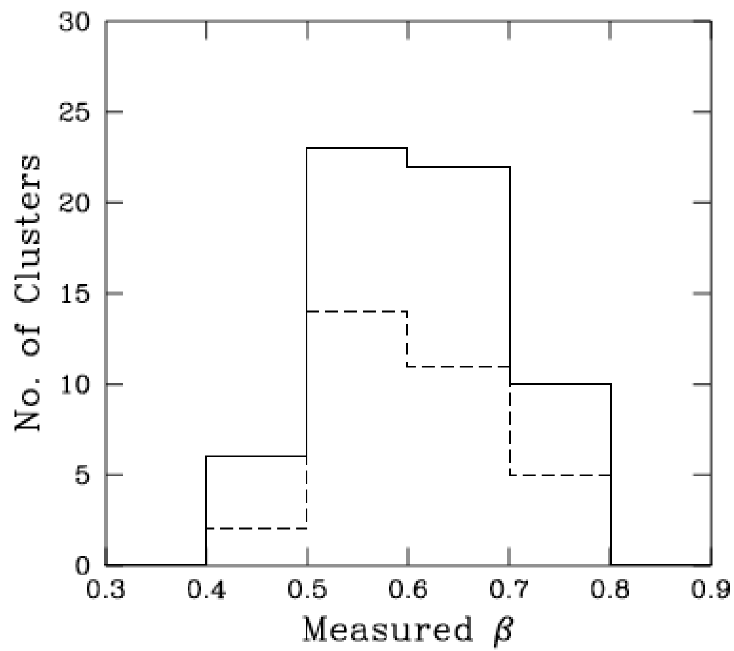


# 'Two' Types of Surface Brightness Profiles

- 'Cored'- ~~the profile is~~ flat in the center
- Central Excess
- Range of core radii and β



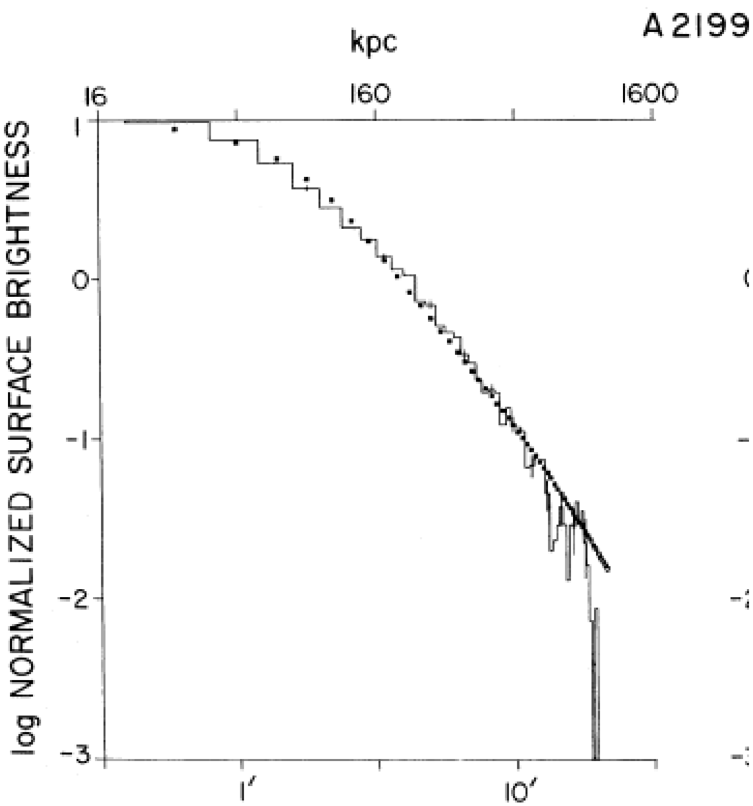



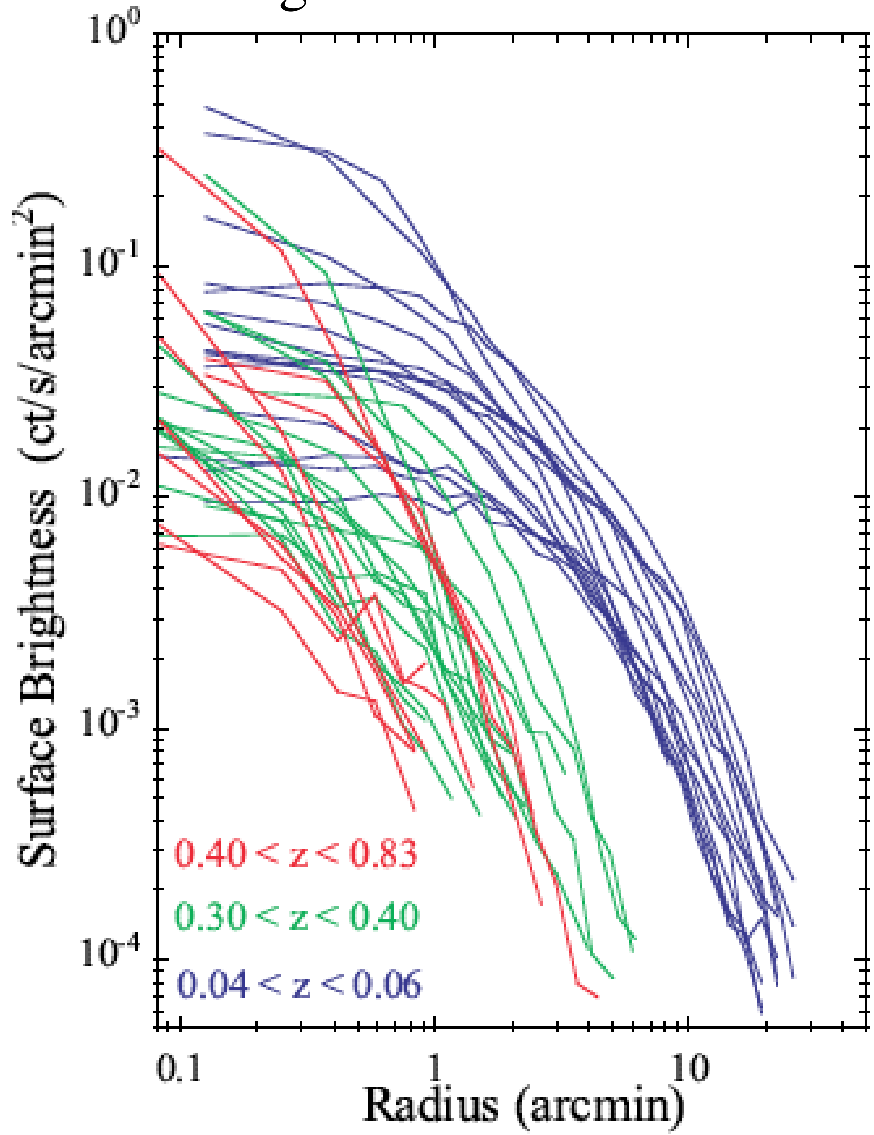



# X-ray Emissivity

- The observed x-ray emissivity is a projection of the density profile

A large set of clusters over a wide range in redshift



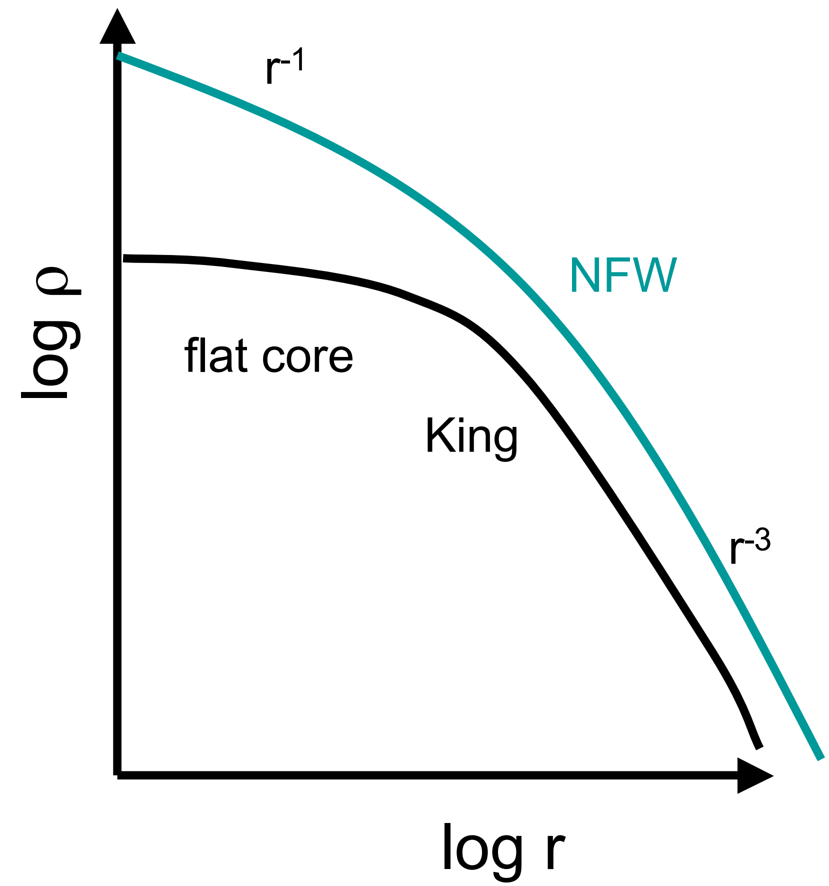



# Cluster Potentials (cont.)

Analytic King Model (approximation to isothermal sphere)

$$\rho_{dm}(r) = \frac{\rho_{dm,0}}{\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{3/2}}$$

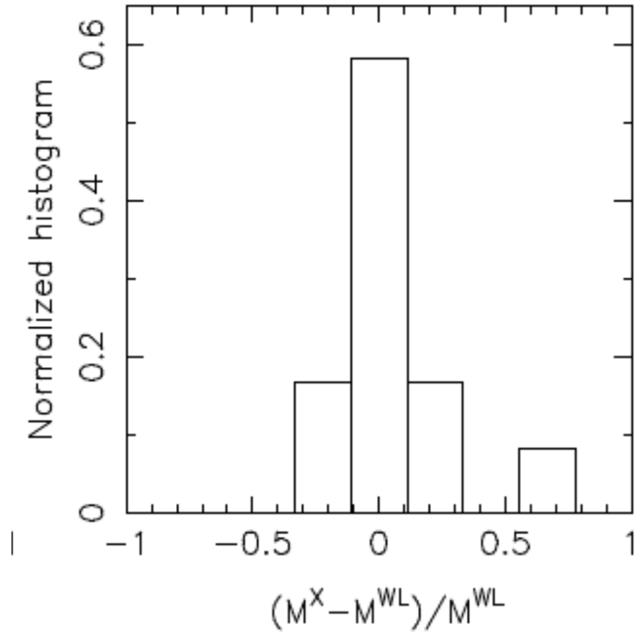
$$r_c \approx r_s / 2 \approx 200\,\mathrm{kpc}$$

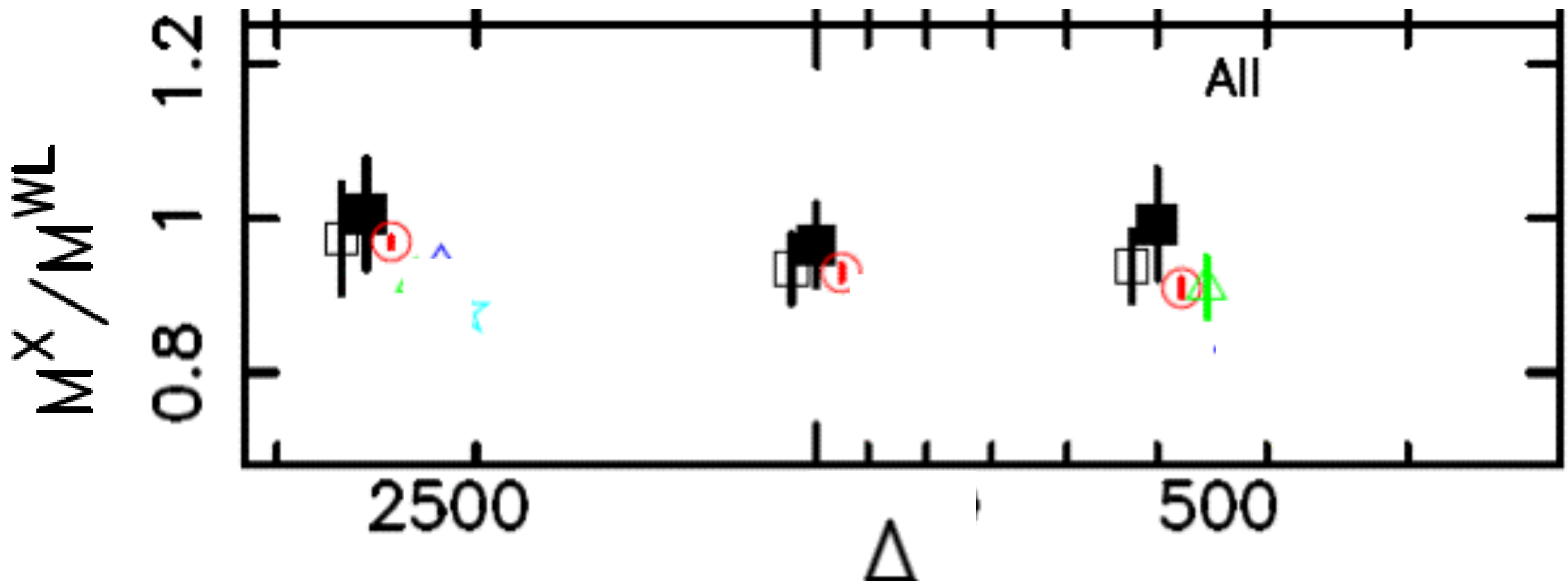



NFW is a potential that comes out of cold dark matter cosmological simulations

# Comparison of Lensing to X-ray Masses

- $\Delta$ is the overdensity of the part of the cluster used for the observations of the cluster mass compared to the critical density of the universe at the redshift of the cluster
- $M_x$ is the mass from x-ray observations and assumption of hydrostatic equilibrium
- $M_L$ is the mass from weak lensing



# Cluster Potentials

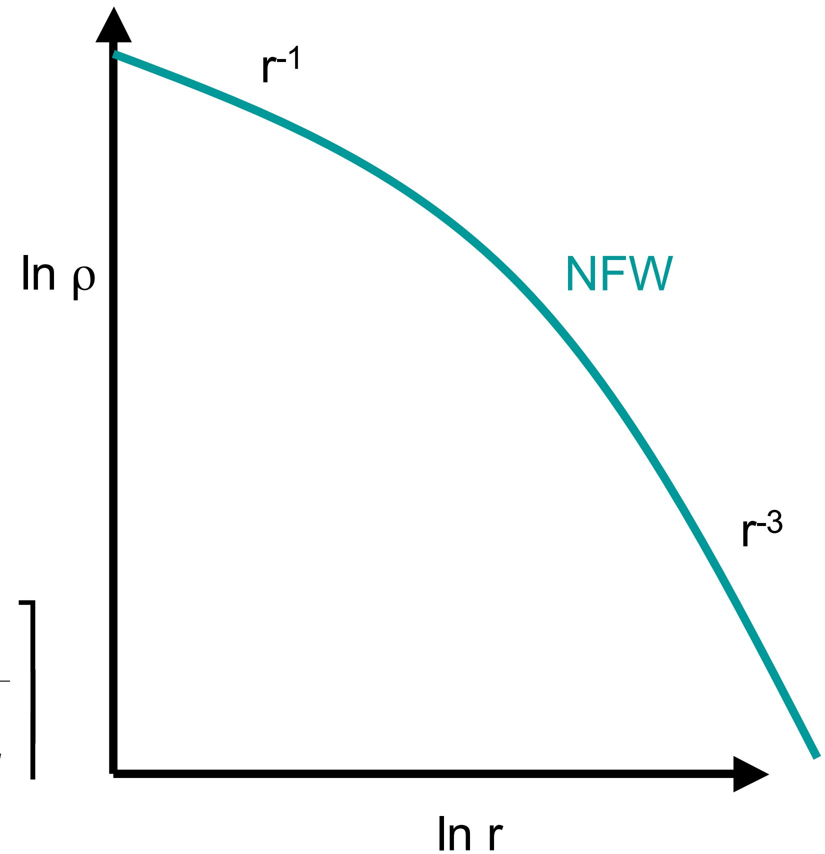
NFW (Navarro, Frenk, & White 1997)

$$\rho_{dm}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

$$c \equiv r_{vir} / r_s \approx 5 \text{ for clusters,}$$

$$r_{vir} \approx 2\,\text{Mpc}, \;\; r_s \approx 400\,\text{kpc}$$

$$M(r) = 4\pi\rho_s r_s^{\,3}\left[\ln(1 + \frac{r}{r_s}) - \frac{r}{r + r_s}\right]$$



# Top Questions on Clusters of Galaxies that Can be Answered by High Energy Astrophysics

- Are clusters fair samples of the Universe ?
- Can we derive accurate and unbiased masses from simple observables such as luminosity and temperature ?
- What is the origin of the metals in the ICM and when were they injected ? What is the origin of the entropy of the ICM ?

# Checking that X-ray Properties Trace Mass

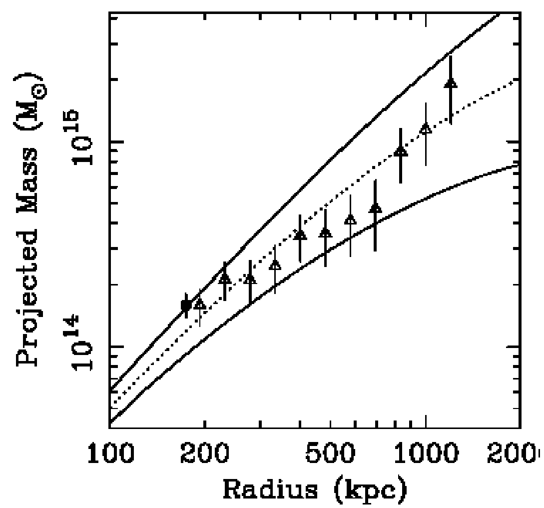
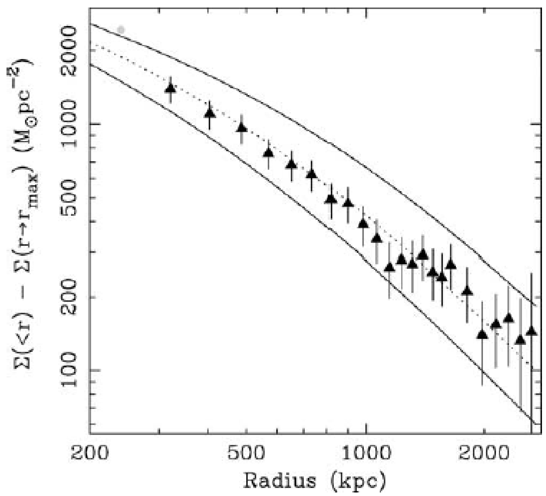
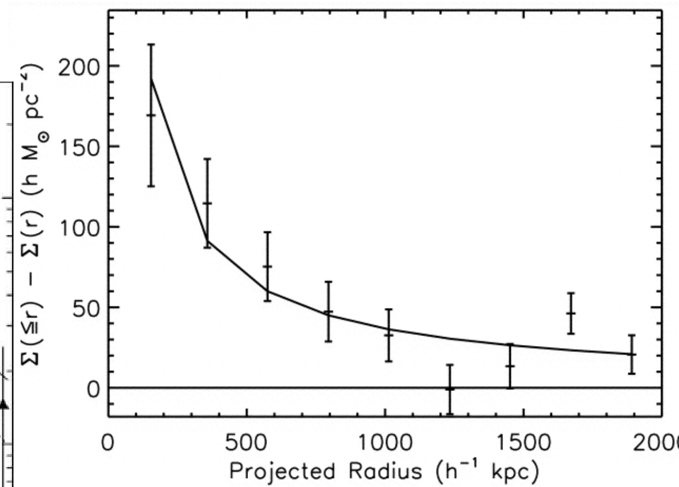



**Figure 8.** A comparison of the projected total mass determined from *Chandra* X-ray data (Section 5) with the strong lensing result of Pierre e (1996; filled circle) and the weak lensing results of Squires et al. (19




projected surface mass density contrast determined from the *Chandra* X-ray data (Section 5) w




Surface mass density for 42 Rosat selected clusters from Sloan lensing analysis

Comparison of cluster mass from lensing and x-ray hydrostatic equilibrium for A2390 and RXJ1340 (Allen et al 2001)

At the relative level of accuracy for smooth relaxed systems the x-ray and lensing mass estimators agree

# 'New' Physics

- The Cooling time ~$\tau$~nkT/E ~8.5x10 $^{10}$yrs$(n/10^{-3})^{-1}T_8^{1/2}$
- For bremmstrahlung but for line emission dominated plasmas it scales as $T_8^{-1/2}$ ;
- That is as the gas gets cooler it cools faster

$\Lambda$=cooling function

- $T_{cool}=5/2nkT/n^2\Lambda \sim t_{Hubble} T_8 \Lambda^{-1}_{-23}\ n_{-2}^{-1}$
- where$T_8$, is the temperature in units of $10^8$, $\Lambda_{-23}$ is the cooling function in units of $10^{-23}$, $n_{-2}$ is the number density in units of $10^{-2}$
- In central regions where the density (n) is large can cool in $t<10^9$ yrs
- 5/2 (the enthalpy) is used instead of 3/2 to take into the compression of  as it cools (and remains in pressure equilibrium)

# Cooling Cores

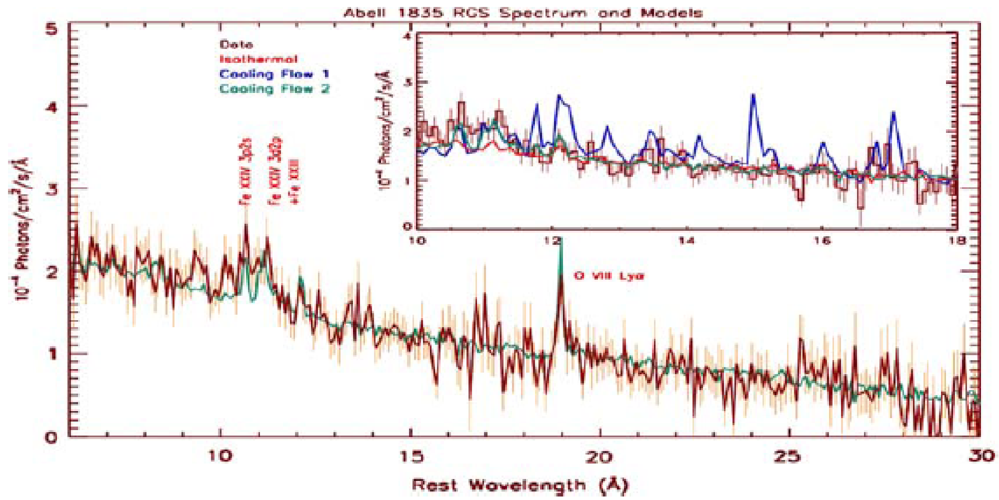
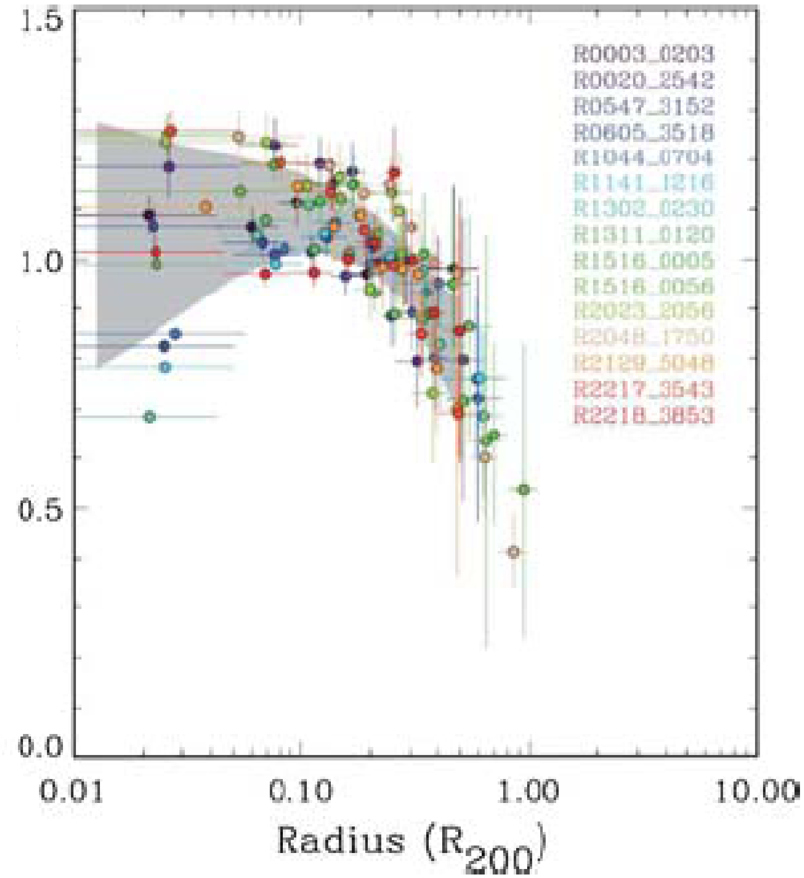
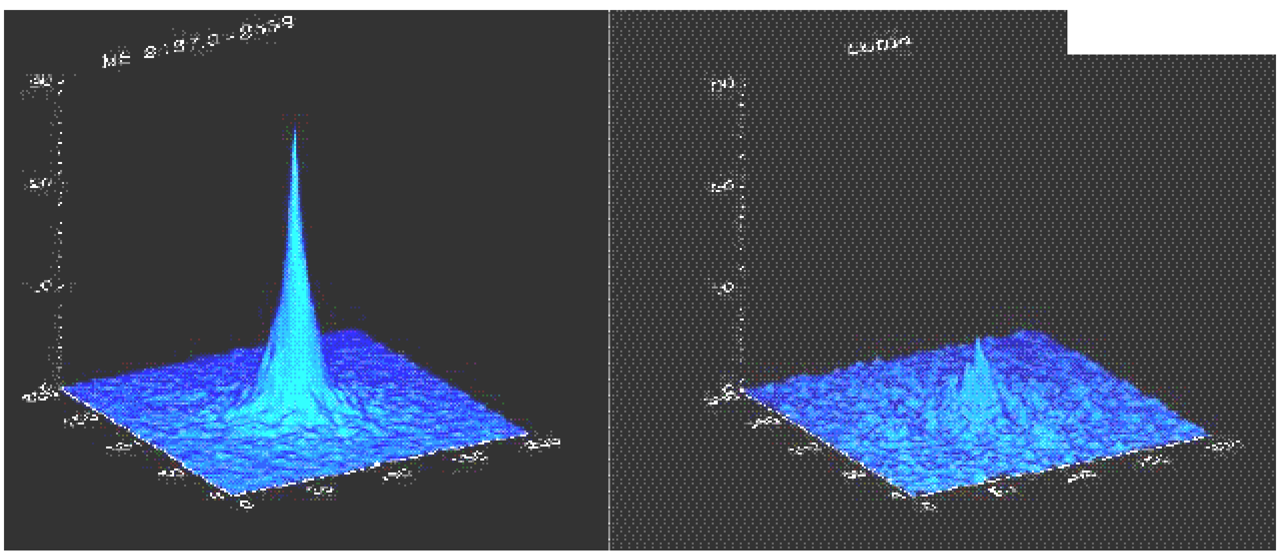



**Fig. 22** *XMM-Newton* RGS spectrum of the central region of the prominent cool core cluster, A1835 com-

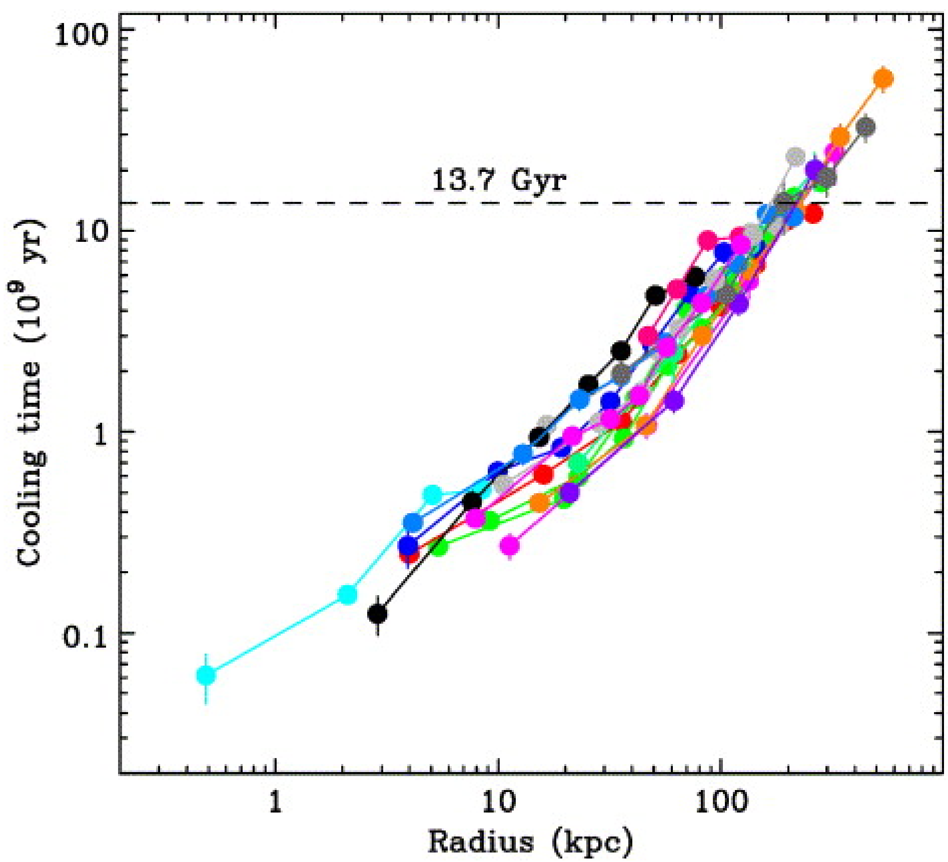
$$t_{cool} = 69\left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1}\left(\frac{T}{10^{8}\ \mathrm{K}}\right)^{1/2}\ \mathrm{Gyr}$$



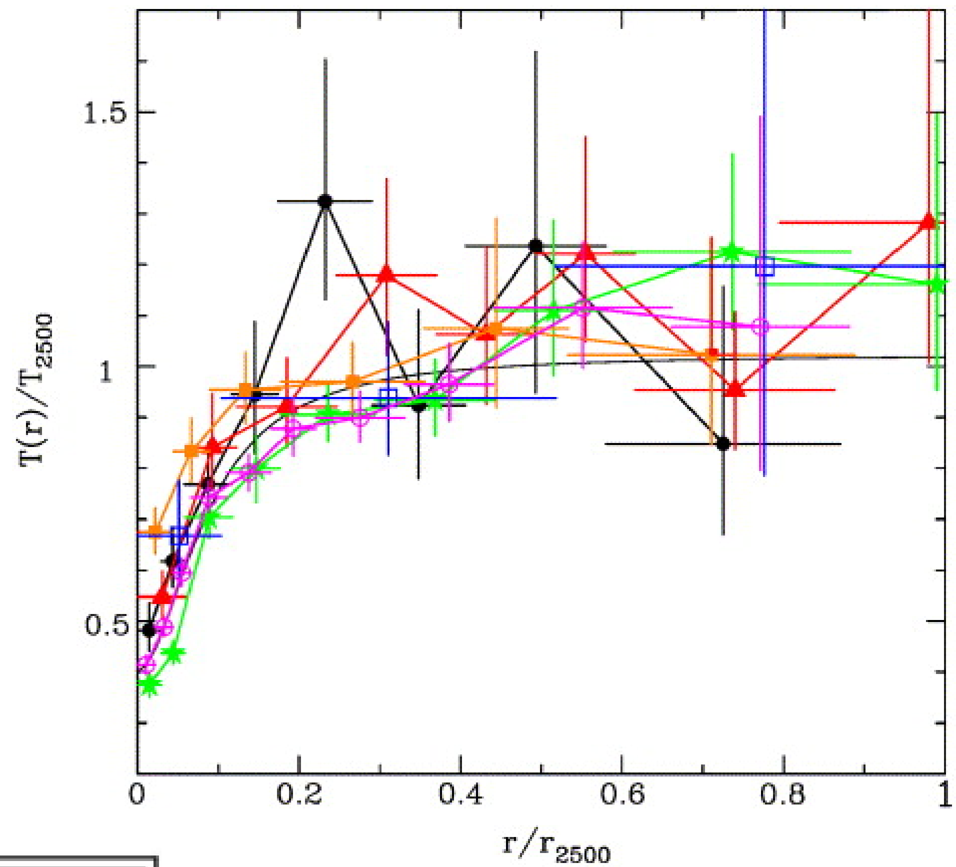
Notice that the central surface brightness of cool core clusters (left panel) is much higher than non-cooling core clusters

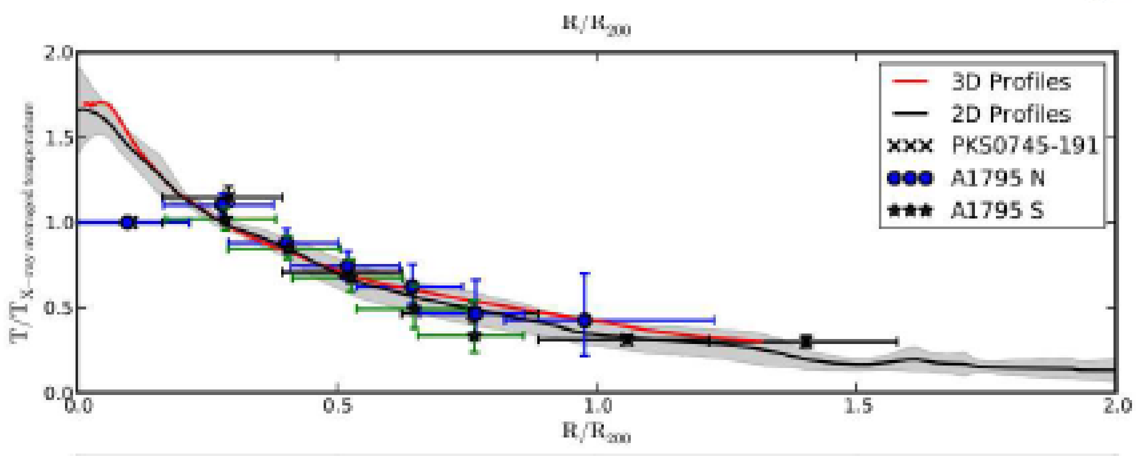
# Cooling Time for a Sample of Clusters



# Observed Temperature Profiles

- If the gas is in equilbrium with the potential (of the NFW form) it should be hotter in the center
- But in many clusters it is cooler




Left panel (from Burns et al 2010) shows the theoretical temperature profile if a NFW potential (in grey) compared to an set of actual cluster temperature profiles