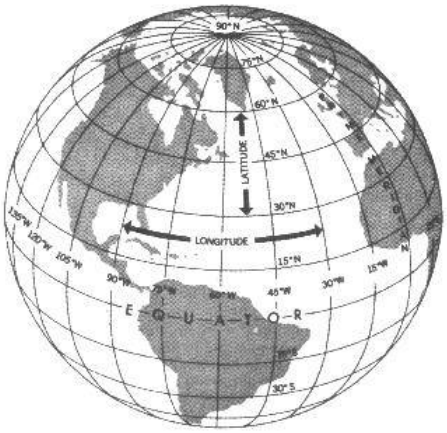
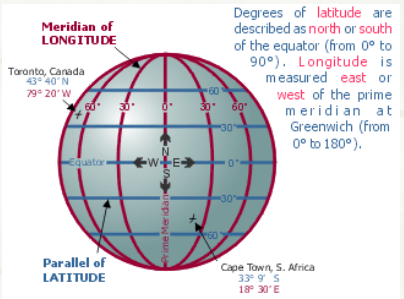
Shortest flight paths are geodesics-

geodesics are defined to be the shortest path between points in the space.

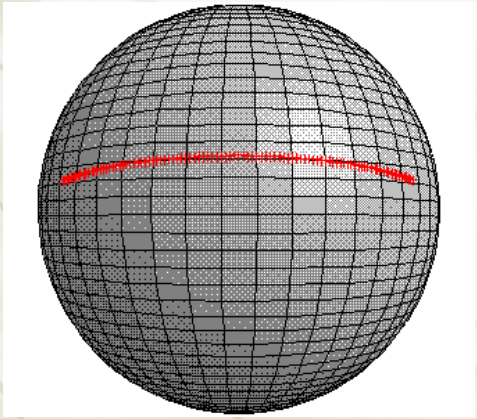


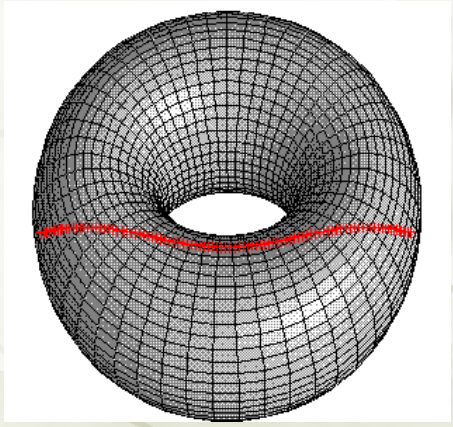
On Globe…



- Constant-longitude lines (meridians) are geodesics
- On the Earth, geodesics are Great Circles, the shortest distance between two points on the surface.
- Constant-latitude lines (parallels) are **not geodesics**

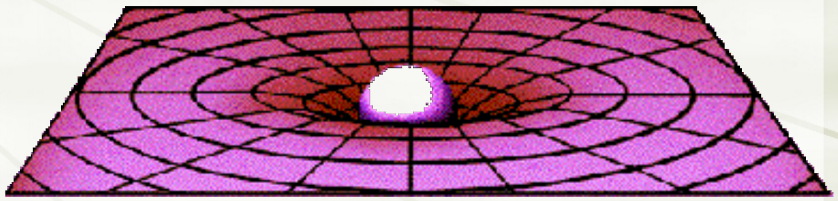
Geodesics on sphere and torus





How does matter "warp" space?

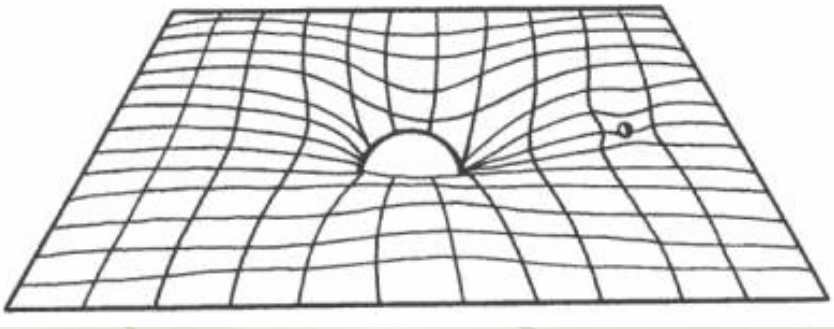
- Use two-dimensional space as an analogy: think of how rubber sheet is affected by weights
- Any weight causes sheet to sag locally
- Amount that sheet sags depends on how heavy weight is



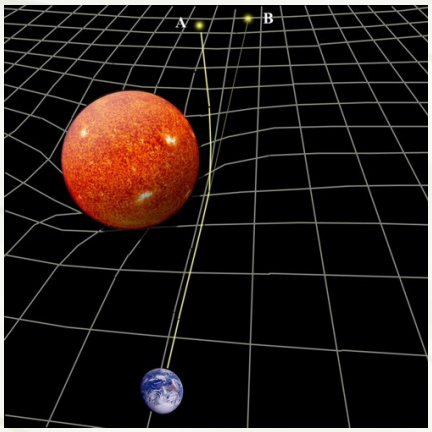
From web site of UCSD

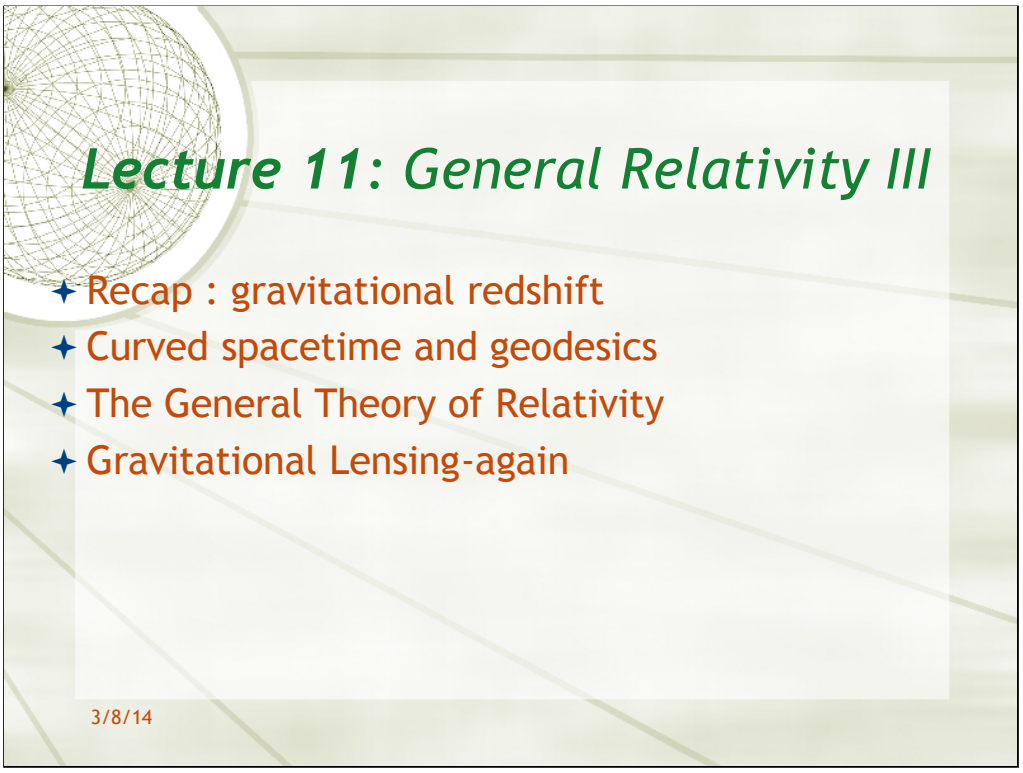
Effect of matter on coordinates

- Lines that would be straight become curved (*to external observer*) when sheet is "weighted"



Is Light or Space 'Bent'?




Lecture 11: General Relativity III
Recap : gravitational redshift
Curved spacetime and geodesics
The General Theory of Relativity
Gravitational Lensing-again
3/8/14

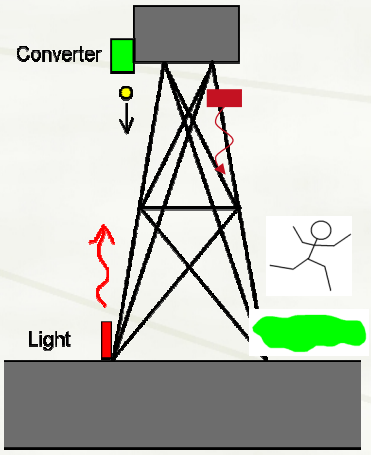
Gravitational Redshift

- To work it out with equations
- For a photon $E=mc^2=h\nu$
- The gravitational potential energy (the energy gained by falling in a gravitational field) $PE=-GMm/r=(-GMh/rc^2)\nu_0$- last lecture we use 'little' g; PE-mgh; where 'little' g is the acceleration at the surface of the earth- these are the same
- So as the photon rises up in the gravitational field
- $h\nu=h\nu_0\{1-(GM/rc^2)\}$; $\nu=\nu_0\{1-(GM/rc^2)\}$; $\Delta\nu/\nu==GM/rc^2$
- (where $\Delta\nu$ is the change in frequency of the photon $\Delta\nu=\nu-\nu_{0)}$
- Since the photon is reduced in frequency (and thus lengthened in wavelength) this is called the gravitational redshift

Resolving the tower problem

- Now consider light ray aimed from top to bottom of tower
- Free-falling(FF) observer sees light ray travel **unaffected** by gravity,since freefall is an inertial frame
- From "Earth's"frame…
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity
  - If FF observer is **supposed to** see a constant frequency light beam,then light must get relatively *blueshifted* as it falls in a gravitational field, to compensate
  - Light beam aimed upward must conversely be increasingly *redshifted* with height
  - ***Gravitational redshifting removes just the right amount of energy to solve the tower paradox!***

(remember that the energy of a photon is $E=h\nu=hc/\lambda$)



Gravitational Redshift

- The conservation of energy-matter: not just the conservation of energy
- 2.'The' equation for the potential energy in a gravitatonal field PE=m*g*h (or more generally PE=GM/r)*
- 3.The energy of a photon $E=h\nu$
- 4.$E=mc^2$
- So the 'trick' is: The energy received must equal the energy emitted minus the energy due to the potential energy

*In GR the potential energy is difficult to define quantity; for a spherical mass it is approximately $PE=m_0c^2\ \mathrm{sqrt}(1-2GM/rc^2)$: where m_0 is the mass of the particle, G is the gravitational constant, M is the mass of the large body (e.g. earth or star and r is the distance from the center of the spherical large body) so this derivation is a little bit of a cheat

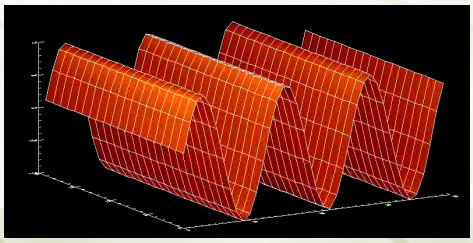
Gravitational Redshift

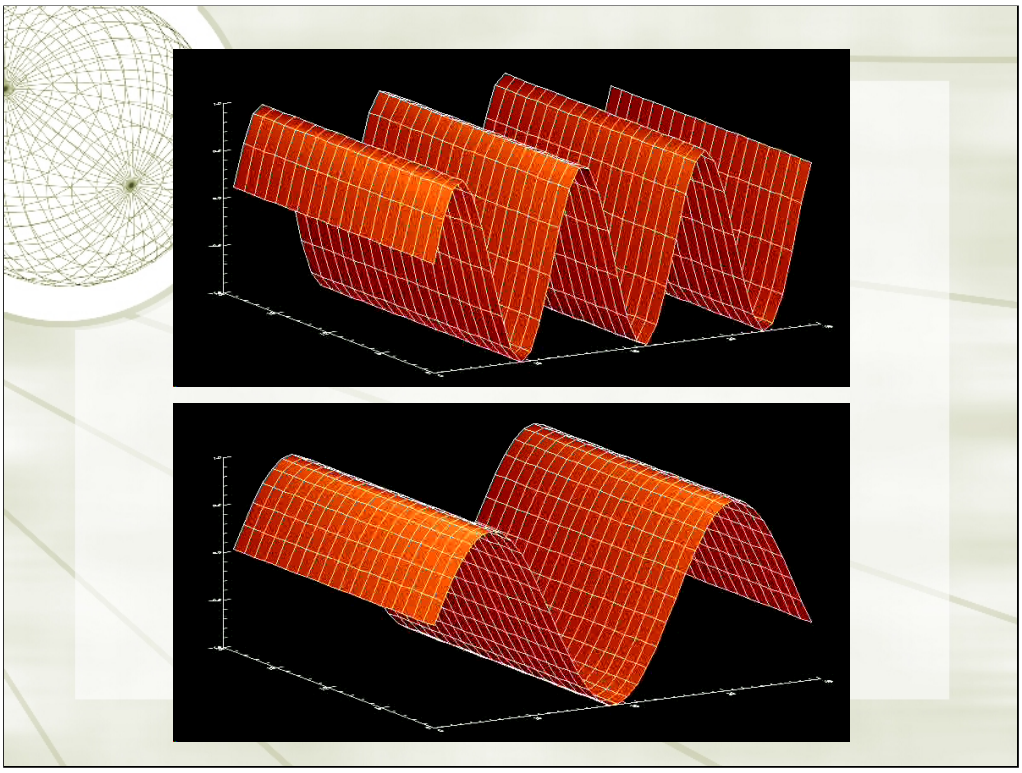
- Finally another way of looking at this
- From the principle of equivalence between gravity and force due to acceleration, the shift in frequency in a gravitational field can be related to the relativistic Doppler shift due to an accelerating light source.

- Derivation:
- for velocities v<<c the Doppler shift formula is $\nu=\nu_0 [1+V/c]$
- For an observer accelerated to velocity V in a time t=L/c (L is some length) V=at=aL/c and then $\nu=\nu_0 [1+aL/c^2]$ and replacing the arbitary acceleration 'a' by gravitational acceleration 'g' $\nu=\nu_0 [1+gL/c^2]$

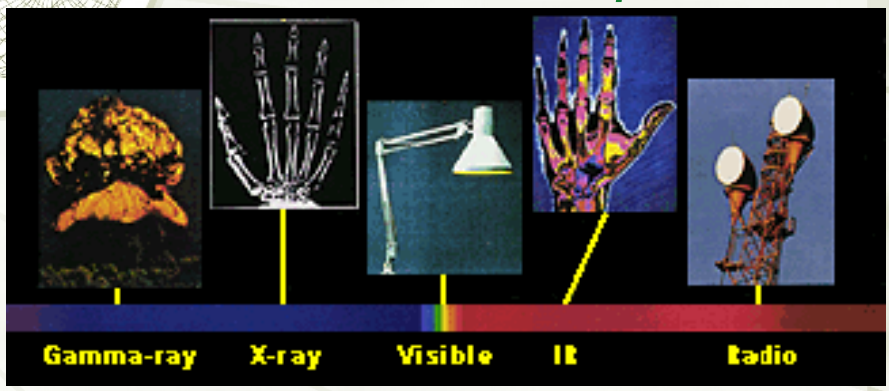
I : GRAVITATIONAL TIME DILATION

- **Recap of waves:**
- Waves characterized by
  - Wavelength (λ) = distance between crests
  - Frequency (f or ν) = number of crests passing a given point per second
- Speed of a crest; $c=\lambda\nu$
- **Energy** of a wave is proportional to frequency ν.





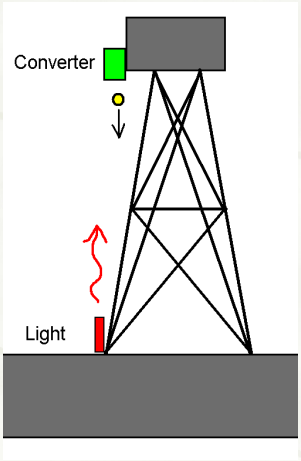
The electromagnetic spectrum



Small wavelength
High Frequency
High energy

Large wavelength
Low frequency
Low energy

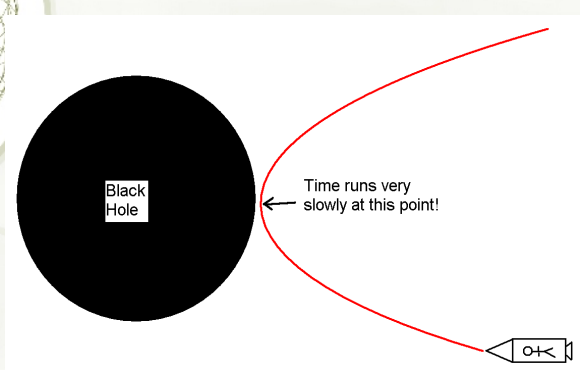
- Remember the tower…
- Laser light must lose energy as it climbs up
  - So…frequency must decrease
  - i.e., light is redshifted.
  - Gravitational redshifting
- Imagine a clock based on frequency of laser light…
  - 1 “tick” = time taken for fixed number of crests to pass
  - Gravitational redshifting slows down the clock.
  - **Clocks in gravitational fields run slowly**



$$t_{grav} \approx \left(1 - \frac{GM}{c^2 r}\right) t_{space}$$

if gravitational field is "weak"

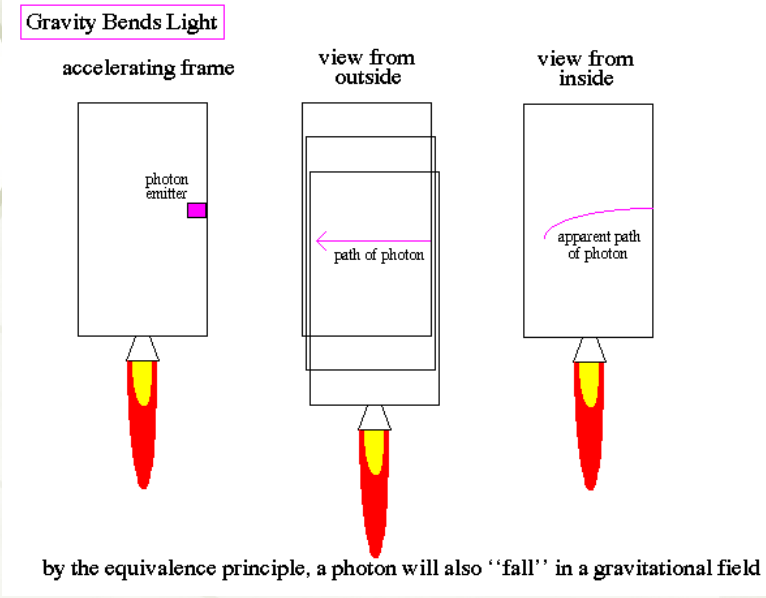
Gravitational Time Dilation



Important where gravity is very strong!

- Observer on Earth would see astronaut's clock running **very** slowly when close to black hole – astronaut would age very slowly.
- (In fact, there are other discomforts from of being near a black hole!)

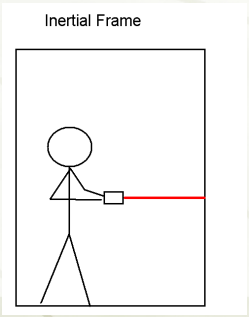
- Bending of light by gravity

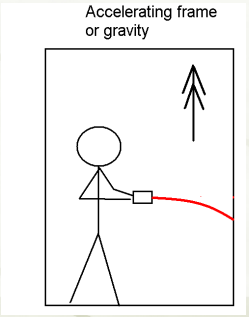


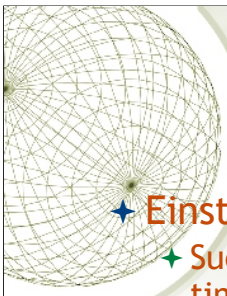
http://abyss.uoregon.edu/~js/cosmo/lectures/lec06.html

What about light? It "falls", too!

- Astronaut in inertial frame with flashlight
  - Inertial frame, so light goes in straight lines
  - It doesn't matter whether this is free fall or far from masses
- What if we now put flashlight in a gravitational field (accelerated frame)?
  - Light beam will bend: it must accelerate at the same rate and direction as the elevator
  - Strong equivalence principle ⇨ frame with gravity acts the same
  - Important conclusion – light "falls" due to gravity!

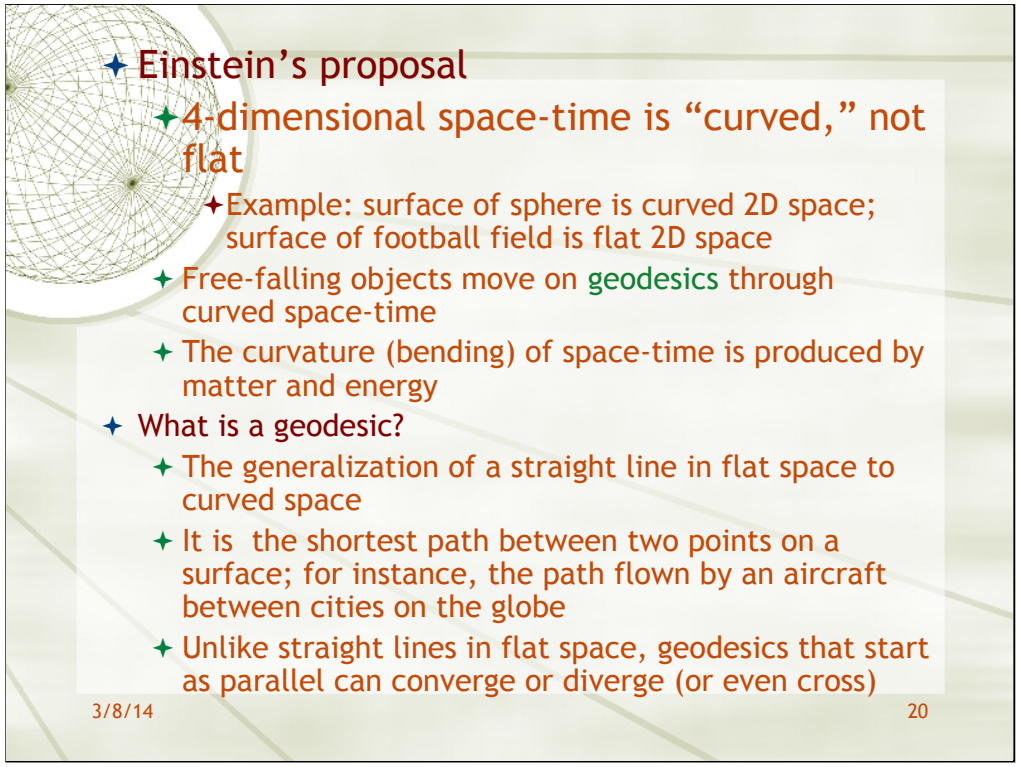






I: CURVED SPACE-TIME

- Einstein pondered several things…
  - Success of Special Relativity showed that space & time were closely linked
  - The "tower thought experiment" suggested that free-fall observers are (locally) free of effects of gravity
  - He wanted to say that gravity was an illusion caused by the fact that we live in an accelerating frame…
  - … but there is no *single* accelerating frame that works! Somehow, you need to stick together frames of reference that are accelerating in different directions


Einstein's proposal
4-dimensional space-time is "curved," not flat
Example: surface of sphere is curved 2D space; surface of football field is flat 2D space
Free-falling objects move on geodesics through curved space-time
The curvature (bending) of space-time is produced by matter and energy
What is a geodesic?
The generalization of a straight line in flat space to curved space
It is the shortest path between two points on a surface; for instance, the path flown by an aircraft between cities on the globe
Unlike straight lines in flat space, geodesics that start as parallel can converge or diverge (or even cross)
3/8/14
20

Anchorage
Vancouver
Seattle
Portland
Eugene
Spokane
Kalispell
Havre
Glasgow
Wolf Point
Sidney
Glendive
Missoula
Great Falls
Lewistown
Helena
Billings
Miles City
Boise
Salt Lake City
Reno
Sacramento
Oakland
San Francisco
San Jose
Honolulu
LAS VEGAS
Burbank
Ontario
Los Angeles
Long Beach
Orange County
San Diego
PHOENIX
Grand Junction
Aspen
Montrose
Telluride
Durango
Farmington
Denver
Colorado Springs
Albuquerque
Tucson
El Paso
Hermosillo
Guaymas
Mazatlán
Los Cabos
Puerto Vallarta
Guadalajara
Manzanillo
Ixtapa/Zihuatanejo
Acapulco
Mexico City
Kingman
Flagstaff / Grand Canyon
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Mobile
Pensacola
New Orleans
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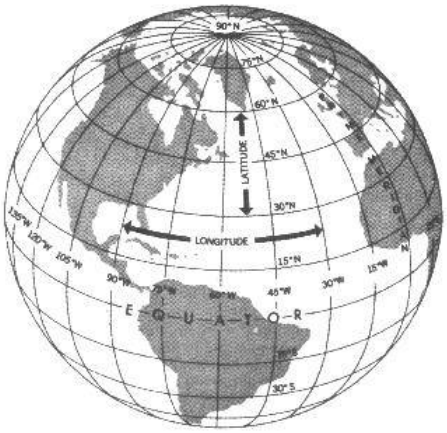
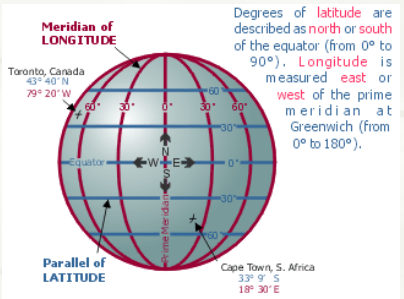
Huntsville
Knoxville
Birmingham
Atlanta
Cleveland
Toronto
Rochester
Buffalo
Syracuse
Albany
Quebec
Montreal
Ottawa
Halifax
Portland
Burlington
Manchester
Boston
Providence
Hartford/Springfield
New York (LGA)
White Plains
Long Island/Islip
Newark
New York (JFK)
Allentown
Harrisburg
Pittsburgh
Philadelphia
Baltimore (BWI)
Washington (National)
Washington (Dulles)
Charleston
Richmond
Norfolk
Greensboro
Raleigh/Durham
Charlotte
Greenville/Spartanburg
Columbia
Charleston
Jacksonville
Orlando
Tampa
Fort Myers
West Palm Beach
Fort Lauderdale/Hollywood
Miami
San Juan, Puerto Rico



On Globe…



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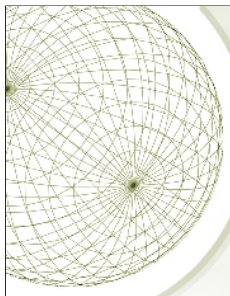
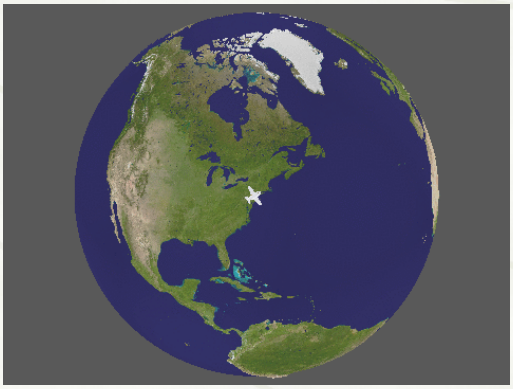
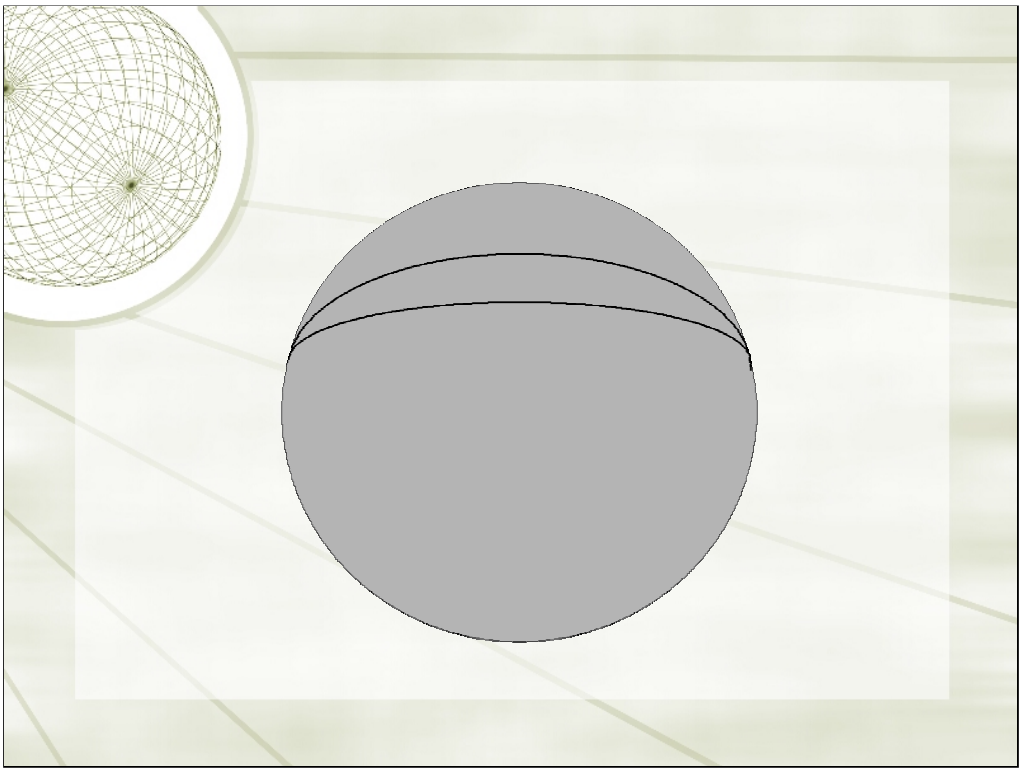
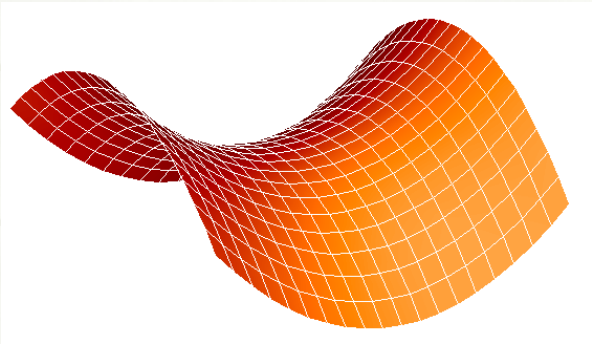


Shortest flight paths are geodesics

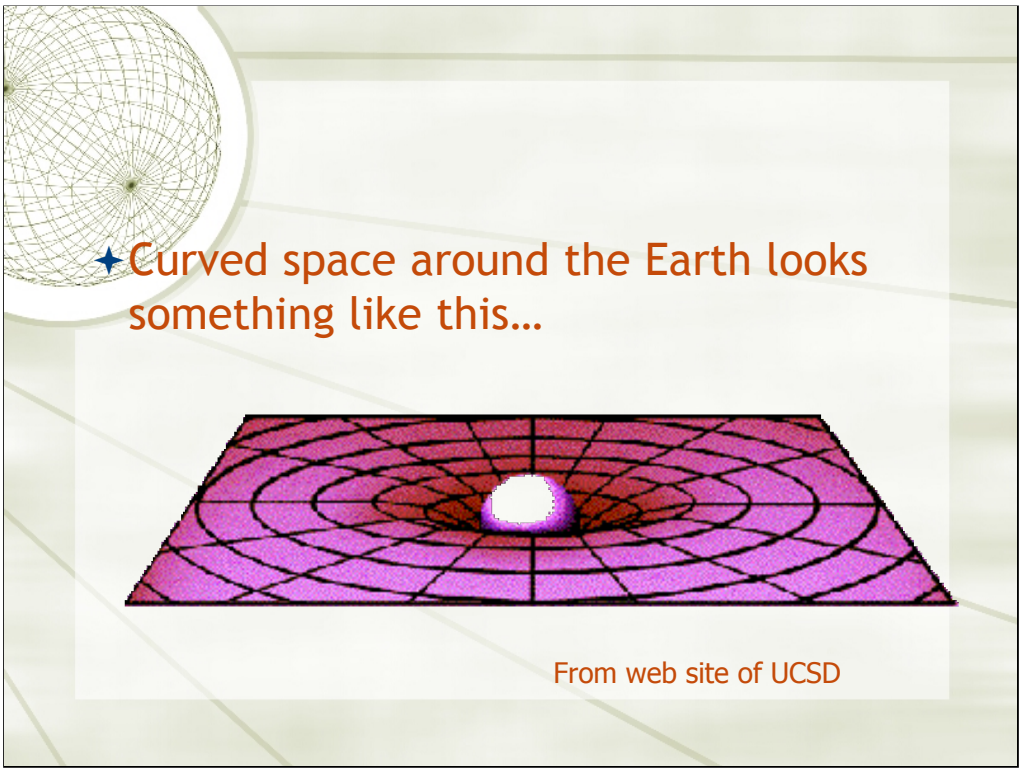


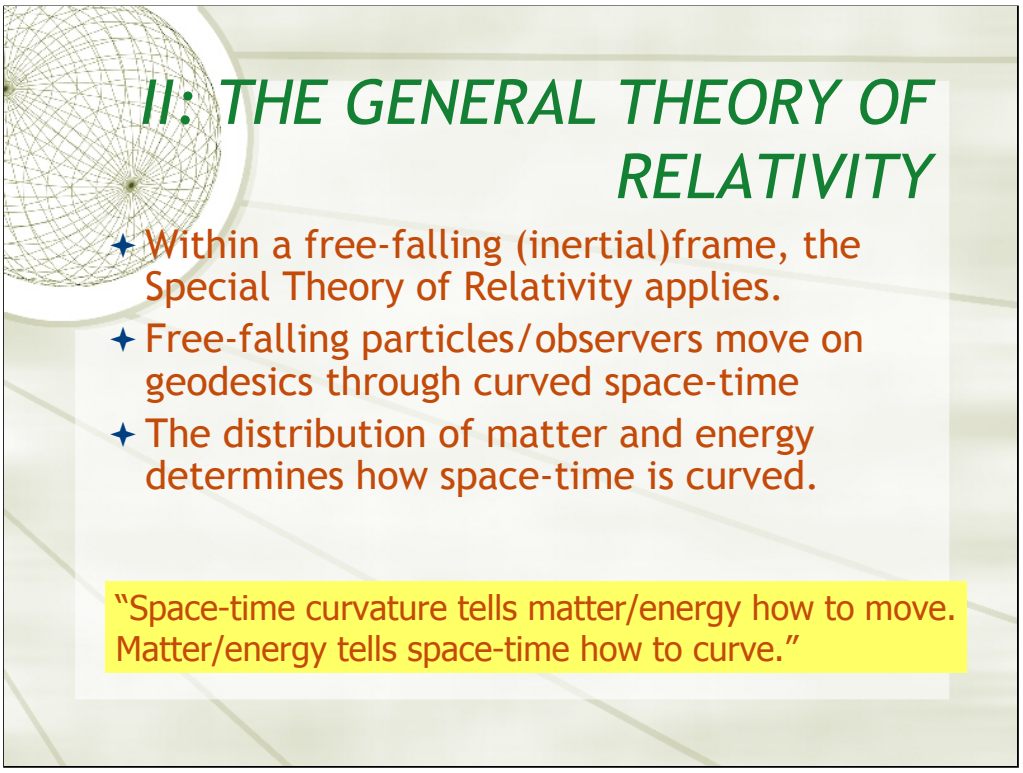


- Another example – a “saddle”

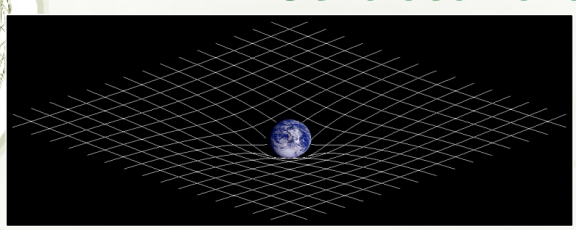


- Geodesics diverge


Curved space around the Earth looks something like this…
From web site of UCSD


II: THE GENERAL THEORY OF RELATIVITY
Within a free-falling (inertial)frame, the Special Theory of Relativity applies.
Free-falling particles/observers move on geodesics through curved space-time
The distribution of matter and energy determines how space-time is curved.
"Space-time curvature tells matter/energy how to move. Matter/energy tells space-time how to curve."

Structure of GR



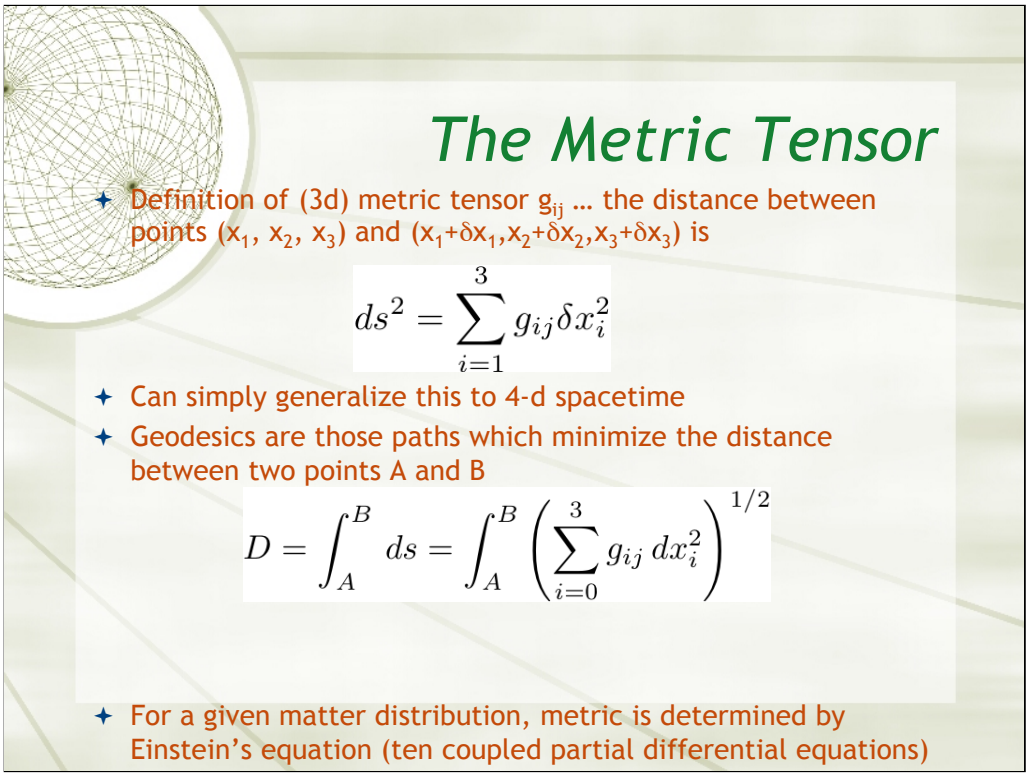
$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

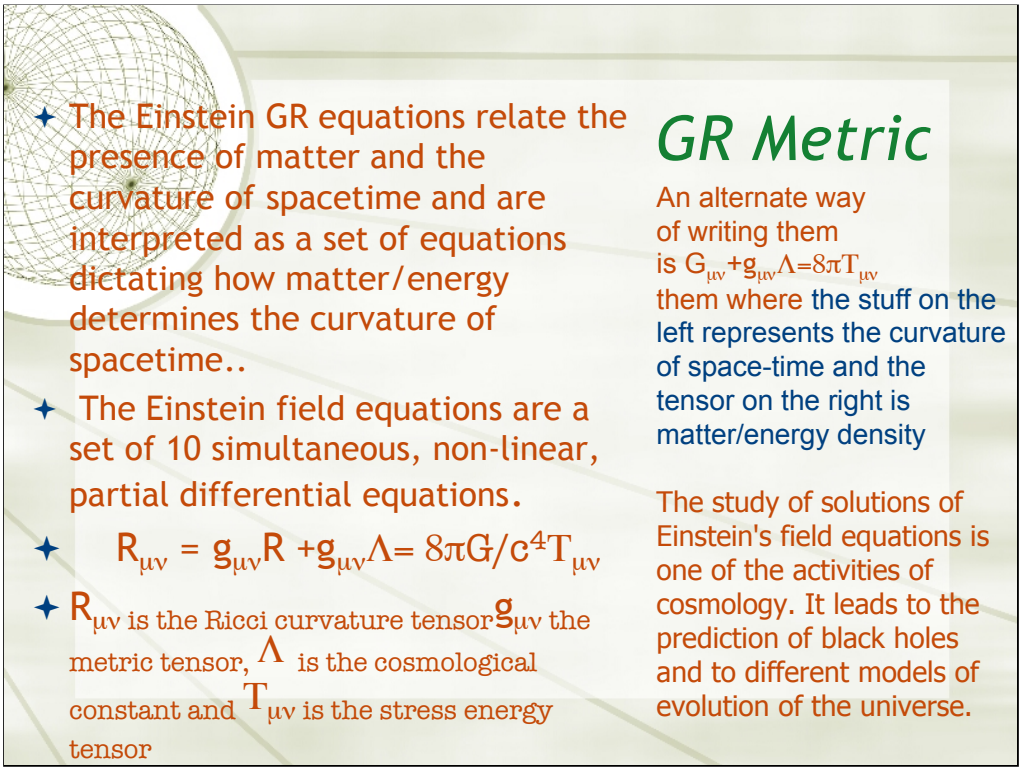
- General Relativity (Einstein 1915)
  - **Gravity is geometry**, not a normal force!
  - Free-fall particles follow "straight-lines" through the 4-d spacetime (Strong Equivalent Principle)… I.e., **spacetime curvature tells matter how to move**
  - Distribution of matter and energy determines curvature of spacetime (Einstein Field Equations). I.e. **matter/energy tells spacetime how to curve**

Einsteins Master Equation

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

- Notes:
  - The Einstein curvature tensor "$\underline{\underline{\mathbf{G}}}$" is mathematical object describing curvature of 4-D space-time.
  - The Stress-Energy tensor "$\underline{\underline{\mathbf{T}}}$" is mathematical object describing distribution of mass/energy.
  - Newton's constant of gravitation "G" and the speed of light "c" appear as fundamental constants in this equation.
  - This is actually a horrendous set of 10 coupled non-linear partial differential equations!!
- For weak gravitational fields, this gives Newton's law of gravitation.


The Metric Tensor
Definition of (3d) metric tensor g_{ij} … the distance between points (x_1, x_2, x_3) and $(x_1+\delta x_1, x_2+\delta x_2, x_3+\delta x_3)$ is
$$ds^2 = \sum_{i=1}^{3} g_{ij}\delta x_i^2$$
Can simply generalize this to 4-d spacetime
Geodesics are those paths which minimize the distance between two points A and B
$$D = \int_A^B ds = \int_A^B \left(\sum_{i=0}^{3} g_{ij}\, dx_i^2\right)^{1/2}$$
For a given matter distribution, metric is determined by Einstein's equation (ten coupled partial differential equations)



- The Einstein GR equations relate the presence of matter and the curvature of spacetime and are interpreted as a set of equations dictating how matter/energy determines the curvature of spacetime..
- The Einstein field equations are a set of 10 simultaneous, non-linear, partial differential equations.
- $R_{\mu\nu} = g_{\mu\nu}R + g_{\mu\nu}\Lambda = 8\pi G/c^4 T_{\mu\nu}$
- $R_{\mu\nu}$ is the Ricci curvature tensor $g_{\mu\nu}$ the metric tensor, Λ is the cosmological constant and $T_{\mu\nu}$ is the stress energy tensor

GR Metric

An alternate way of writing them is $G_{\mu\nu}+g_{\mu\nu}\Lambda=8\pi T_{\mu\nu}$ them where the stuff on the left represents the curvature of space-time and the tensor on the right is matter/energy density

The study of solutions of Einstein's field equations is one of the activities of cosmology. It leads to the prediction of black holes and to different models of evolution of the universe.

Testing of GR

- The metric of GR (the set of equations describing space-time) is mathematically complex.
- However one can express it approximately as a algebraic series where the first time is 'Newtonian' and the higher order terms represent deviations from Newton's laws.

- $ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \ldots)c^2dt^2 - (1 - 2\gamma\phi_N + \ldots)dx^2$.
- (Remember back in Special relativity we defined ds^2 as the space time distance)
- In GR $\beta,\gamma = 1$ and different values would represent a deviation from GR.
- β,γ have been measured in the solar system and elsewhere that deviate by $< 3 \times 10^{-5}$ from 1.