Basics of Accretion

Is there a limit on accretion?

If the accreting material is exposed to the radiation it is producing it receives a force due to radiation pressure

The minimum radiation pressure is
(Flux/c)xσ (σ is the relevant cross section)
Or
$L\sigma_T/4\pi r^2 m_p c$ (σ_T is the Thompson cross section (6.6×10^{-25} cm^2) m_p is the mass of the proton)

The gravitational force on the proton is
GM_x/R^2

Equating the two gives the Eddington limit

$L_{Edd}=4\pi M_x G m_p c/\sigma_T = 1.3\times10^{38} M_{sun}$ erg/sec

Frank, King & Raine, "Accretion Power in Astrophysics",

Eddington Limit

- Assumes spherical geometry
- Assumes cross section is Thompson cross section.
- Other geometries ...

Accretion -Basic idea see Melia ch 7

- Viscosity/friction moves angular momentum outward
  - allowing matter to spiral inward
  - Accreting onto the compact object at center

gravitational potential energy is converted by *friction* to heat

Some fraction is radiated as light

Very efficient process Energy $\sim GM/R = 1.7\times10^{16}\,(R/10\mathrm{km})^{-1}$ J/kg

$\sim 1/2mc^2$

Nuclear burning releases $\sim 7\times10^{14}$J/kg

(0.4% of mc^2)

Gravitational potential of spherically symmetric mass M of radius R

$$\Phi = -\frac{GM}{r} \qquad (r > R)$$

Acceleration of gravity

$$\mathrm{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{r}$$

Particles freely falling from $r \rightarrow \infty$ to r:

$$E_{\mathrm{K}} = \frac{1}{2}v^2 \qquad \text{(kinetic energy per unit mass)}$$

Energy conservation: $E_{\mathrm{K}} + \Phi = E = \mathrm{cst.}$

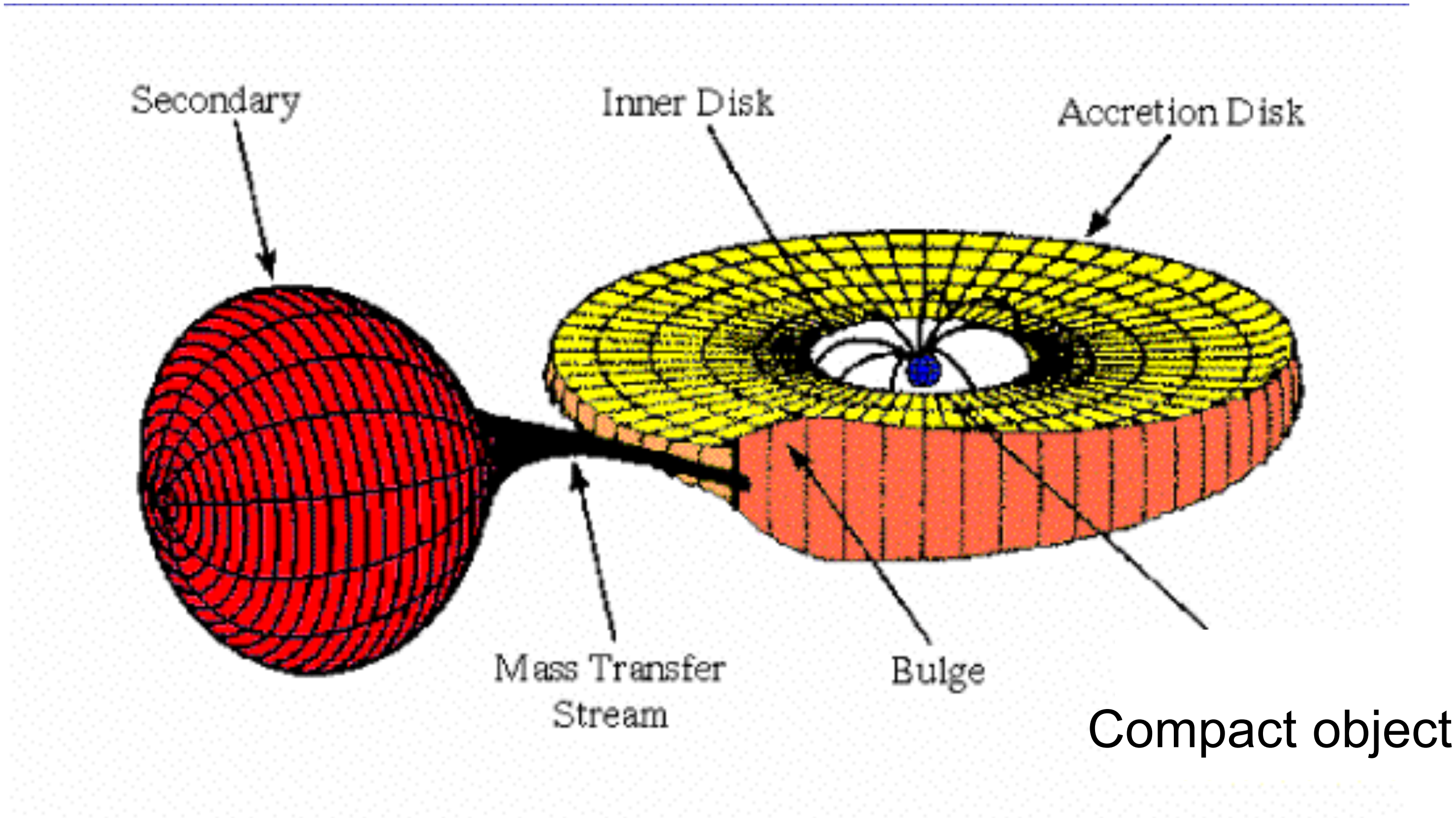
At r: $\quad v^2 = \dfrac{2GM}{r} \qquad$ (free-fall or escape speed)

Viral temperature T_{viral}=GM/kr; for a NS M~1.4M_{sun}, R~10 km

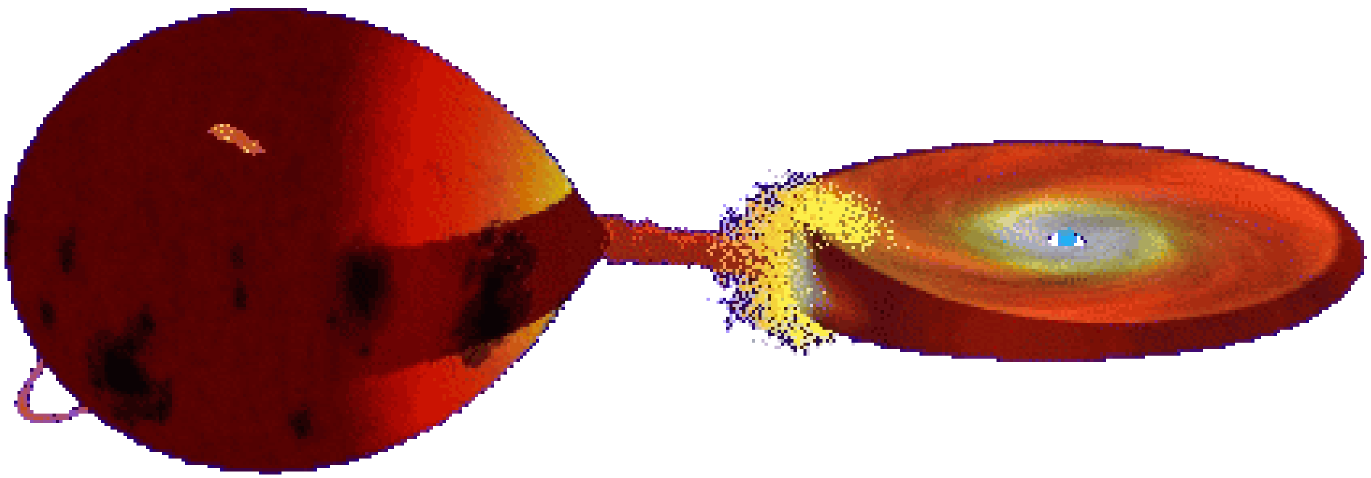
T~10^{12}k

(H. Spruit)

Basic Geometry

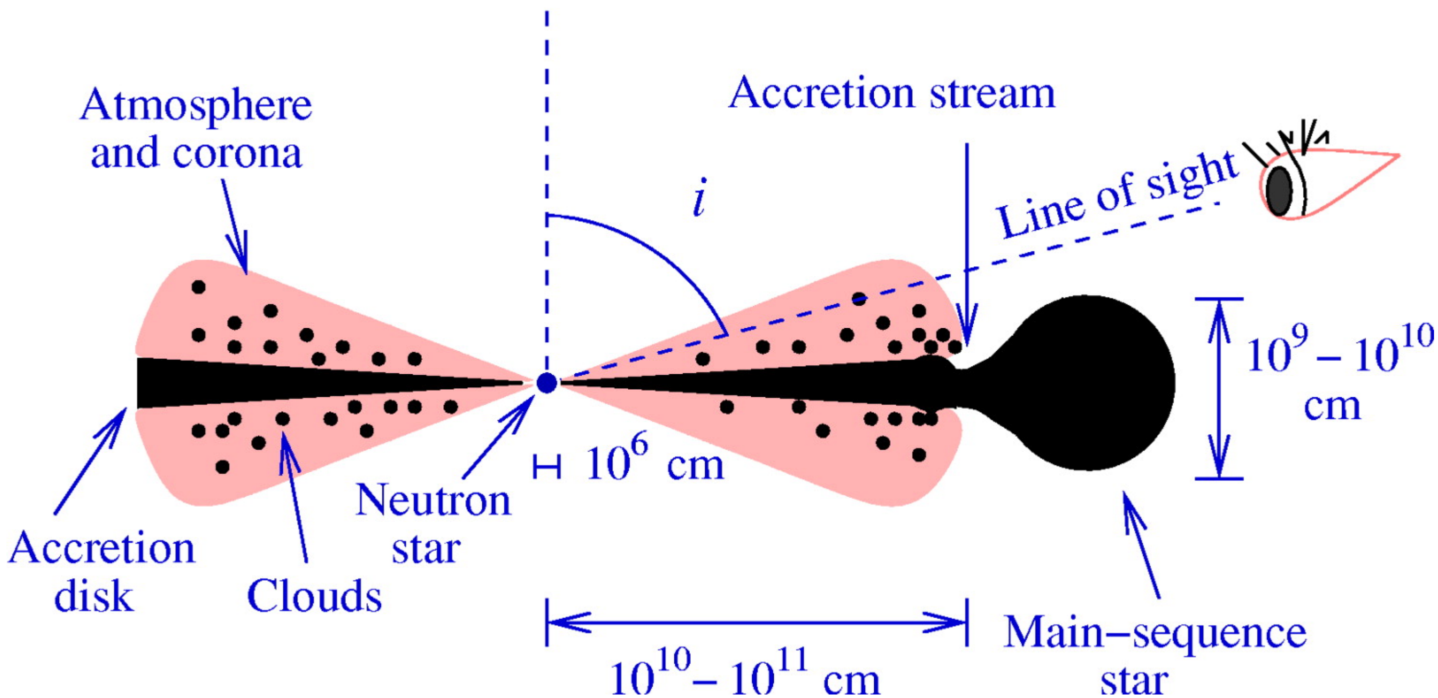


Accretion from a Dwarf Companion

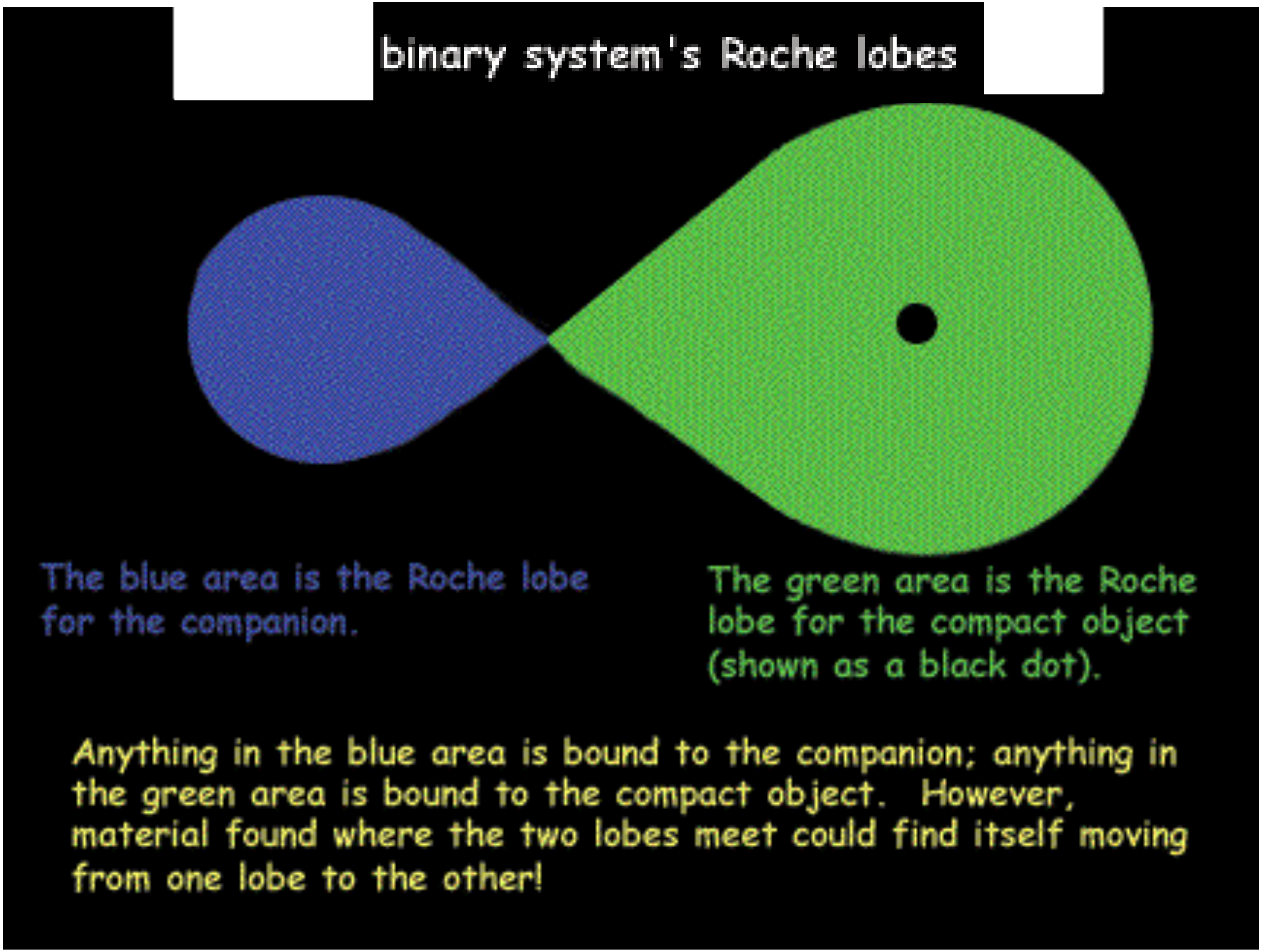


- http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif

Geometry of heated accretion disk + coronal in LMXB



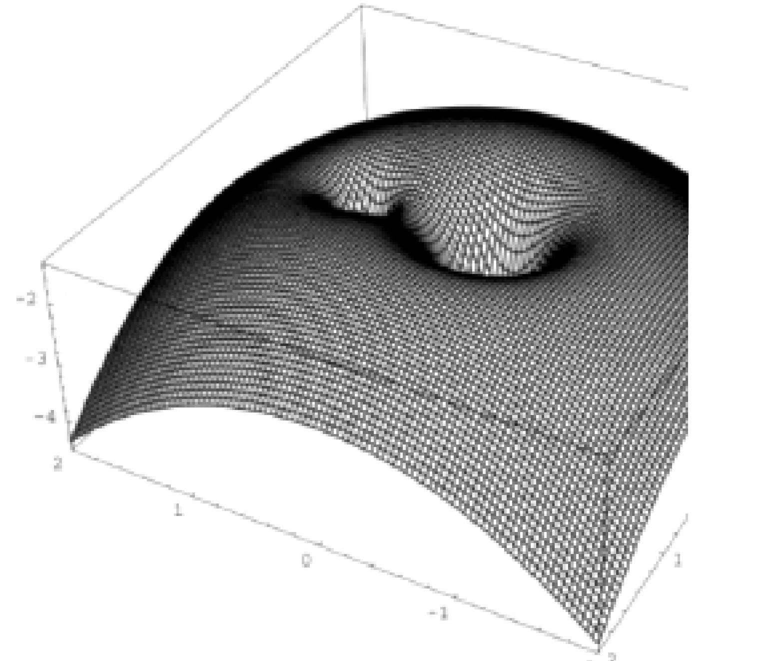
Jimenez-Garate et al. 2002


binary system's Roche lobes
The blue area is the Roche lobe for the companion.
The green area is the Roche lobe for the compact object (shown as a black dot).
Anything in the blue area is bound to the companion; anything in the green area is bound to the compact object. However, material found where the two lobes meet could find itself moving from one lobe to the other!

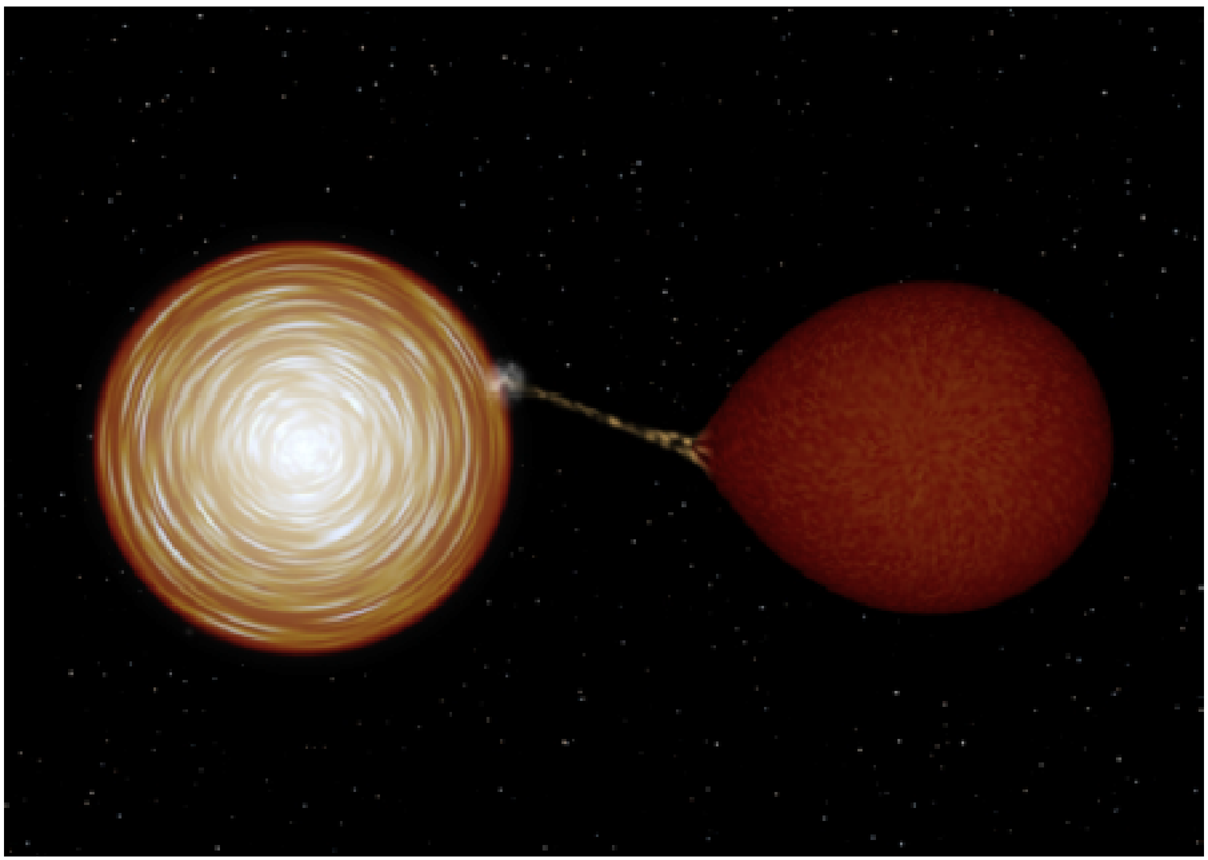
Roche Lobe Overflow Systems

Sample

- Almost all LMXBs
- Small fraction of HMXBs



From Frank et al., 2002, Accretion P
Astrophysics

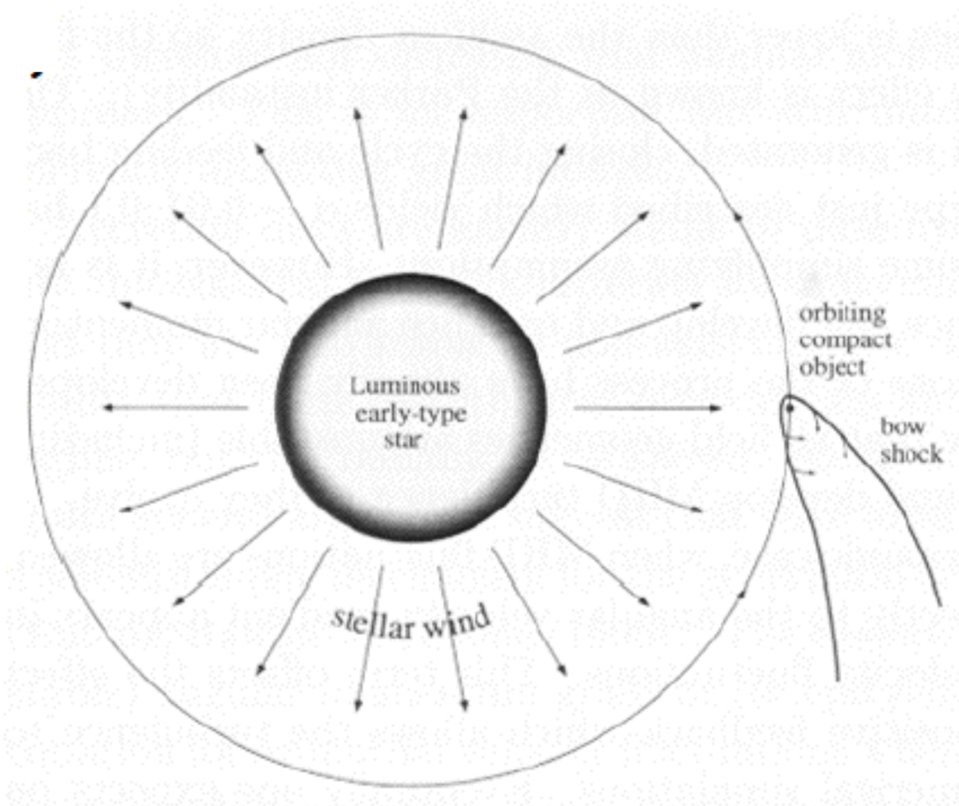


R. Hynes

Wind Fed Systems

High Mass X-ray Binaries

R. Hynes

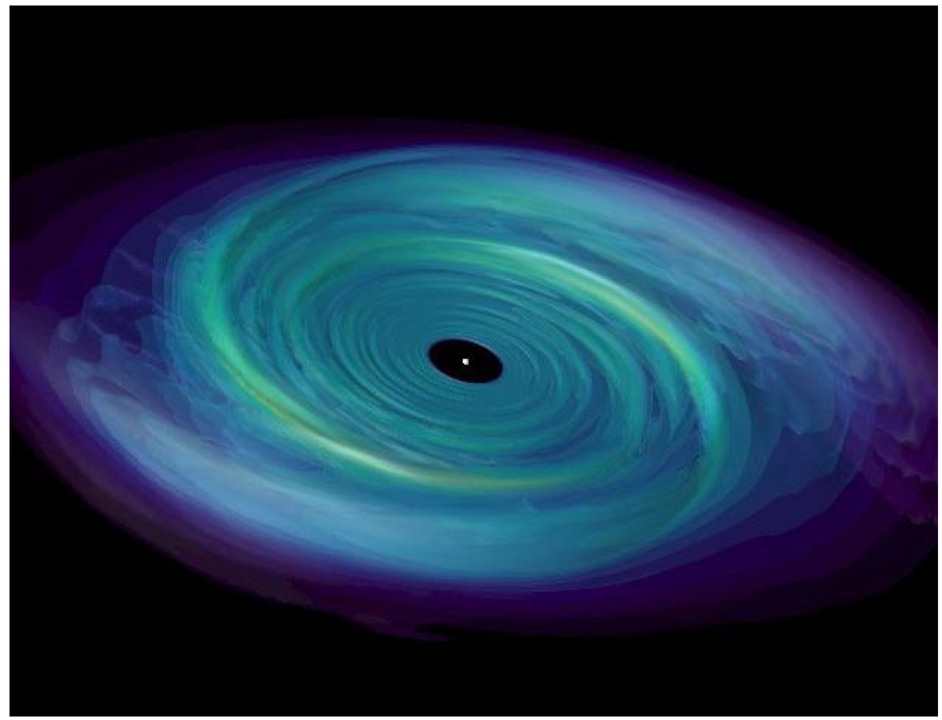


From Frank et al., 2002, Accretion Power in Astrophysics

Accretion disks

- Accretion disks form due to angular-momentum of incoming gas
- Once in circular orbit, specific angular momentum (i.e., per unit mass) is
- So, gas must shed its angular momentum for it to actually accrete…
- Releases gravitational potential energy in the process!
- Generic family of simplistic disks
- Axisymmetric disk from R_{in} to R_{out}
- Local spectrum assumed to be black body
-

$$J = vr = \sqrt{GMr}$$



How much energy is released by an accretion disk?

- Consider 1kg of matter in the accretion disk. Further, assume that…
  - The matter orbits in circular paths (will always be approximately true)
  - Centripetal acceleration is mainly due to gravity of central object (i.e., radial pressure forces are negligible… will be true if the disk is <u>thin</u>)

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

- Energy of 1 kg of matter in the accretion disk is..

$$E = \frac{1}{2}v^2 - \frac{GM}{r} = -\frac{GM}{2r}$$

- So, the total luminosity liberated by accreting a flow of matter is

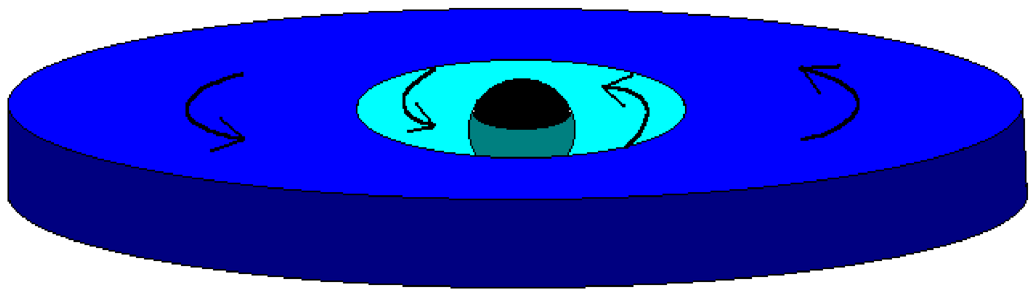
$$L = \left[0 - \left(-\frac{GM}{2r_{\rm in}}\right)\right]\dot{M} = \frac{GM\dot{M}}{2r_{\rm in}}$$

Initial energy (at infinity)

Final energy

Mass flow rate

•Total luminosity of disk depends on inner radius of dissipative part of accretion disk



- For a disk that extends down to the innermost stable orbit ISCO for a non-rotating black hole, simple Newtonian calculation gives…

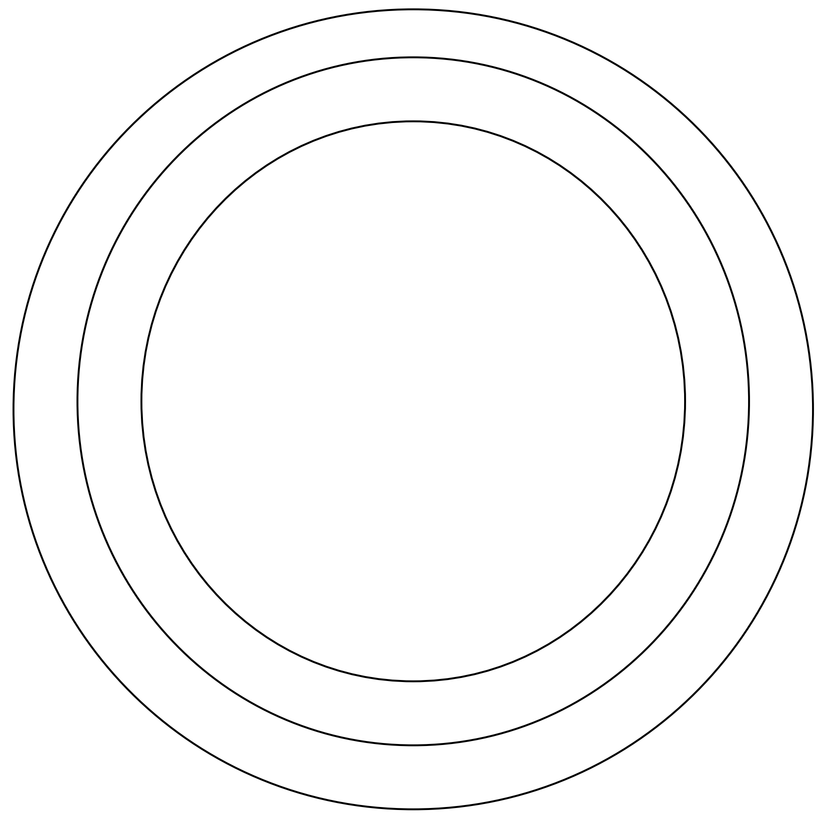
$$L = \frac{GM\dot{M}}{2(6GM/c^2)} = \frac{1}{12}\dot{M}c^2$$

- More detailed relativistic calculation gives…

$$L = \left(1 - \sqrt{\frac{8}{9}}\right)\dot{M}c^2 \approx 0.06\dot{M}c^2$$

- Consider two consecutive rings of the accretion disk.
- The torque exerted by the outer ring on the inner ring is

$$G(R) = 2\pi R h \nu \rho R^2 \frac{\partial \Omega}{\partial r}$$



$$\begin{aligned} h &= \text{thickness of disk at radius R} \\ \nu &= \text{coefficient of (kinematic) viscosity} \\ \rho &= \text{density of disk at radius R} \\ \Omega &= \text{angular velocity of disk orbit} \end{aligned}$$

- Viscous dissipation per unit area of disk surface is given by

$$D(R) = \frac{G}{4\pi R}\frac{\partial\Omega}{\partial R} = \frac{1}{2}\nu\Sigma\left(R\frac{\partial\Omega}{\partial R}\right)^2$$

- Evaluating for circular Newtonian orbits (i.e., “Keplerian” orbits),

$$D(R) = \frac{8}{9}\nu\Sigma\frac{GM}{R^3}$$

What gives rise to viscosity?

- Normal “molecular/atomic” viscosity fails to provide required angular momentum transport by many orders of magnitude! (Rossweg and Bruggen eq. 8.37-8.39)

- Source of <u>anomalous viscosity</u> was a major puzzle in accretion disk studies!
- Long suspected to be due to some kind of turbulence in the gas… then can guess that:

$$\nu = \alpha c_s h$$

- 20 years of accretion disk studies were based on this “alpha-prescription” (c_s= sound speed)…
- But what drives this turbulence? What are its properties?

The magnetorotational instability

- Major breakthrough in 1991… Steve Balbus and John Hawley (re)-discovered a powerful magneto-hydrodynamic (MHD) instability
  - Called magnetorotational instability (MRI)
  - MRI will be effective at driving turbulence
  - Turbulence transports angular momentum in just the right way needed for accretion
- Two satellites connected by a weak spring provide an excellent analogy for understanding the MRI.

THE ASTROPHYSICAL JOURNAL, 376:214–222, 1991 July 20

A POWERFUL LOCAL SHEAR INSTABILITY IN WEAKLY MAGNETIZED DISKS. I. LINEAR ANALYSIS

STEVEN A. BALBUS AND JOHN F. HAWLEY
Virginia Institute for Theoretical Astronomy, Department of Astronomy, University of Virginia, P.O. Box 3818, Charlottesville, VA 22903
Received 1990 November 1; accepted 1991 January 16

ABSTRACT

In this paper and a companion work, we show that a broad class of astrophysical accretion disk is dynamically unstable to axisymmetric disturbances in the presence of a weak magnetic field. Because of the ubiquity of magnetic fields, this result bears upon gaseous differentially rotating systems quite generally. This work presents a linear analysis of the instability. (The companion work presents the results of nonlinear numerical simulations.) The instability is local and extremely powerful. The maximal growth rate is of order the angular rotation velocity and is *independent* of the strength of the magnetic field, provided only that the energy density in the field is less than the thermal energy density. Unstable axisymmetric disturbances require the presence of a poloidal field component, and are indifferent to the presence of a toroidal component. The instability also requires that the angular velocity be decreasing outward. In the absence of a powerful dissipation process, there are no other requirements for instability. Fluid motions associated with the instability directly generate both poloidal and toroidal field components. We discuss the physical interpretation of the instability in detail. Conditions under which saturation occurs are suggested. The *nonemergence* of the classical Rayleigh criterion for shear instability in the limit of vanishing field strength is noted and explained. The instability is sensitive neither to disk boundary conditions nor to the constituative fluid properties. Its existence precludes the possibility of internal (noncompressive) wave propagation in a disk. If present in astrophysical disks, the instability, which has the character of an interchange, is very likely to lead to generic and efficient angular momentum transport, thereby resolving an outstanding theoretical puzzle.
Subject headings: accretion — hydrodynamics — hydromagnetics — instabilities

1. INTRODUCTION

A long-standing challenge to the theory of accretion disks has been to show from first principles a mechanism capable of generating a turbulent viscosity, since the angular momentum transport resulting from the action of ordinary molecular viscosity is extremely inefficient (Pringle 1981). In this work and a companion paper (Hawley & Balbus 1991, hereafter II), we show that accretion disks are subject to a very powerful shearing instability mediated by a *weak* magnetic field of any plausible astrophysical strength. We suggest that this instability is of some relevance to understanding the origin of turbulent viscosity in accretion disks.

It is of course widely appreciated that magnetic fields can play an important role in accretion disk dynamics (e.g., Blandford 1989). In their seminal paper, Shakura & Sunyaev (1973) noted that magnetic turbulance could act as a viscous couple, but argued that nonlinear perturbations would be required to disrupt laminar flow. Magnetic fields have also been invoked, for example, as a source

The First Physical Disk Model

- The first physical model of a disk was developed by Shakura and Sunyaev in 1973
- They made a set of reasonable assumptions which have proved to be reasonable.
- The disk is optically thick
- The local emission should consist of a sum of quasi–blackbody spectra

- What allows the accreting gas to lose its angular momentum?

- Suppose that there is some kind of “viscosity” in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material

- Process producing this “viscosity” might also be dissipative… could turn gravitational potential energy into heat (and eventually radiation)

A Simple Disk
C. Done IAC winter school

- The underlying physics of a Shakura-Sunyaev accretion disc (a very simple derivation just conserving energy -rather than the proper derivation which conserves energy and angular momentum).
- A mass accretion rate $\mathcal{M}$ spiraling inwards from R to R-dR liberates potential energy at a rate dE/dt = $L_{po}t = (GM\mathcal{M}/R^2)xdR$.
- The virial theorem* says that only half of this can be Kinetic energy and so radiated, so dLrad = $GM\mathcal{M}\, dR/(2R^2)$. (2T+U=0)
- If this thermalises to a blackbody then dL = (dA)xkT^4 where k is the Stephan-Boltzman constant and area of the annulus dA = 2 x 2πRxdR (where the factor 2 comes from the fact that there is a top and bottom face of the ring).
- Then the luminosity from the annulus $dL_{rad} = GM\mathcal{M}\, dR/(2R^2) = 4\pi dRkdRT^4$ or $kT^4(R) = (GM\mathcal{M}/8\pi R^3)$
- This is only out by a factor $3(1-(R_{in}/R)^{1/2})$ which comes from a full analysis including angular momentum
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

- Energy released by an element of mass in going from r+dr to r

Gravitational potential energy is

E_p =-GMm/2r so energy released is

E_g =-GMmdr/r^2.

the luminosity of this annulus, for an accretion rate $\mathcal{M}$, is

dL ~ GM$\mathcal{M}$ dr/r^2.

assuming the annulus radiates its energy as a blackbody?

For a

- blackbody, L = σAT^4. The area of the annulus is 2πrdr, and since
- L=M$\mathcal{M}$ dr/r^2 we have
- T^4 ~M$\mathcal{M}r^{-3}$, or
- T(r) ~(M$\mathcal{M}$/r^3)$^{1/4}$

See Rossweg and Bruggen 8.22-8.47 for more details

Full accretion disk spectrum

N. Brandt

exponential cutoff at high energies $E > kT_{inner}$

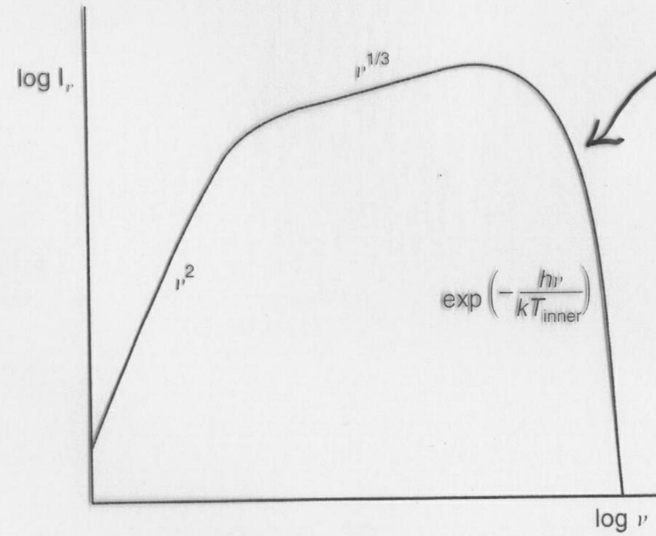


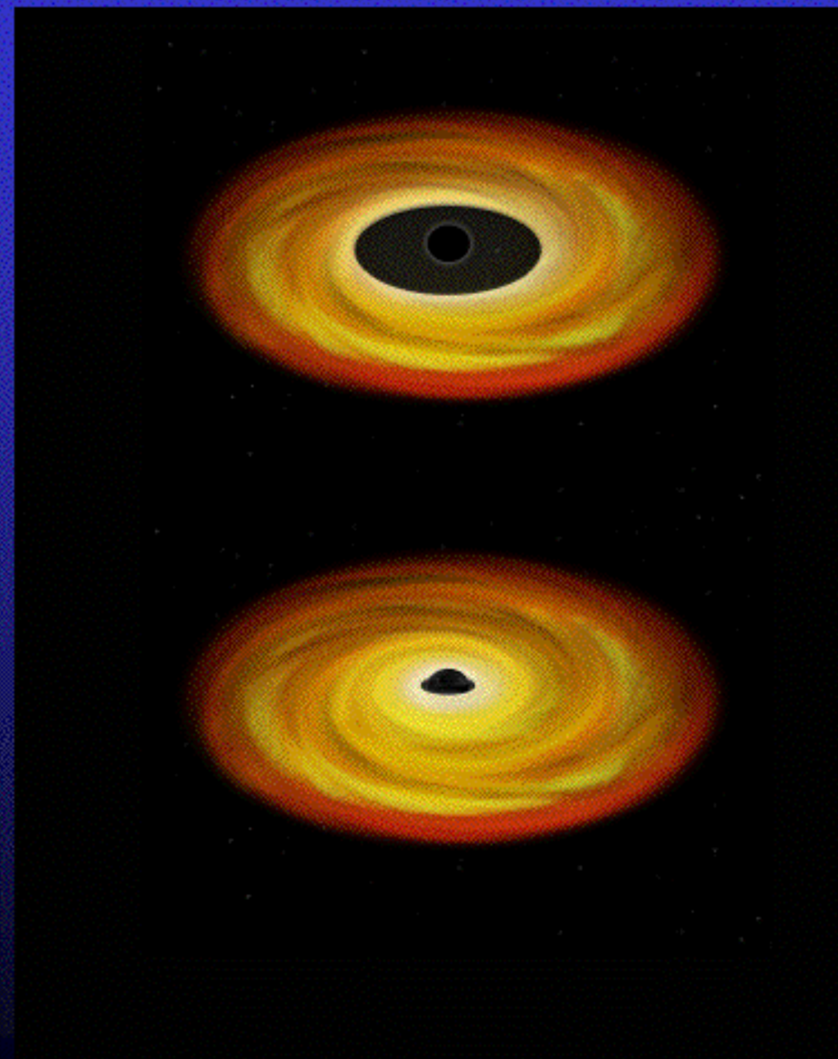
Figure 16.7. A schematic representation of the emission spectrum of an optically thick accretion disc. The exponential cut-off at high frequencies occurs at frequency $\nu = kT_{inner}/h$, where T_{inner} is the temperature of the innermost layers of the thin accretion disc.

$\nu^{1/3}$ dependence between freq. corresponding to r_{inner} and r_{outer}

Fig 7.4 Melia

Spectra of accretion flow: disc

- Differential Keplerian rotation
- Viscosity B: gravity → heat
- Thermal emission: $L = A\sigma T^4$
- Temperature increases inwards until minimum radius R_{IN}



Need to integrate a black body, I_ν over the disk $2h\nu^3/c^2(\exp(h\nu/kT(r))-1)$

$F(\nu)=\int I_\nu(R)RdR$

see Melia 7.2.2

skipping over a lot of algebra one finds that $F \sim \nu^{3/2-n}$ where n is the radial temperature distribution since $T \sim r^{-3/4}$ n =3/4 which gives $F \sim \nu^{1/3}$ with an exponential cutoff corresponding to the temperature at R_{IN}

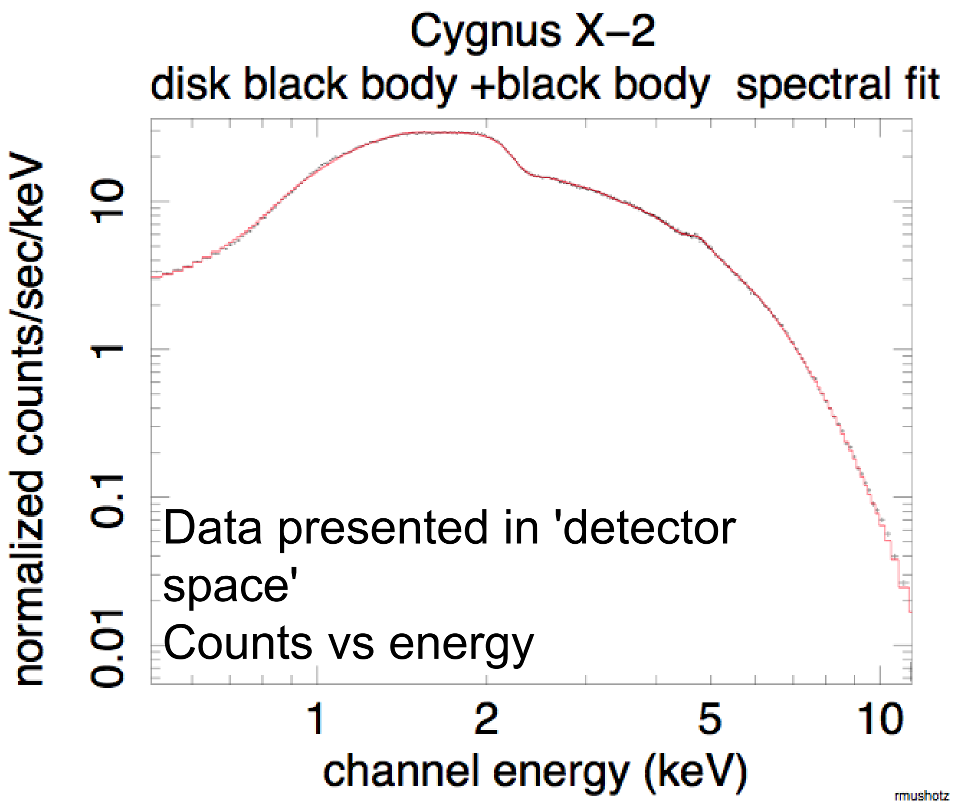
Total Spectrum

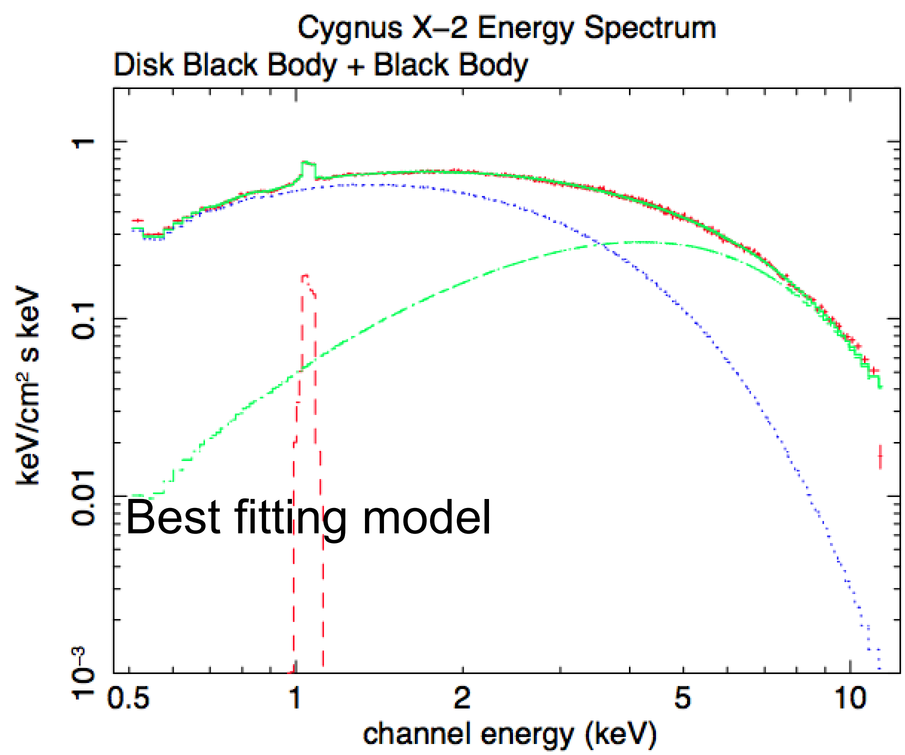
- If each annulus radiates like a black body the temperature scales as $T \sim r^{-3/4}$
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$
- At lower frequencies the spectrum has a Raleigh-Jeans ν^2 shape and at higher energies has a exponential cutoff corresponding to the maximum temperature ($e^{-h\nu/kT_{inner}}$)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

What sets the inner edge of the accretion disk?

- Accretion disk around a star…
  - Inner edge set by radius of star, R
  - Luminosity of disk is $GM\dot{M}/2R_*$
  - Additional $GM\dot{M}/2R_*$ liberated in boundary layer between disk and star
- Accretion disk around a black hole
  - Inner edge often set by the "innermost stable circular orbit" (ISCO)
  - GR effects make circular orbits within the ISCO unstable… matter rapidly spirals in
  - $R_{isco}=6GM/c^2$ for a non-rotating black hole

Fit to Real Data

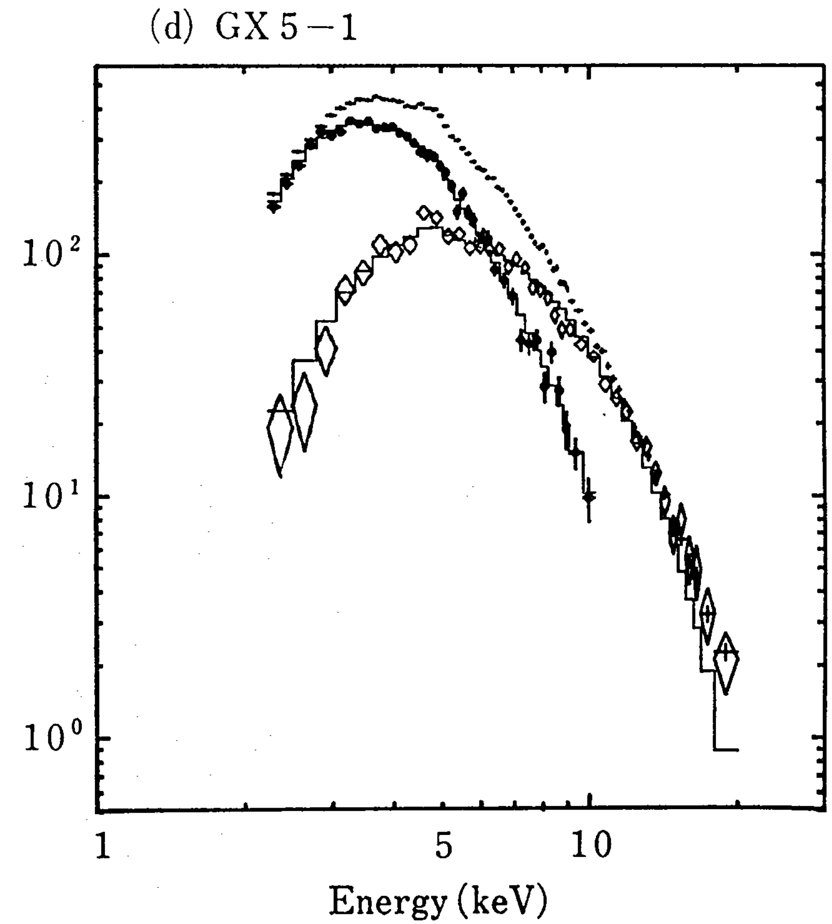




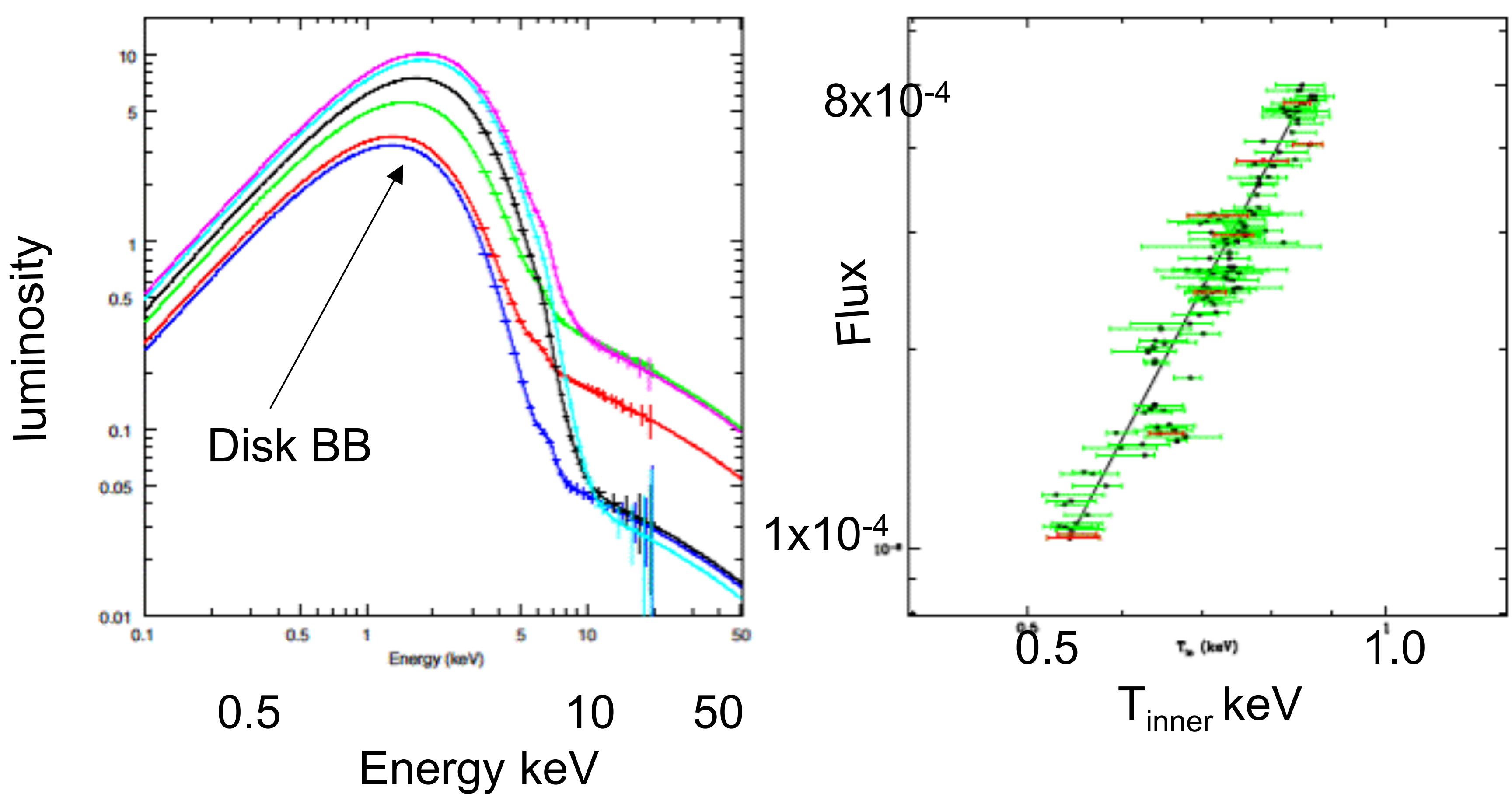
The data is of very high signal to noise
Simple spectral form fits well over a factor of 20 in energy
Emitted energy peaks over broad range from 2-6 kev

Actual Neutron Star X-ray Spectra

- Low Mass x-ray binaries (NS with a 'weak' magnetic field) have a 2 component spectrum
  - The low energy component is well described by a multi-color disk black body spectrum
  - And the hotter temperature black body is related to the boundary layer
    - However the observed temperatures disagree with simple theory due to three effects
      - General relativity
      - The 'non-black body' nature of the radiation
      - Reprocessing of the radiation of the central regions by the outer regions and then re-emission



Do They Really Look Like That



- X-ray spectrum of accreting Neutron star at various intensity levels
- Right panel is $T(r_{in})$ vs flux - follows the T^4 law

X-ray Pulsars

- The rate of change of the pulse period can
  - measure the orbital period of the source
  - The accreted angular momentum (e.g. the amount of material accreted)
  - $(dP/dt)/P \sim 3x10^{--5f}(/1sec)(L/10_{37})^{6/7}$ yr^{-1} (Ghosh and Lamb 1978)

The Known Galactic Black holes

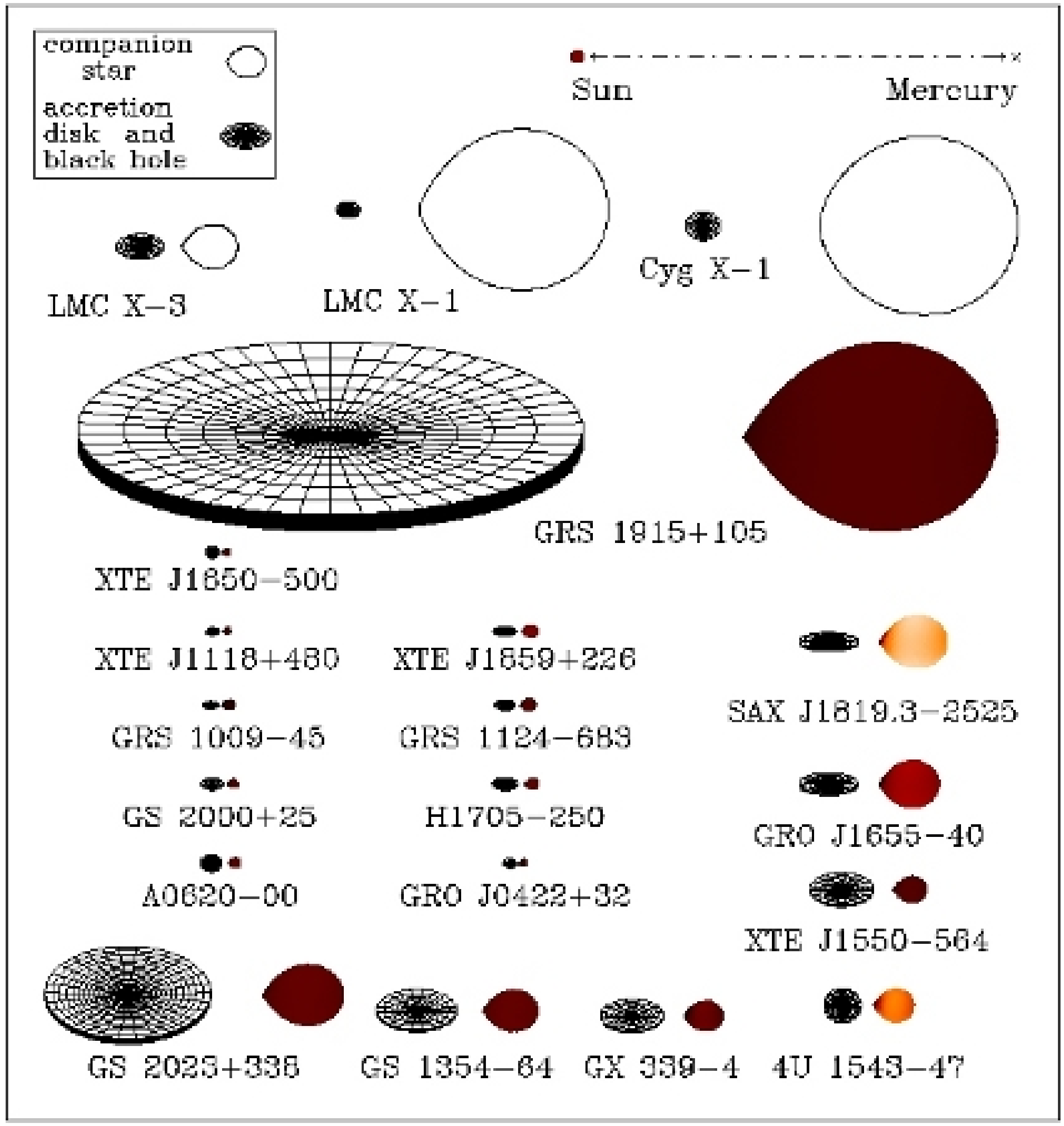


Figure by Jerome Arthur Orosz)