Cosmology

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Abstract
In these lectures we first concentrate on the cosmological problems which, hopefully, have to do with the new physics to be probed at the LHC: the nature and origin of dark matter and generation of matter-antimatter asymmetry. We give several examples showing the LHC cosmological potential. These are WIMPs as cold dark matter, gravitinos as warm dark matter, and electroweak baryogenesis as a mechanism for generating matter-antimatter asymmetry. In the remaining part of the lectures we discuss the cosmological perturbations as a tool for studying the epoch preceeding the conventional hot stage of the cosmological evolution.

1 Introduction

The more we learn about our Universe, the better we understand that it is full of mysteries. These fall into three broad classes. One major mystery is dark energy, which deserves a separate class. We briefly discuss dark energy in Section 5, although, honestly speaking, we do not have much to say about it. The second class most likely has to do with the early hot epoch of the cosmological evolution, and the third one with an even earlier stage which preceeded the hot epoch. Along with dark energy, we encounter mysteries of the second class when studying the present composition of the Universe. It hosts matter but not antimatter, and after 40 years after it was understood that this is a problem, we do not have an established theory explaining this asymmetry. The Universe hosts dark matter, and we do not know what it is made of. In this context, one of the key players is the Large Hadron Collider. Optimistically, the LHC experiments may discover dark matter particles and their companions, and establish physics behind the matter-antimatter asymmetry. Otherwise they will rule out some very plausible scenarios; this will also have profound impact on our understanding of the early Universe. Let us mention also exotic hypotheses on physics beyond the Standard Model, like TeV scale gravity; their support by the LHC will have an effect on the early cosmology, which is hard to overestimate.

In the first part of these lectures we concentrate on a few examples showing the LHC cosmological potential. Before coming to that, we briefly introduce the basic notions of cosmology that are useful for our main discussion. We then turn to dark matter, and present the WIMP scenario for cold dark matter, which is currently the most popular one — for good reason. We also consider light gravitino scenario for warm dark matter. Both are probed by the LHC, as they require rather particular new physics in the LHC energy range. We then discuss electroweak baryogenesis — a mechanism for the generation of matter-antimatter asymmetry that may have operated at temperature of order $100\,\mathrm{GeV}$ in the early Universe. This mechanism also needs new physics at energies $100 - 300\,\mathrm{GeV}$, so it will be confirmed or ruled out by the LHC.

The third class of mysteries is related to cosmological perturbations, i.e., inhomogeneities in energy density and associated gravitational potentials and, possibly, relic gravity waves. As we explain in the second part of these lectures, the observed properties of density perturbations show that they were generated at some epoch that preceeded the hot stage of the cosmological evolution. Obviously, the very fact that we are confident about the existence of such an epoch is a fundamental result of theoretical and observational cosmology. The most plausible hypothesis on that epoch is cosmological inflation, though the observational support of this hypothesis is presently not particularly strong, and alternative scenarios have not been ruled out. We will briefly discuss the potential of future cosmological observations in discriminating between different options.

These lectures are meant to be self-contained, but we necessarily omit numerous details, while trying to make clear the basic ideas and results. More complete accounts of particle physics aspects of cosmology may be found in reviews [1]. Dark matter, including various hypotheses about its particles, is reviewed in Ref. [2]. Electroweak baryogenesis is discussed in detail in reviews [3]. For reviews on dark energy, see, e.g., Ref. [4]. The theory and observations of cosmological perturbations are presented in Ref. [5].

2 Homogeneous isotropic Universe

2.1 Friedmann–Lemaître–Robertson–Walker metric

Two basic facts about our visible Universe are that it is *homogeneous and isotropic* at large spatial scales, and that it *expands*.

There are three types of homogeneous and isotropic three-dimensional spaces. These are[1] three-sphere, flat (Euclidean) space and three-hyperboloid. Accordingly, one speaks about closed, flat and open Universe; in the latter two cases the spatial size of the Universe is infinite, whereas in the former the Universe is compact.

The homogeneity and isotropy of the Universe mean that its hypersurfaces of constant time are either three-spheres or Euclidean spaces or three-hyperboloids. The distances between points may (and, indeed, do) depend on time, i.e., the interval has the form

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \ , \tag{1}$$

where $d\mathbf{x}^2$ is the distance on unit three-sphere/Euclidean space/hyperboloid. The metric (1) is usually called Friedmann–Lemaître–Robertson–Walker (FLRW) metric, and $a(t)$ is called the scale factor. In our Universe $\dot{a} \equiv \frac{da}{dt} > 0$, which means that the distance between points of fixed spatial coordinates $\mathbf{x}$ grows, $dl^2 = a^2(t)d\mathbf{x}^2$. The space stretches out; the Universe expands.

The coordinates $\mathbf{x}$ are often called comoving coordinates. It is straightforward to check that $\mathbf{x} =$ const is a time-like geodesic, so a galaxy put at a certain $\mathbf{x}$ at zero velocity will stay at the same $\mathbf{x}$. Furthermore, as the Universe expands, non-relativistic objects loose their velocities $\dot{\mathbf{x}}$, i.e., they get frozen in the comoving coordinate frame.

Observational data set strong constraints on the spatial curvature of the Universe. They tell that to a very good approximation our Universe is spatially flat, i.e., our *3-dimensional space is Euclidean*. In what follows $d\mathbf{x}^2$ is simply the line interval in the Euclidean 3-dimensional space.

2.2 Redshift

Like the distances between free particles in the expanding Universe, the photon wavelength increases too. We will always label the present values of time-dependent quantities by subscript 0: the present wavelength of a photon is thus denoted by λ_0, the present time is t_0, the present value of the scale factor is $a_0 \equiv a(t_0)$, etc. If a photon was emitted at some moment of time t in the past, and its wavelength at the moment of emission was λ_e, then we receive today a photon whose physical wavelength is longer,

$$\frac{\lambda_0}{\lambda_e} = \frac{a_0}{a(t)} \equiv 1 + z \ .$$

Here we introduced the redshift z. The redshift of an object is directly measurable. λ_e is fixed by physics of the source, say, it is the wavelength of a photon emitted by an excited hydrogen atom. So,

[1] Strictly speaking, this statement is valid only locally: in principle, homogeneous and isotropic Universe may have complex global properties. As an example, spatially flat Universe may have topology of three-torus. There is some discussion of such a possibility in literature, and fairly strong limits have been obtained by the analyses of cosmic microwave background radiation [6].

one identifies a series of emission or absorption lines, thus determining λ_e, and measures their actual wavelengths λ_0. These spectroscopic measurements give accurate values of z even for distant sources. On the other hand, the redshift is related to the time of emission, and hence to the distance to the source.

Let us consider a "nearby" source, for which $z \ll 1$. This corresponds to relatively small $(t_0 - t)$. Expanding $a(t)$, one writes

$$a(t) = a_0 - \dot{a}(t_0)(t_0 - t) \ . \tag{2}$$

To the leading order in z, the difference between the present time and the emission time is equal to the distance to the source r (the speed of light is set equal to 1). Let us define the Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

and denote its present value by H_0. Then Eq. (2) takes the form $a(t) = a_0(1 - H_0 r)$, and we get for the redshift, again to the leading non-trivial order in z,

$$1 + z = \frac{1}{1 - H_0 r} = 1 + H_0 r \ .$$

In this way we obtain the Hubble law,

$$z = H_0 r \ , \quad z \ll 1 \ . \tag{3}$$

Traditionally, one tends to interpret the expansion of the Universe as runaway of galaxies from each other, and redshift as the Doppler effect. Then at small z one writes $z = v$, where v is the radial velocity of the source with respect to the Earth, so H_0 is traditionally measured in units "velocity per distance". Observational data give [8]

$$H_0 = [71.0 \pm 2.5] \, \frac{\text{km/s}}{\text{Mpc}} \approx (14 \cdot 10^9 \ \text{yrs})^{-1} \ , \tag{4}$$

where $1 \ \text{Mpc} = 3 \cdot 10^6 \ \text{light yrs} = 3 \cdot 10^{24} \ \text{cm}$ is the distance measure often used in cosmology. Traditionally, the present value of the Hubble parameter is written as

$$H_0 = h \cdot 100 \, \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \ . \tag{5}$$

Thus $h \approx 0.71$. We will use this value in further estimates.

Let us point out that the interpretation of redshift in terms of the Doppler shift is actually not adequate, especially for large enough z. In fact, there is no need in this interpretation at all: the "radial velocity" enters neither theory nor observations, so this notion may be safely dropped. Physically meaningful quantity is redshift z itself.

A final comment is that H_0^{-1} has dimension of time, or length, as indicated in Eq. (4). Clearly, this quantity sets the cosmological scales of time and distance at the present epoch.

2.3 Hot Universe

Our Universe is filled with cosmic microwave background (CMB). Cosmic microwave background as observed today consists of photons with excellent black-body spectrum of temperature

$$T_0 = 2.726 \pm 0.001 \ \text{K} \ . \tag{6}$$

The spectrum has been precisely measured by various instruments and does not show any deviation from the Planck spectrum [7].

Thus, the present Universe is "warm". Earlier Universe was warmer; it cooled down because of the expansion. While the CMB photons freely propagate today, it was not so at early stage. When the Universe was hot, the usual matter (electrons and protons with rather small admixture of light nuclei) was in the plasma phase. At that time photons strongly interacted with electrons due to the Thomson scattering and protons interacted with electrons via Coulomb force, so all these particles were in thermal equilibrium. As the Universe cooled down, electrons "recombined" with protons into neutral hydrogen atoms, and the Universe became transparent to photons. The temperature scale of recombination is, very crudely speaking, determined by the ionisation energy of hydrogen, which is of order 10 eV. In fact, recombination occured at lower temperature[2], $T_{rec} \approx 3000$ K. An important point is that the duration of the period of recombination was considerably shorter than the Hubble time at that epoch; to a reasonable approximation, recombination occured instantaneously.

The importance of the recombination epoch (more precisely, the epoch of photon last scattering; we will use the term recombination for brevity) is that the CMB photons travel freely after it: the density of hydrogen atoms was so small (about 250 cm^{-3} right after recombination) that the gas was transparent to photons. So, CMB photons give the photographic picture of the Universe at recombination, i.e., at redshift and age

$$z_{rec} = 1090 \; , \qquad t_{rec} = 370\,000 \text{ years} \; . \tag{7}$$

It is worth noting that even though after recombination photons no longer were in thermal equilibrium with anything, the shape of the photon distribution function has not changed, except for overall redshift. Indeed, the thermal distribution function for *ultra-relativistic* particles, the Planck distribution, depends only on the ratio of frequency to temperature, $f_{Planck}(p, T) = f\,(\omega_p/T)$, $\omega_p = |p|$. As the Universe expands, the photon momentum gets redshifted, $p(t) = p(t_{rec}) \cdot \frac{a(t_{rec})}{a(t)}$, the frequency is redshifted in the same way, but the shape of the spectrum remains Planckian, with redshifted temperature. Hence, the Planckian form of the observed spectrum is no surprise. Generaly speaking, this property does not hold for massive particles.

At even earlier times, the temperature of the Universe was even higher. The earliest time at the hot stage which has been observationally probed so far is the Big Bang Nucleosynthesis epoch; that epoch began at temperature of order 1 MeV, when the lifetime of the Universe was about 1 s. At that time the weak processes like

$$e^- + p \longleftrightarrow n + \nu_e$$

switched off, and the comoving number density of neutrons freezed out. Somewhat later, these neutrons combined with protons into light elements in thermonuclear reactions

$$\begin{aligned} p + n &\to {}^2H + \gamma \; , \\ {}^2H + p &\to {}^3He + \gamma \; , \\ {}^3He +^2 H &\to {}^4He + p \; , \end{aligned} \tag{8}$$

etc., up to ${}^7Li$. Comparison of the Big Bang Nucleosynthesis theory with the observational determination of the composition of cosmic medium gives us confidence that we understand the Universe at that epoch. Notably, we are convinced that the cosmological expansion was governed by General Relativity.

2.4 Properties of components of cosmic medium

Let us come back to photons. Their effective temperature after recombination scales as

$$T(t) \propto a^{-1}(t) \; . \tag{9}$$

[2]The reason is that the number density of electrons and protons is small compared to the number density of photons. At temperature above 3000 K, a hydrogen atom formed in an electron-proton encounter is quickly destroyed by absorbing a photon from the high energy tail of the Planck distribution, and after that the electron/proton lives long time before it meets proton/electron and forms a hydrogen atom again. In thermodynamical terms, at temperatures above 3000 K there is large entropy per electron/proton, and recombination is not thermodynamically favourable because of entropy considerations.

This behaviour is characteristic of *ultra-relativistic* free species (at zero chemical potential). The same formula is valid for ultra-relativistic particles (at zero chemical potential) which are in thermal equilibrium. Thermal equilibrium means adiabatic expansion; during adiabatic expansion, the temperature of ultra-relativistic gas scales as the inverse size of the system, according to usual thermodynamics. The energy density of ultra-relativistic gas scales as $\rho \propto T^4$, and pressure is $p = \rho/3$.

Both for free photons, and for photons in thermal equilibrium, the number density behaves as follows,

$$n_\gamma = \text{const} \cdot T^3 \propto a^{-3} ,$$

and the energy density is given by the Stefan–Boltzmann law,

$$\rho_\gamma = \frac{\pi^2}{30} \cdot 2 \cdot T^4 \propto a^{-4} , \tag{10}$$

where the factor 2 accounts for two photon polarizations. The present number density of relic photons is

$$n_{\gamma,0} = 410 \ \text{cm}^{-3} , \tag{11}$$

and their energy density is

$$\rho_{\gamma,0} = 2.7 \cdot 10^{-10} \frac{\text{GeV}}{\text{cm}^3} . \tag{12}$$

An important characteristic of the early Universe is the entropy density of cosmic plasma in thermal equilibrium. It is given by

$$s = \frac{2\pi^2}{45} g_* T^3 , \tag{13}$$

where g_* is the number of degrees of freedom with $m \lesssim T$, that is, the degrees of freedom which are relativistic at temperature T; each spin state counts as an independent degree of freedom, and fermions contribute to g_* with a factor of $7/8$. The point is that the entropy density scales *exactly* as a^{-3},

$$sa^3 = \text{const} , \tag{14}$$

while temperature scales *approximately* as a^{-1}. The property (14) is nothing but the reflection of the fact that the Universe expands relatively slowly, and the evolution is adiabatic (barrig fairly exotic scenarios with strong entropy generation at some early cosmological epoch). The temperature would scale as a^{-1} if the number of relativistic degrees of freedom would be independent of time. This is not the case, however. Indeed, the value of g_* depends on temperature: at $T \sim 10\,\text{MeV}$ relativistic species are photons, neutrinos, electrons and positrons, while at $T \sim 1\,\text{GeV}$ four flavors of quarks, gluons, muons and τ-leptons are relativistic too. The number of degrees of freedom in the Standard Model at $T \gtrsim 100\,\text{GeV}$ is

$$g_*(100\,\text{GeV}) \approx 100 .$$

The present value of the entropy density (taking into account neutrinos as if they were massless) is

$$s_0 \approx 3000\,\text{cm}^{-3} . \tag{15}$$

The parameter g_* determines not only the entropy density but also the energy density of the cosmic plasma in thermal equilibrium. The Stefan–Boltzmann law gives

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4 , \tag{16}$$

where subscript *rad* indicates that we are talking about the relativistic component (radiation in broad sense).

Let us now turn to non-relativistic particles: baryons, massive neutrinos, dark matter particles, etc. If they are not destroyed during the evolution of the Universe (that is, they are stable and do not annihilate), their number density merely gets diluted,

$$n \propto a^{-3} \; . \tag{17}$$

This means, in particular, that the baryon-to-photon ratio stays constant in time (we consider for definiteness the late Universe, $T \lesssim 100\,\text{keV}$),

$$\eta_B \equiv \frac{n_B}{n_\gamma} = \text{const} \approx 6.1 \cdot 10^{-10} \; . \tag{18}$$

The numerical value here is determined by two independent methods: one is Big Bang Nucleosynthesis theory and measurements of the light element abundances, and another is the measurements of the CMB temperature anisotropy. It is reassuring that these methods give consistent results (with comparable precision).

The energy density of non-relativistic particles scales as

$$\rho(t) = m \cdot n(t) \propto a^{-3}(t) \; , \tag{19}$$

in contrast to more rapid fall off (10) characteristic of relativistic species.

Finally, dark energy density does not decrease in time as fast as in Eqs. (10) or (19). In fact, to a reasonable approximation dark energy density does not depend on time at all,

$$\rho_\Lambda = \text{const} \; . \tag{20}$$

Dark energy with exactly time-independent energy density is the same thing as the cosmological constant, or Λ-term.

2.5 Composition of the present Universe

The basic equation governing the expansion rate of the Universe is the Friedmann equation, which we write for the case of spatially flat Universe,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho \; , \tag{21}$$

where dot denotes derivative with respect to time t, ρ is the *total* energy density in the Universe and G is Newton's gravity constant; in natural units $G = M_{Pl}^{-2}$ where $M_{Pl} = 1.2 \cdot 10^{19}\,\text{GeV}$ is the Planck mass. The Friedmann equation is nothing but the (00)-component of the Einstein equations of General Relativity,

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi T_{00} \; ,$$

specified to FLRW metric.

Let us introduce the parameter

$$\rho_c = \frac{3}{8\pi G} H_0^2 \approx 5 \cdot 10^{-6} \, \frac{\text{GeV}}{\text{cm}^3} \; . \tag{22}$$

According to Eq. (21), it is equal to the sum of all forms of energy density in the present Universe. As a side remark, we note that the latter statement would not be true if our Universe were not spatially flat. However, according to observations, spatial flatness holds to a very good precision, corresponding to less than 1 per cent deviation of the total energy density from ρ_c [9].

As we now discuss, the cosmological data correspond to a very weird composition of the Universe.

Before proceeding, let us introduce a notion traditional in the analysis of the composition of the present Universe. For every type of matter i with the present energy density $\rho_{i,0}$, one defines the parameter

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c} \ .$$

Then Eq. (21) tells that $\sum_i \Omega_i = 1$ where the sum runs over all forms of energy. Let us now discuss contributions of different species to this sum.

We begin with **baryons**. The result (18) gives

$$\rho_{B,0} = m_B \cdot n_{B,0} \approx 2.4 \cdot 10^{-7} \, \frac{\text{GeV}}{\text{cm}^3} \ . \tag{23}$$

Comparing this result with the value of ρ_c given in Eq. (22), one finds

$$\Omega_B = 0.045 \ .$$

Thus, baryons constiute rather small fraction of the present energy density in the Universe.

Photons contribute even smaller fraction, as is clear from Eq. (12), namely $\Omega_\gamma \approx 5 \cdot 10^{-5}$. From electric neutrality, the number density of **electrons** is about the same as that of baryons, so electrons contribute negligible fraction to the total mass density. The remaining known stable particles are **neutrinos**. Their number density is calculable in Hot Big Bang theory and these calculations are nicely confirmed by Big Bang Nucleosynthesis. The present number density of each type of neutrinos is

$$n_{\nu_\alpha,0} = 110 \text{ cm}^{-3} \ ,$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$ (more appropriately, ν_α are neutrino mass eigenstates). Direct limit on the mass of electron neutrino, $m_{\nu_e} < 2\,\text{eV}$, together with the observations of neutrino oscillations suggest that every type of neutrino has mass smaller than 2 eV (neutrinos with masses above 0.05 eV must be degenerate, according to neutrino oscillation data). The energy density of all types of neutrinos is thus smaller than ρ_c:

$$\rho_{\nu,total} = \sum_\alpha m_{\nu_\alpha} n_{\nu_\alpha} < 3 \cdot 2\,\text{eV} \cdot 110 \, \frac{1}{\text{cm}^3} \sim 6 \cdot 10^{-7} \, \frac{\text{GeV}}{\text{cm}^3} \ ,$$

which means that $\Omega_{\nu,total} < 0.12$. This estimate does not make use of any cosmological data. In fact, cosmological observations give stronger bound

$$\Omega_{\nu,total} \lesssim 0.014 \ . \tag{24}$$

This bound is mostly due to the analysis of the structures at relatively small length scales, and has to do with streaming of neutrinos from the gravitational potential wells at early times when neutrinos were moving fast. In terms of the neutrino masses the bound (24) reads [10, 11]

$$\sum m_{\nu_\alpha} < 0.6\,\text{eV} \ ,$$

so every neutrino must be lighter than 0.2 eV. It is worth noting that the atmospheric neutrino data, as well as K2K, Minos and T2K experiments tell us that the mass of at least one neutrino must be larger than about 0.05 eV. Comparing these numbers, one sees that it may be feasible to measure neutrino masses by cosmological observations (!) in the future.

Coming back to our main topic here, we conclude that most of the energy density in the present Universe is not in the form of known particles; most energy in the present Universe must be in “something unknown”. Furthermore, this “something unknown” has two components: clustered (dark matter) and unclustered (dark energy).

Clustered dark matter consists presumably of new stable massive particles. These make clumps of energy (mass) which constitute most of the mass of galaxies and clusters of galaxies. There are various ways of estimating the contribution of non-baryonic dark matter into the total energy density of the Universe (see Ref. [2] for details):

– Composition of the Universe affects the angular anisotropy of cosmic microwave background. Quite accurate measurements of the CMB anisotropy, available today, enable one to estimate the total mass density of dark matter.

– Composition of the Universe, and especially the density of non-baryonic dark matter, is crucial for structure formation of the Universe. Comparison of the results of numerical simulations of structure formation with observational data gives reliable estimate of the mass density of non-baryonic clustered dark matter.

The bottom line is that the non-relativistic component constitutes about 27 per cent of the total present energy density, which means that non-baryonic dark matter has

$$\Omega_{DM} \approx 0.22 \; , \tag{25}$$

the rest is due to baryons.

There is direct evidence that dark matter exists in the largest gravitationally bound objects – clusters of galaxies. There are various methods to determine the gravitating mass of a cluster, and even mass distribution in a cluster, which give consistent results. To name a few:

– One measures velocities of galaxies in galactic clusters, and makes use of the gravitational virial theorem,

$$\text{Kinetic energy of a galaxy} = \frac{1}{2}\,\text{Potential energy} \; .$$

In this way one obtains the gravitational potential, and thus the distribution of the total mass in a cluster.

– Another measurement of masses of clusters makes use of intracluster gas. Its temperature obtained from X-ray measurements is also related to the gravitational potential.

– Fairly accurate reconstruction of mass distributions in clusters is obtained from the observations of gravitational lensing of background galaxies by clusters.

These methods enable one to measure mass-to-light ratio in clusters of galaxies. Assuming that this ratio applies to all matter in the Universe[3], one arrives at the estimate for the mass density of clumped matter in the present Universe. Remarkably, this estimate agrees with Eq. (25).

Finally, dark matter exists also in galaxies. Its distribution is measured by the observations of rotation velocities of distant stars and gas clouds around a galaxy.

Thus, cosmologists are confident that much of the energy density in our Universe consists of new stable particles. We will see that there is good chance for the LHC to produce these particles.

Unclustered dark energy. Non-baryonic clustered dark matter is not the whole story. Making use of the above estimates, one obtains an estimate for the energy density of all particles, $\Omega_\gamma + \Omega_B + \Omega_{\nu,total} + \Omega_{DM} \approx 0.27$. This implies that 73 per cent of the energy density is unclustered. This component is called dark energy; it has the properties similar to those of vacuum. We will briefly discuss dark energy in Section 5.

All this fits nicely all cosmological observations, but does not fit to the Standard Model of particle physics. It is our hope that the LHC will shed light at least on some of the properties of the Universe.

[3]This is a fairly strong assumption, since only about 10 per cent of galaxies are in clusters.

2.6 Regimes of cosmological expansion

The cosmological expansion at the present epoch is determined mostly by dark energy, since its contribution to the right hand side of the Friedmann equation (21) is the largest,

$$\Omega_\Lambda = 0.73 \ .$$

Non-relativistic matter (dark matter and baryons) is also non-negligible,

$$\Omega_M = 0.27 \ , \tag{26}$$

while the energy density of relativistic matter (photons and neutrinos, if one of the neutrino species is massless or very light) is negligible today. This was not always the case. Making use of Eq. (10) for photons and relativistic neutrinos, Eq. (17) for non-relativistic matter, and assuming for definiteness that dark energy density is constant in time, we can rewrite the Friedmann equation (21) in the following form

$$\begin{aligned} H^2(t) &= \frac{8\pi}{3M_{Pl}^2}[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)] \\ &= H_0^2 \left[\Omega_\Lambda + \Omega_M \left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a(t)}\right)^4 \right] \ . \end{aligned} \tag{27}$$

It is appropriate for our purposes to treat neutrinos as massless particles; including their contribution to Ω_{rad} one has

$$\Omega_{rad} = 8.4 \cdot 10^{-5} \ . \tag{28}$$

Equation (27) tells that at early times, when the scale factor $a(t)$ was small, the expansion was dominated by relativistic matter ("radiation"), later on there was long period of domination of the non-relativistic matter, and in future the expansion will be dominated by dark energy,

$$\ldots \Longrightarrow \text{Radiation domination} \Longrightarrow \text{Matter domination} \Longrightarrow \Lambda\text{–domination} \ .$$

Dots here denote some cosmological epoch preceding the hot stage of evolution; as we discuss in Section 6, we are confident that such an epoch existed, but do not quite know what it was. Making use of (26) and (28), it is straightforward to find the redshift at radiation–matter equality, when the first two terms in (27) are equal,

$$1 + z_{eq} = \frac{a_0}{a(t_{eq})} = \frac{\Omega_M}{\Omega_{rad}} \approx 3000 \ ,$$

and using the Friedmann equation one finds the age of the Universe at equality

$$t_{eq} \approx 60\,000 \text{ years} \ .$$

Note that recombination occured at matter domination, but rather soon after equality, see (7).

It is useful for what follows to find the evolution of the scale factor at the radiation domination epoch. At that time the energy density is given by Eq. (16), so that the Friedmann equation can be written as follows

$$H = \frac{T^2}{M_{Pl}^*} \ , \tag{29}$$

where $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*})$. Now, we neglect for simplicity the dependence of g_* on temperature, and hence on time, and recall that in this case the temperature scales as a^{-1}, see Eq. (14). Hence, we obtain

$$\frac{\dot{a}}{a} = \frac{\text{const}}{a^2} \ .$$

This gives the desired evolution law

$$a(t) = \text{const} \cdot \sqrt{t} \ . \tag{30}$$

The constant here does not have physical significance, as one can rescale the coordinates $\mathbf{x}$ at some fixed moment of time, thus changing the normailzation of a.

There are several points to note regarding the result (30). First, the expansion *decelerates*:

$$\ddot{a} < 0 \ .$$

This property holds also for the matter dominated epoch, but, as we see momentarily, it does not hold for domination of the dark energy.

Second, time $t = 0$ is the Big Bang singularity (assuming erroneously that the Universe starts being radiation dominated). The expansion rate

$$H(t) = \frac{1}{2t}$$

diverges as $t \to 0$, and so does the energy density $\rho(t) \propto H^2(t)$ and temperature $T \propto \rho^{1/4}$. Of course, the classical General Relativity and usual notions of statistical mechanics (e.g., temperature itself) are not applicable very near the singularity, but our result suggests that in the picture we discuss (hot epoch right after the Big Bang), the Universe starts its classical evolution in a very hot and dense state, and its expansion rate is very high in the beginning. It is customary to assume for illustrational purposes that the relevant quantities in the beginning of the classical expansion take the Planck values, $\rho \sim M_{Pl}^4$, $H \sim M_{Pl}$, etc.

Third, at a given moment of time the size of a causally connected region is finite. Consider signals emitted right after the Big Bang and travelling with the speed of light. These signals travel along the light cone with $ds = 0$, and hence $a(t)dx = dt$. So, the coordinate distance that a signal travels from the Big Bang to time t is

$$x = \int_0^t \frac{dt}{a(t)} \equiv \eta \ . \tag{31}$$

In the radiation dominated Universe

$$\eta = \text{const} \cdot \sqrt{t} \ .$$

The physical distance from the emission point to the position of the signal is

$$l_{H,t} = a(t)x = a(t) \int_0^t \frac{dt}{a(t)} = 2t \ .$$

As expected, this physical distance is finite, and it gives the size of a causally connected region at time t. It is called the horizon size (more precisely, the size of particle horizon). A related property is that an observer at time t can see only the part of the Universe whose current physical size is $l_{H,t}$. Both at radiation and matter domination one has, modulo numerical constant of order 1,

$$l_{H,t} \sim H^{-1}(t) \ .$$

To give an idea of numbers, the horizon size at the present epoch is

$$l_{H,t_0} \approx 15 \text{ Gpc} \simeq 4.5 \cdot 10^{28} \text{ cm} \ .$$

One property of the Universe that starts its expansion from rafiation domination is puzzling. Using Eq. (31) one sees that the size of the observable Universe increases in time. For example, the coordinate size of the present horizon is about 50 times larger that the coordinate size of the horizon at recombination. Hence, when performing CMB observations we see 50^2 regions on the sphere of last scattering

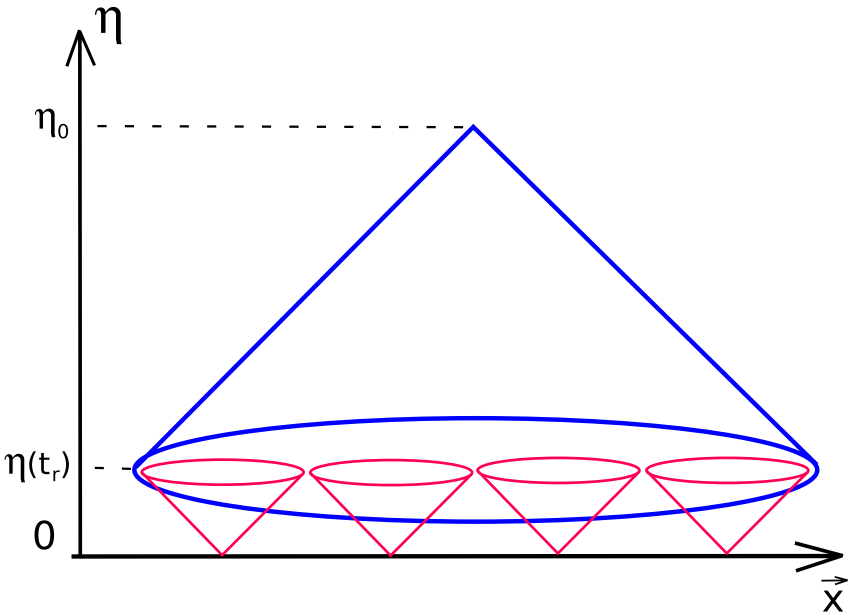

Fig. 1: Causal structure of space-time in hot Big Bang theory

which were causally disconnected at the recombination epoch, see Fig. 1. Yet they look exactly the same! Clearly, this is a problem for the hot Big Bang theory, which is called horizon problem. We will see in Section 6 that this problem has a somewhat different side, which unambiguously shows that the hot Big Bang theory is not the whole story: the hot epoch was preceeded by some other, very different epoch of the cosmological evolution.

To end up this section, let us note that the properties of the Universe dominated by dark energy are quite different. Assuming for definiteness that ρ_Λ is independent of time, we immediately find the solution to the Friedmann equation for the Λ-dominated Universe:

$$a(t) = \text{const} \cdot \text{e}^{H_\Lambda t} \; , \tag{32}$$

where $H_\Lambda = \sqrt{8\pi\rho_\Lambda/3M_{Pl}^2}$. The cosmological expansion *accelerates*,

$$\ddot{a} > 0 \; .$$

The dark energy was introduced precisely for explaining the accelerated expansion of the Universe at the present epoch.

3 Dark matter

Dark matter is characterized by the mass-to-entropy ratio,

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} \approx \frac{0.22 \cdot 5 \cdot 10^{-6}\,\text{GeV} \cdot \text{cm}^{-3}}{3000\ \text{cm}^{-3}} = 4 \cdot 10^{-10}\,\text{GeV} \; . \tag{33}$$

This ratio is constant in time since the freeze out of dark matter density: both number density of dark matter particles n_{DM} (and hence their mass density $\rho_{DM} = m_{DM}n_{DM}$) and entropy density dilute exactly as a^{-3}.

Dark matter is crucial for our existence, for the following reason. Density perturbations in baryon-electron-photon plasma before recombination do not grow because of high pressure, which is mostly due to photons; instead, perturbations are sound waves propagating in plasma with time-independent amplitudes. Hence, in a Universe without dark matter, density perturbations in baryonic component would start to grow only after baryons decouple from photons, i.e., after recombination. The mechanism of the growth is pretty simple: an overdense region gravitationally attracts surrounding matter; this matter falls into the overdense region, and the density contrast increases. In the expanding matter dominated Universe this gravitational instability results in the density contrast growing like $(\delta\rho/\rho)(t) \propto a(t)$. Hence, in a Universe without dark matter, the growth factor for baryon density perturbations would be at most[4]

$$\frac{a(t_0)}{a(t_{rec})} = 1 + z_{rec} = \frac{T_{rec}}{T_0} \approx 10^3 \ . \tag{34}$$

The initial amplitude of density perturbations is very well known from the CMB anisotropy measurements, $(\delta\rho/\rho)_i = 5 \cdot 10^{-5}$. Hence, a Universe without dark matter would still be pretty homogeneous: the density contrast would be in the range of a few per cent. No structure would have been formed, no galaxies, no life. No structure would be formed in future either, as the accelerated expansion due to dark energy will soon terminate the growth of perturbations.

Since dark matter particles decoupled from plasma much earlier than baryons, perturbations in dark matter started to grow much earlier. The corresponding growth factor is larger than Eq. (34), so that the dark matter density contrast at galactic and sub-galactic scales becomes of order one, perturbations enter non-linear regime and form dense dark matter clumps at $z = 5 - 10$. Baryons fall into potential wells formed by dark matter, so dark matter and baryon perturbations develop together soon after recombination. Galaxies get formed in the regions where dark matter was overdense originally. The development of perturbations in our Universe is shown in Fig. 2. For this picture to hold, dark matter particles must be non-relativistic early enough, as relativistic particles fly through gravitational wells instead of being trapped there. This means, in particular, that neutrinos cannot constitute a considerable part of dark matter, hence the bound (24).

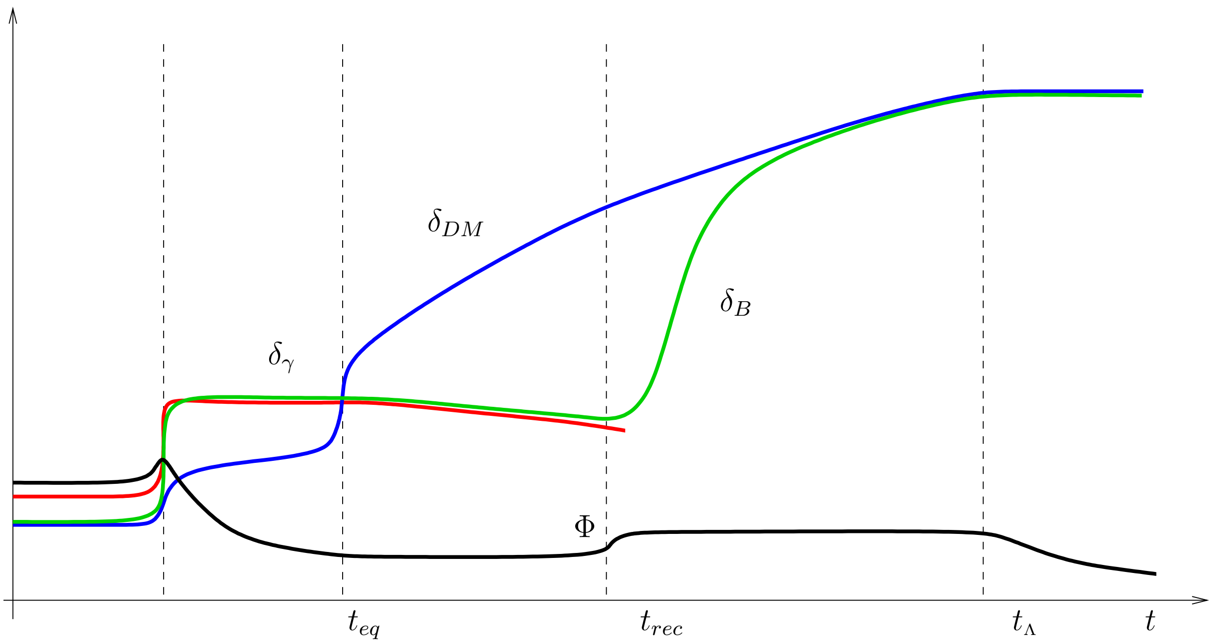

Fig. 2: A sketch of the time dependence, in the linear regime, of density contrasts of dark matter, baryons and photons, $\delta_{DM} \equiv \delta\rho_{DM}/\rho_{DM}$, δ_B and δ_γ, respectively, as well as the Newtonian potential Φ. t_{eq} and t_Λ correspond to the transitions from radiation domination to matter domination, and from decelerated expansion to accelerated expansion, t_{rec} refers to the recombination epoch.

[4]Because of the presence of dark energy, the growth factor is even somewhat smaller.

Depending on the mass of the dark matter particles and mechanism of their production in the early Universe, dark matter may be *cold* (CDM) and *warm* (WDM). Roughly speaking, CDM consists of heavy particles, while the masses of WDM particles are smaller,

$$\text{CDM} : \quad m_{DM} \gtrsim 100\,\text{keV} , \tag{35a}$$

$$\text{WDM} : \quad m_{DM} = 3 \; - \; 30\,\text{keV} . \tag{35b}$$

This assumes that the dark matter particles were in thermal (kinteic) equilibrium at some early times, or, more generally, that their kinetic energy was comparable to temperature. This need not be the case for very weakly interacting particles; a well known example is axions which are *cold* dark matter candidates despite their very small mass. Likewise, very weakly interacting warm dark matter particles may be much heavier than Eq. (35b) suggests.

We will discuss warm dark matter option later on, and now we move on to CDM.

3.1 WIMPS: best guess for cold dark matter

There is a simple mechanism of the dark matter generation in the early Universe. It applies to *cold* dark matter. Because of its simplicity and robustness, it is considered by many as a very likely one, and the corresponding dark matter candidates — weakly interacting massive particles, WIMPs — as the best candidates. Let us describe this mechanism in some detail.

Let us assume that there exists a heavy stable neutral particle Y, and that Y-particles can only be destroyed or created via their pair-annihilation or creation, with annihilation products being the particles of the Standard Model. We will see that the overall cosmological behaviour of Y-particles is as follows. At high temperatures, $T \gg m_Y$, the Y-particles are in thermal equilibrium with the rest of cosmic plasma; there are lots of Y-particles in the plasma, which are continuously created and annihilate. As the temperature drops below m_Y, the equilibrium number density decreases. At some "freeze-out" temperature T_f the number density becomes so small, that Y-particles can no longer meet each other during the Hubble time, and their annihilation terminates. After that the number density of survived Y's decreases like a^{-3}, and these relic particles contribute to the mass density in the present Universe. Our purpose is to estimate the range of properties of Y-particles, in which they serve as dark matter.

Assuming thermal equilibrium, elementary considerations of mean free path of a particle in gas give for the lifetime of a non-relativistic Y-particle in cosmic plasma, τ_{ann},

$$\sigma_{ann} \cdot v \cdot \tau_{ann} \cdot n_Y \sim 1 \; ,$$

where v is the velocity of Y-particle, σ_{ann} is the annihilation cross section at velocity v and n_Y is the equilibrium number density given by the Boltzmann law at zero chemical potential,

$$n_Y = g_Y \cdot \left(\frac{m_Y T}{2\pi} \right)^{3/2} \mathrm{e}^{-\frac{m_Y}{T}} \; ,$$

where g_Y is the number of spin states of Y-particle. Note that we consider non-relativistic regime, $m_Y \ll T$. Let us introduce the notation

$$\sigma_{ann} v = \sigma_0$$

(in fact, the left hand side is to be understood as thermal average). If the annihilation occurs in s-wave, then σ_0 is a constant independent of temperature, for p-wave it is somewhat suppressed at $T \ll m_Y$. One should compare the lifetime with the Hubble time, or annihilation rate $\Gamma_{ann} \equiv \tau_{ann}^{-1}$ with the expansion rate H. At $T \sim m_Y$, the equilibrium density is of order $n_Y \sim T^3$, and $\Gamma_{ann} \gg H$ for not too small σ_0. This means that annihilation (and, by reciprocity, creation) of Y-pairs is indeed rapid, and Y-particles are indeed in thermal equilibrium with the plasma. At very low temperature, on the other hand, the equilibrium number density $n_Y^{(eq)}$ is exponentially small, and the equilibrium rate is small,

$\Gamma_{ann}^{(eq)} \ll H$. At low temperatures we cannot, of course, make use of the equilibrium formulas: Y-particles no longer annihilate (and, by reciprocity, are no longer created), there is no thermal equilibrium with respect to creation–annihilation processes, and the number density n_Y gets diluted only because of the cosmological expansion.

The freeze-out temperature T_f is determined by the relation

$$\tau_{ann}^{-1} \equiv \Gamma_{ann} \sim H \ ,$$

where we can still use the equilibrium formulas, as Y-particles are in thermal equilibrium (with respect to annihilation and creation) just before freeze-out. Making use of the relation (29) between the Hubble parameter and temperature at radiation domination, we obtain

$$\sigma_0(T_f) \cdot n_Y(T_f) \sim \frac{T_f^2}{M_{Pl}^*} \ , \tag{36}$$

or

$$\sigma_0(T_f) \cdot g_Y \cdot \left(\frac{m_Y T_f}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{m_Y}{T_f}} \sim \frac{T_f^2}{M_{Pl}^*} \ .$$

The latter equation gives the freeze-out temperature, which, up to loglog corrections, is

$$T_f \approx \frac{m_Y}{\ln(M_{Pl}^* m_Y \sigma_0)} \tag{37}$$

(the possible depndence of σ_0 on temperature is irrelevant in the right hand side: we are doing the calculation in the leading-log approximation anyway). Note that this temperature is somewhat lower than m_Y, if the relevant microscopic mass scale is much below M_{Pl}. This means that Y-particles freeze out when they are indeed non-relativistic, hence the term “cold dark matter”. The fact that the annihilation and creation of Y-particles terminate at relatively low temperature has to do with rather slow expansion of the Universe, which should be compensated for by the smallness of the number density n_Y.

At the freeze-out temperature, we make use of Eq. (36) and obtain

$$n_Y(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0(T_f)} \ .$$

Note that this density is inversely proportional to the annihilation cross section (modulo logarithm). The reason is that for higher annihilation cross section, the creation–annihilation processes are longer in equilibrium, and less Y-particles survive.

Up to a numerical factor of order 1, the number-to-entropy ratio at freeze-out is

$$\frac{n_Y}{s} \simeq \frac{1}{g_*(T_f) M_{Pl}^* T_f \sigma_0(T_f)} \ . \tag{38}$$

This ratio stays constant until the present time, so the present number density of Y-particles is $n_{Y,0} = s_0 \cdot (n_Y/s)_{freeze-out}$, and the mass-to-entropy ratio is

$$\frac{\rho_{Y,0}}{s_0} = \frac{m_Y n_{Y,0}}{s_0} \simeq \frac{\ln(M_{Pl}^* m_Y \sigma_0)}{g_*(T_f) M_{Pl}^* \sigma_0(T_f)} \simeq \frac{\ln(M_{Pl}^* m_Y \sigma_0)}{\sqrt{g_*(T_f)} M_{Pl} \sigma_0(T_f)} \ ,$$

where we made use of Eq. (37). This formula is remarkable. The mass density depends mostly on one parameter, the annihilation cross section σ_0. The dependence on the mass of Y-particle is through the logarithm and through $g_*(T_f)$; it is very mild. The value of the logarithm here is between 30 and 40, depending on parameters (this means, in particular, that freeze-out occurs when the temperature drops 30 to 40 times below the mass of Y-particle). Plugging in other numerical values ($g_*(T_f) \sim 100$,

$M_{Pl}^* \sim 10^{18}$ GeV), as well as numerical factor omitted in Eq. (38), and comparing with Eq. (33) we obtain the estimate

$$\sigma_0(T_f) \equiv \langle \sigma v \rangle (T_f) = (1 \div 2) \cdot 10^{-36} \text{ cm}^2 \ . \tag{39}$$

This is a weak scale cross section, which tells us that the relevant energy scale is TeV. We note in passing that the estimate (39) is quite precise and robust.

If the annihilation occurs in s-wave, the annihilation cross section may be parametrized as $\sigma_0 = \alpha^2/M^2$ where α is some coupling constant, and M is a mass scale (which may be higher than m_Y). This parametrization is suggested by the picture of Y pair-annihilation via the exchange by another particle of mass M. With $\alpha \sim 10^{-2}$, the estimate for the mass scale is roughly $M \sim 1$ TeV. Thus, with very mild assumptions, we find that the non-baryonic dark matter may naturally originate from the TeV-scale physics. In fact, what we have found can be understood as an approximate equality between the cosmological parameter, mass-to-entropy ratio of dark matter, and the particle physics parameters,

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \ .$$

Both are of order 10^{-10} GeV, and it is very tempting to think that this is not a mere coincidence. If it is not, the dark matter particle should be found at the LHC.

Of course, the most prominent candidate for WIMP is neutralino of the supersymmetric extensions of the Standard Model. The situation with neutralino is somewhat tense, however. The point is that the pair-annihilation of neutralinos often occurs in p-wave, rather than s-wave. This gives the suppression factor in $\sigma_0 \equiv \langle \sigma_{ann} v \rangle$, proportional to $v^2 \sim T_f/m_Y \sim 1/30$. Hence, neutralinos tend to be overproduced in most of the parameter space of MSSM and other models. Yet neutralino remains a good candidate, especially at high $\tan\beta$.

3.2 Warm dark matter: light gravitinos

The cold dark matter scenario successfully describes the bulk of the cosmological data. Yet, there are clouds above it. First, according to numerical simulations, CDM scenario tends to overproduce small objects — dwarf galaxies: it predicts hundreds of satellite dwarf galaxies in the vicinity of a large galaxy like Milky Way, whereas only dozens of satellites have been observed so far. Second, again according to simulations, CDM tends to produce too high densities in galactic centers (cusps in density profiles); this feature is not confirmed by observations either. There is no strong discrepancy yet, but one may be motivated to analyse a possibility that dark matter is not that cold.

An alternative to CDM is warm dark matter whose particles decouple being relativistic. Let us assume for definiteness that they are in kinetic equilibrium with cosmic plasma when their number density freezes out (thermal relic). After kinetic equilibrium breaks down, and WDM particles decouple completely, their spatial momenta decrease as a^{-1}, i.e., the momenta are of order T all the time after decoupling. WDM particles become non-relativistic at $T \sim m$, where m is their mass. Only after that the WDM perturbations start to grow[5]: as we mentioned above, relativistic particles escape from gravitational potentials, so the gravitational potentials get smeared out instead of getting deeper. Before becoming non-relativistic, WDM particles travel the distance of the order of the horizon size; the WDM perturbations therefore are suppressed at those scales. The horizon size at the time t_{nr} when $T \sim m$ is of order

$$l(t_{nr}) \simeq H^{-1}(T \sim m) = \frac{M_{Pl}^*}{T^2} \sim \frac{M_{Pl}^*}{m^2} \ .$$

Due to the expansion of the Universe, the corresponding length at present is

$$l_0 = l(t_{nr}) \frac{a_0}{a(t_{nr})} \sim l(t_{nr}) \frac{T}{T_0} \sim \frac{M_{Pl}}{m T_0} \ , \tag{40}$$

[5]The situation in fact is somewhat more complicated, but this simplified picture will be sufficient for our estimates.

where we neglected (rather weak) dependence on g_*. Hence, in WDM scenario, structures of sizes smaller than l_0 are less abundant as compared to CDM. Let us point out that l_0 refers to the size of the perturbation as if it were in the linear regime; in other words, this is the size of the region from which matter collapses into a compact object.

The present size of a dwarf galaxy is a few kpc, and the density is about 10^6 of the average density in the Universe. Hence, the size l_0 for these objects is of order 100 kpc $\simeq 3 \cdot 10^{23}$ cm. Requiring that perturbations of this size, but not much larger, are suppressed, we obtain from Eq. (40) the estimate (35b) for the mass of WDM particles.

Among WDM candidates, light gravitino is probably the best motivated. The gravitino mass is of order

$$m_{3/2} \simeq \frac{F}{M_{Pl}} ,$$

where $\sqrt{F}$ is the supersymmetry breaking scale. Hence, gravitino masses are in the right ballpark for rather low supersymmetry breaking scales, $\sqrt{F} \sim 10^6 - 10^7\,\text{GeV}$. This is the case, e.g., in gauge mediation scenario. With so low mass, gravitino is the lightest supersymmetric particle (LSP), so it is stable in many supersymmetric extensions of the Standard Model. From this viewpoint gravitinos can indeed serve as dark matter particles. For what follows, important parameters are the widths of decays of other superpartners into gravitino and the Standard Model particles. These are of order

$$\Gamma_{\tilde{S}} \simeq \frac{M_{\tilde{S}}^5}{F^2} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} , \tag{41}$$

where $M_{\tilde{S}}$ is the mass of the superpartner.

One mechanism of the gravitino production in the early Universe is decays of other superpartners. Gravitino interacts with everything else so weakly, that once produced, it moves freely, without interacting with cosmic plasma. At production, gravitinos are relativistic, hence they are indeed *warm* dark matter candidates. Let us assume that production in decays is the dominant mechanism and consider under what circumstances the present mass density of gravitinos coincides with that of dark matter.

The rate of gravitino production in decays of superpartners of the type $\tilde{S}$ in the early Universe is

$$\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s}\Gamma_{\tilde{S}} ,$$

where $n_{3/2}$ and $n_{\tilde{S}}$ are number densities of gravitinos and superpartners, respectively, and s is the entropy density. For superpartners in thermal equilibrium, one has $n_{\tilde{S}}/s = \text{const} \sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, and $n_{\tilde{S}}/s \propto \exp(-M_{\tilde{S}}/T)$ at $T \ll M_{\tilde{S}}$. Hence, the production is most efficient at $T \sim M_{\tilde{S}}$, when the number density of superpartners is still large, while the Universe expands most slowly. The density of gravitinos produced in decays of $\tilde{S}$'s is thus given by

$$\frac{n_{3/2}}{s} \simeq \left(\frac{d(n_{3/2}/s)}{dt} \cdot H^{-1} \right)_{T \sim M_{\tilde{S}}} \simeq \frac{\Gamma_{\tilde{S}}}{g_*} H^{-1}(T \sim M_{\tilde{S}}) \simeq \frac{1}{g_*} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \cdot \frac{M_{Pl}^*}{M_{\tilde{S}}^2} .$$

This gives the mass-to-entropy ratio today:

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \sum_{\tilde{S}} \frac{M_{\tilde{S}}^3}{g_*^{3/2} M_{Pl} m_{3/2}} , \tag{42}$$

where the sum runs over all superpartner species *which have ever been relativistic in thermal equilibrium*. The correct value (33) is obtained for gravitino masses in the range (35b) at

$$M_{\tilde{S}} = 100 - 300\,\text{GeV} . \tag{43}$$

Thus, the scenario with gravitino as warm dark matter particle requires light superpartners, which are to be discovered at the LHC.

A few comments are in order. First, decays of superpartners is not the only mechanism of gravitino production: gravitinos may also be produced in scattering of superpartners. To avoid overproduction of gravitinos in the latter processes, one has to assume that the maximum temperature in the Universe (reached after post-inflationary reheating stage) is quite low, $T_{max} \sim 1 - 10\,\text{TeV}$. This is not a particularly plausible assumption, but it is consistent with everything else in cosmology and can indeed be realized in some models of inflation. Second, existing constraints on masses of strongly interacting superpartners (gluinos and squarks) suggest that their masses exceed Eq. (43). Hence, these particles should not contribute to the sum in Eq. (42), otherwise WDM gravitinos would be overproduced. This is possible, if masses of squarks and gluinos are larger than T_{max}, so that they were never abundant in the early Universe. Third, gravitino produced in decays of superpartners is *not* a thermal relic, as it was never in thermal equilibrium with the rest of cosmic plasma. Nevertheless, since gravitinos are produced at $T \sim M_{\tilde{S}}$ and at that time have energy $E \sim M_{\tilde{S}} \sim T$, our estimate (40) does apply. Finally, the decay into gravitino and the Standard Model particles is the only decay channel for the next-to-lightest superpartner (NLSP). Hence, the estimate for the total width of NLSP is given by Eq. (41), so that

$$c\tau_{NLSP} = \text{a few} \cdot \text{mm} - \text{a few} \cdot 100 \text{ m}$$

for $m_{2/3} = 3 - 30\,\text{keV}$ and $M_{NLSP} = 100 - 300\,\text{GeV}$. Thus, NLSP should either be visible in a detector, or fly it through.

Needless to say, the warm gravitino scenario is a lot more contrived than the WIMP option. It is reassuring, however, that it can be ruled out or confirmed at the LHC.

3.3 Discussion

If dark matter particles are indeed WIMPs, and the relevant energy scale is of order 1 TeV, then the Hot Big Bang theory will be probed experimentally up to temperature of (a few) $\cdot\,(10 - 100)$ GeV and down to age $10^{-9} - 10^{-11}$ s in relatively near future (compare to 1 MeV and 1 s accessible today through Big Bang Nucleosynthesis). With microscopic physics to be known from collider experiments, the WIMP density will be reliably calculated and checked against the data from observational cosmology. Thus, WIMP scenario offers a window to a very early stage of the evolution of the Universe.

If dark matter particles are gravitinos, the prospect of probing quantitatively so early stage of the cosmological evolution is not so bright: it would be very hard, if at all possible, to get an experimental handle on the gravitino mass; furthermore, the present gravitino mass density depends on an unknown reheat temperature T_{max}. On the other hand, if this scenario is realized in Nature, then the whole picture of the early Universe will be quite different from our best guess on the early cosmology. Indeed, gravitino scenario requires low reheat temperature, which in turn calls for rather exotic mechanism of inflation.

The mechanisms discussed here are by no means the only ones capable of producing dark matter, and WIMPs and gravitinos are by no means the only candidates for dark matter particles. Other dark matter candidates include axions, sterile neutrinos, Q-balls, very heavy relics produced towards the end of inflation, etc. Hence, even though there are grounds to hope that the dark matter problem will be solved by the LHC, there is no guarantee at all.

4 Baryon asymmetry of the Universe

In the present Universe, there are baryons and almost no antibaryons. The number density of baryons today is characterized by the ratio η_B, see Eq. (18). In the early Universe, the appropriate quantity is

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s} \; ,$$

where $n_{\bar{B}}$ is the number density of antibaryons, and s is the entropy density. If the baryon number is conserved, and the Universe expands adiabatically, Δ_B is constant, and its value, up to a numerical factor, is equal to η (cf. Eqs. (11) and (15)). More precisely,

$$\Delta_B \approx 0.8 \cdot 10^{-10} .$$

At early times, at temperatures well above 100 MeV, cosmic plasma contained many quark-antiquark pairs, whose number density was of the order of the entropy density,

$$n_q + n_{\bar{q}} \sim s ,$$

while the baryon number density was related to densities of quarks and antiquarks as follows (baryon number of a quark equals $1/3$),

$$n_B = \frac{1}{3}(n_q - n_{\bar{q}}) .$$

Hence, in terms of quantities characterizing the very early epoch, the baryon asymmetry may be expressed as

$$\Delta_B \sim \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} .$$

We see that there was one extra quark per about 10 billion quark-antiquark pairs! It is this tiny excess that is responsible for the entire baryonic matter in the present Universe: as the Universe expanded and cooled down, antiquarks annihilated with quarks, and only the excessive quarks remained and formed baryons.

There is no logical contradiction to suppose that the tiny excess of quarks over antiquarks was built in as an initial condition. This is not at all satisfactory for a physicist, however. Furthermore, inflationary scenario does not provide such an initial condition for the hot Big Bang epoch; rather, inflation theory predicts that the Universe was baryon-symmetric just after inflation. Hence, one would like to explain the baryon asymmetry dynamically.

The baryon asymmetry may be generated from initially symmetric state only if three necessary conditions, dubbed Sakharov's conditions, are satisfied. These are

(i) baryon number non-conservation;

(ii) C- and CP-violation;

(iii) deviation from thermal equilibrium.

All three conditions are easily understood. (i) If baryon number were conserved, and initial net baryon number in the Universe was zero, the Universe today would still be symmetric. (ii) If C or CP were conserved, then the rate of reactions with particles would be the same as the rate of reactions with antiparticles, and no asymmetry would be generated. (iii) Thermal equilibrium means that the system is stationary (no time dependence at all). Hence, if the initial baryon number is zero, it is zero forever, unless there are deviations from thermal equilibrium.

There are two well understood mechanisms of baryon number non-conservation. One of them emerges in Grand Unified Theories and is due to the exchange of super-massive particles. It is similar, say, to the mechanism of charm non-conservation in weak interactions, which occurs via the exchange of heavy W-bosons. The scale of these new, baryon number violating interactions is the Grand Unification scale, presumably of order $M_{GUT} \simeq 10^{16}$ GeV. It is rather unlikely that the baryon asymmetry was generated due to this mechanism: the relevant temperature would be of order M_{GUT}, while so high reheat temperature after inflation is difficult to obtain.

Another mechanism is non-perturbative [12] and is related to the triangle anomaly in the baryonic current (a keyword here is "sphaleron" [13, 14]). It exists already in the Standard Model, and, possibly with slight modifications, operates in all its extensions. The two main features of this mechanism, as applied to the early Universe, is that it is effective over a wide range of temperatures, $100\,\text{GeV} < T < 10^{11}$ GeV, and that it conserves $(B-L)$.

Let us pause here to discuss the physics behind electroweak baryon and lepton number non-conservation in little more detail, though still at a qualitative level. A detailed analysis can be found in the book [15] and in references therein.

The first object to consider is the baryonic current,

$$B^\mu = \frac{1}{3} \cdot \sum_i \bar{q}_i \gamma^\mu q_i \ ,$$

where the sum runs over quark flavors. Naively, the baryonic current is conserved, but at the quantum level its divergence is non-zero, the effect called triangle anomaly (similar effect goes under the name of axial anomaly in the context of QED and QCD),

$$\partial_\mu B^\mu = \frac{1}{3} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \ ,$$

where $F^a_{\mu\nu}$ and g are the field strength of the $SU(2)_W$ gauge field and the $SU(2)_W$ gauge coupling, respectively. Likewise, each leptonic current ($n = e, \mu, \tau$) is anomalous,

$$\partial_\mu L^\mu_n = \frac{g^2}{32\pi^2} \cdot \epsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \ .$$

A non-trivial fact is that there exist large field fluctuations, $F^a_{\mu\nu}(\mathbf{x}, t) \propto g^{-1}$ which have

$$Q \equiv \int d^3x dt \, \frac{g^2}{32\pi^2} \cdot \epsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \neq 0 \ .$$

Furthermore, for any such fluctuation the value of Q is integer. Suppose now that a fluctuation with non-vanishing Q has occured. Then the baryon numbers in the end and beginning of the process are different,

$$B_{fin} - B_{in} = \int d^3x dt \, \partial_\mu B^\mu = 3Q \ . \tag{44}$$

Likewise

$$L_{n,\,fin} - L_{n,\,in} = Q \ . \tag{45}$$

This explains the selection rule mentioned above: B is violated, $(B-L)$ is not.

At zero temprature, the large field fluctuations that induce baryon and lepton number violation are vacuum fluctuations, called instantons, which to a certain extent are similar to virtual fields that emerge and disappear in vacuum of quantum field theory at the perturbative level. The peculiarity is that instantons are *large* field fluctuations. The latter property results in the exponential suppression of the probability of their emergence, and hence the rate of baryon number violating processes, by a factor

$$\mathrm{e}^{-\frac{16\pi^2}{g^2}} \sim 10^{-165} \ .$$

On the other hand, at high temperatures there are large *thermal* fluctuations ("sphalerons") whose rate is not necessarily small. And, indeed, B-violation in the early Universe is rapid as compared to the cosmological expansion at sufficiently high temperatures, provided that

$$\langle\phi\rangle_T < T \ , \tag{46}$$

where $\langle\phi\rangle_T$ is the Higgs expectation value at temperature T.

One may wonder how baryon number may be not conserved even though there are no baryon number violating terms in the Lagrangian of the Standard Model. To understand what is going on, let us consider a massless *left handed* fermion field in the background of the $SU(2)$ gauge field $\mathbf{A}(\mathbf{x}, t)$, which depends on space-time coordinates in a non-trivial way. As a technicality, we set the temporal component of the gauge field equal to zero, $A_0 = 0$, by the choice of gauge. One way to understand the behavior of the fermion field in the gauge field background is to study the system of eigenvalues of the Dirac Hamiltonian $\{\omega(t)\}$. The Hamiltonian is defined in the standard way

$$H_{Dirac}(t) = i\alpha^i \left(\partial_i - igA_i(\mathbf{x}, t)\right) \frac{1-\gamma_5}{2} ,$$

where $\alpha^i = \gamma^0\gamma^i$, so that the Dirac equation has the Schrödinger form,

$$i\frac{\partial \psi}{\partial t} = H_{Dirac}\psi .$$

We are going to discuss the eigenvalues $\omega_n(t)$ of the operator $H_{Dirac}(t)$, treating t as a parameter. These eigenvalues are found from

$$H_{Dirac}(t)\psi_n = \omega_n(t)\psi_n .$$

At $\mathbf{A} = 0$ the system of levels is shown schematically in Fig. 3. Importantly, there are both positive- and negative-energy levels. According to Dirac, the lowest energy state (Dirac vacuum) has all negative energy levels occupied, and all positive energy levels empty. Occupied positive energy levels (three of them in Fig. 3) correspond to real fermions, while empty negative energy levels describe antifermions (one in Fig. 3). Fermion-antifermion annihilation in this picture is a jump of a fermion from a positive energy level to an unoccupied negative energy level.

As a side remark, this original Dirac picture is, in fact, equivalent to the more conventional (by now) procedure of the quantization of fermion field, which does not make use of the notion of negative energy levels. The discussion that follows can be translated into the conventional language; however, the original Dirac picture turnd out to be a lot more transparent in our context. This is a nice example of the complementarity of various approaches in quantum field theory.

Let us proceed with the discussion of the fermion energy levels in gauge field backgrounds. In weak background fields, the energy levels depend on time (move), but nothing dramatic happens. For adiabatically varying background fields, the fermions merely sit on their levels, while fast changing fields generically give rise to jumps from, say, negative- to positive-energy levels, that is, creation of fermion-antifermion pairs. Needless to say, fermion number $(N_f - N_{\bar{f}})$ is conserved.

The situation is entirely different for the background fields with non-zero Q. The levels of left-handed fermions move as shown in the left panel of Fig. 4. Some levels necessarily cross zero, and the net number of levels crossing zero from below equals Q. This means that the number of left-handed fermions is not conserved: for adiabatically varying gauge field $\mathbf{A}(\mathbf{x}, t)$ the motion of levels shown in the left panel of Fig. 4 corresponds to the case in which the initial state of the fermionic system is vacuum (no fermions at all) whereas the final state contains Q real fermions (two in the particular case shown). If the evolution of the gauge field is not adiabatic, the result for the fermion number non-conservation is the same: there may be jumps from negative energy levels to positive energy levels, or vice versa. These correspond to creation or annihilation of fermion-antifermion pairs, but the net change of the fermion number (number of fermions minus number of antifermions) remains equal to Q. Importantly, the initial and final field configurations of the gauge field may be trivial, $\mathbf{A} = 0$ (up to gauge transformation), so that fermion number non-conservation may occur due to a fluctuation that begins and ends in the gauge field vacuum[6]. This is precisely an instanton-like vacuum fluctuation. At finite temperatures, processes of this type occur due to thermal fluctuations, sphalerons.

[6]A subtlety here is that in four-dimensional gauge theories, this is impossible for *Abelian* gauge fields, so fermion number non-conservation is inherent in *non-Abelian* gauge theories only.

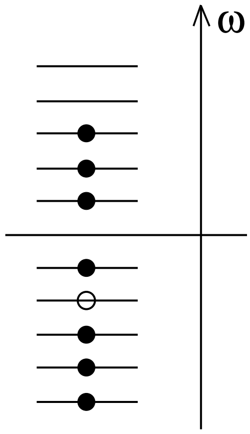

Fig. 3: Fermion energy levels at zero background gauge field.

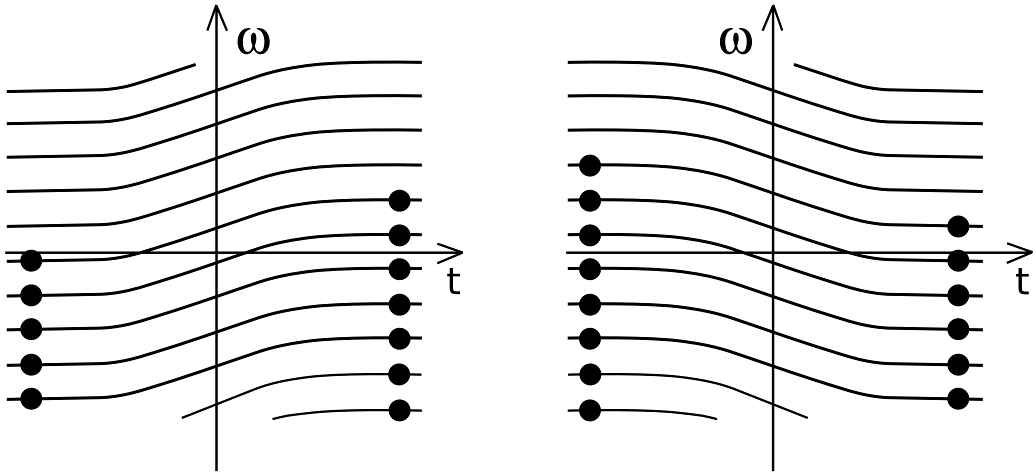

Fig. 4: Motion of fermion levels in background gauge fields with non-vanishing Q (shown is the case $Q = 2$). Left panel: left-handed fermions. Right panel: right-handed fermions.

If the same gauge field interacts also with right-handed fermions, the motion of the levels of the latter is opposite to that of left-handed fermions. This is shown in the right panel of Fig. 4. The change in the number of right-handed fermions is equal to $(-Q)$. So, if the gauge interaction is vector-like, the total fermion number $N_{left} + N_{right}$ is conserved, while chirality $N_{left} - N_{right}$ is violated even for massless fermions. This explains why there is no baryon number violation in QCD. On the other hand, non-perturbative violation of chirality in QCD in the limit of massless quarks has non-trivial consequences, which are indeed confirmed by phenomenology. In this sense anomalous non-conservation of fermion quantum numbers is an experimentally established fact.

In electroweak theory, right-handed fermions do not interact with $SU(2)_W$ gauge field, while left-handed fermions do. Therefore, fermion number is not conserved. Since fermions of each $SU(2)_W$-doublet interact with the $SU(2)_W$ gauge bosons (essentially W and Z) in one and the same way, they are equally created in a process involving a gauge field fluctuation with non-zero Q. This again leads to the relations (44) and (45), i.e., to the selection rules $\Delta B = \Delta L$, $\Delta L_e = \Delta L_\mu = \Delta L_\tau$.

It is tempting to use this mechanism of baryon number non-conservation for explaining the baryon asymmetry of the Universe. There are two problems, however. One is that CP-violation in the Standard Model is too weak: the CKM mechanism alone is insufficient to generate the realistic value of the baryon asymmetry. Hence, one needs extra sources of CP-violation. Another problem has to do with departure from thermal equilibrium that is necessary for the generation of the baryon asymmetry. At temperatures well above 100 GeV electroweak symmetry is restored, the expectation value of the Higgs field ϕ is zero[7], the relation (46) is valid, and the baryon number non-conservation is rapid as compared to the cosmological expansion. At temperatures of order 100 GeV the relation (46) may be violated, but the Universe expands very slowly: the cosmological time scale at these temperatures is

$$H^{-1} = \frac{M_{Pl}^*}{T^2} \sim 10^{-10} \text{ s} , \tag{47}$$

which is very large by the electroweak physics standards. The only way in which strong departure from thermal equilibrium at these temperatures may occur is through the first order phase transition.

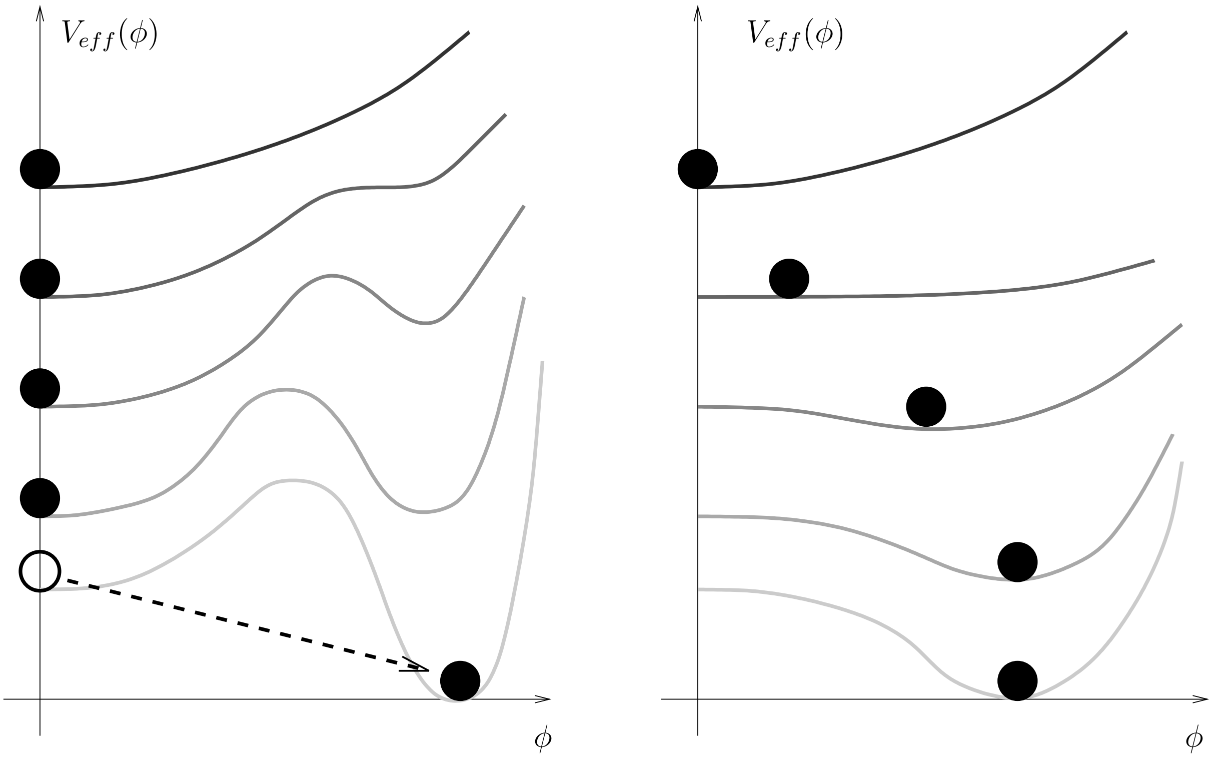

Fig. 5: Effective potential as function of ϕ at different temperatures. Left: first order phase transition. Right: second order phase transition. Upper curves correspond to higher temperatures.

The property that at temperatures well above 100 GeV the expectation value of the Higgs field is zero, while it is non-zero in vacuo, suggests that there may be a phase transition from the phase with $\langle\phi\rangle = 0$ to the phase with $\langle\phi\rangle \neq 0$. The situation is pretty subtle here, as ϕ is not gauge invariant, and hence cannot serve as an order parameter, so the notion of phases with $\langle\phi\rangle = 0$ and $\langle\phi\rangle \neq 0$ is vague. In fact, neither electroweak theory nor most of its extensions have a gauge-invariant order parameter, so there is no real distinction between these “phases”. This situation is similar to that in liquid-vapor system, which does not have an order parameter and may or may not experience vapor-liquid phase transition as temperature decreases, depending on other parameters characterizing this system, e.g., pressure. In the Standard Model the role of such a parameter is played by the Higgs self-coupling λ or, in other words, the Higgs boson mass.

Continuing to use somewhat sloppy terminology, we observe that the interesting case for us is the first order phase transition. In this case the effective potential (free energy density as function of

[7]There are subtleties at this point, see below.

ϕ) behaves as shown in the left panel of Fig. 5. At high temperatures, there exists one minimum of V_{eff} at $\phi = 0$, and the expectation value of the Higgs field is zero. As the temperature decreases, another minimum appears at finite ϕ, and then becomes lower than the minimum at $\phi = 0$. However, the probability of the transition from the phase $\phi = 0$ to the phase $\phi \neq 0$ is very small for some time, so the system gets overcooled. The transition occurs when the temperature becomes sufficiently low, as shown schematically by an arrow in Fig. 5. This is to be contrasted to the case, e.g., of the second order phase transition with the behavior of the effective potential shown in the right panel of Fig. 5. In the latter case, the field slowly evolves, as the temperature decreases, from zero to non-zero vacuum value, and the system remains very close to the thermal equilibrium at all times.

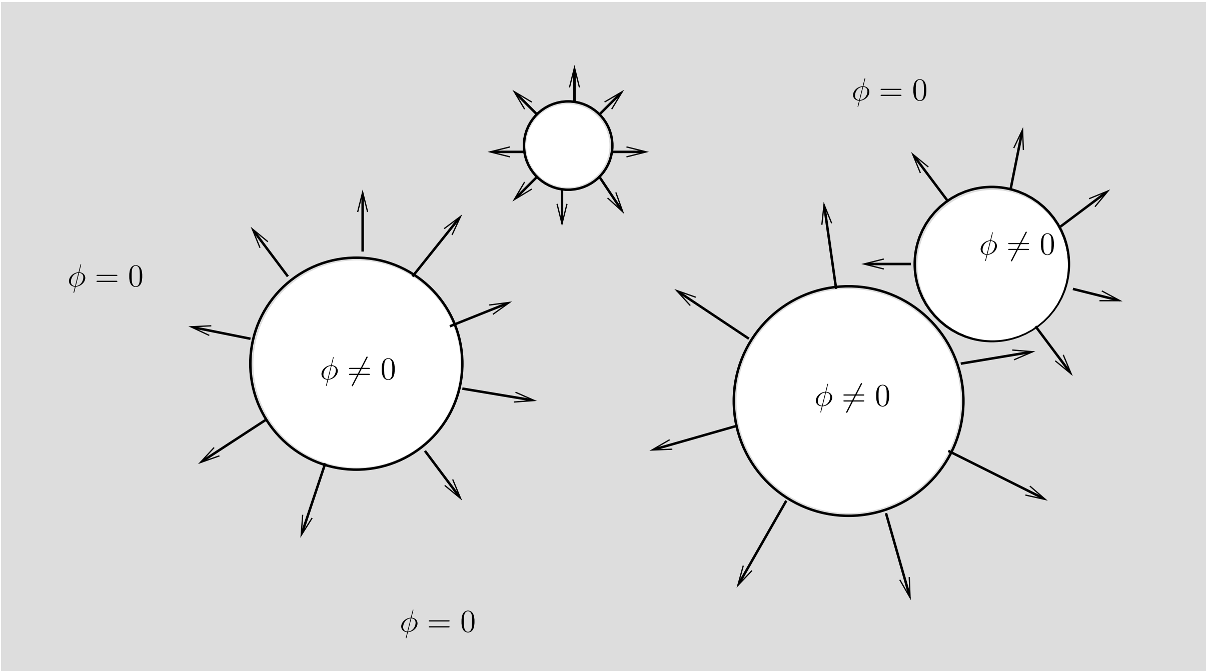

Fig. 6: First order phase transition: boiling Universe.

The first order phase transition occurs via spontaneous creation of bubbles of the new phase inside the old phase. These bubbles then grow, their walls eventually collide, and the new phase finally occupies entire space. The Universe boils, as shown schematically in Fig. 6. In the cosmological context, this process happens when the bubble nucleation rate per Hubble time per Hubble volume is of order 1, $\Gamma_{nucl} \sim H^{-4}$. The velocity of the bubble wall in the relativistic cosmic plasma is roughly of the order of the speed of light (in fact, it is somewhat smaller, from $0.1\ c$ to $0.01\ c$), simply because there are no relevant dimensionless parameters characterizing the system. Hence, the bubbles grow large before their walls collide: their size at collision is roughly of order of the Hubble size. While at nucleation the bubble is microscopic — its size is dictated by the elecroweak scale and is roughly of order $(100\,\text{GeV})^{-1} \sim 10^{-16}$ cm — its size at collision of walls is macroscopic, $H^{-1} \sim$ a few cm, as follows from Eq. (47). Clearly, boiling is a highly inequilibrium process, and one may hope that the baryon asymmetry may be generated at that time. And, indeed, there exist mechanisms of the generation of the baryon asymmetry, which have to do with interactions of quarks and leptons with moving bubble walls. The value of the resulting baryon asymmetry may well be of order 10^{-10}, as required by observations, provided that there is enough CP-violation in the theory.

A necessary condition for the electroweak generation of the baryon asymmetry is that the inequality (46) must be violated *just after* the phase transition. Indeed, in the opposite case the electroweak baryon number violating processes are fast after the transition, and the baryon asymmetry, generated during the transition, is washed out afterwards. Hence, the phase transition must be of strong enough first order. This is *not* the case in the Standard Model. To see why this is so, and to get an idea in which extensions of the Standard Model the phase transition may be of strong enough first order, let us consider the effective potential in some detail. At zero temperature, the Higgs potential has the standard fom,

$$V(\phi) = -\frac{m^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4 .$$

Here

$$|\phi| \equiv \left(\phi^\dagger \phi\right)^{1/2} \tag{48}$$

is the length of the Higgs doublet ϕ, $m^2 = \lambda v^2$ and $v = 247\,\text{GeV}$ is the Higgs expectation value in vacuo. The Higgs boson mass is related to the latter as follows,

$$m_H = \sqrt{2\lambda} v . \tag{49}$$

Now, to the leading order of perturbation theory, the finite temperature effects modify the effective potential into

$$V_{eff}(\phi, T) = \frac{\alpha}{2}|\phi|^2 - \frac{\beta}{3}T|\phi|^3 + \frac{\lambda}{4}|\phi|^4 , \tag{50}$$

with $\alpha(T) = -m^2 + \hat{g}^2 T^2$, where $\hat{g}^2$ is a positive linear combination of squares of coupling constants of all fields to the Higgs field (in the Standard Model, a linear combination of g^2, ${g'}^2$ and y_i^2, where g and g' are gauge couplings and y_i are Yukawa couplings), while β is a positive linear combination of cubes of coupling constants of all *bosonic* fields to the Higgs field. In the Standard Model, β is a linear combination of g^3 and ${g'}^3$, i.e., a linear combination of M_W^3/v^3 and M_Z^3/v^3,

$$\beta = \frac{1}{2\pi} \frac{2M_W^3 + M_Z^3}{v^3} . \tag{51}$$

The cubic term in Eq. (50) is rather peculiar: in view of Eq. (48) it is not analytic in the original Higgs field ϕ. Yet this term is crucial for the first order phase transition: for $\beta = 0$ the phase transition would be of the second order. The origin of the non-analytic cubic term can be traced back to the enhancement of the Bose–Einstein thermal distribution at low momenta, $p, m \ll T$,

$$f_{Bose}(p) = \frac{1}{\mathrm{e}^{\frac{\sqrt{p^2+m_a^2}}{T}} - 1} \simeq \frac{T}{\sqrt{p^2 + m_a^2}} ,$$

where $m_a \simeq g_a|\phi|$ is the mass of the boson a that is generated due to the non-vanishing Higgs field, and g_a is the coupling constant of the field a to the Higgs field. Clearly, at $p \ll g_a|\phi|$ the distribution function is non-analytic in ϕ,

$$f_{Bose}(p) \simeq \frac{T}{g_a|\phi|} .$$

It is this non-analyticity that gives rise to the non-analytic cubic term in the effective potential. Importantly, the Fermi–Dirac distribution,

$$f_{Fermi}(p) = \frac{1}{\mathrm{e}^{\frac{\sqrt{p^2+m_a^2}}{T}} + 1} ,$$

is analytic in m_a^2, and hence $\phi^\dagger \phi$, so fermions do not contribute to the cubic term.

With the cubic term in the effective potential, the phase transition is indeed of the first order: at high temperatures the coefficient α is positive and large, and there is one minimum of the effective potential at $\phi = 0$, while for α small but still positive there are two minima. The phase transition ocurs at $\alpha \approx 0$; at that moment

$$V_{eff}(\phi, T) \approx -\frac{\beta T}{3}|\phi|^3 + \frac{\lambda}{4}|\phi|^4 \ .$$

We find from this expression that immediately after the phase transition the minimum of V_{eff} is at

$$\phi \simeq \frac{\beta T}{\lambda} \ .$$

Hence, the necessary condition for successfull electroweak baryogenesis, $\phi > T$, translates into

$$\beta > \lambda \ . \tag{52}$$

According to Eq. (49), λ is proportional to m_H^2, whereas in the Standard Model β is proportional to $(2M_W^3 + M_Z^3)$. Therefore, the relation (52) holds for small Higgs boson masses only; in the Standard Model one makes use of Eqs. (49) and (51) and finds that this happens for $m_H < 50$ GeV, which is ruled out[8].

This discussion indicates a possible way to make the electroweak phase transition strong. What one needs is the existence of new bosonic fields that have large enough couplings to the Higgs field(s), and hence provide large contriutions to β. To have an effect on the dynamics of the transition, the new bosons must be present in the cosmic plasma at the transition temperature, $T \sim 100$ GeV, so their masses should not be too high, $M \lesssim 300$ GeV. In supersymmetric extensions of the Standard Model, the natural candidate for long time has been stop (superpartner of top-quark) whose Yukawa coupling to the Higgs field is the same as that of top, that is, large. The light stop scenario for electroweak baryogenesis would indeed work, as has been shown by the detailed analysis in Ref. [16].

Yet another issue is CP-violation, which has to be strong enough for successfull electroweak baryogenesis. As the asymmetry is generated in the interactions of quarks and leptons (and their superpartners in supersymmetric extensions) with the bubble walls, CP-violation must occur at the walls. Recall now that the walls are made of the Higgs field(s). This points towards the necessity of CP-violation in the Higgs sector, which may only be the case in a theory with more than one Higgs fields.

To summarize, electroweak baryogenesis requires a considerable extension of the Standard Model, with masses of new particles in the range $100 - 300$ GeV. Hence, this mechanism will definitely be ruled out or confirmed by the LHC. We stress, however, that electroweak baryogenesis is not the only option at all: an elegant and well motivated competitor is leptogenesis [17]; several other mechanisms have been proposed that may be responsible for the baryon asymmetry of the Universe.

5 Dark energy

Dark energy, the famous "substance", does not clump, unlike dark matter. It gives rise to the accelerated expansion of the Universe. As we see from Eq. (32), the Universe with constant energy density should expand exponentially; if the energy density is almost constant, the expansion is almost exponential. Let us make use of the first law of thermodynamics, which for the adiabatic expansion reads

$$dE = -pdV \ ,$$

and apply it to comoving volume, $E = \rho V$, $V = a^3$. We obtain for dark energy

$$d\rho_\Lambda = -3\frac{da}{a}(\rho_\Lambda + p_\Lambda) \ ,$$

[8]In fact, in the Standard Model with $m_H > 114$ GeV, there is no phase transition at all; the electroweak transition is smooth crossover instead. The latter fact is not visible from the expression (50), but that expression is the lowest order perturbative result, while the perturbation theory is not applicable for describing the transition in the Standard Model with large m_H.

or

$$\frac{d\rho_\Lambda}{\rho_\Lambda} = -3\frac{da}{a}(1+w) \ ,$$

where we introduced the equation of state parameter w such that

$$p_\Lambda = w\rho_\Lambda \ .$$

Thus, (almost) time-independent dark energy density corresponds to $w \approx -1$, i.e., effective pressure of dark energy is negative. We emphasize that pressure is by definition a spatial component of the energy-momentum tensor, which in the homogeneous and isotropic situation has the general form

$$T_{\mu\nu} = \text{diag}\ (\rho, p, p, p) \ .$$

Dark energy density does not depend on time at all, if $p_\Lambda = -\rho_\Lambda$, i.e.,

$$T_{\mu\nu} = \rho_\Lambda \eta_{\mu\nu} \ ,$$

where $\eta_{\mu\nu}$ is the Minkowski tensor. This is characteristic of vacuum, whose energy-momentum tensor must be Lorentz-covariant. Observationally, w is close to -1 to reasonably good precision. The most accurate detrmination, which, however, does not include systematic errors in supernovae data and possible time-dependence of w, is [9]

$$w = -0.98 \pm 0.05 \ . \tag{53}$$

So, the dark energy density is almost time-independent, indeed.

The problem with dark energy is that its present value is extremely small by particle physics standards,

$$\rho_{DE} \approx 4\,\text{GeV/m}^3 = (2 \times 10^{-3}\,\text{eV})^4 \,.$$

In fact, there are two hard problems. One is that particle physics scales are much larger than the scale relevant to the dark energy density, so the dark energy density is zero to an excellent approximation. Another is that it is non-zero nevertheless, and one has to understand its energy scale. To quantify the first problem, we recall the known scales of particle physics and gravity,

$$\begin{aligned} \text{Strong interactions :} \quad & \Lambda_{QCD} \sim 1\,\text{GeV}\,, \\ \text{Electroweak :} \quad & M_W \sim 100\,\text{GeV}\,, \\ \text{Gravitational :} \quad & M_{pl} \sim 10^{19}\,\text{GeV}\,. \end{aligned}$$

In principle, vacuum should contribute to ρ_Λ, and there is absolutely no reason for vacuum to be as light as it is. The discrepancy here is huge, as one sees from the above numbers.

To elaborate on this point, let us note that the action of gravity plus, say, the Standard Model has the general form

$$S = S_{EH} + S_{SM} - \rho_{\Lambda,0} \int \sqrt{-g}\ d^4x \ ,$$

where $S_{EH} = -(16\pi G_N)^{-1} \int R\, \sqrt{-g}\ d^4x$ is the Einstein–Hilbert action of General Relativity, S_{SM} is the action of the Standard Model and $\rho_{\Lambda,0}$ is the bare cosmological constant. In order that the vacuum energy density be almost zero, one needs fantastic cancellations between the contributions of the Sandard Model fields into the vacuum energy density, on the one hand, and $\rho_{\Lambda,0}$ on the other. For example, we know that QCD has a complicated vacuum structure, and one would expect that the energy density of QCD combined with $\rho_{\Lambda,0}$ should be of order $(1\,\text{GeV})^4$. Nevertheless, it is not, so at least for QCD, one needs a cancellation on the order of 10^{-44}. If one goes further and considers other interactions, the numbers get even worse.

What are the hints from this "first" cosmological constant problem? There are several options, though not many. One is that the Universe could have a very long prehistory. Extremely long. This option has to do with relaxation mechanisms. Suppose that the original vacuum energy density is indeed large, say, comparable to the particle physics scales. Then there must be a mechanism which can relax this value down to an acceptably small number. It is easy to convince oneself that this relaxation could not happen in the history of the Universe we know of. Instead, the Universe should have a very long prehistory during which this relaxation process might occur. At that prehistoric time, the vacuum in the Universe must have been exactly the same as our vacuum, so the Universe in its prehistory must have been exactly like ours, or almost exactly like ours. Only in that case could a relaxation mechanism work. There are concrete scenarios of this sort [18]. However, at the moment it seems that these scenarios are hardly testable, since this is prehistory.

Another possible hint is towards anthropic selection. The argument that goes back to Weinberg and Linde [19, 20] is that if the cosmological constant were larger, say, by a factor of 100, we simply would not exist: the stars would not have formed because of the fast expansion of the Universe. So, the vacuum energy density may be selected anthropically. The picture is that the Universe may be much, much larger than what we can see, and different large regions of the Universe may have different properties. In particular, vacuum energy density may be different in different regions. Now, we are somewhere in the place where one can live. All the rest is empty of human beings, because there the parameters such as vacum energy density are not suitable for their existence. This is disappointing for a theorist, as this point of view allows for arbitrary tuning of fundamental parameters. It is hard to disprove this option, on the other hand. We do exist, and this is an experimental fact. The anthropic viewpoint may, though hopefully will not, get more support from the LHC, if no or insufficient new physics is found there. Indeed, another candidate for an environmental quantity is the electroweak scale.

Let us recall in this regard the gauge hierarchy problem: the electroweak scale $M_W \sim 100\,\mathrm{GeV}$ is much lower than the natural scale in gravitational physics, the Planck mass, $M_{Pl} \sim 10^{19}\,\mathrm{GeV}$. The electroweak scale in the Standard Model is unprotected from large contributions due to high energy physics, and in this sense it is very similar to the cosmological constant. There are various anthropic arguments showing that the electroweak scale must be small. A simple example is that if one makes it larger without touching other parameters, then quarks would be too heavy. Neutron would be the lightest baryon, and proton would be unstable. There would be no stable hydrogen, and that is presumably inconsistent with our existence. Hence, one of the "solutions" to the gauge hierarchy problem is anthropic.

An interesting part of the story is that unlike the cosmological constant, there are natural ways to make the electroweak scale small and render it small in extensions of the Standard Model, like low energy supersymmetry. All these extensions require new physics at TeV energies. So we are in a situation where the experiment has to say its word. If it says that none of these extensions is there in Nature, then we will have to take the anthropic viewpoint much more seriously than before.

Turning to the "second" cosmological constant problem, we note that the scale $10^{-3}\,\mathrm{eV}$ may be associated with some new light field(s), rather than with vacuum. This implies, in general, that ρ_Λ depends on time, i.e., $w \neq -1$ and w may well depend on time itself. "Normal" field (called quitessence in this context) has $w > -1$, but there are examples (rather contrived) of fields with $w < -1$ (called phantom fields). Current data are compatible with time-independent w equal to -1, but their precision is not particularly high. We conclude that future cosmological observations may shed new light on the field content of fundamental theory.

6 Cosmological perturbations and the very early Universe

With Big Bang nucleosynthesis theory and observations, we are confident of the theory of the early Universe at temperatures up to $T \simeq 1\,\mathrm{MeV}$, that corresponds to age of $t \simeq 1$ second. With the LHC, we hope to be able to go up to temperatures $T \sim 100\,\mathrm{GeV}$ and age $t \sim 10^{-10}$ second. The question is: are we going to have a handle on even earlier epoch?

The key issue in this regard is cosmological perturbations. These are inhomogeneities in the energy density and assoiated gravitational potentials, in the first place. This type of inhomogeneities is called scalar perturbations, as they are described by 3-scalars. There may exist perturbations of another type, called tensor; these are primordial gravity waves. We will mostly concentrate on scalar perturbations, since they are observed; tensor perturbations are important too, and we comment on them later on. It is worth pointing out that perturbations of the present size below ten Megaparsec have large amplitudes today and are non-linear, but in the past their amplitudes were small, and they can be described within the linearized theory. Indeed, CMB temperature anisotropy tells us that the perturbations at recombination epoch were roughly at the level

$$\delta \equiv \frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5} \ .$$

Thus, the linearized theory works very well before recombination and somewhat later.

Properties of scalar perturbations are mesured in various ways. Perturbations of large spatial scales leave their imprint in CMB temperature anisotropy and polarization, so we have very detailed knowledge of them. Shorter wavelength perturbations are studied by analysing distributions of galaxies and quasars at present and in relatively near past. There are several other methods, some of which can probe even shorter wavelengths. As we discuss in more detail below, scalar perturbations in the linear regime are actually Gaussian random field, and the first thing to measure is its power spectrum. Overall, independent methods give consistent results, see Fig. 7.

Cosmic medium in our Universe has several components that interact only gravitationally: baryons, photons, neutrinos, dark matter. Hence, there may be and, in fact, there are perturbations in each of these components. As we pointed out in the beginning of Section 3, electromagnetic interactions between baryons, electrons and photons were strong before recombination, so these species made single fluid, and it is appropriate to talk about perturbations in this fluid. After recombination, baryons and photons evolved independently.

The main point of this part of lectures is that by analysing the density perturbations, we have already learned a number of very important things. To appreciate what they are, it is instructive to consider first the baryon-electron-photon fluid before recombination. Perturbations in this fluid are nothing but sound waves; they obey a wave equation. So, let us turn to the wave equation in the expanding Universe.

6.1 Wave equation in expanding Universe. Subhorizon and superhorizon regimes.

The actual system of equations for density perturbations in the baryon-electron-photon fluid and associated gravitational potentials is fairly cumbersome. So, let us simplify things. Instead of writing and then solving the equations for sound waves, let us consider a toy example, the case of massless scalar field. The general properties of density perturbations are similar to this case, although there are a few places in which they differ; we comment on the differences in due course.

The action for the massless scalar field is

$$S = \frac{1}{2}\int d^4x \,\sqrt{-g}\; g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \int d^3x dt\; a^3\; \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2a^2}(\partial_i\phi)^2\right] \ ,$$

where we specified to FLRW metric in the second expression. The field equation thus reads:

$$-\frac{d}{dt}(a^3\dot{\phi}) + a\partial_i\partial_i\phi = 0,$$

i.e.,

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\partial_i^2\phi = 0 \ , \tag{54}$$

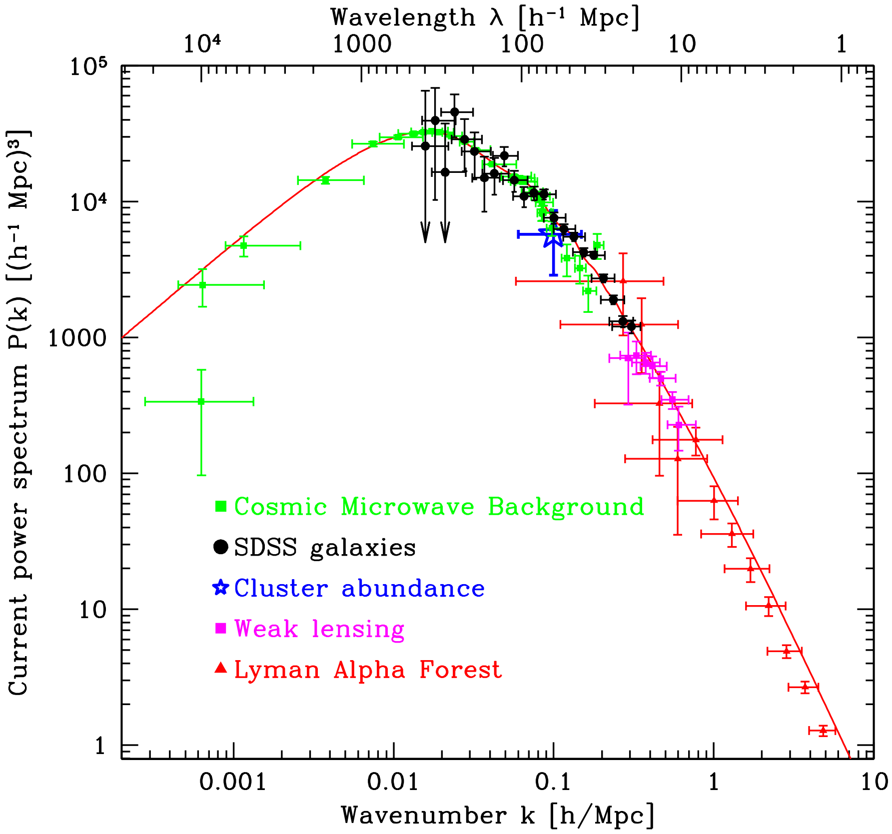

Fig. 7: Power spectrum of density perturbations, measured by various methods and translated to the present epoch by using the linearized theory [21]. k is the present wavenumber, and $h \approx 0.7$ is the dimensionless Hubble parameter at the present epoch.

where $H \equiv \dot{a}/a$ is again the Hubble parameter. This equation is linear in ϕ and homogeneous in space, so it is natural to represent ϕ in terms of the Fourier harmonics,

$$\phi(\mathbf{x}, t) = \int e^{i\mathbf{k}\mathbf{x}} \phi_{\mathbf{k}}(t) \, d^3k.$$

Clearly, the value of $\mathbf{k}$ for a given Fourier mode is constant in time. However, k is not the physical wavenumber (physical momentum), since x is not the physical distance. k is called conformal momentum, while physical momentum equals $q \equiv 2\pi/\lambda = 2\pi/(a(t)\Delta x) = k/a(t)$. Δx here is time-independent comoving wavelength of perturbation, and λ is the physical wavelength; the latter grows due to the expansion of the Universe. Accordingly, as the Universe expands, the physical momentum of a given mode decreases (gets redshifted), $q(t) \propto a^{-1}(t)$. For a mode of given conformal momentum k, Eq. (54) gives:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0. \tag{55}$$

Besides the redshift of momentum, the cosmological expansion has the effect of inducing the second term, "Hubble friction".

Equation (55) has two time-dependent parameters of the same dimension: k/a and H. Let us consider two limiting cases: $k/a \ll H$ and $k/a \gg H$. In cosmological models with conventional equation of state of the dominant component (e.g., matter-dominated or radiation-dominated Universe),

H^{-1} is of the order of the size of the cosmological horizon, see Section 2.6. So, the regime $k/a \ll H$ is the regime in which the physical wavelength $\lambda = 2\pi a/k$ is greater than the horizon size (this is called superhorizon regime), while for $k/a \gg H$ the physical wavelength is smaller than the horizon size (subhorizon regime). The time when the wavelength of the mode coincides with the horizon size is called horizon crossing. In what follows we denote this time by the symbol $\times$. Both at radiation- and matter-dominated epochs, the ratio $k/(aH)$ grows. Indeed, in the radiation-dominated epoch $a \propto \sqrt{t}$, while $H \propto t^{-1}$, so $k/(aH) \propto \sqrt{t}$. This means that every mode was at some early time superhorizon, and later on it becomes subhorizon, see Fig. 8. It is straightforward to see that for all cosmlogically interesting wavelengths, horizon crossing occurs much later than 1 s after the Big Bang, i.e., at the time we are confident about. So, there is no guesswork at this point.

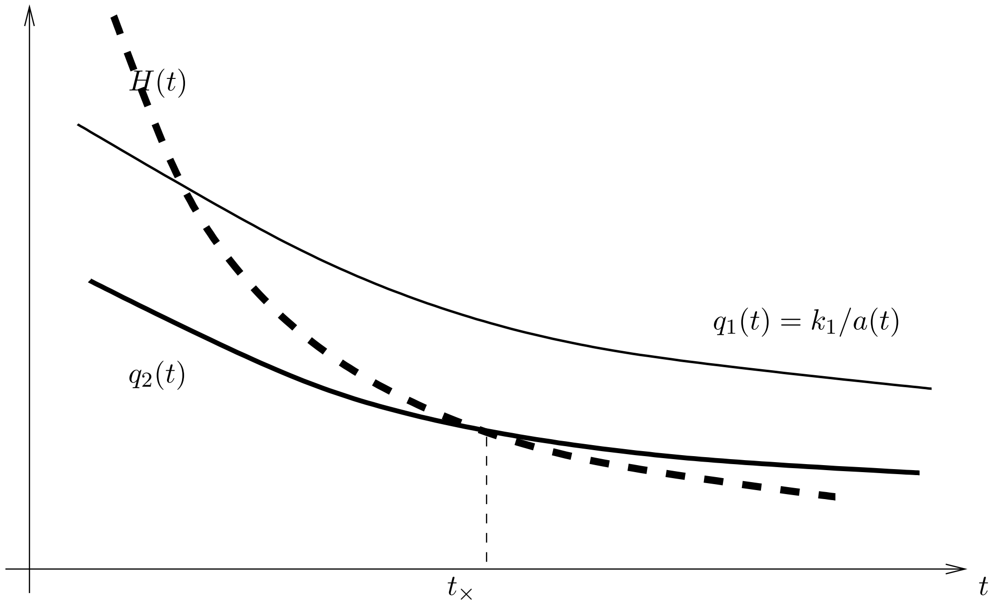

Fig. 8: Physical momenta (solid lines, $k_2 < k_1$) and Hubble parameter (dashed line) at radiation- and matter-dominated epochs. $t_\times$ is the horizon entry time.

Now we can address the question of the origin of density perturbations. By causality, any mechanism of their generation that operates at the radiation- and/or matter-dominated epoch, can only work after the horizon entry time $t_\times$. Indeed, no physical process can create a perturbation whose wavelength exceeds the size of an entire causally connected region. So, in that case the perturbation modes were never superhorizon. On the other hand, if modes were ever superhorizon, they have to exist already in the beginning of the hot epoch. Hence, in the latter situation one has to conclude that there existed another epoch *before* the hot stage: that was the epoch of the generation of primordial density perturbations.

Observational data, notably (but not only) on CMB temperature anisotropy and polarization, disentangle these two possibilities. They unambiguously show that density perturbations *were* superhorizon at radiation and matter domination!

To understand how this comes about, let us see what is special about a perturbation which was superhorizon at the hot stage. For a superhorizon mode, we can neglect the term $\phi \cdot k^2/a^2$ in Eq. (54). Then the field equation, e.g., in the radiation–dominated Universe ($a \propto t^{1/2}, H = 1/2t$), becomes

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} = 0 \; . \tag{56}$$

The general solution to this equation is

$$\phi(t) = A + \frac{B}{\sqrt{t}} \ , \tag{57}$$

where A and B are constants. This behavior is generic for all cosmological perturbations at the hot stage: there is a constant mode (A in our case) and a mode that decays in time. If we extrapolate the decaying mode $B/\sqrt{t}$ back in time, we get very strong (infinite in the limit $t \to 0$) perturbation. For density perturbations (and also tensor perturbations) this means that this mode corresponds to strongly inhomogeneous early Universe. Therefore, the consistency of the cosmological model dictates that the decaying mode has to be absent for actual perturbations. Hence, for given $\mathbf{k}$, the solution is determined by a single parameter, the initial amplitude A of the mode $\phi_{\mathbf{k}}$.

After entering the subhorizon regime, the modes oscillate — these are the analogs of conventional sound waves. In the subhorizon regime one makes use of the WKB aproximation to solve the complete equation

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} + \frac{k^2}{a^2(t)}\phi = 0 \ . \tag{58}$$

The general solution in the WKB approximation reads

$$\phi(t) = \frac{A'}{a(t)} \cos\left(\int_0^t \frac{k}{a(t')}dt' + \psi_0\right) \ , \tag{59}$$

with the two constants being the amplitude A' and the phase ψ_0. The amplitude A' of these oscillations is determined by the amplitude A of the superhorizon initial perturbation, while *the phase ψ_0 of these oscillations is uniquely determined by the condition of the absence of the decaying mode*, $B = 0$. Imposing this condition yields

$$\phi(t) = cA\frac{a_\times}{a(t)} \sin\left(\int_0^t \frac{k}{a(t')}dt'\right) \ , \tag{60}$$

where the constant c is of order 1 and can be evaluated by solving the complete equation (58). The decreasing amplitude of oscillations $\phi(t) \propto 1/a(t)$ and the particular phase $\psi_0 = -\pi/2$ in Eq. (59) are peculiar properties of the wave equation (54), as well as the radiation-dominated cosmological expansion. However, *the fact that the phase of oscillations is uniquely determined by the requirement of the absence of the superhorizon decaying mode is generic*.

The perturbations in the baryon–photon medium before recombination — sound waves — behave in a rather similar way. Their evolution is as follows:

$$\delta_\gamma \equiv \frac{\delta\rho_\gamma}{\rho_\gamma} = \begin{cases} \text{const}, & \text{outside horizon}, \\ \text{const} \cdot \cos\left(\int\limits_0^t v_s \frac{k}{a(t')}dt'\right), & \text{inside horizon}, \end{cases} \tag{61}$$

where $v_s \equiv \sqrt{dp/d\rho}$ is the sound speed. The baryon–photon medium before recombination is almost relativistic[9], since $\rho_{\rm B} < \rho_\gamma$. Therefore, $v_s \approx 1/\sqrt{3}$. Let us reiterate that the phase of the oscillating solution in (61) is uniquely defined.

6.2 Oscillations in CMB angular spectrum

CMB gives us the photographic picture of the Universe at recombination (photon last scattering), see Fig. 9. Waves of different momenta k are at different phases at recombination. At that epoch, oscillations in time in Eq. (61) show up as oscillations in momentum. This in turn gives rise to the observed oscillations in the CMB angular spectrum.

[9] This does not contradict the statement that the Universe is in matter-dominated regime at recombination. The dominant component at this stage is dark matter.

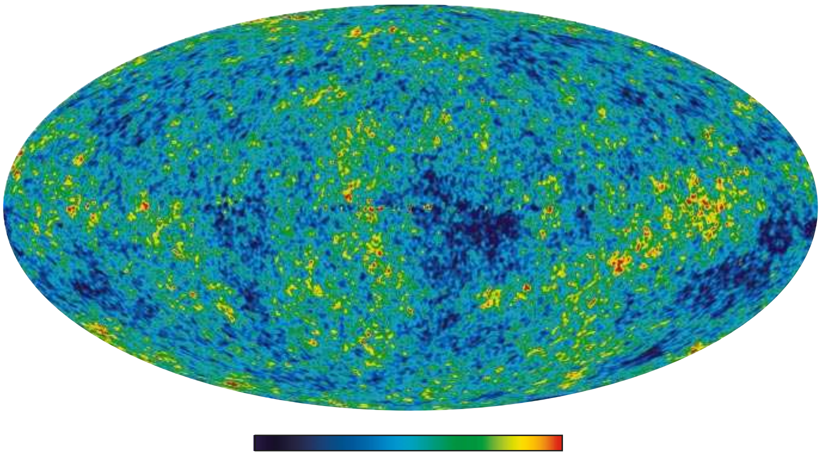

Fig. 9: CMB sky as seen by WMAP experiment.

In more detail, at the time of last scattering t_{rec} we have

$$\delta_\gamma \equiv \frac{\delta\rho_\gamma}{\rho_\gamma} = A(k)\cdot\cos\left(\int_0^{t_{rec}} v_s \frac{k}{a(t')}dt'\right) = A(k)\cdot\cos kr_s \ , \tag{62}$$

where $A(k)$ is linearly related to the initial amplitude of the superhorizon perturbation and is a non-oscillatory function of k, and

$$r_s = \int_0^{t_{rec}} v_s \frac{dt'}{a(t')}$$

is the comoving size of the sound horizon at recombination, while its physical size equals $a(t_{rec})r_s$. So, we see that the density perturbation at recombination indeed oscillates as a function of wavenumber. The period of this oscillation is determined by r_s, which is a straighforwardly calculable quantity.

Omitting details, the fluctuation of the CMB temperature is partially due to the density perturbation in the baryon-photon medium at recombination. The relevant place is the point where the photons last scatter before coming to us. This means that the temperature fluctuation of photons coming from the direction $\mathbf{n}$ in the sky is, to a reasonable accuracy,

$$\delta T(\mathbf{n}) \propto \delta_\gamma(\mathbf{x_n}, \eta_{rec}) + \delta T_{smooth}(\mathbf{n}) \ ,$$

where $T_{smooth}(\mathbf{n})$ corresponds to the non-oscillatory part of the CMB angular spectrum, and

$$\mathbf{x_n} = -\mathbf{n}(\eta_0 - \eta_{rec}) \ .$$

Here the variable η is defined in (31), and η_0 is its present value, so that $(\eta_0 - \eta_{rec})$ is the coordinate distance to the sphere of photon last scattering, and $\mathbf{x_n}$ is the coordinate of the place where the photons coming from the direction $\mathbf{n}$ scatter last time. $T_{smooth}(\mathbf{n})$ originates from the gravitational potential generated by the dark matter perturbation; dark matter has zero pressure at all times, so there are no sound waves in this component, and there are no oscillations at recombination as a function of momentum.

One expands the temperature variation on celestial sphere in spherical harmonics:

$$\delta T(\mathbf{n}) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi).$$

The multipole number l characterizes the temperature fluctuations at the angular scale $\Delta\theta = \pi/l$. The sound waves of momentum k are seen roughly at an angle $\Delta\theta = \Delta x/(\eta_0 - \eta_{rec})$, where $\Delta x = \pi/k$ is coordinate half-wavelength. Hence, there is the correspondence

$$l \longleftrightarrow k(\eta_0 - \eta_{rec}) \; .$$

Oscillations in momenta in (62) thus translate into oscillations in l, and these are indeed observed, see Fig. 10.

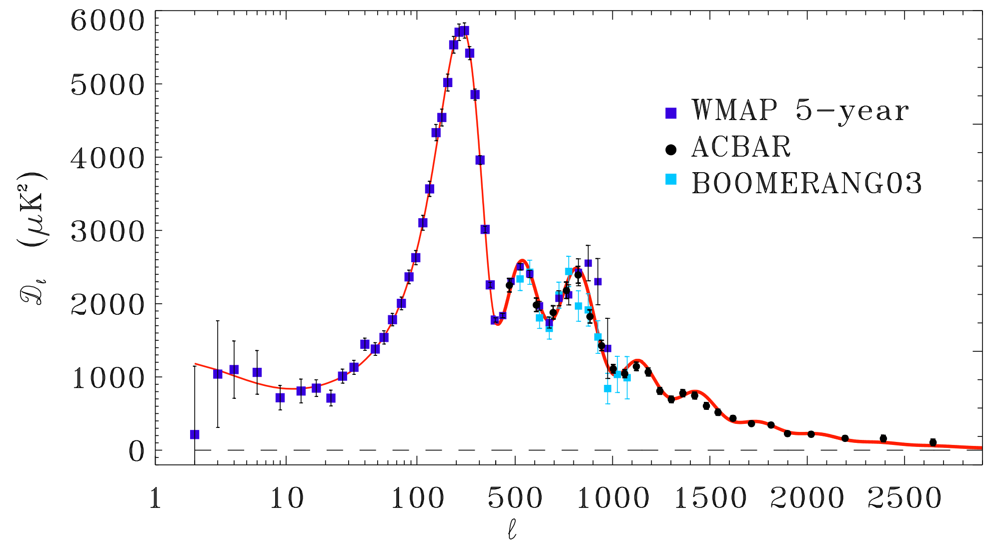

Fig. 10: The angular spectrum of the CMB temperature anisotropy [22]. The quantity in vertical axis is D_l defined in Eq. (65).

To understand what is shown in Fig. 10, we note that all observations today support the hypothesis that a_{lm} *are independent Gaussian random variables*. Gaussianity means that

$$P(a_{lm})da_{lm} = \frac{1}{\sqrt{2\pi C_l}} e^{-\frac{a_{lm}^2}{2C_l}} da_{lm}, \tag{63}$$

where $P(a_{lm})$ is the probability density for the random variable a_{lm}. For a hypothetical ensemble of Universes like ours, the average values of products of the coefficients a_{lm} would obey

$$\langle a_{lm} a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'} . \tag{64}$$

This gives the expression for the temperature fluctuation:

$$\langle [\delta T(\mathbf{n})]^2 \rangle = \sum_l \frac{2l+1}{4\pi} C_l \approx \int \frac{dl}{l} D_l \; ,$$

where

$$D_l = \frac{l(l+1)}{2\pi} C_l \; . \tag{65}$$

It is the latter quantity that is usually shown in plots, in particular, in Fig. 10. Note the unconventional scale on the horizontal axis, aimed at showing both small l region (large angular scales) and large l region.

The fact that the CMB angular spectrum has oscillatory behavior unambiguously tells us that *density perturbations were indeed superhorizon* at hot cosmological stage. If these perturbations were generated by some causal mechanism after horizon entry, there would be no reason for the phase ψ_0 in (59) (better to say, in the analog of (59) for density perturbations) to take a very definite value. Instead, one would expect that this phase is a random function of $\mathbf{k}$, so there would be no oscillations in l in the CMB angular spectrum at all. This is indeed the case in concrete causal models aimed at generating the density perturbations at the hot stage, which make use, e.g., of topological defects (strings, textures, etc.), see Fig. 11.

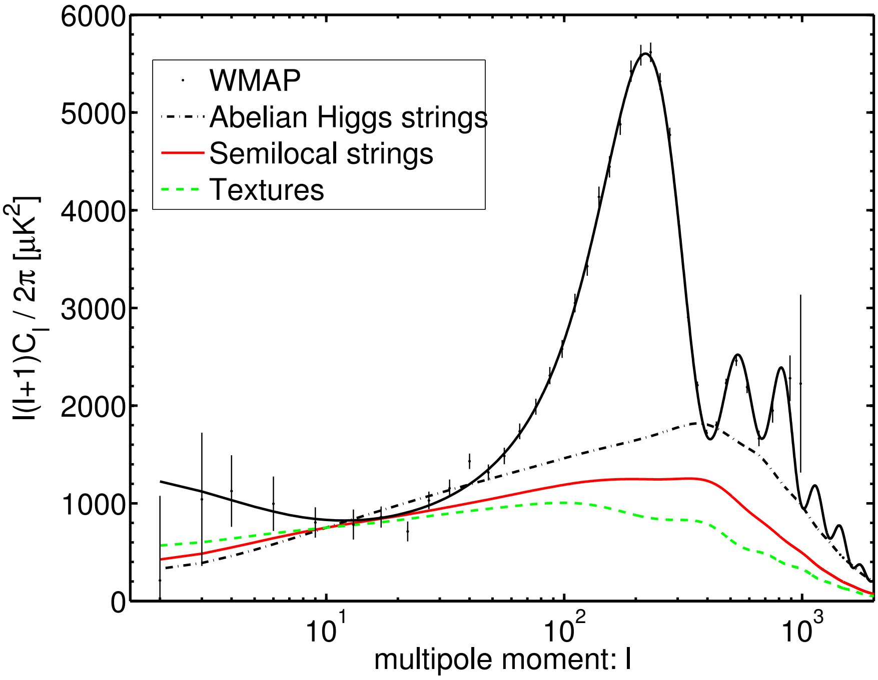

Fig. 11: The angular spectrum of the CMB temperature anisotropy in causal models that generate the density perturbations at the hot stage (non-oscillatory lines) versus data (sketched by oscillatory line) [23].

Another point to note is that the CMB measurements show that at recombination, there were density perturbations which were still superhorizon at that time. These correspond to low multipoles, $l \lesssim 50$. Perturbations of these wavelengths cannot be produced at the hot stage before recombination, and, indeed, causal mechanisms produce small power at low multipoles. This is also seen in Fig. 11.

6.3 Baryon acoustic oscillations

Another manifestation of the well defined phase of sound waves in baryon-photon medium before recombination is baryon acoustic oscillations. Right after recombination, baryons decouple from photons, the sound speed in the baryon component becomes essentially zero, and the spatial distribution of the baryon density freezes out. Since just before recombination baryons, together with photons, have energy distribution (61) which is oscillatory function of k, there is oscillatory component in the Fourier spectrum of the total matter distribution after recombination. This oscillatory component persists until today, and shows up as oscillations in the matter power spectrum $P(k)$. This is a small effect, since the dominant component at late times is dark matter,

$$\rho_M(\mathbf{k}) = \rho_{DM}(\mathbf{k}) + \rho_B(\mathbf{k}) ,$$

and only ρ_B oscillates as function of k

$$\delta\rho_B(\mathbf{k}) \approx \rho_B \delta_\gamma(\mathbf{k}) = \rho_B \cdot A(k) \cdot \cos kr_s \tag{66}$$

(as we already noticed, dark matter has zero pressure at all times, so there are no sound waves in this component). Nevertheless, this effect has been observed in large galaxy surveys, see Fig. 12.

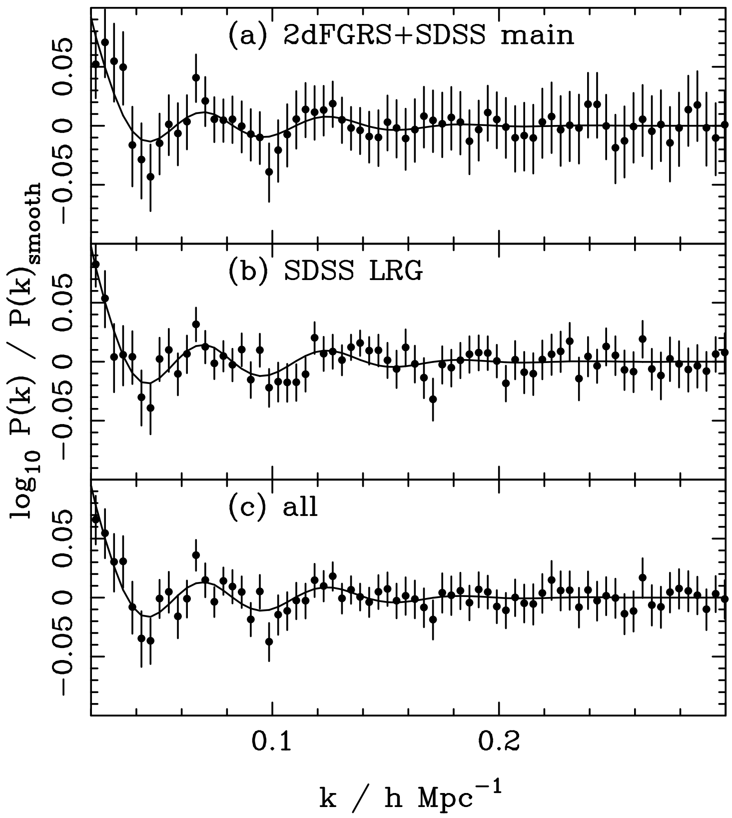

Fig. 12: Baryon acoustic oscillations in matter power spectrum detected in galaxy surveys [24].

There is a simple interpretation of the effect. As we discuss below, the overdensities in the baryon-photon medium and in the dark matter are at the same place before horizon entry (adiabatic mode). But before recombination the sound speed in baryon-photon plasma is of the order of the speed of light, while the sound speed in dark matter is basically zero. So, the overdensity in baryons generates an outgoing density wave after horizon crossing. This wave propagates until recombination, and then freezes out. On the other hand, the overdensity in the dark matter remains in its original place. The current distance from the overdensity in dark matter to the front of the baryon density wave equals 150 Mpc. Hence, there is an enhanced correlation between matter perturbations at this distance scale, which shows up as a feature in the correlation function[10], see Fig. 13. In the Fourier space, this feature produces oscillations (66).

6.4 "Side" remarks

Before proceeding to further discussion of primordial perturbations, let us make a couple of miscellaneous remarks.

[10]Notice that the separation at Fig. 13 is given in h^{-1} Mpc, where $h = H_0/100\,\mathrm{km/(s \cdot Mpc)} \approx 0.7$. Hence, 100 h^{-1} Mpc roughly corresponds to 150 Mpc.

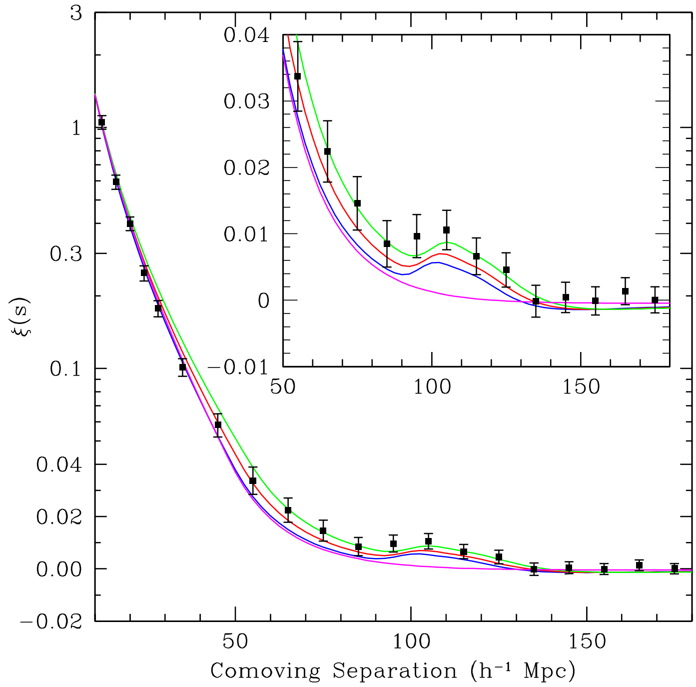

Fig. 13: Baryon acoustic oscillations in galaxy correlation function [25].

6.4.1 Cosmic variance.

We can measure only one Universe, and the best one can do is to *define* the angular spectrum C_l obtained from the data as

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2 \ .$$

This is not the same thing as C_l defined in (64), as the latter definition involves averaging over an ensemble of Universes. For given l, there are $(2l+1)$ independent coefficients a_{lm} only, so there exists an irreducible statistical uncertainty of order $\delta C_l / C_l \sim 1/\sqrt{2l+1}$, called cosmic variance. It is particularly pronounced at small l and, indeed, it is much larger than the experimental errors in this part of the angular spectrum (as an example, error bars in the left part of Fig. 10 are precisely due to the cosmic variance).

6.4.2 Measuring the cosmological parameters.

The angular spectrum of CMB temperature anisotropy and polarization, as well as other cosmological data, encodes information on the cosmological parameters. As an example, the sound horizon at recombination is a good standard ruler back at that epoch. It is seen at an angle that depends on the geometry of 3-dimensional space (an interval is seen at larger angle on a sphere than on a plane) and on the dark energy density (since dark energy affects the distance to the sphere of photon last scattering). This is shown in Fig. 14.

Likewise, the baryon acoustic oscillations provide a standard ruler at relatively late times (low redshifts $z \sim 0.2 - 0.4$). A combination of their measurement with CMB anisotropy give quite precise determination of both spatial curvature (which is found to be zero within error bars) and dark energy density. Notably, this determination of ρ_Λ is in good agreement with various independent data, notably, with the data on SNe 1a, which were the first unambiguous evidence for dark energy [27, 28].

There are many other ways in which the cosmological parameters, including Ω_B and Ω_{DM}, affect

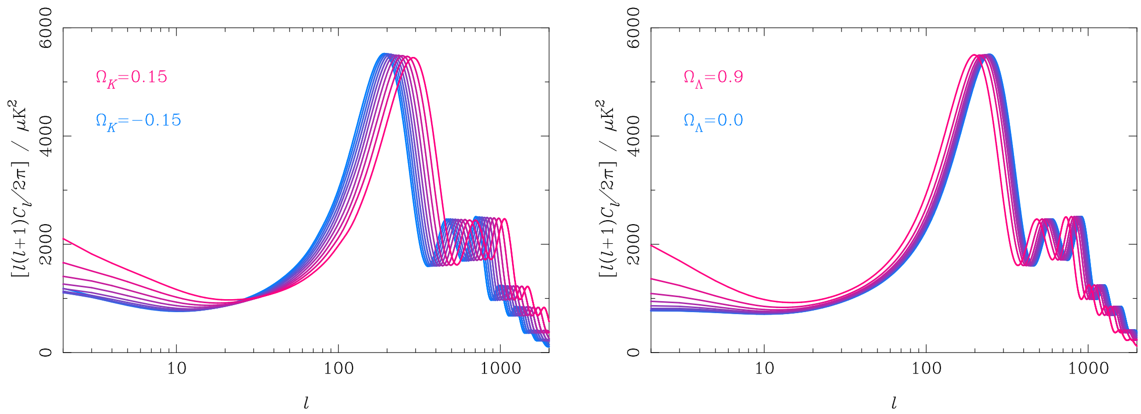

Fig. 14: Effect of spatial curvature (left) and dark energy (right) on the CMB temperature angular spectrum [26]. $\Omega_k = \pm(RH_0)^2$ is the relative contribution of spatial curvature to the Friedmann equation, with R being the radius of spatial curvature. Negative sign corresponds to 3-sphere. As Ω_k decreases, the curves in the left plot move left. Likewise, the curves on the right plot move left as Ω_Λ increases.

the CMB anisotropies. In particular, the hights of the acoustic peaks in the CMB temperature angular spectrum are very sensitive to the baryon-to-photon ratio η_B (and hence to Ω_B), the overall shape of the curve in Fig. 10 stronfly depends on Ω_{DM}, etc. By fitting the CMB data and combining them with the results of other cosmological observations, one is able to obtain quite precise knowledge of our Universe.

6.5 Properties of primordial density perturbations — hints about the earliest cosmological epoch

As we emphasized above, the density perturbations were generated at a very early, pre-hot epoch of the cosmological evolution. Obviously, it is of fundamental importance to figure out what precisely that epoch was. One of its properties is clear right away: it must be such that the cosmologically relevant wavelengths, including the wavelengths of the present horizon scale, were subhorizon early at that epoch. Only in that case the perturbations of these wavelengths could be generated in a causal manner at the pre-hot epoch. Notice that this is another manifestation of the horizon problem discussed in Section 2.6: we know from the observational data on density perturbtions that our entire visible Universe was causally connected by the beginning of the hot stage.

An excellent hypothesis on the pre-hot stage is inflation, the epoch of nearly exponential expansion,

$$a(t) = \mathrm{e}^{\int H dt} , \qquad H \approx \text{const} .$$

Originally [29], inflation was designed to solve the problems of the hot Big Bang cosmology, such as the horizon problem, as well as the flatness, entropy and other problems. It does this job very well: the horizon size at inflation is at least

$$l_H(t) = a(t) \int_{t_i}^{t} \frac{dt'}{a(t')} = H^{-1} \mathrm{e}^{H(t-t_i)} ,$$

where t_i is the time inflation begins, and we set $H =$ const for illustrational purposes. This size is huge for $t - t_i \gg H^{-1}$, so the entire visible Universe is naturally causally connected.

From the viewpoint of perturbations, the physical momentum $q(t) = k/a(t)$ decreases (gets red-shifted) at inflation, while the Hubble parameter stays almost constant. So, every mode is first subhorizon ($q(t) \gg H(t)$), and later superhorizon ($q(t) \ll H(t)$) at inflation. This situation is opposite to what happens at radiation and matter domination, see Fig. 15; this is precisely the pre-requisite for generating the density perturbations. In fact, inflation does generate primordial density perturbations [30], whose properties are consistent with everything we know about them.

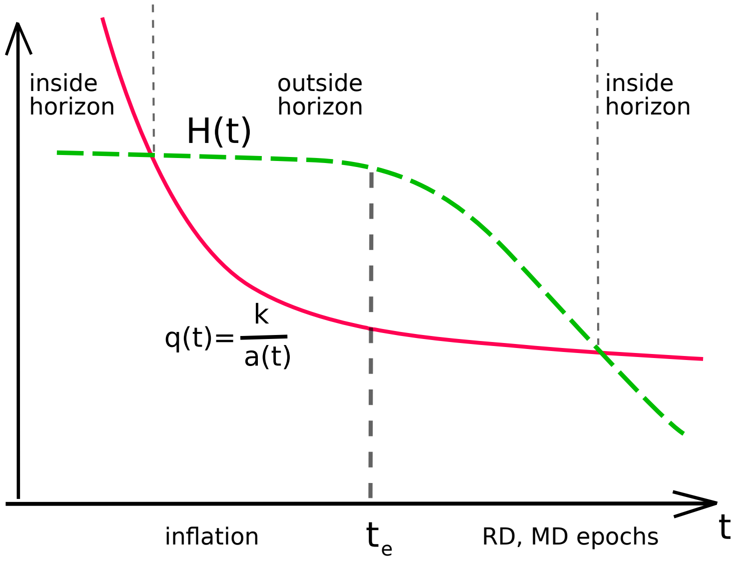

Fig. 15: Physical momentum and Hubble parameter at inflation and later. t_e is the time of the inflation end.

Inflation is not the only hypothesis proposed so far, however. One option is the bouncing Universe scenario, which assumes that the cosmological evolution begins from contraction, then the contracting stage terminates at some moment of time (bounce) and is followed by expansion. A version is the cycling Universe scenario with many cycles of contraction–bounce–expansion. Another scenario is that the Universe starts out from nearly flat and static state and then speeds up its expansion. Theoretical realizations of these scenarios are more difficult than inflation, but they are not impossible, as became clear recently. So, one of the major purposes of cosmology is to choose between various hypotheses on the basis of observational data. The properties of cosmological perturbations are the key issue in this regard.

There are several things which we already know about the primordial density perturbations. By "primordial" we mean the perturbations deep in the superhorizon regime at the radiation-domination epoch. As we already know, perturbations are time-independent in this regime. They set the initial conditions for further evolution, and this evolution is well understood, at least in the linear regime. Hence, using observational data, one is able to measure the properties of primordial perturbations. Of course, since the properties we know of are established by observations, thery are valid within certain error bars. Conversely, deviations from the results listed below, if observed, would be extremely interesting.

First, density perturbations are **adiabatic**. This means that there are perturbations in the energy density, but *not in composition*. More precisely, the baryon-to-entropy ratio and dark matter-to-entropy ratio are constant in space,

$$\delta\left(\frac{n_B}{s}\right) = \text{const} , \qquad \delta\left(\frac{n_{DM}}{s}\right) = \text{const} . \tag{67}$$

This is consistent with the generation of the baryon asymmetry and dark matter at the hot cosmological epoch: in that case, all partciles were at thermal equilibrium early at the hot epoch, the temperature completely characterized the whole cosmic medium at that time, and as long as physics behind the baryon asymmetry and dark matter generation is the same everywhere in the Universe, the baryon and dark matter abundance (relative to the entropy density) is necessarily the same everywhere. In principle, there may exist *entropy* (or isocurvature) perturbations, such that at the early hot epoch energy density (dominated by relativistic matter) was homogeneous, while the composition was not. This would give initial conditions for the evolution of density perturbations, which would be entirely different from those characteristic of the adiabatic perturbations. As a result, the angular spectrum of the CMB temperature anisotropy would be entirely different, see Fig. 16. No admixture of the entropy perturbations have been detected so far, but it is worth emphasizing that even small admixture will show that the most popular mechanisms for generating dark matter and/or baryon asymmetry (including those discussed in Sections 3 and 4) have nothing to do with reality. One would have to think, instead, that the baryon asymmetry and/or dark matter were generated before the beginning of the hot stage.

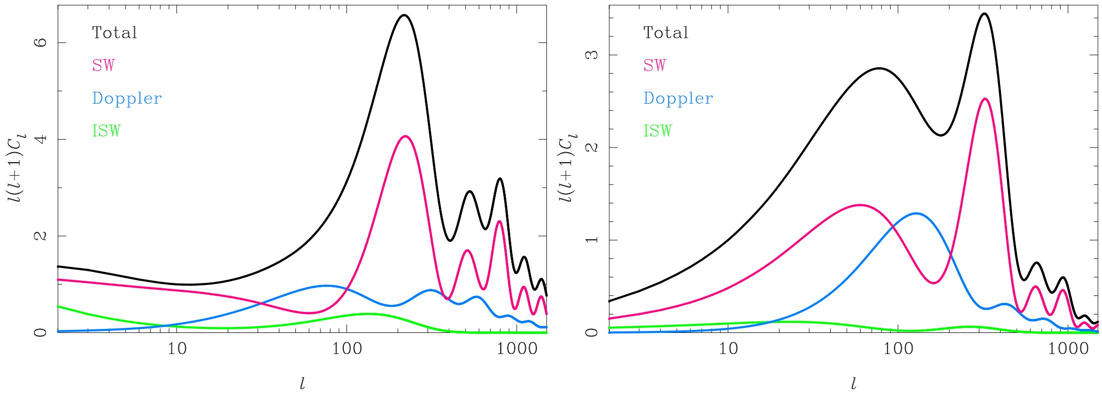

Fig. 16: Angular spectrum of the CMB temperature anisotropy for adiabatic perturbations (left) and entropy perturbations (right) [26].

Second, the primordial density perturbations are **Gaussian random field**. Gaussianity means that the three-point (and all odd) correlation function vanishes, while the four-point function and all higher order even correlation functions are expressed through the two-point function via Wick's theorem:

$$\begin{aligned}\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0\\ \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle\cdot\langle\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle\\ &\quad + \text{permutations of momenta}\ ,\end{aligned}$$

while all odd correlation functions vanish. A technical remark is in order. As a varibale characterizing the primordial adiabatic perturbations we use here

$$\delta \equiv \delta\rho_{rad}/\rho_{rad} = \delta\rho/\rho$$

deep at the radiation-dominated epoch. This variable is not gauge-invariant, so we implicitly have chosen the conformal Newtonian gauge. In cosmological literature, other, gauge-invariant quantities are commonly in use, ζ and $\mathcal{R}$. In the conformal Newtonian gauge, there is a simple relationship, valid in the superhorizon regime at radiation domination:

$$\mathcal{R} = \zeta = \frac{3}{4}\delta\ .$$

We will continue to use δ as the basic variable.

Coming back to Gaussianity, we note that this property is characteristic of *vacuum fluctuations of non-interacting (linear) quantum fields.* Hence, it is quite likely that the density perturbations originate from the enhanced vacuum fluctuations of non-interacting or weakly interacting quantum field(s). Free quantum field has the general form

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{k}\mathbf{x}}\left(f_{\mathbf{k}}^{(+)}(t)a_{\mathbf{k}}^{\dagger} + e^{i\mathbf{k}\mathbf{x}} f_{\mathbf{k}}^{(-)}(t)a_{\mathbf{k}}\right)\ ,$$

where $a_{\mathbf{k}}^{\dagger}$ and $a_{\mathbf{k}}$ are creation and annihilation operators. For the field in Minkowski space-time one has $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$, while enhancement, e.g. due to the evolution in time-dependent background, means that $f_{\mathbf{k}}^{(\pm)}$ are large. But in any case, Wick's theorem is valid, provided that the state of the system is vacuum, $a_{\mathbf{k}}|0\rangle = 0$.

Inflation does the job very well: fluctuations of all light fields get enhanced greatly due to the fast expansion of the Universe. This is true, in particular, for the field that dominates the energy density at inflation, called inflaton. Enhanced vacuum fluctuations of the inflaton are nothing but perturbations in the energy density at inflationary epoch in the simplest inflationary models, which are reprocessed into perturbations in the hot medium after the end of inflation. The generation of the density perturbations is less automatic in scenarios alternative to inflation, but there are various examples showing that this is not a particularly difficult problem.

Non-Gaussianity is an important topic of current research. It would show up as a deviation from Wick's theorem. As an example, the three-point function (bispectrum) may be non-vanishing,

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\; G(k_i^2;\; \mathbf{k}_1\mathbf{k}_2;\; \mathbf{k}_1\mathbf{k}_3) \neq 0 \; .$$

The shape of $G(k_i^2;\; \mathbf{k}_1\mathbf{k}_2;\; \mathbf{k}_1\mathbf{k}_3)$ is different in different models, so this shape is a potential discriminator. In some models the bispectrum vanishes, e.g., due to symmetries. In that case the trispectrum (connected 4-point function) may be measurable instead. Non-Gaussianity is very small in the simplest inflationary models, but it can be sizeable in more contrived models of inflation and in alternatives to inflation. It is worth emphasizing that non-Gaussianity has not been detected yet.

Another important property is that the primordial power spectrum of density perturbations **is flat** (or almost flat). A convenient definition of the power spectrum for homogeneous and anisotropic Gaussian random field is[11]

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = \frac{1}{4\pi k^3}\mathcal{P}(k)\delta(\mathbf{k}+\mathbf{k}') \; . \tag{68}$$

The power spectrum $\mathcal{P}(k)$ defined in this way determines the fluctuation in a logarithmic interval of momenta,

$$\langle \delta^2(\mathbf{x})\rangle = \int_0^\infty \frac{dk}{k}\,\mathcal{P}(k) \; .$$

By definition, the flat spectrum is such that $\mathcal{P}$ is independent of k. It is worth noting that the flat spectrum was conjectured by E. Harrison [31] and Ya. Zeldovich [32] in the beginning of 1970's, long before realistic mechanisms of the generation of density perturbations have been proposed.

In view of the approximate flatness, a natural parametrization is

$$\mathcal{P}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \; , \tag{69}$$

where A_s is the amplitude, $(n_s - 1)$ is the tilt and k_* is a fiducial momentum, chosen at one's convenience. The flat spectrum in this parametrization has $n_s = 1$. Cosmological data favor the value $n_s \approx 0.96$ (i.e., slightly smaller than 1), see below, but it is fair to say that $n_s = 1$ is still consistent with observations.

The flatness of the power spectrum calls for some symmetry behind this property. In inflationary theory this is the symmetry of the de Sitter space-time, which is the space time of constant Hubble rate,

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\mathbf{x}^2 \; , \qquad H = \text{const} \; .$$

This metric is invariant under spatial dilatations supplemented by time translations,

$$\mathbf{x} \to \lambda\mathbf{x} \; , \quad t \to t - \frac{1}{2H}\log\lambda \; .$$

[11] Note that the the definition of the power spectrum used in Figs. 7 and 12 is different from (68).

At inflation, H is almost constant in time, and the de Sitter symmetry is an approximate symmetry. For this reason inflation automatically generates nearly flat power spectrum.

The de Sitter symmetry is not the only candidate symmetry behind the flatness of the power spectrum. One possible alternative is conformal symmetry [33, 34]. The point is that the conformal group includes dilatations, $x^\mu \to \lambda x^\mu$. This property indicates that the relevant part of the theory possesses no scale, and has good chance for producing the flat spectrum. Model-building in this direction has begun recently [34].

6.6 What's next?

Thus, only very basic facts about the primordial denisty perturbations are observationally established. Even though very suggestive, these facts by themselves are not sufficient for unambiguously establishing the properties of the Universe at the pre-hot epoch of its evolution. In coming years, new properties of cosmological perturbations will hopefully be discovered, which will shed much more light on this pre-hot epoch. Let us discuss some of the potential observables.

6.6.1 Tensor perturbations = relic gravity waves

The simplest, and hence most plausible models of inflation predict sizeable tensor perturbations, which are perturbations of the metric independent of perturbations in the energy density. After entering the horizon, tensor perturbations are nothing but gravity waves. The reason for their generation at inflation is that the exponential expansion of the Universe enhances vacuum fluctuations of all fields, including the gravitational field itself. In inflationary theory, the primordial tensor perturbations are Gaussian random field with nearly flat power spectrum

$$\mathcal{P}_T = A_T \left(\frac{k}{k_*} \right)^{n_T} , \tag{70}$$

where the inflationary prediction is $n_T \approx 0$ (the reason for different definitions of the tensor spectral index n_T in (70) and scalar spectral index n_s in (69) is purely historical).

On the other hand, there seems to be no way of generating nearly flat tensor power spectrum in alternatives to inflation. In fact, most, if not all, alternative scenarios predict unobservably small amplitude of tensor perturbations. Thus, the discovery of tensor modes would be the strongest possible argument in favor of inflation. It is worth noting that *non-observation* of tensor perturbations would not rule inflation out: there are numerous models of inflation which predict tensor modes of very small amplitude.

The tensor power is usually characterized by the tensor-to-scalar ratio

$$r = \frac{A_T}{A_s} .$$

The simplest inflationary models predict, roughly speaking, $r \sim 0.1 - 0.3$. The current situation is summarized in Fig. 17. Clearly, there is an indication for the negative scalar tilt $(n_s - 1)$ or non-zero tensor amplitude, or both, though it is premature to say that the flat scalar spectrum with no tensor modes (the Harrison–Zeldovich point) is ruled out.

For the time being, the most sensitive probe of the tensor perturbations is the CMB temperature anisotropy. However, the most promising tool is the CMB polarization. The point is that a certain class of polarization patterns (called B-mode) is generated by tensor perturbations, while scalar perturbations are unable to create it. Hence, the Planck experiment, and especially dedicated experiments aiming at measuring the CMB polarization may well discover the tensor perturbations, i.e., relic gravity waves. Needless to say, this would be a profound discovery. To avoid confusion, let us note that the CMB polarization has been already observed, but it belongs to another class of patterns (so called E-mode) and is consistent with the existence of the scalar perturbations only.

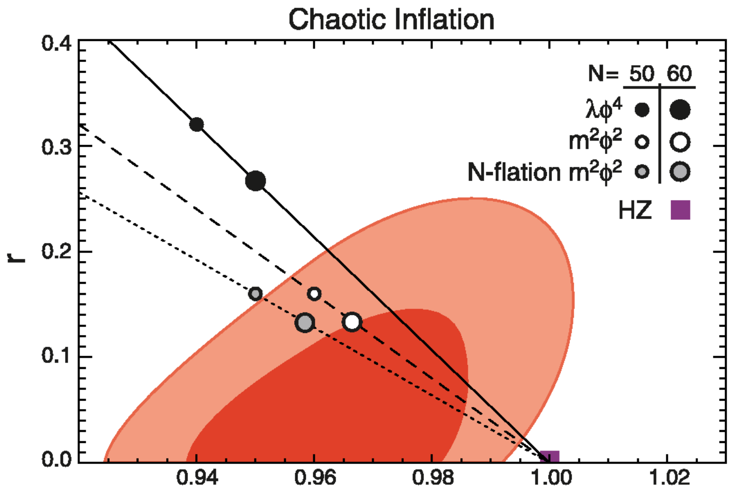

Fig. 17: Allowed regions (at 68% and 95% CL) in the plane (n_s, r), where n_s is the scalar spectral index and r is the tensor-to-scalar ratio. Rectangle shows the Harrison–Zeldovich point (flat scalar spectrum, no tensor modes). Points on the lines show predictions of popular inflationary models.

6.6.2 Scalar tilt.

Inflationary models and their alternatives will be constrained by the precise determination of the scalar tilt $(n_s - 1)$ and its dependence on momentum k. It appears, however, that the information on $n_s(k)$ that will be obtained in reasonably near future will be of limited significance from the viewpoint of discriminating between different (and even grossly different) scenarios.

6.6.3 Non-Gaussianity.

As we pointed out already, non-Gaussianity of density perturbations is very small in the simplest inflationary models. Hence, its discovery will signalize that either inflation and inflationary generation of density perturbations occured in a rather complicated way, or an alternative scenario was realized. Once the non-Gaussianity is discovered, and its shape is revealed even with moderate accuracy, many concrete models will be ruled out, while at most a few will get strong support.

6.6.4 Statistical anisotropy.

In principle, the power spectrum of density perturbations may depend on the direction of momentum, e.g.,

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_0(k)\left(1 + w_{ij}(k)\frac{k_i k_j}{k^2} + \dots\right)$$

where w_{ij} is a fundamental tensor in our part of the Universe (odd powers of k_i would contradict commutativity of the Gaussian random field $\delta(\mathbf{k})$, see Eq. (68)). Such a dependence would definitely imply that the Universe was anisotropic at the pre-hot stage, when the primordial perturbations were generated. This statistical anisotropy is rather hard to obtain in inflationary models, though it is possible in inflation with strong vector fields [35]. On the other hand, statistical anisotropy is natural in some other scenarios, including conformal models [36]

The statistical anisotropy would show up in correlators [37]

$$\langle a_{lm} a_{l'm'} \rangle \qquad \text{with } \; l' \neq l \; \text{ and/or } \; m' \neq m$$

At the moment, the situation with observational data is controversial [38], and the new data, notably from the Planck experiment, will hopefully clear it up.

6.6.5 *Admixture of entropy perturbations.*

As we explained above, even small admixture of entropy perturbations would force us to abandon the most popular scenarios of the generation of baryon asymmetry and/or dark matter, which assumed that it happened at the hot epoch. The WIMP dark matter would no longer be well motivated, while other, very weakly interacting dark matter candidates, like axion or superheavy relic, would be prefered. This would make the direct searches for dark matter rather problematic.

7 Conclusion

We are at the eve of new era not only in particles physics, but also in cosmology. There is reasonably well justified expectation that the LHC will shed light on long-standing cosmological problems of the origin of the baryon asymmetry and nature of dark matter in our Universe. The ideas we discussed in these lectures in this regard may well be not the right ones: we can only hypothesize on physics beyond the Standard Model and its role in the early Universe.

In fact, the TeV scale physics may be dramatically different from physics we get used to. As an example, it is not excluded that TeV is not only electroweak, but also gravitational scale. This is the case in models with large extra dimensions, in which the Planck scale is related to the fundamental gravity scale in a way that involves the volume of extra dimensions, and hence the fundamental scale can be much below M_{Pl} (for a review see, e.g., Ref. [39]). If the LHC will find that, indeed, the fundamental gravity scale is in the TeV range, this would have most profound consequences for both microscopic physics and cosmology. On the microscopic physics side, this would enable one to study at colliders quantum gravity and its high-energy extension — possibly string theory, while on the cosmological side, the entire picture of the early Universe would have to be revised. Inflation, if any, would have to occur either at low energy density or in the regime of strong quantum gravity effects. The highest temperatures in the usual expansion history would be at most in the TeV range, so dark matter and baryon asymmetry would have to be generated either below TeV temperatures or in quantum gravity regime. Even more intriguing will be the study of quantum gravity cosmological epoch, with hints from colliders gradually coming. This, probably, is too bright a prospective to be realistic.

It is more likely that the LHC will find something entirely new, something theorists have not thought about. Or, conversely, find so little that one will have to get serious about anthropic principle. In any case, the LHC results will definitely change the landscape of fundamental physics, cosmology included.

The observational data unequivocally tell us that the hot stage of the cosmological evolution was preceeded by some other epoch, at which the cosmological perturbations were generated. The best guess for this epoch is inflation, but one should bear in mind that there are alternative possibilities. It is fascinating that with new observational data, there is good chance to learn what precisely that pre-hot epoch was. It may very well be that in this way we will be able to probe physics at the energy, distance and time scales well beyond the reach of the LHC.

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