General Introduction to Galaxy Clusters: Scaling Relations

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With thanks to Helen Russell

Astronomical Units & Terminology

$$r_{2500} = 500\,kpc\left(\frac{T}{5\,keV}\right)^{\frac{1}{2}}$$

$$M_{2500} = 2x10^{14}\,M_{\oplus}\left(\frac{T}{5\,keV}\right)^{\frac{1}{2}}$$

scaled to critical density of Universe

Kiloparsec	Distance	1000 parsecs = $3x10^{21}$ cm
Light year	Distance	3.26 parsecs

Milky Way Diameter = 100,000 ly = 30 kpc

10^{53} erg	Energy	supernova explosion/GRB
keV	Temperature	1 keV = 1.16×10^{7} K
dynes/cm^2	Pressure	cluster center $P=10^{-10}$ dyne/cm^2
		10^{-16} Atm

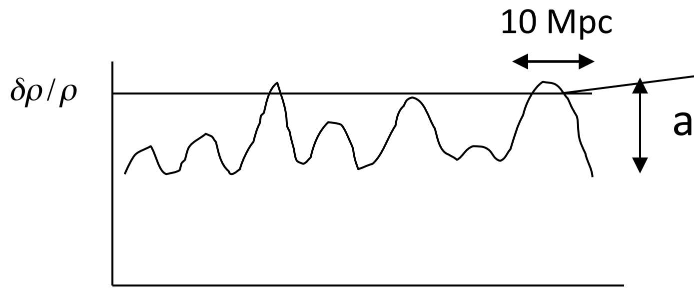
Why Study Galaxy Clusters?

- Cosmology
  - evolve by gravity: largest structures evolve slowly--dominated by dark matter, sensitive to matter and energy density of the Universe
  - fair sample hypothesis: baryons, dark matter, baryon fraction
  - observational goal: measuring masses; observational proxies for mass (L_x, T, etc.)

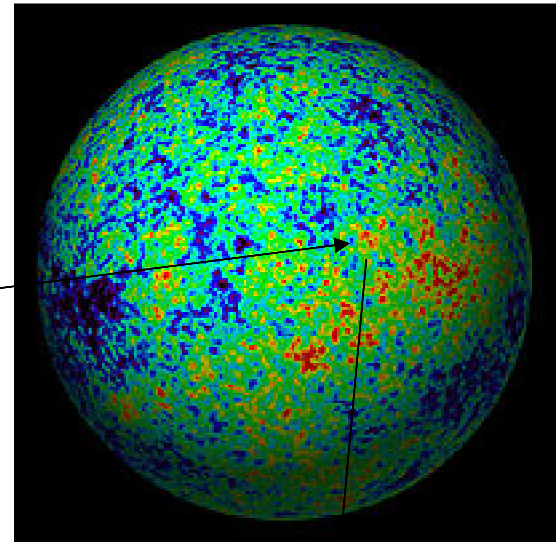
- Galaxy formation & Evolution
  - closed boxes: retain byproducts of stellar evolution; history of star formation
  - galaxies at same redshift: uniform sampling in space & time
  - merging & perturbation: dynamical evolution of galaxies

- Cosmic cooling problem
  - only few percent of baryons have condensed in clusters
  - cooling flows/feedback

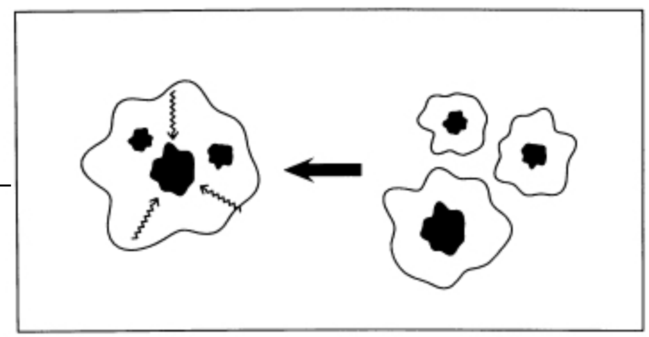
Gaussian density perturbations

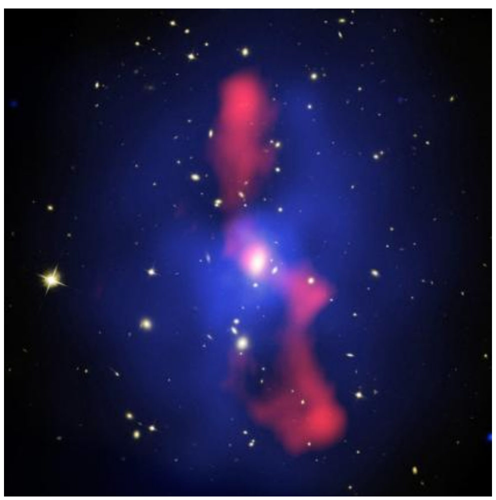


$N_{cl} \propto a$

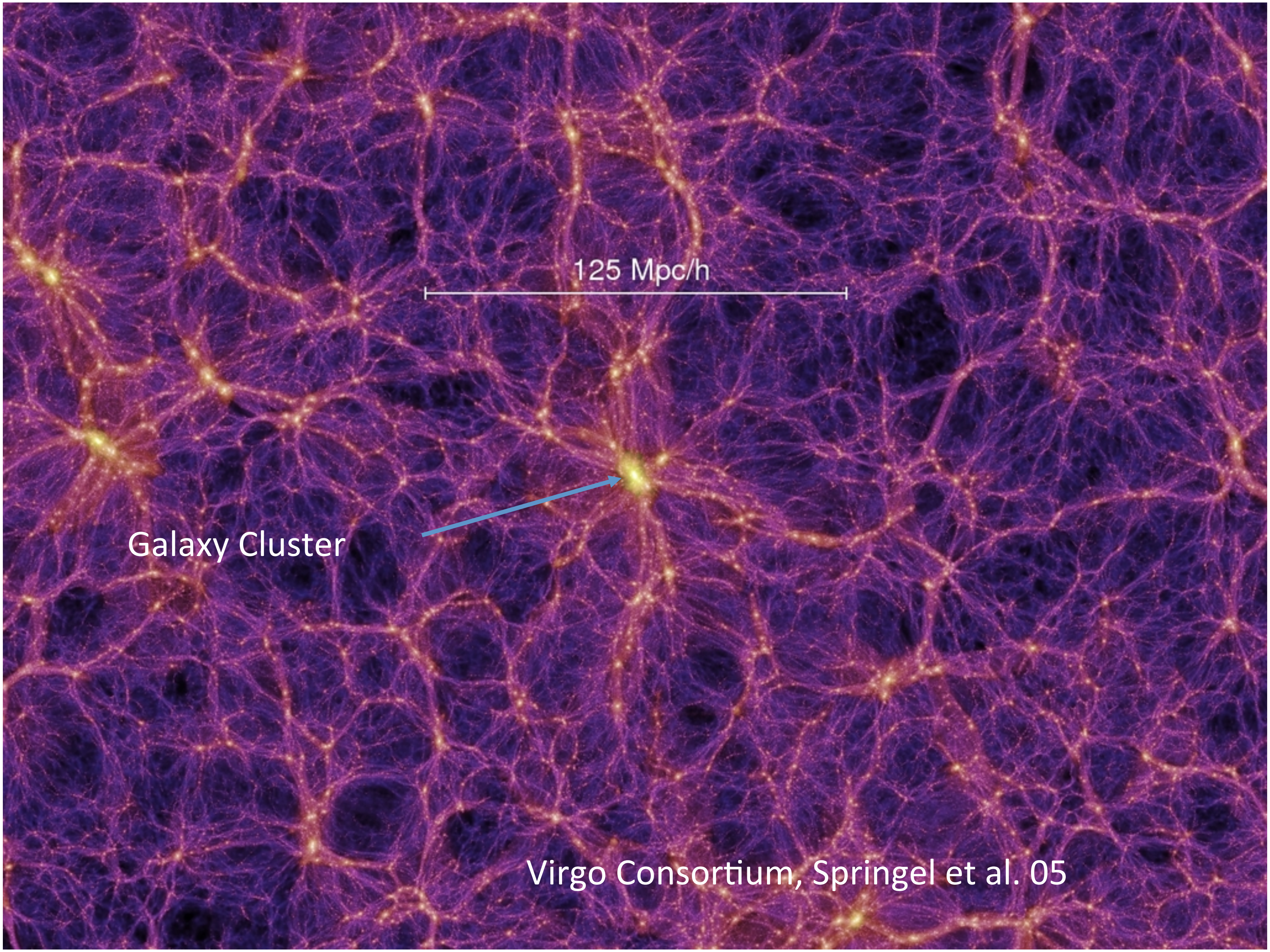


Hierarchical clustering



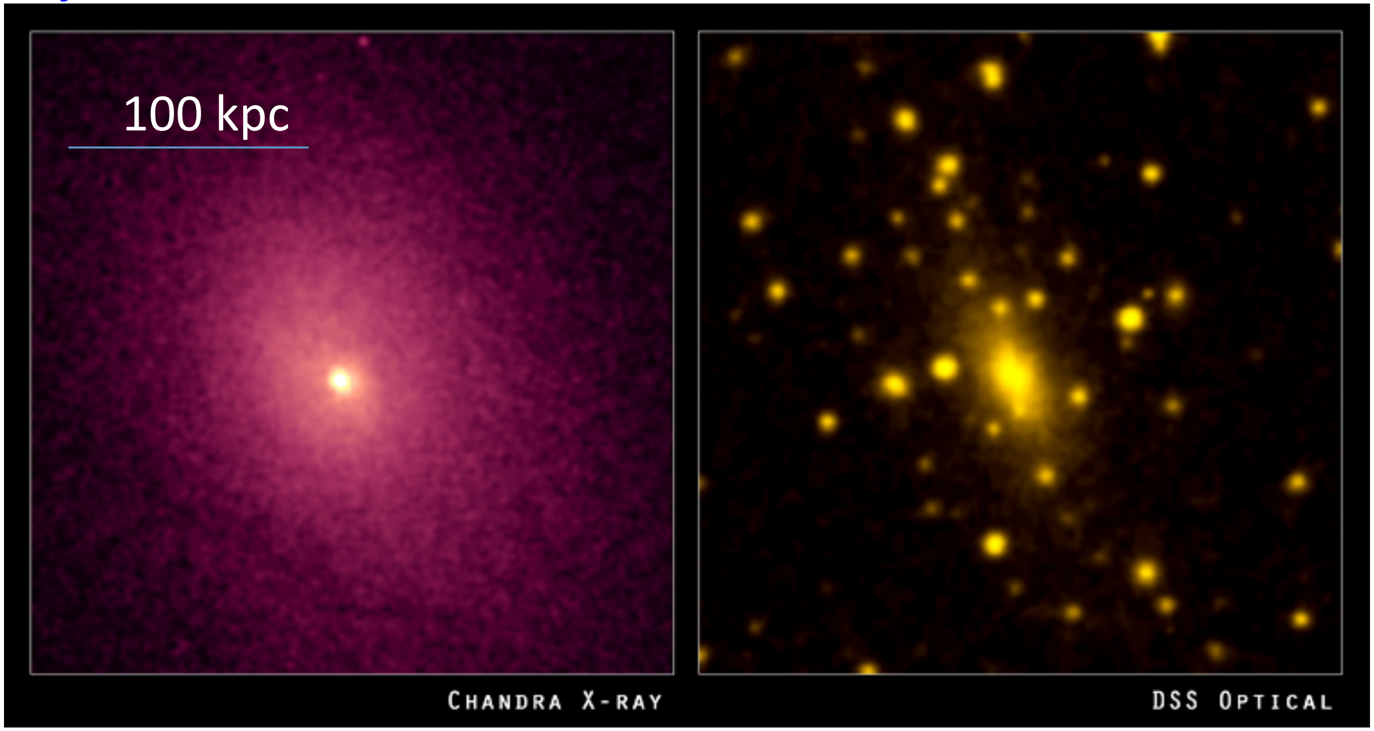


Number density of clusters: amplitude of perturbations, *a*
Degree of clustering: Ω_m


125 Mpc/h
Galaxy Cluster
Virgo Consortium, Springel et al. 05

Galaxy Cluster at X-ray & Visual wavelengths

X-ray visual



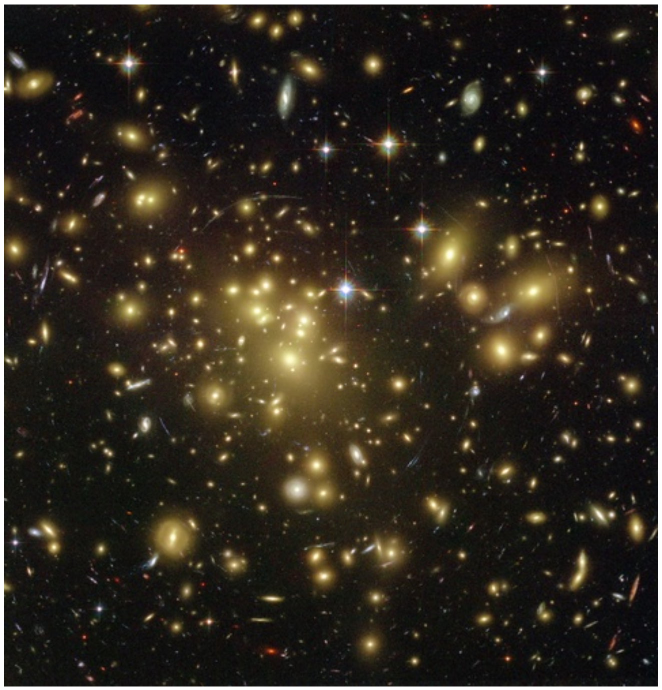
L_x = 10^{43} - 10^{45} erg s^{-1} $kT_{gas} \approx \mu m_p \sigma^2$ T_{gas} = 10^7 - 10^8 K

$\rho_{gas} \approx 10^{-1} - 10^{-4} cm^{-3}$ Bremsstrahlung emissivity: $\varepsilon \propto n_e n_i \sqrt{T} e^{-h\nu/kT}$

X-ray emission ~n_e^2 unambiguous tracer of deep potential well ie., clusters

What is a galaxy Cluster?

Abell 1689 RC =4



<u>Richness Classes</u>

$M_3 + 2$

0	30-49
1	50-79
2	80-129
3	130-199
4	200-299
5	300>

Sizes 1-3 Mpc

$$\sigma \approx 300 - 1000\,km\,s^{-1}$$

$$M_{cl} \approx 10^{13} - 10^{15}\,M_{\oplus}$$

$$M_{vir} \approx 3\sigma^2 r/G \approx 10^{15}\,h^{-1}M_{\Theta}$$

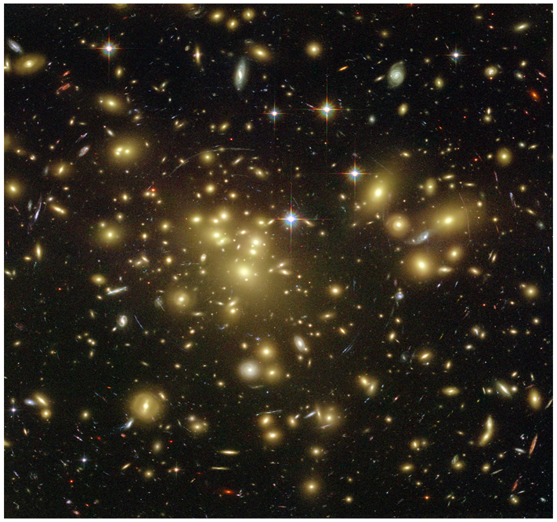
for $\sigma \approx 1000\,km\,s^{-1}$

Gravitationally bound systems

- Crossing time for the galaxies:

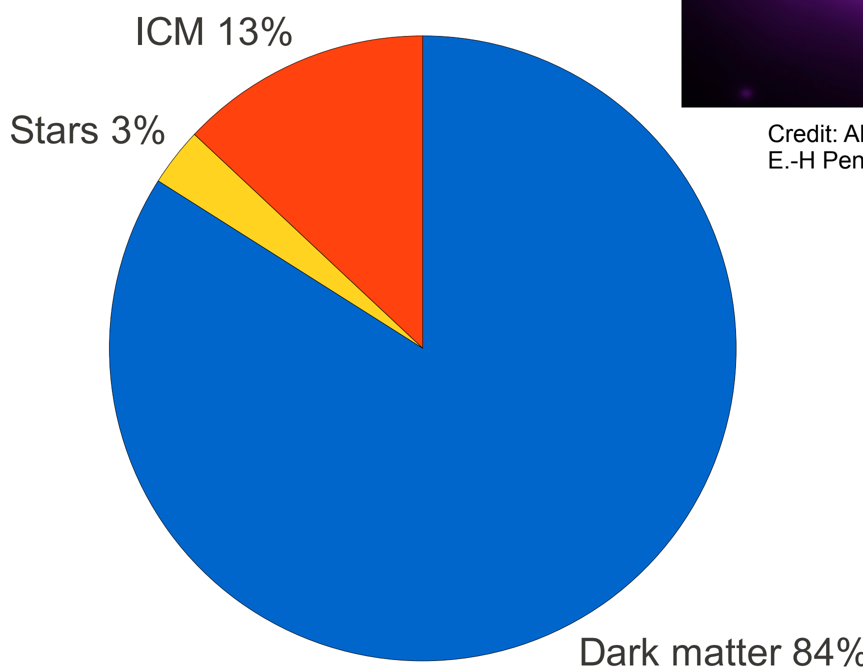
$$t_{cr,gal} = \frac{R}{\langle v^2 \rangle^{1/2}}$$

$$t_{cr,gal} \approx 2\times 10^9\ yr$$



→ clusters are gravitationally bound

Galaxy cluster mass partitioning



Credit: Abell 1689, X-ray: NASA/CXC/MIT/
E.-H Peng et al.; Optical: NASA/STScI

?

Virial Theorem

- For gravitationally bound systems:

$$2\mathrm{E}_K + E_P = 0$$

$$M_{tot} = \frac{\langle v^2 \rangle R_{tot}}{G}$$

- Assuming random uncorrelated velocity vectors:

$$M_{tot} = \frac{3\sigma_r^2 R_{tot}}{G}$$

- For velocities 1000km/s and R = 1Mpc => M_{tot} = 10^{15}Msun

Sound crossing time

- Sound crossing time for X-ray gas:

$$t_{cr,gas} = \frac{D}{c_s}$$

- For an ideal gas the adiabatic sound speed is:

$$c_s^2 = \frac{dP}{d\rho} = \gamma \frac{P}{\rho} = \gamma \frac{k_B T}{\mu m_H}$$

- For a monatomic gas the adiabatic index is 5/3.

$$t_{cr,gas} \approx few \times 10^8 \, yr$$

- Much shorter than age of cluster ~10^{10}yrs

Hydrostatic Equilibrium

- Hydrostatic equilibrium: compression due to gravity is balanced by a pressure gradient force.

$$\nabla P = -\rho \nabla \Phi \ , \qquad \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)}{r^2} \ ,$$

- Assumptions:
  - Sound crossing time much less than the age of the cluster
  - Timescale for elastic collisions much shorter than timescale for heating or cooling

$$t_s \equiv \frac{D}{c_s} \approx 6.6 \times 10^8 \,\mathrm{yr} \left(\frac{T}{10^8\,\mathrm{K}}\right)^{-1/2} \left(\frac{D}{1\,\mathrm{Mpc}}\right)$$

Kinetic Equilibrium

Timescales for electrons and ions to form a Maxwellian distribution electrons via Coulomb collisions:

$$t_{\rm eq}(e,e) \approx 3\times10^5\,{\rm yr}\left(\frac{T}{10^8\,{\rm K}}\right)^{3/2}\left(\frac{n_e}{10^{-3}\,{\rm cm}^{-3}}\right)^{-1}\,{\rm yr}$$

$$t_{eq}(p,p) \approx (m_p/m_e)^{1/2}\, t_{eq}(e,e) \sim 43 t_{eq}(e,e)$$

$$t_{eq}(p,e) \approx (m_p/m_e)\; t_{eq}(e,e) \sim 1870 t_{eq}(e,e)$$

For typical conditions

$$t_{\rm eq}(e,e) \sim 10^5\ {\rm yr},\ t_{\rm eq}(p,p) \sim 4\times10^6\ {\rm yr},\ {\rm and}\ t_{\rm eq}(p,e) \sim 2\times10^8\ {\rm yr}$$

Electrons & ions in equipartition, such that $T = T_e = T_p$

Detecting Clusters with X-rays

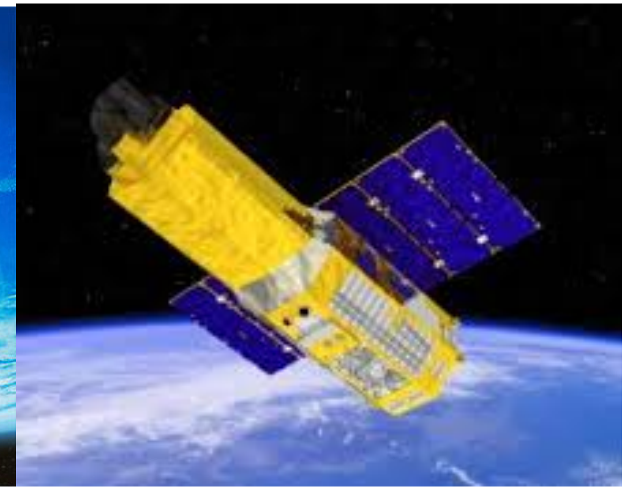
Chandra

E = 0.5-10 keV
EA = 600 cm^2 @1.5 keV
FOV = 16.9x16.9 arcmin
PSF = 0.5 arcsec FWHM

XMM-Newton

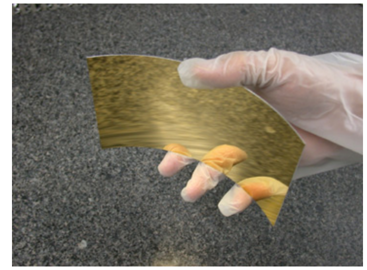
E = 0.1-15 keV
EA = 922 cm^2 @ 1 keV
FOV = 33 x 33 arcmin
PSF ~ 4 arcsec FWHM

Suzaku



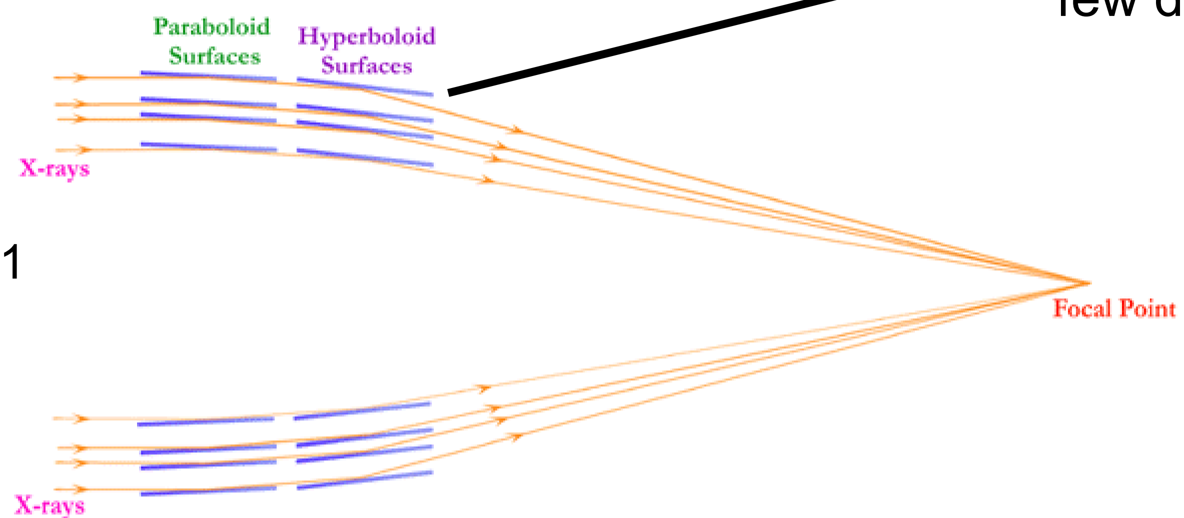
E = 0.4-600 keV
EA = 900 cm^2 @ 1.5 keV
FOV = 20 arcmin @1keV
PSF ~ 2 arcmin (HPD)

Chandra's mirrors



Grazing incidence optics

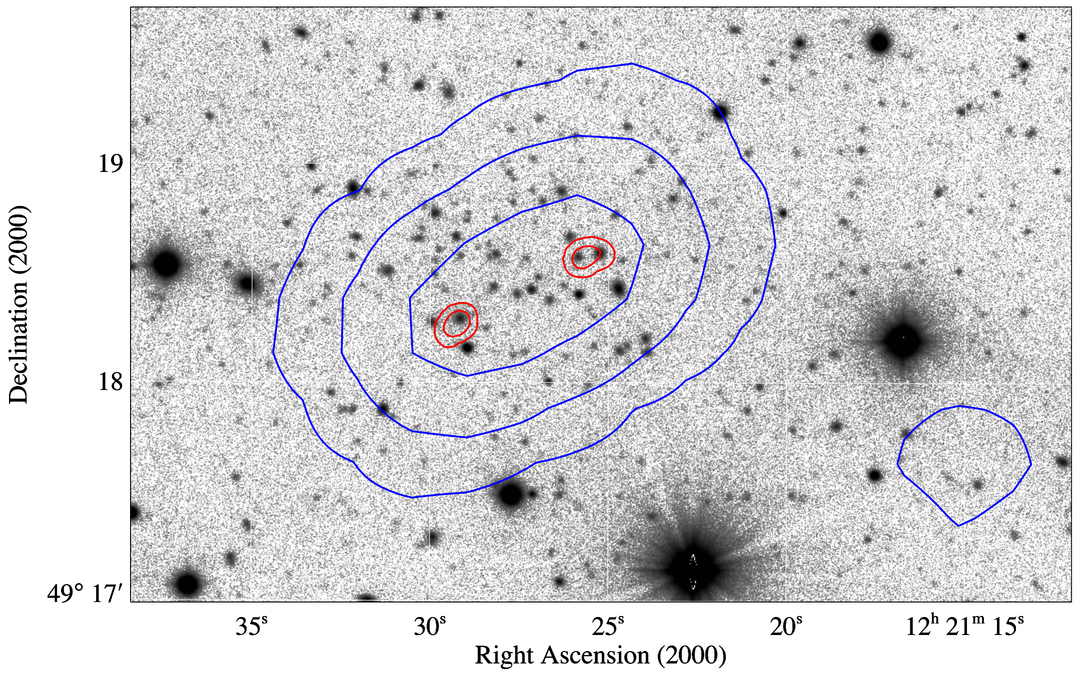
critical angle
few degrees



Wolter Type 1

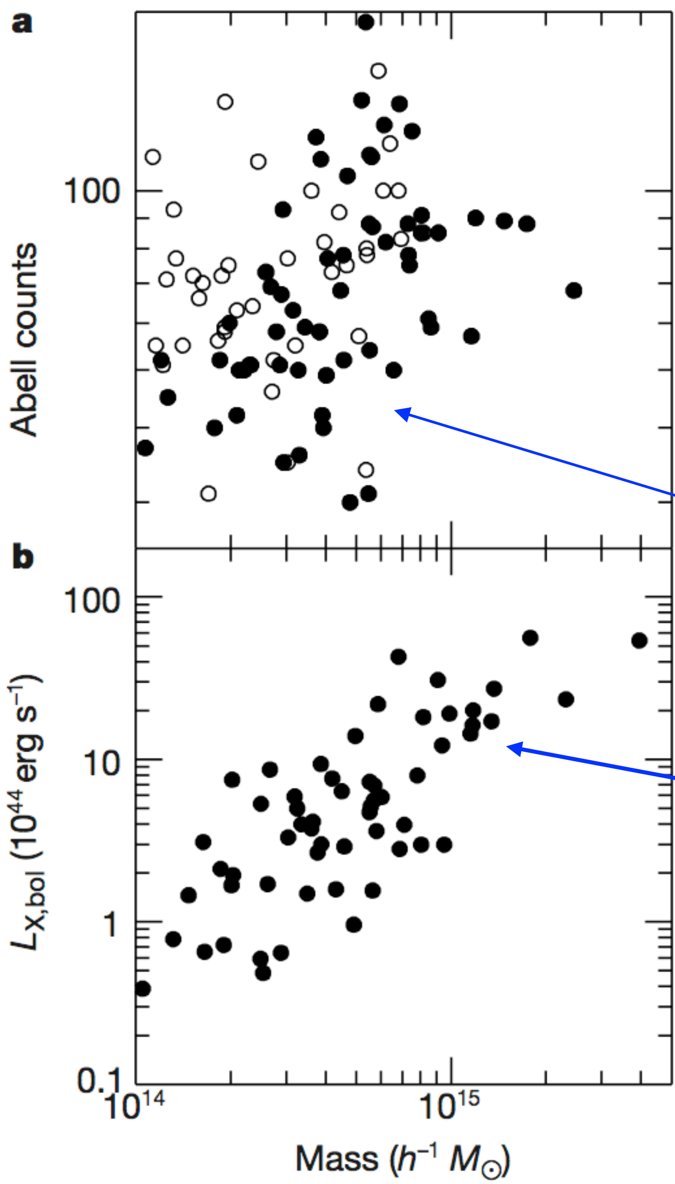
Hundreds of Clusters Detected with *ROSAT*

NRAO-VLA Sky Survey (NVSS)
ROSAT X-ray Imaging

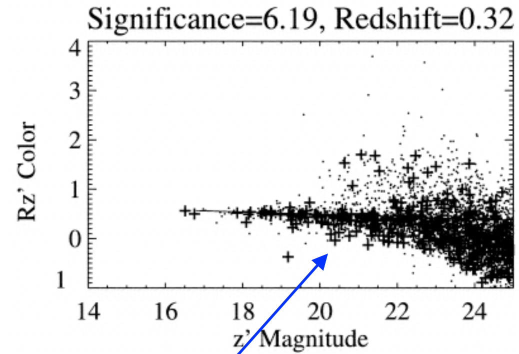


J1221+4918
z = 0.7
Lx = 1.2×10^{45} erg s^{-1}
kT = 6.5 keV

Ma + 11



Borgani & Guzzo 01



"red sequence"

Gladders & Yee 05

Optical: good for finding distant clusters
poor measure of cluster properties

X-ray: good measure of cluster properties
poor tracer beyond z ~ 1

Thermal Bremsstrahlung

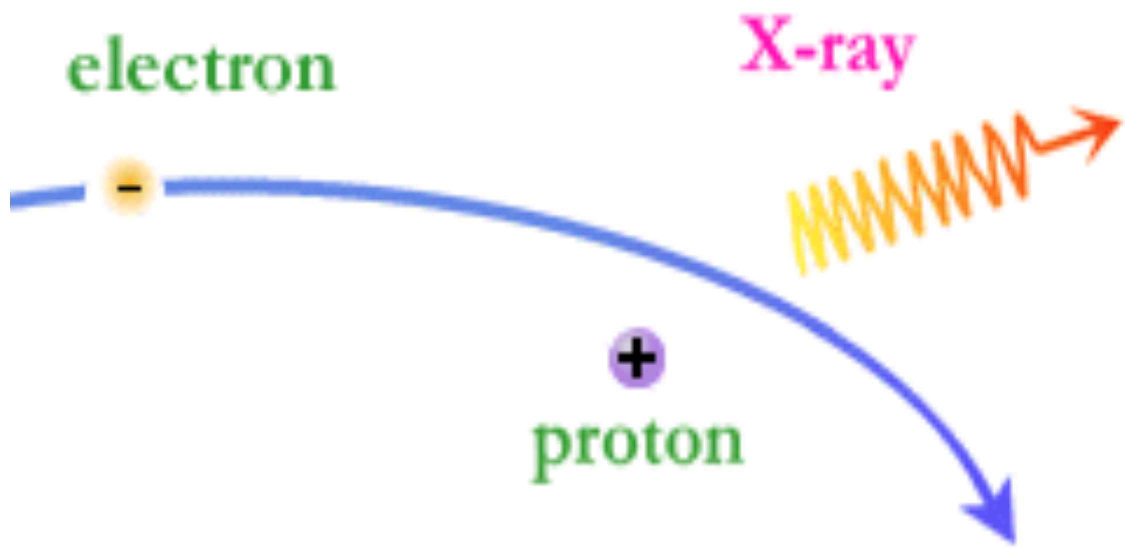
Primary source of X-ray emission from clusters due to diffuse intracluster plasma at 10^8K

Power radiated by accelerating charge:

Larmor's equation

$$P = \frac{2e^2a^2}{3c^3}$$

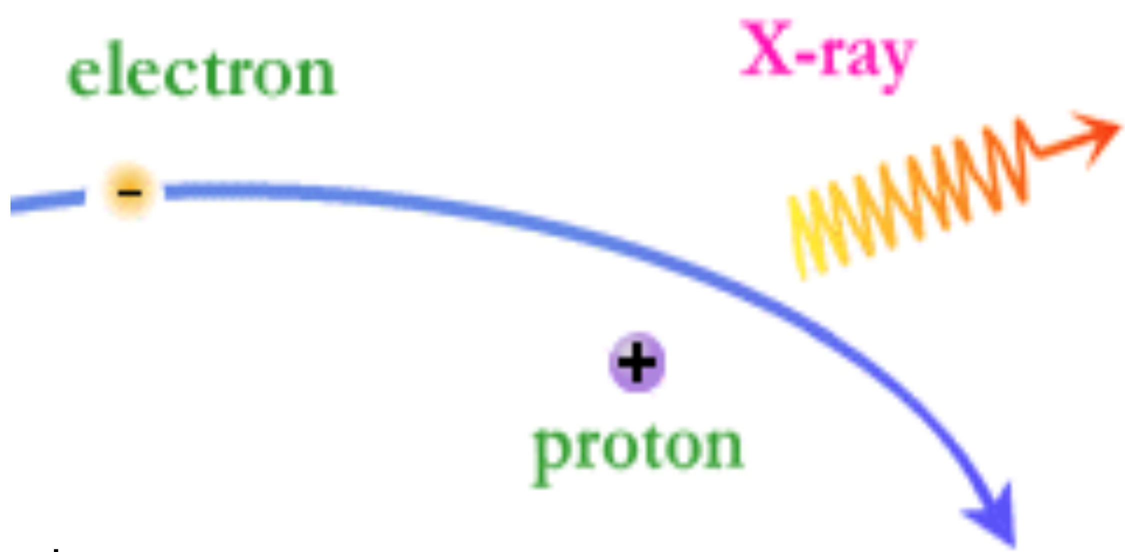
a = acceleration



Thermal Bremsstrahlung

- Power per unit frequency and per unit volume:

$$\varepsilon^{ff}(\nu,i) = \frac{2^5 \pi e^6}{3 m_e c^3}\left(\frac{2\pi}{3 k_B m_e}\right)^{1/2} T^{-1/2} Z_i^2 g_{ff}(Z_i,T,\nu) n_e n_i e^{-h\nu/kT}$$



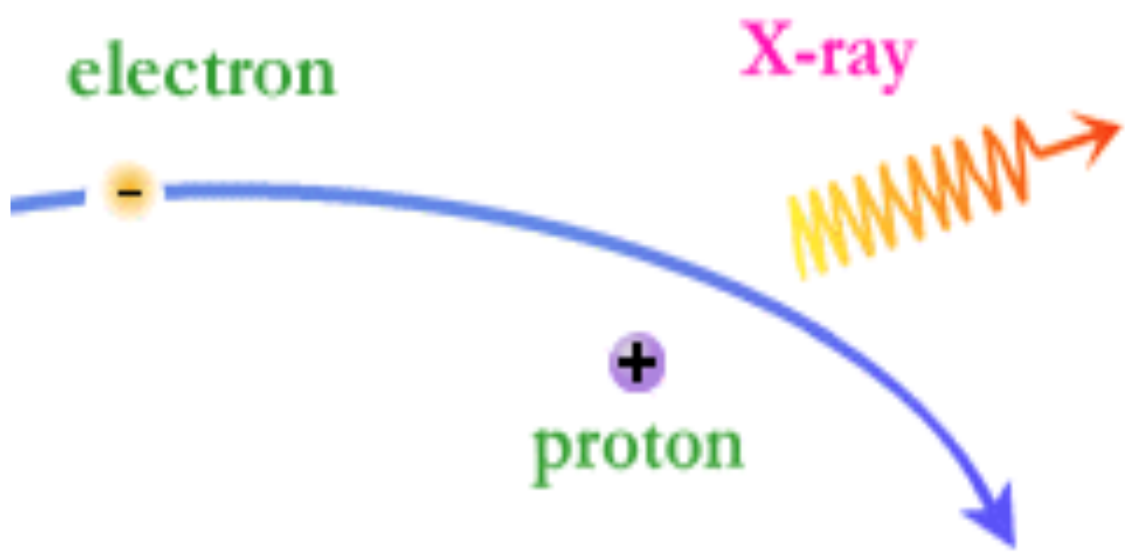
See Rybicki + Lightman or Longair

Thermal Bremsstrahlung Total Power

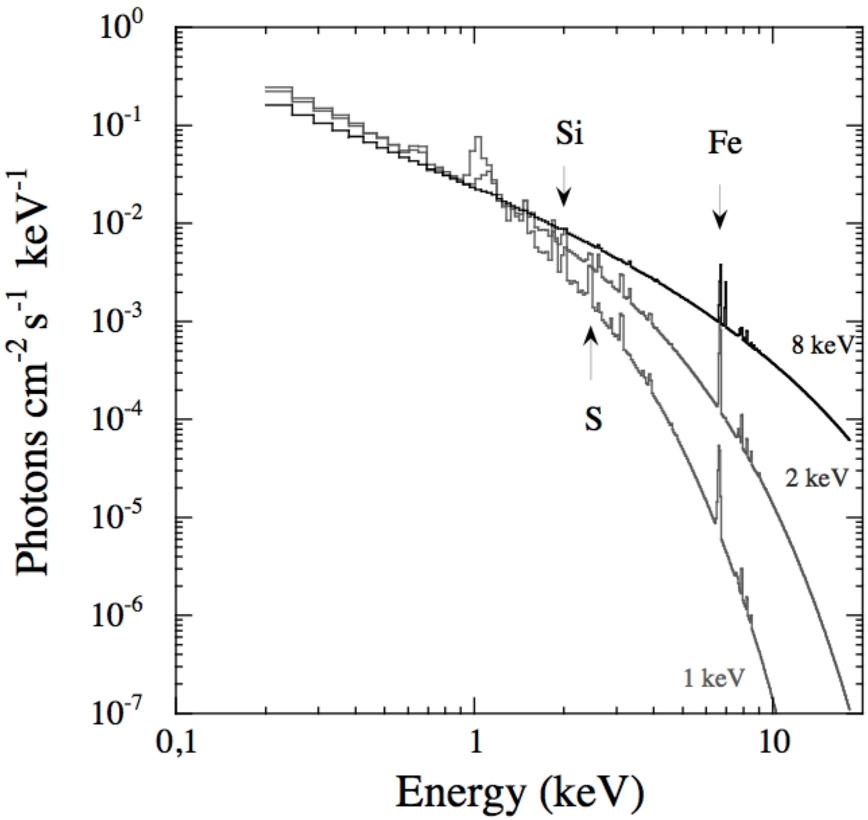
- Radiated power per unit volume:

$$L \propto n_e n_H T^{1/2}$$

- Two body collision → power strongly dependent on density^2

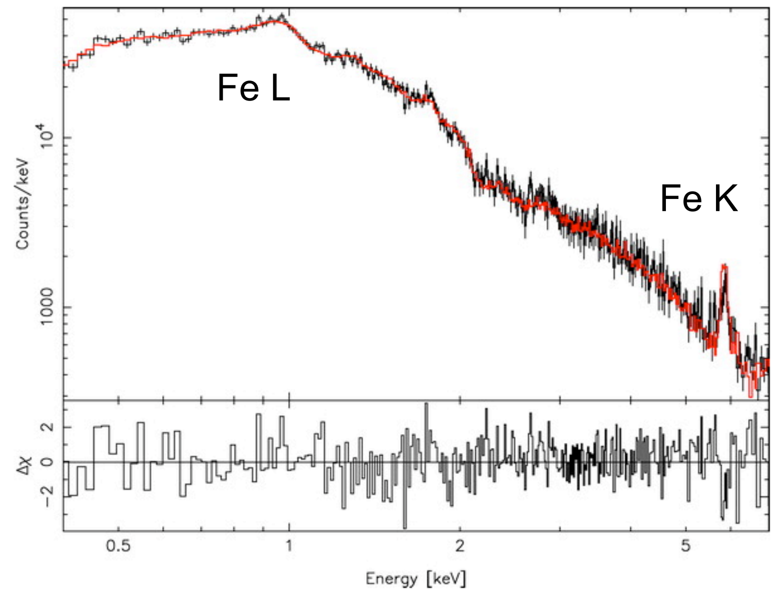


emitted spectrum



$$\varepsilon \propto g(E,T,Z)T^{-1/2}\exp(-E/kT)$$

observed spectrum



$$s \propto \varepsilon(E,T,Z) \otimes R(E,\phi)$$

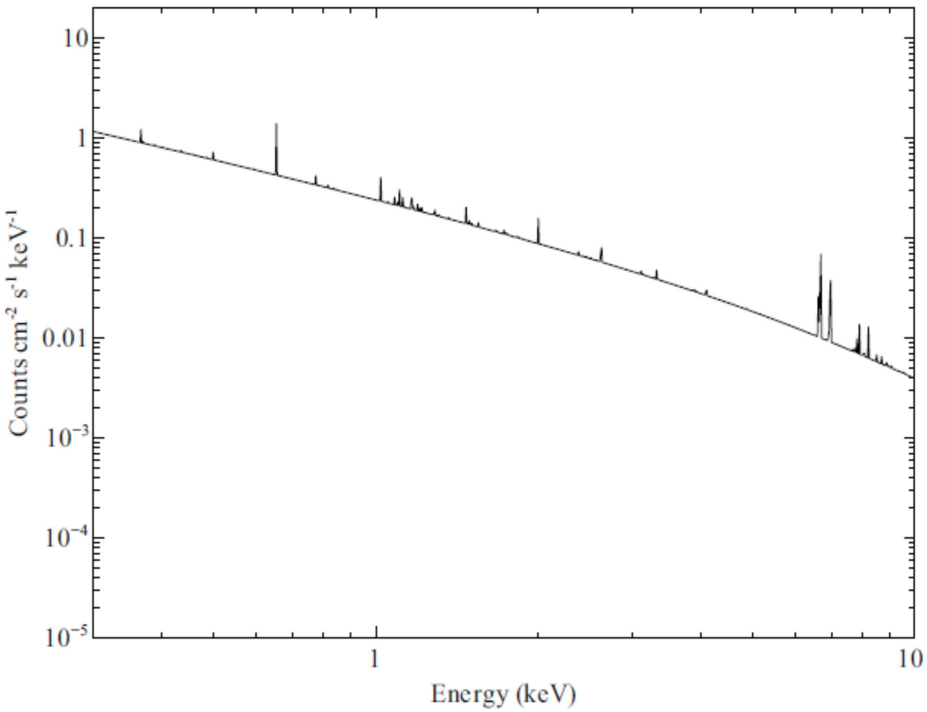
- ionization equilibrium
- temperature sensitivity
- metallicity sensitivity
- Z ~ 1/3 solar

Normalization: $EM \equiv \int n_p n_e dV$

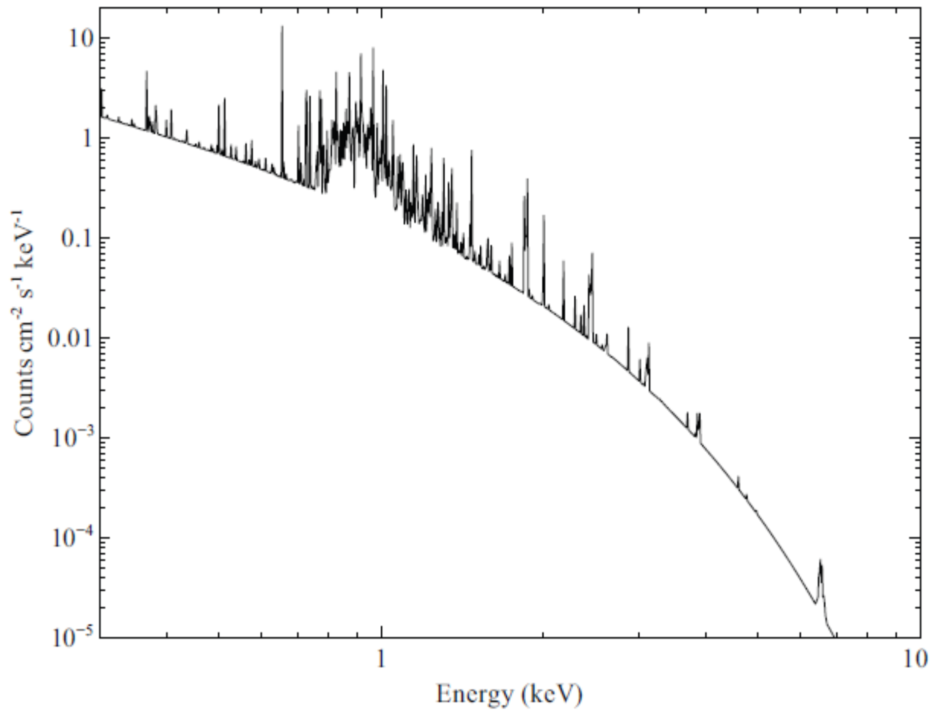
Arnaud 05

Thermal Bremsstrahlung Spectrum

- Model cluster spectrum, 1/3 solar abundance



8 keV cluster



1 keV group

$$\varepsilon^{ff}(\nu, i) = \frac{2^5 \pi e^6}{3 m_e c^3} \left(\frac{2\pi}{3 k_B m_e} \right)^{1/2} T^{-1/2} Z_i^2 g_{ff}(Z_i, T, \nu) n_e n_i e^{-h\nu/kT}$$

Line Emission

Emissivity from collisionally excited line (electrons)

$$\int \epsilon_\nu^{\rm line} {\rm d}\nu = n(X^i) n_e \frac{h^3 \nu \Omega(T) B}{4\omega_{gs}(X^i)} \left[\frac{2}{\pi^3 m_e^3 kT}\right]^{1/2} e^{-\Delta E/kT}$$

dE- excitation energy of line above ground state
B – branching ratio (probability that excited state decays to given state)
Omega – Collision strength
omega_gs- statistical weight of ground state

Luminosity per frequency

$$L_\nu = \Lambda_\nu(T, \text{Abundances}) \int n_e n_p \, {\rm d}V$$

Emissivity:

$$\varepsilon_\nu = \Lambda_\nu(z,T) n_e n_p \qquad \text{(erg/cm}^3\text{)}$$

Significant Soft X-ray Lines

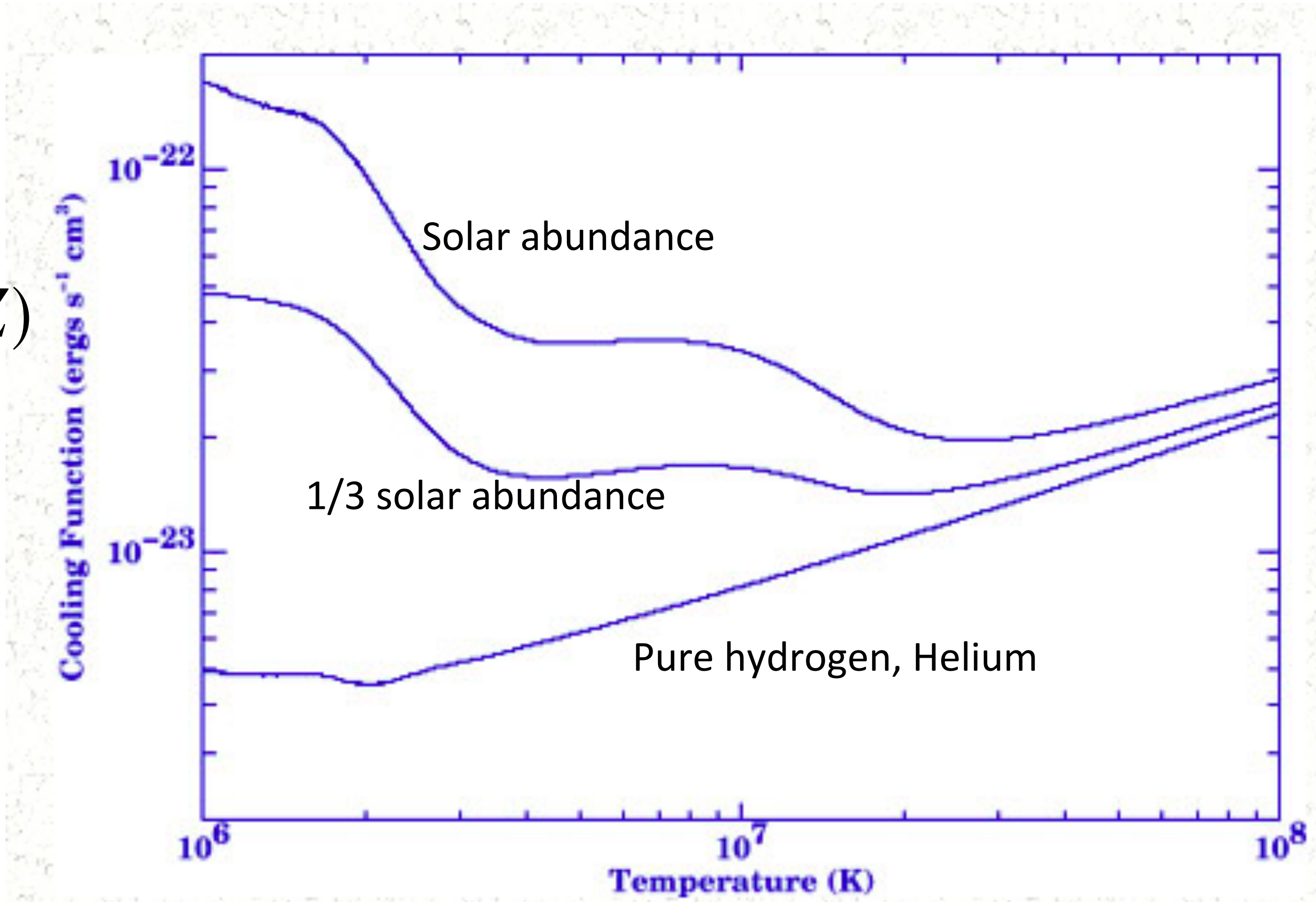
IMPORTANT LINE BLENDS IN CLUSTER SOFT X-RAY SPECTRA

Ion	Wavelengths	Temperature	Ion	Wavelengths	Temperature
	Å	keV			
Fe XXIV	10.6, 11.2	$0.9 \rightarrow 4.0$	O VIII	19.0, 16.0	>0.2
Fe XXIII	11.0, 11.4	$0.8 \rightarrow 2.0$	O VII	21.6, 22.0	$0.1 \rightarrow 0.2$
	12.2		Si XIV	6.2	>1.0
Fe XXII	11.8, 12.2	$0.6 \rightarrow 1.5$	Si XIII	6.6, 6.7	$0.2 \rightarrow 1.0$
Fe XXI	12.2, 12.8	$0.5 \rightarrow 1.0$	Al XIII	7.2	>1.2
Fe XX	12.8, 13.5	$0.4 \rightarrow 1.0$	Al XII	7.8, 7.9	$0.3 \rightarrow 1.2$
Fe XIX	13.5, 12.8	$0.3 \rightarrow 0.9$	Mg XII	8.4	>0.7
Fe XVIII	14.2, 16.0	$0.3 \rightarrow 0.8$	Mg XI	9.2, 9.3	$0.1 \rightarrow 0.6$
Fe XVII	15.0, 17.1	$0.2 \rightarrow 0.6$	Ne X	12.2	>0.4
	15.3, 16.8		Ne IX	13.5, 13.7	$0.1 \rightarrow 0.3$
N VII	24.8	>0.1			
C VI	33.7	>0.1			

Peterson + 01

X-ray cooling function

$\Lambda(T,Z)$

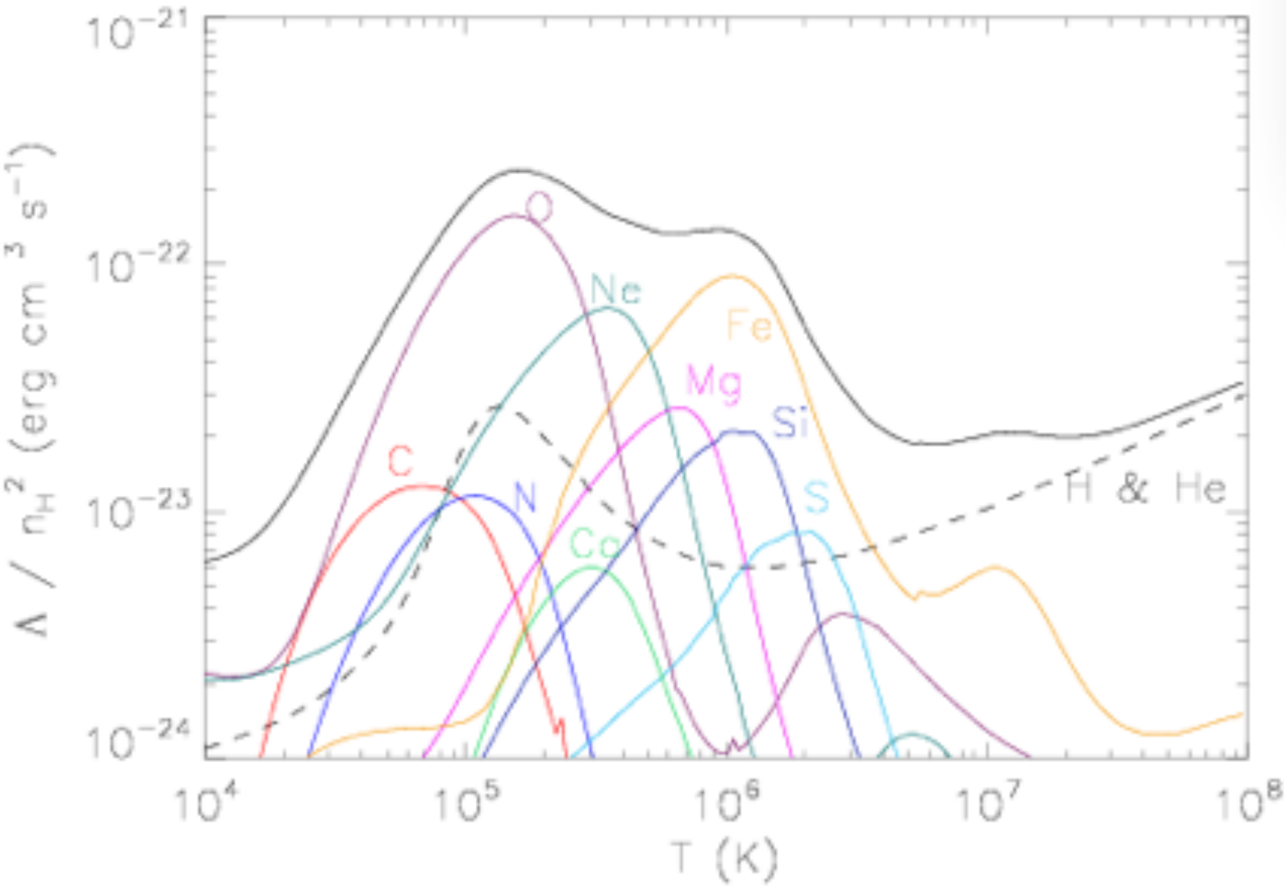


$$\varepsilon_\nu = \Lambda_\nu(z,T)n_e n_p$$

Peterson & Fabian 06

X-ray cooling flunction

$$\Lambda(T,Z)$$



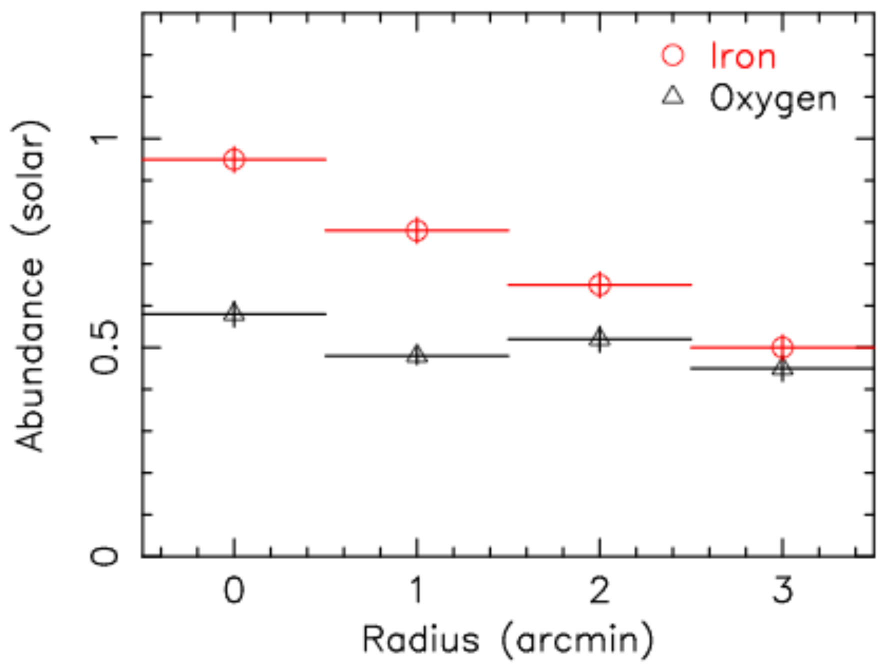
$$\varepsilon_\nu = \Lambda_\nu(z,T)n_e n_p$$

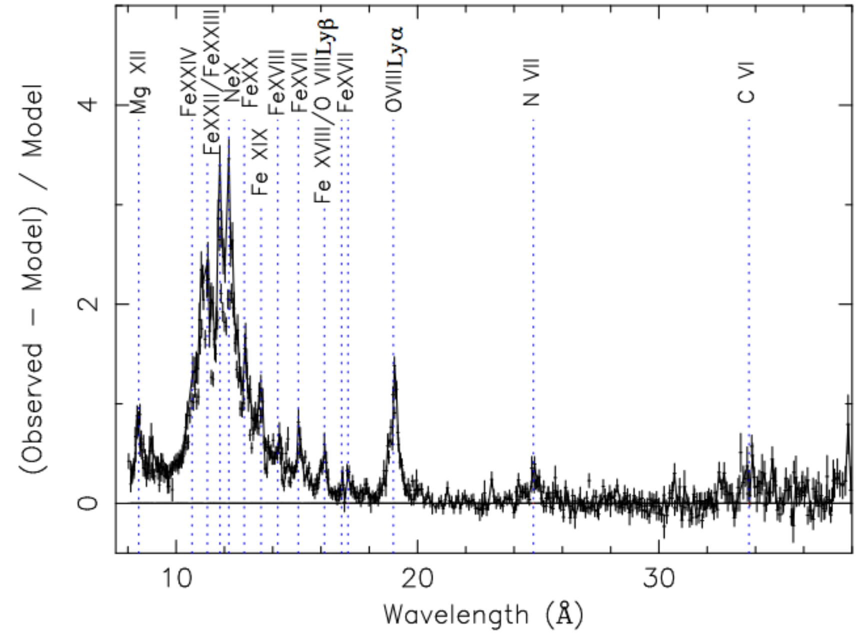
Abundances from X-ray emission

- Line emissivity proportional n_e^2 and elemental abundance
- Thermal bremsstrahlung proportional to n_e^2
- ratio of line emission to brems continuum indep of density
- Temperature sensitive to brems shape and line ratios (low T)
- Hence, line equivalent width gives elemental abundance

M87 Observed with the Reflection Grating Spectrometer

Early metal enrichment by core collapse SN, ongoing Sn Ia

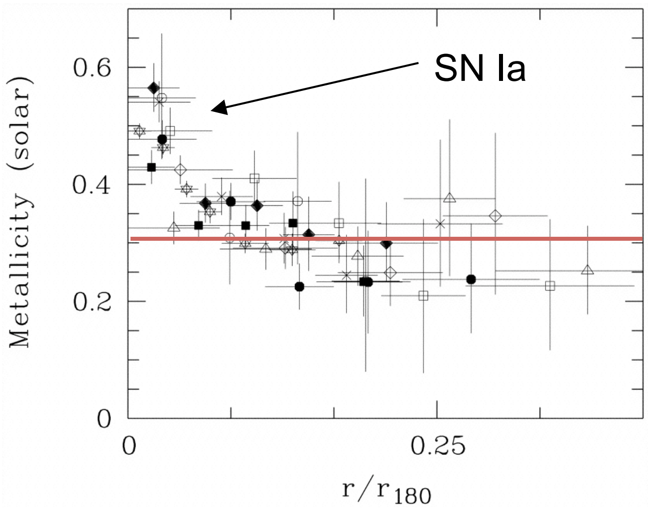


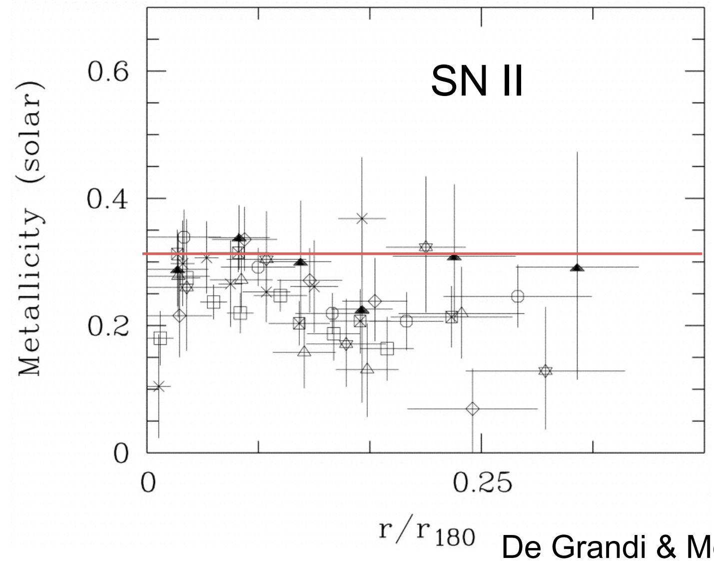


Werner .. Kaastra, De Plaa…06

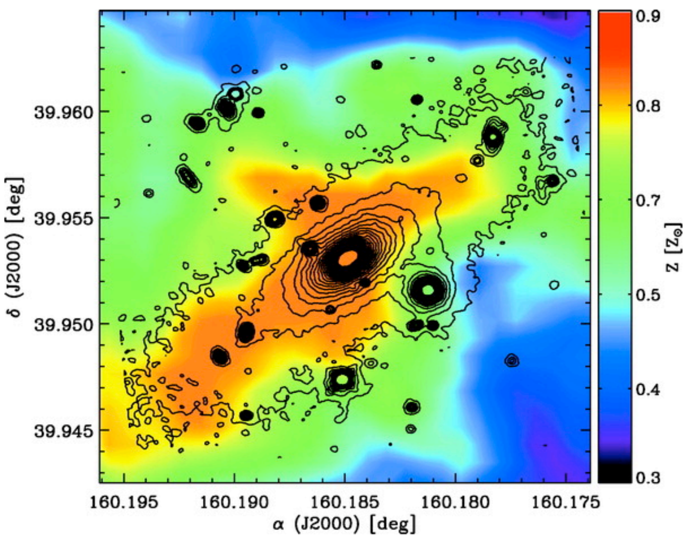
60% core collapse
40% Sn Ia

Metallicity Distribution





De Grandi & Molendi 01

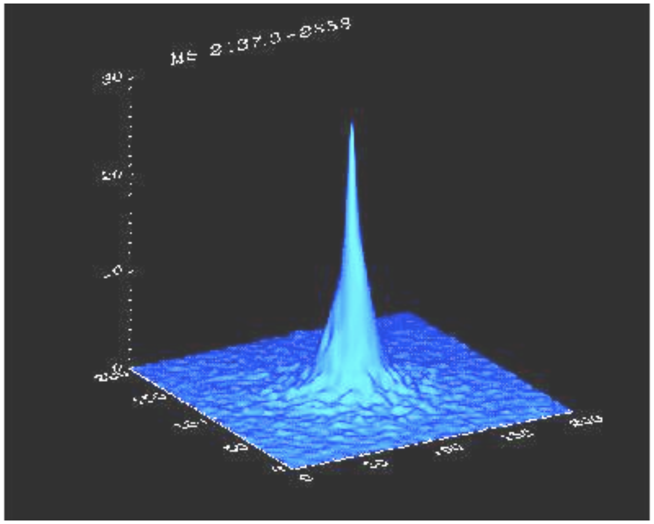


Wise et al. 04

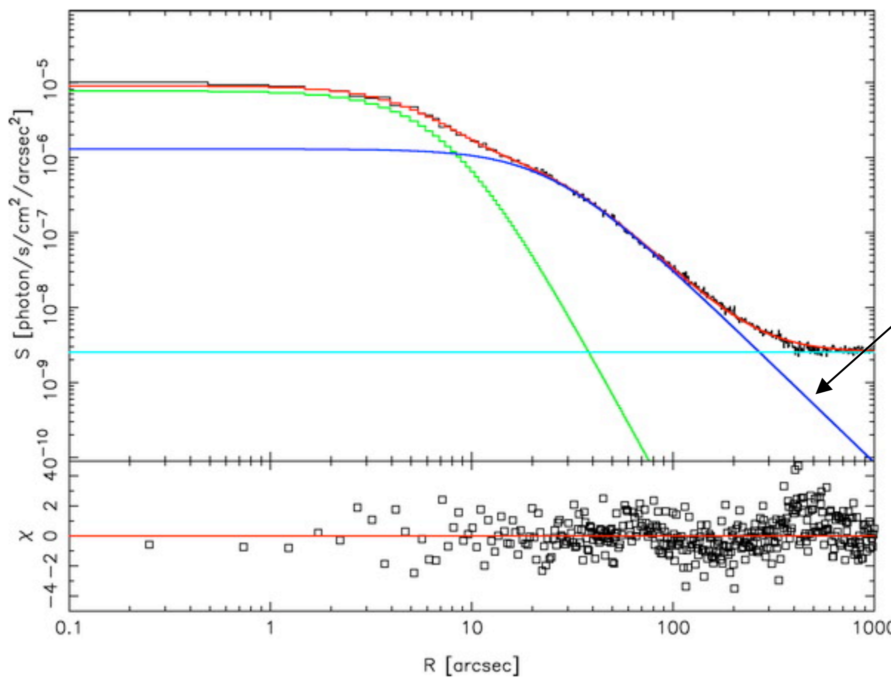
α element enhanced (Si, S, Ne, Mg)

early enrichment z >> 0.5

top-heavy IMF?



X-ray surface brighness profile



$I_x \propto r^{-3}$

Isothermal Beta Model: Cavaliere & Fusco-Femiano (76)

surface brightness: $I_x \propto [1+(r/r_c)]^{-3\beta}$ $\quad \beta = \dfrac{\mu\, m_p\, \sigma_r^2}{kT}$ $\quad \beta \approx 2/3$

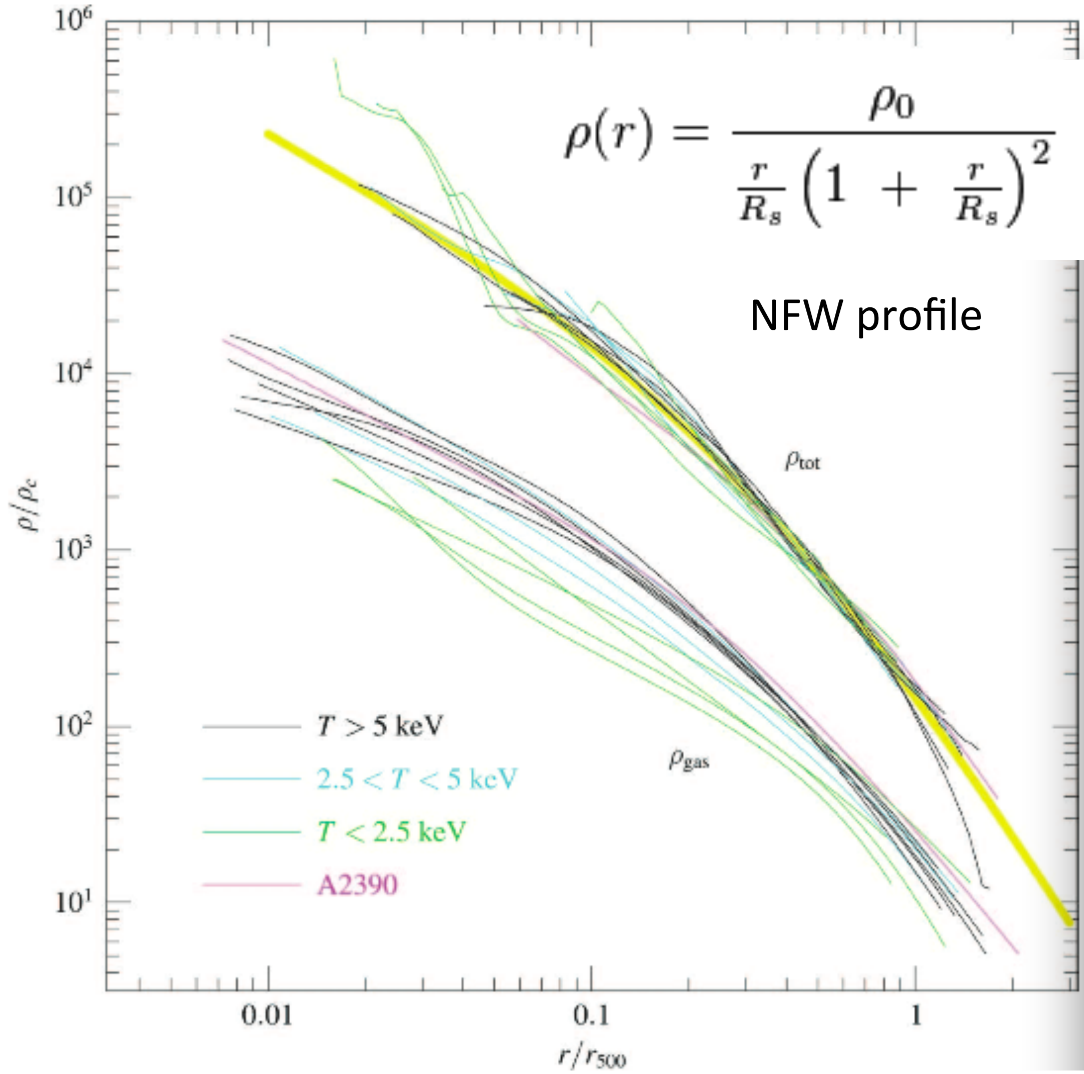
ratio of energy per unit mass in galaxies to that in gas

electron density: $n_e(r) = n_0[1+(r/r_c)^2]^{-3\beta/2}$

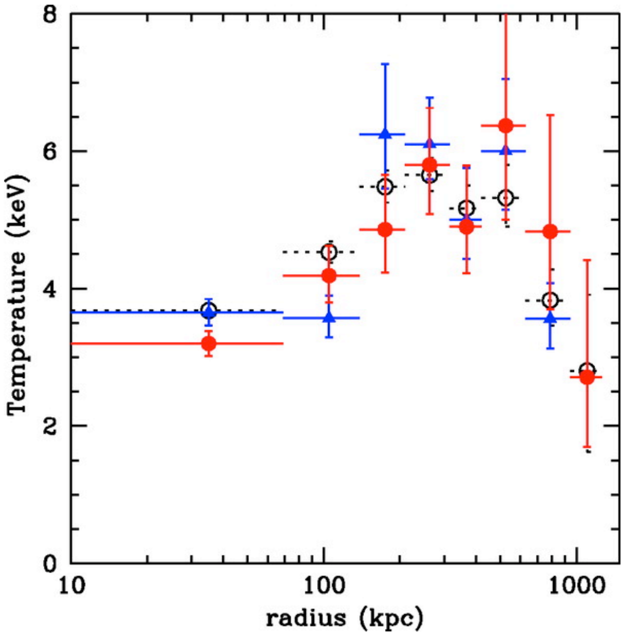
$I_c = 0.35\, I_0$ $\quad n_c = 0.5\, n_0$

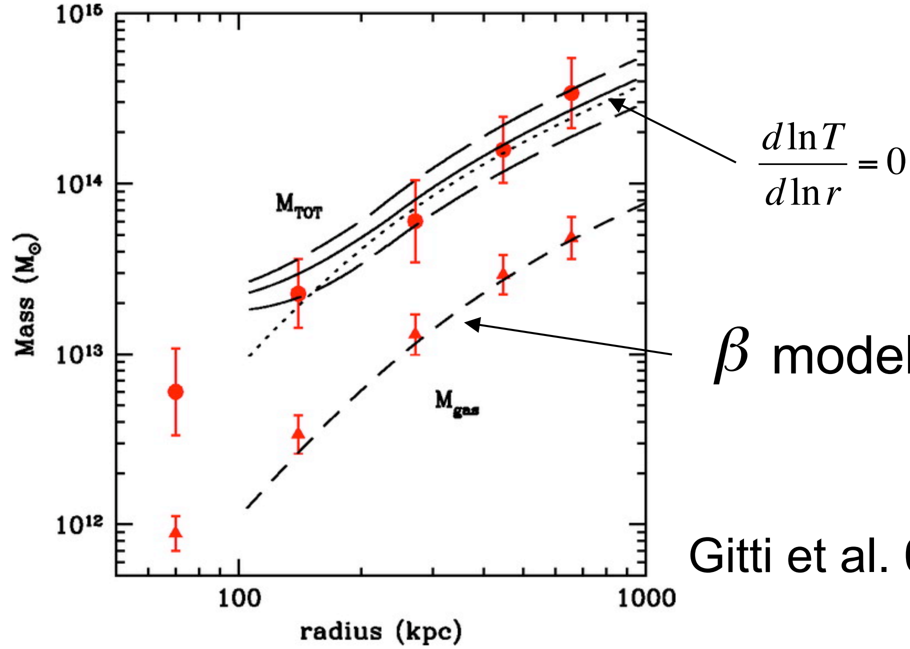
See Vikhlinin + 06 for more sophisticated models

Cluster Density Profiles - Vikhlinin 06

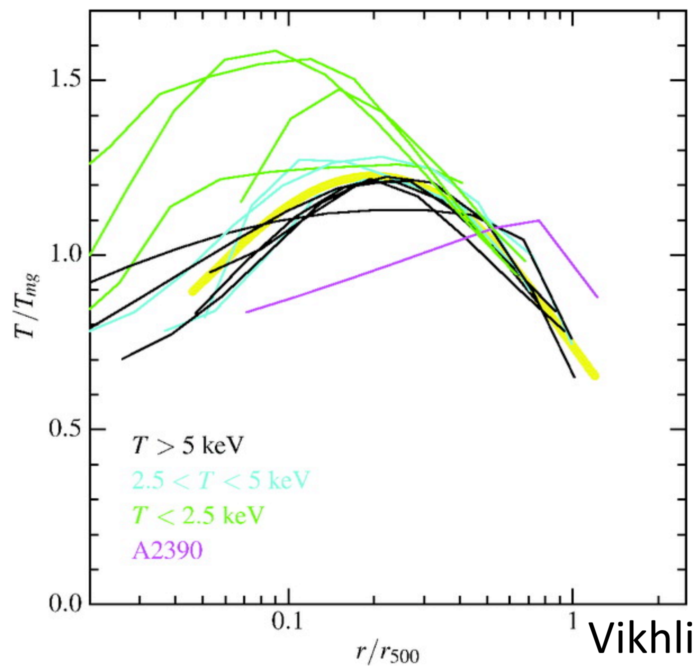


Mass Profiles





Gitti et al. 07



Vikhlinin + 05

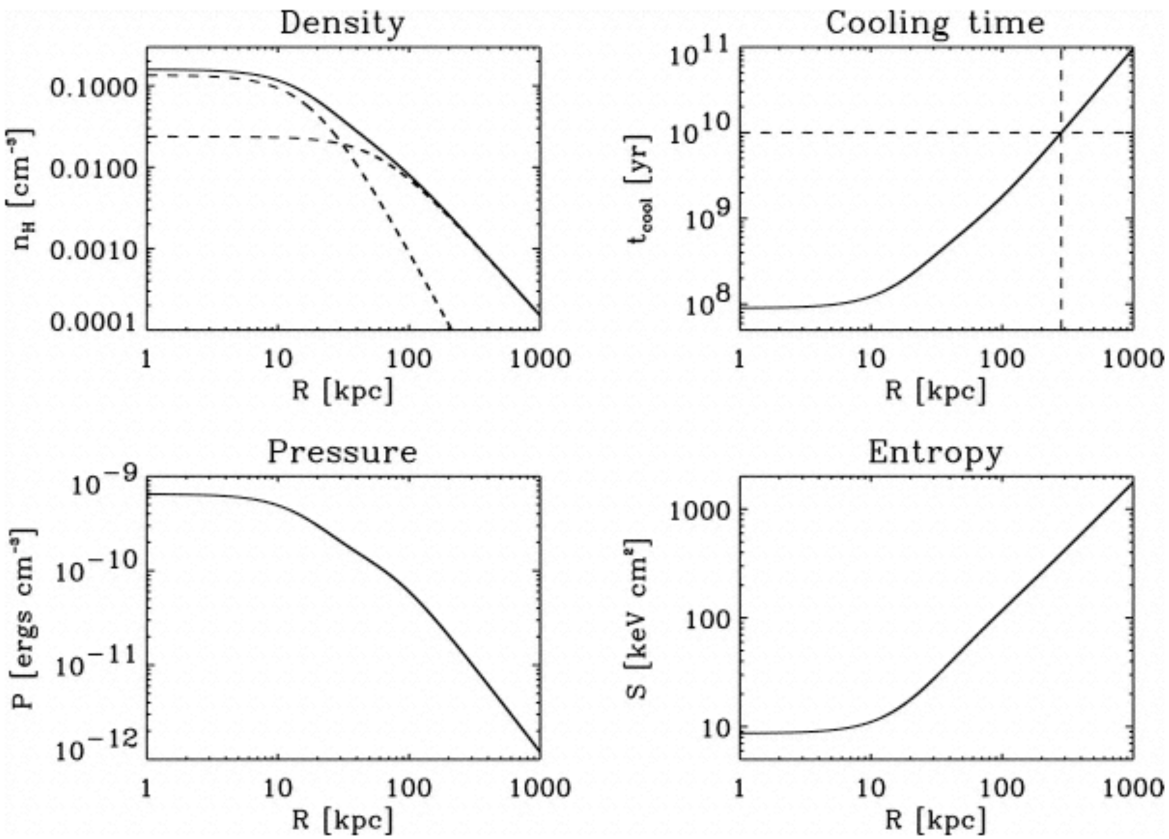
$$M_{tot}(<r) = \frac{kTr}{G\mu m_p}\left(\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right)$$

$$\rho(r) = \rho_0\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta/2}$$

$$M_{tot}(<r) = \frac{kr^2}{G\mu m_p}\left(\frac{3\beta r T}{r^2 + r_c^2} - \frac{dT}{dr}\right)$$

Assume
- hydrostatic equil.
- spherical symmetry
- only gas pressure

Typcal Gas Profiles



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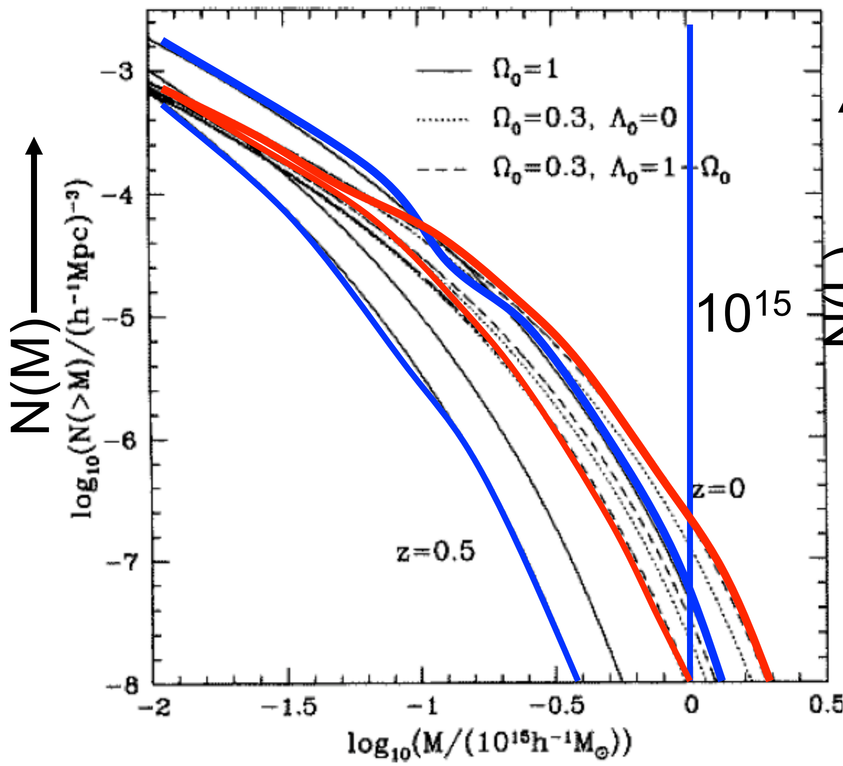
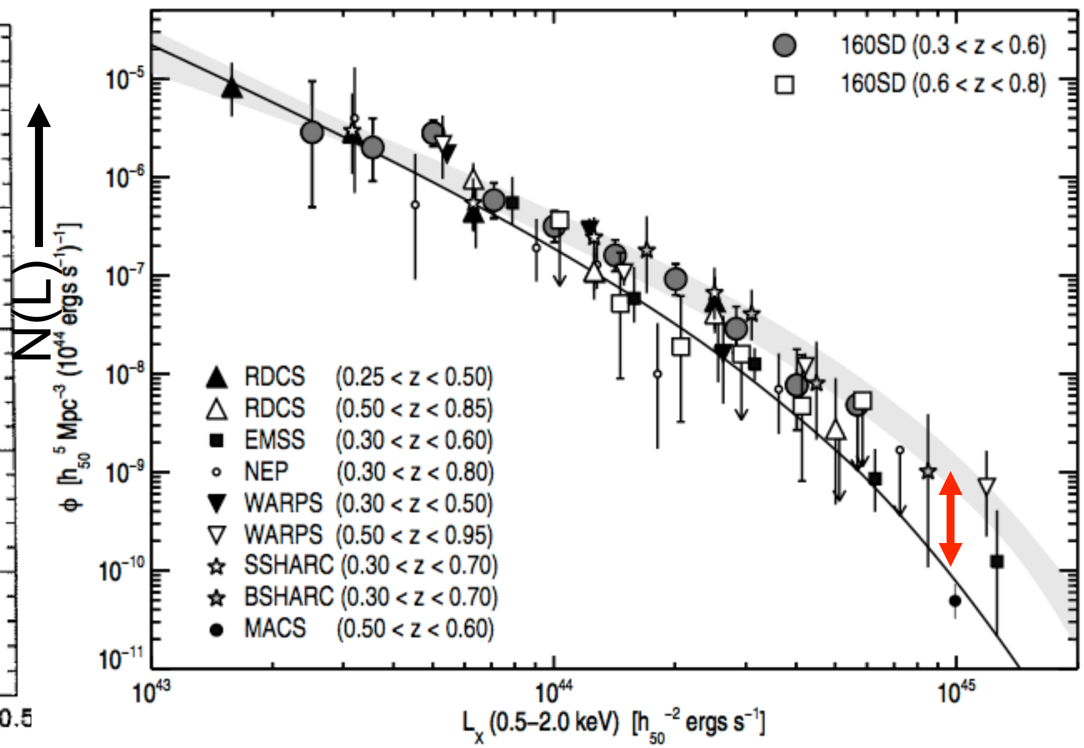
Clusters & Cosmology

mass & luminosity function sensitivity to $\Omega_{m,\Lambda}$



M →

Eke et al. 92



L_x →

Mullis et al. 04

$\Omega_m = 1$

$\Omega_m = 0.3, \Omega_\Lambda = 0.7$

Evolution: $L_x > 5\times10^{44}$ erg s^{-1} z > 0.5

Problem: L_x to M ? Need L-T, M-T, relations

L-T – baryon physics

Cluster Cosmology requires: Simulations + Observations

Is cluster matter distribution only a function of Mass and Redshift?

→ Compare observations to scaling relations

Deviations: models deal primarily with DM mass
observations deal with luminous baryons
simulated & observed parameters (eg. L,T) do not always probe same quantity

Mass Function constrains Ω_m, σ_8

mean matter density in units of critial density, amplitude of primordial density fluctuations

strategy: find observational proxies for mass

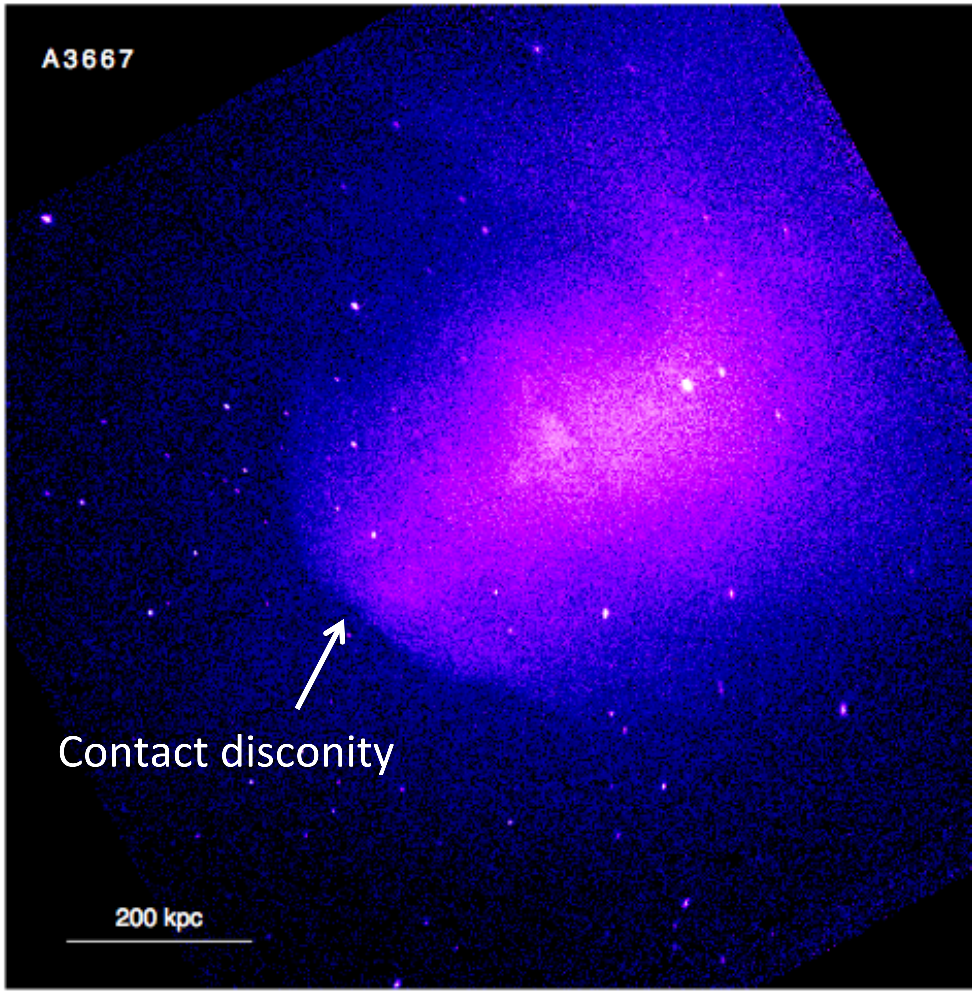
L_x - easy to measure, sensitive to non-gravitational processes

T_x - difficult to measure except for brightest objects

M - even tougher

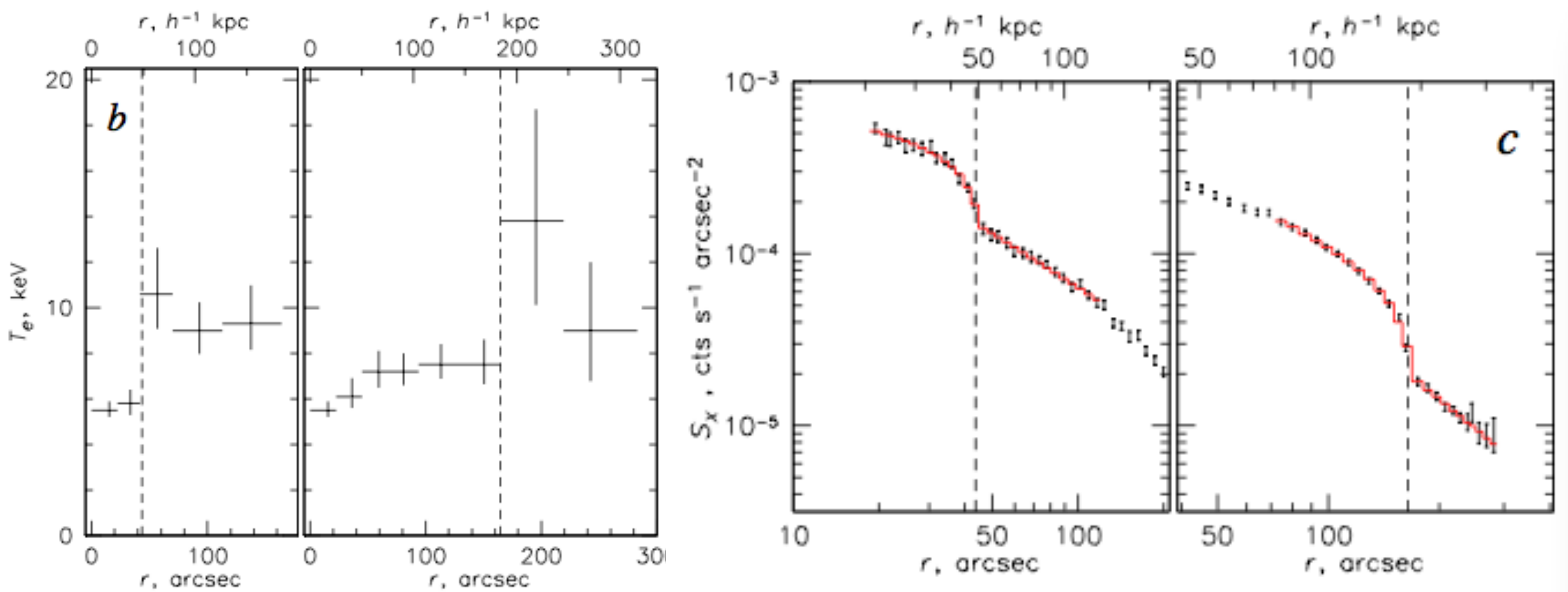

Clusters Grow by Mergers

Cold Fronts – mildly transonic mergers/sloshing



Markevich & Vikhlinin 06

Contact Discontinuity: Abell 2142



Markevich & Vikhlinin 06

Cluster Scaling Relations

Simple assumptions: gravity, adiabatic compression, shocks
self similarity: scale free, cluster defined by its mass and redshift

$$M \propto T_x^{3/2}(1+z)^{-1}$$

$$L_x \propto T_x^2(1+z)^{3/2}$$

$$L_x \propto M^{4/3}(1+z)^{7/2}$$

Implications:

Same internal structure, but distant clusters denser, smaller, more luminous

Observables in reverse order of difficulty: L_x, T_x, M

Departures from scaling relations: additional physics (cooling, feedback, etc.)

Evolution of scaling laws: Ettori et al. 04

Are clusters self-similar ? Do they scale as expected? Do they evolve as expected?

Relationship between Mass, Temperature, Luminosity

examples:

$$M \propto TR$$

$$L_x \propto \rho^2 V \Lambda$$ ← cooling

$$R \propto T^{1/2}$$

$$M \propto \rho V \propto M_{gas}$$

$$\Lambda \propto T^{1/2}$$ ($T > 2$ keV)

$$M \propto T^{3/2}$$

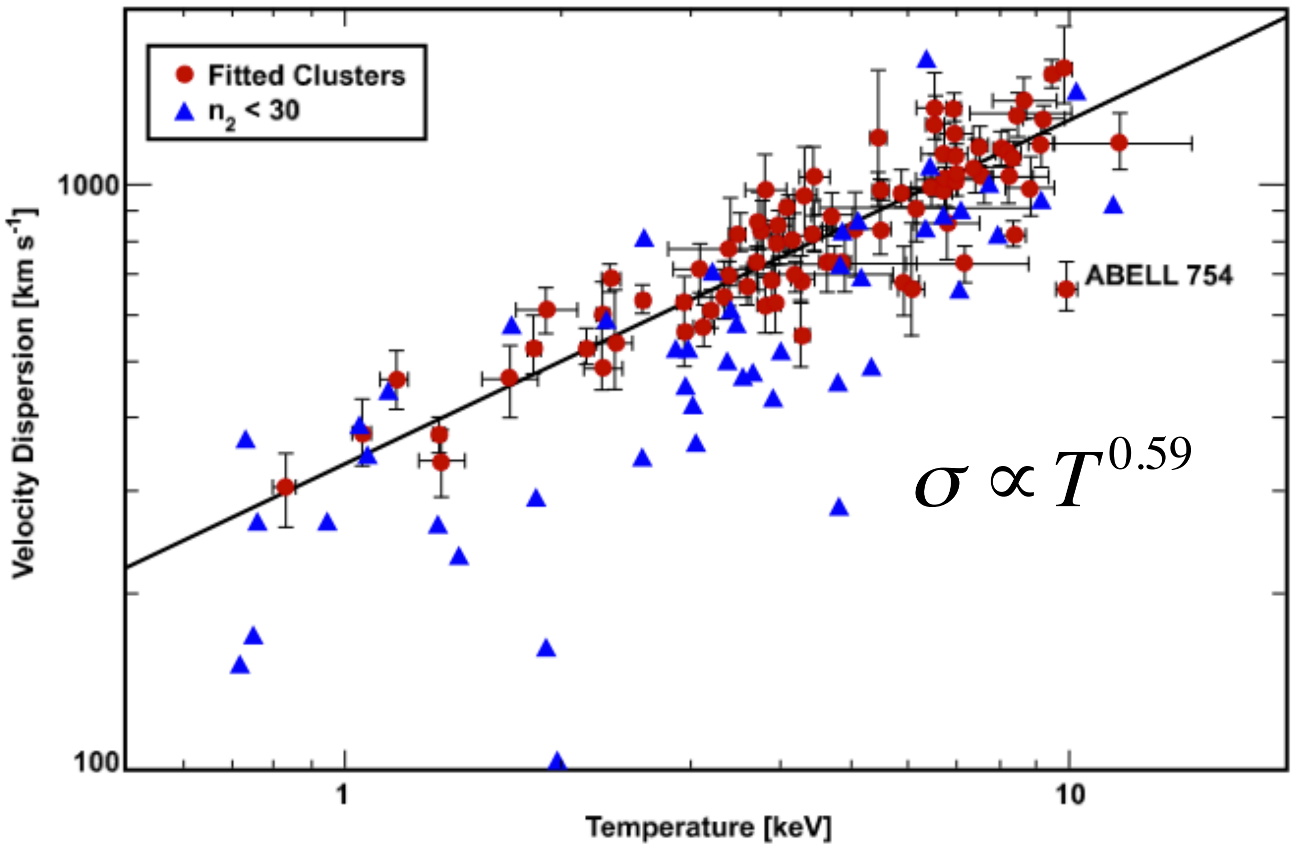
$$L_x \propto M \rho T^{1/2}$$

$$M \propto T^{1.5}$$

$$L \sim M^{4/3}$$

$$L_x \propto T^2$$

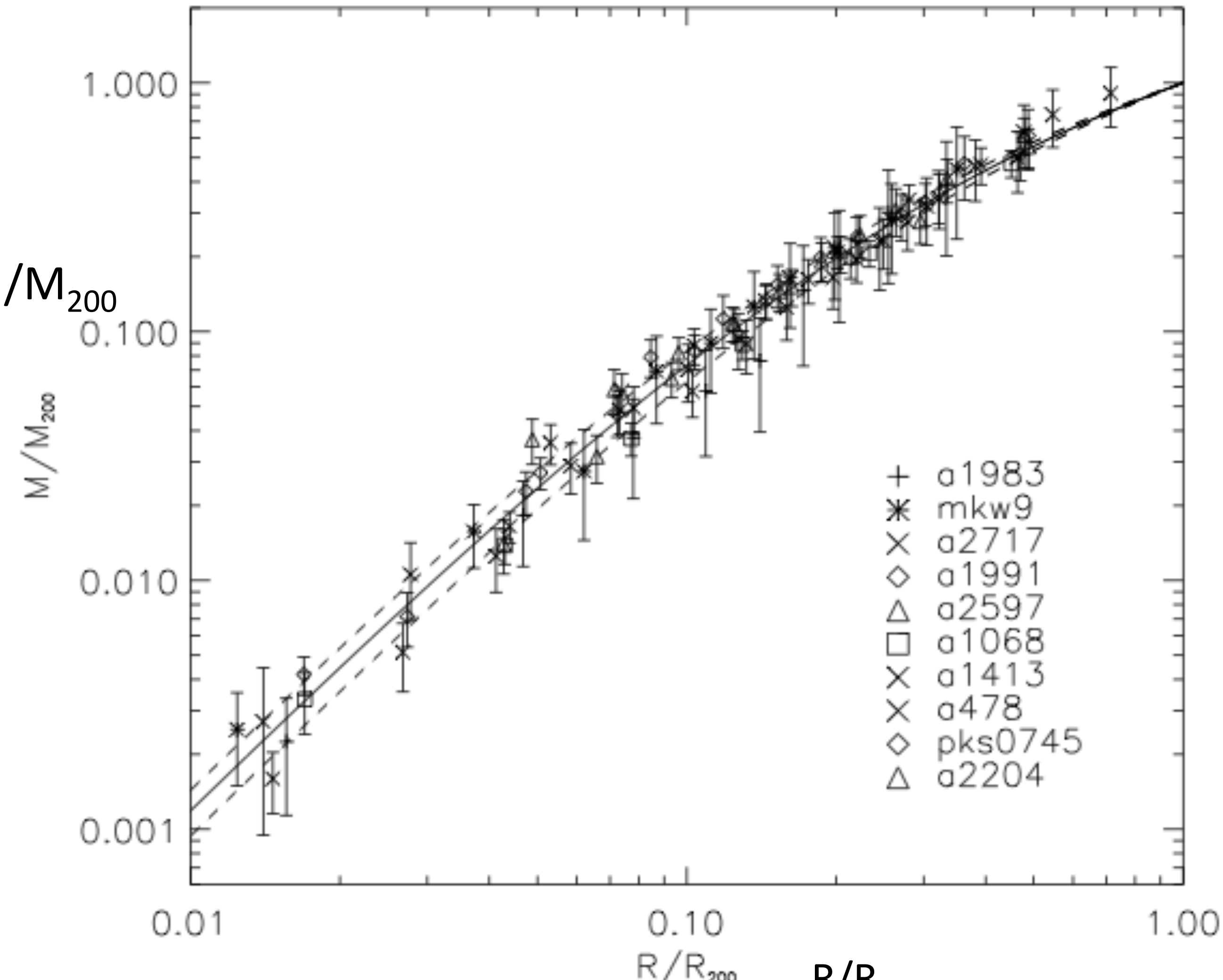
Gas Temperature is related to velocity dispersion, Mass



Mushotzky 2004

Scaling relations $\sigma \propto T^{1/2}$ Gas temperature => GM/R

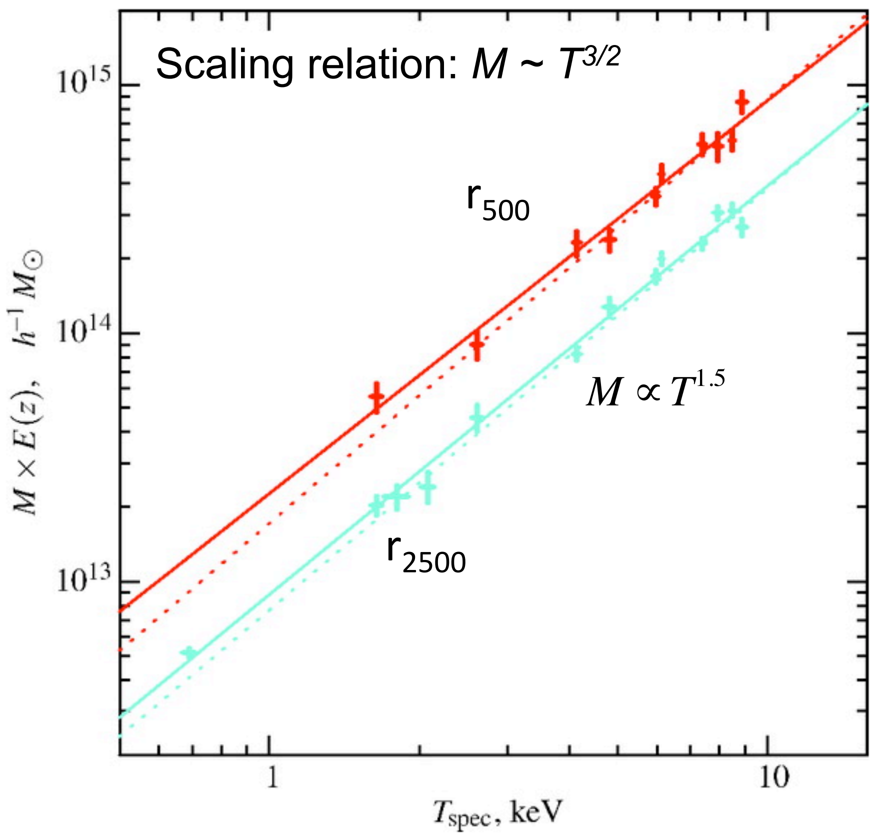
Cluster Mass Profiles closely follow Universal NFW Profile
Good News for Cluster Cosmology



M/M_{200}

R/R_{200}

Mass-Temperature Relation: Temperature good Mass proxy
Problem: Hard to measure in abundance

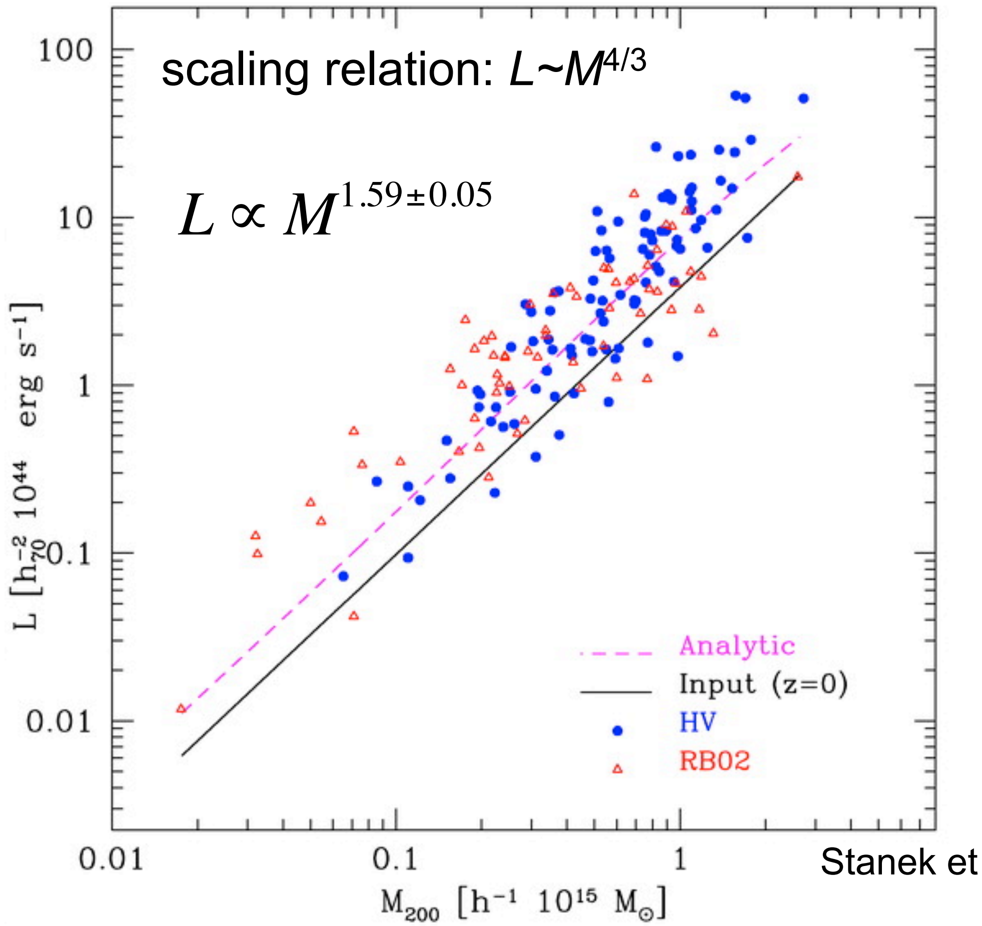


Vikhlinin et al. 06
Ettori et al. 04
Arnaud 05

Issues:
- Beta model fits
- inner core
- reference radius
- redshift range

•small deviations from self similarity for cool clusters
•normalization varies by ~30% between studies
•least affected by non-gravitational physics
•consistent with self-similar evolution -Ettori et al. 04

Mass-luminosity Relation: clusters too luminous for their masses

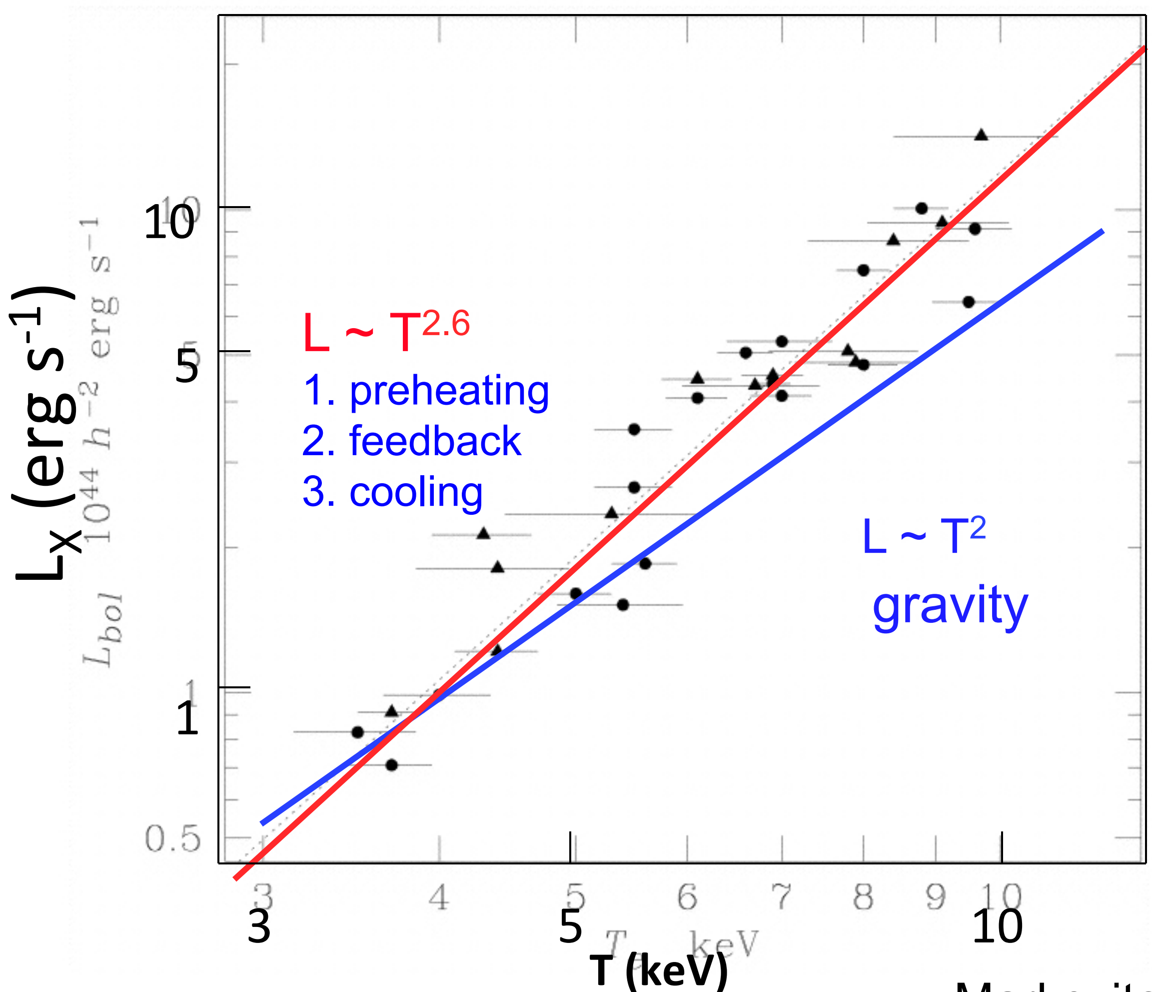


Stanek et al. 06

$L \propto M^p$ P = 1.59 +/- 0.05 Stanek et al. 06 too steep!

= 1.88 +/- 0.43 Ettori et al. 04

Luminosity-Temperature Relation: too steep



EM ~ $T^{1/2}$ SS

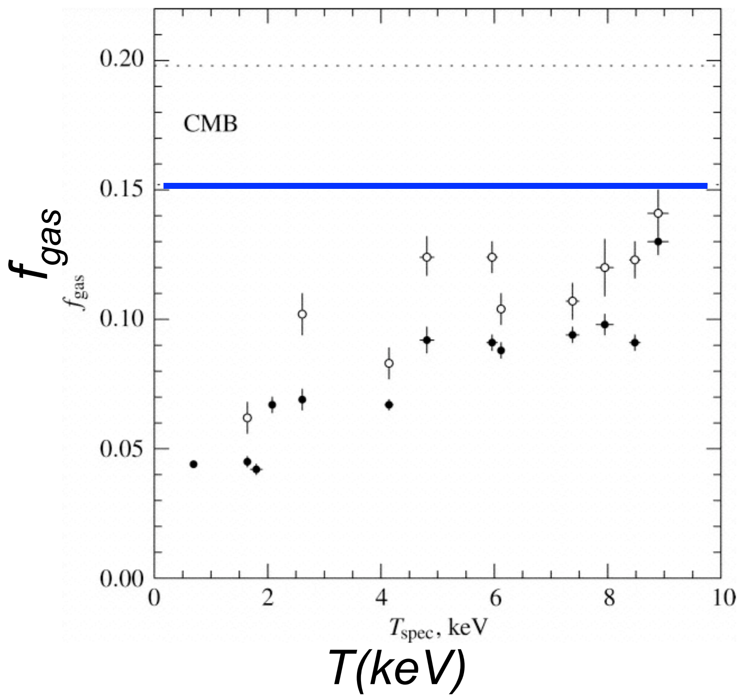
EM ~ $T^{1.38}$ observed

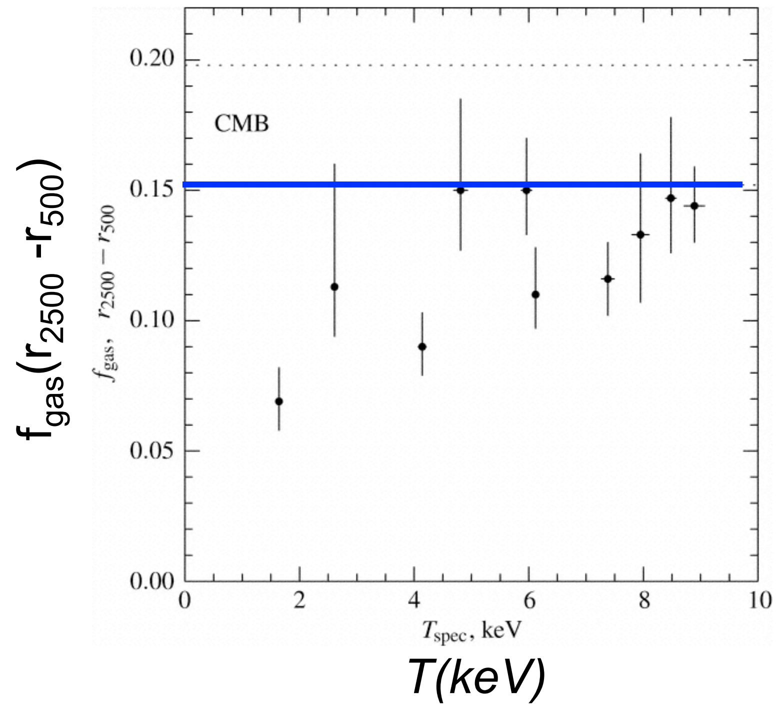
steepens L-T

Markevitch 1998

Evolution see Branchesi, Gioia et al. 07

Baryon fraction as cosmological probe





Vikhlinin et al. 06
Ettori & Fabian 99
Ettori et al. 03

assumption: universal f_b

$$f_b = \Omega_b / \Omega_m$$

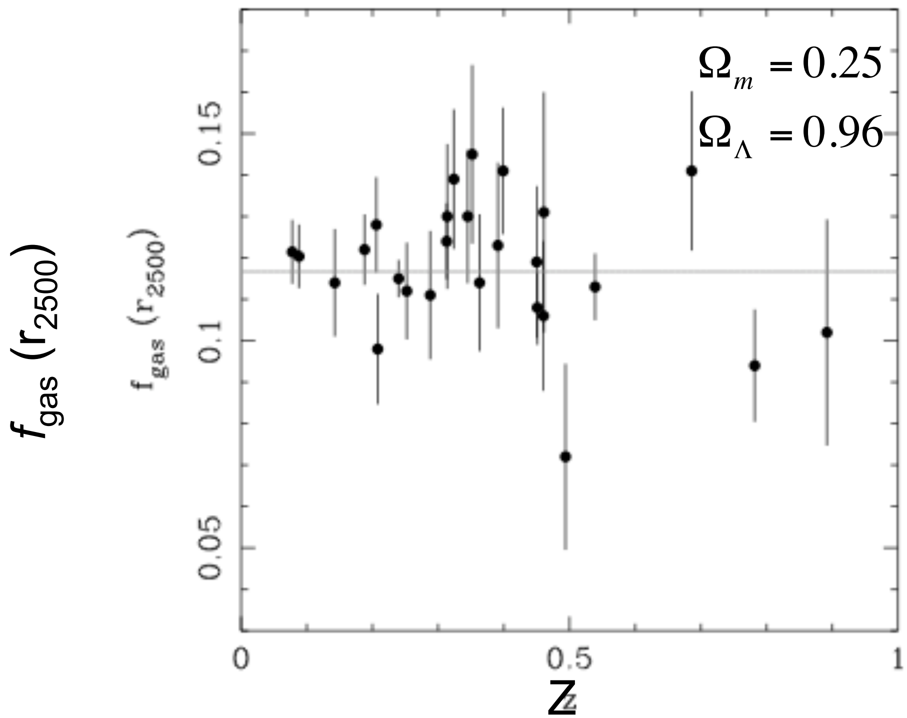
$f_{gal} \sim 0.016 \quad \therefore f_{gas} / f_{gal} \approx 10$

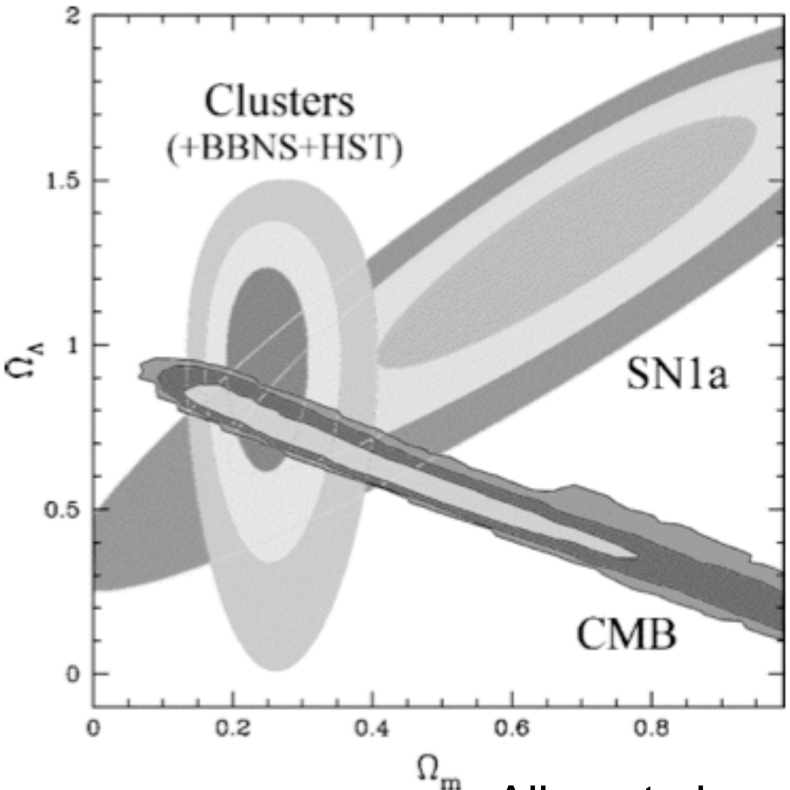
$$f_b = f_{gas} + f_{gal}$$

$\Omega_b = 0.045 h_{70}^{-2}$ (from BBN & WMAP)

$$\Omega_m = \Omega_b / f_b = 0.30^{+0.04}_{-0.03}$$

Constraints on Ω_m, Ω_Λ from f_{gas}





Allen et al.

assume f_{gas} = constant

$$f_{gas} \propto (1+z)^2 d(h(z))_A^{3/2}$$

$$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

- geometrical test
- precise mass estimates over z
- biases in aperture size, *f* constancy
- priors: H_0, $\Omega_b h^2$

Summary

Clusters are a rich area of studies at all wavelengths

- Clusters close to self-similar population (M-T)
- Dark matter profile universal: theory solid
- Gas scaling steeper than predicted:

problem: baryon physics: cooling, heating