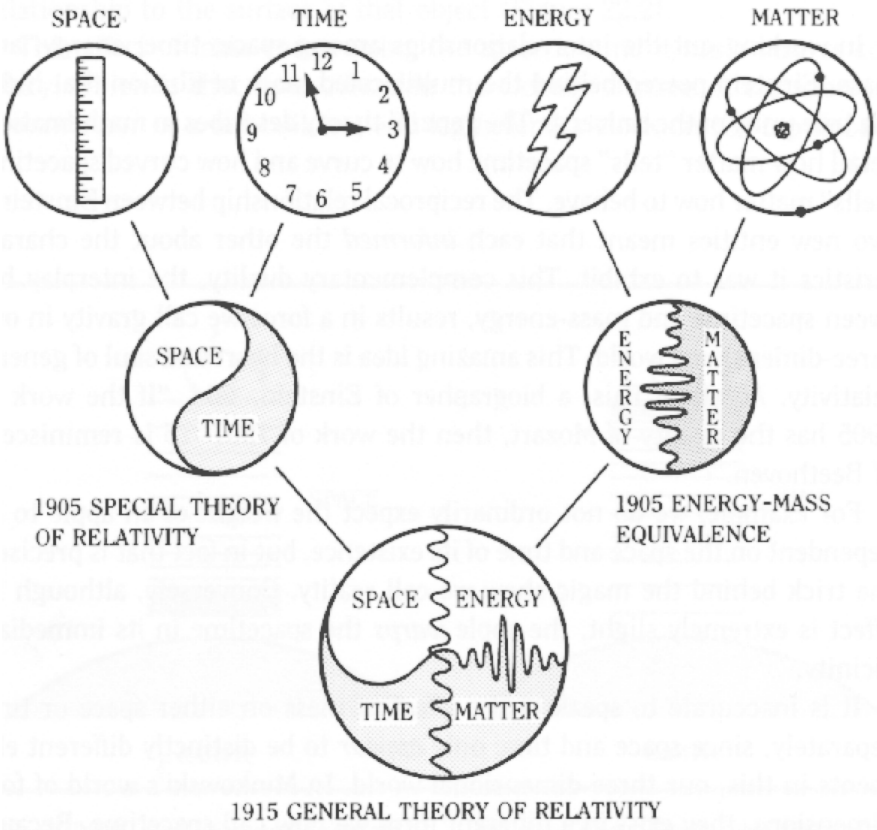
Lecture 11: General Relativity III

- Recap : gravitational redshift
- Curved spacetime and geodesics
- The General Theory of Relativity
- Black Holes



Gravitational Redshift

- To work it out with equations
- For a photon $E=mc^2=h\nu$
- The gravitational potential energy (the energy gained by falling in a gravitational field) $PE=-GMm/r=(-GMh/rc^2)\nu_0$- last

lecture we use 'little' g; PE=mgh; where 'little' g is the acceleration at the surface of the earth- these are the same

- So as the photon rises up in the gravitational field
- $h\nu=h\nu_0\{1-(GM/rc^2)\}$; $\nu=\nu_0\{1-(GM/rc^2)\}$; $\Delta\nu/\nu=GM/rc^2$
  where $\Delta\nu$ is the change in frequency of the photon $\Delta\nu=(\nu-\nu_0)$
- Since the photon is reduced in frequency (and thus lengthened in wavelength) this is called the gravitational redshift

Gravitational Redshift

- The conservation of energy-matter: not just the conservation of energy
- 'The' equation for the potential energy in a gravitational

field PE=m*g*h (or more generally PE=GM/r)

- The energy of a photon $E=h\nu=mc^2$

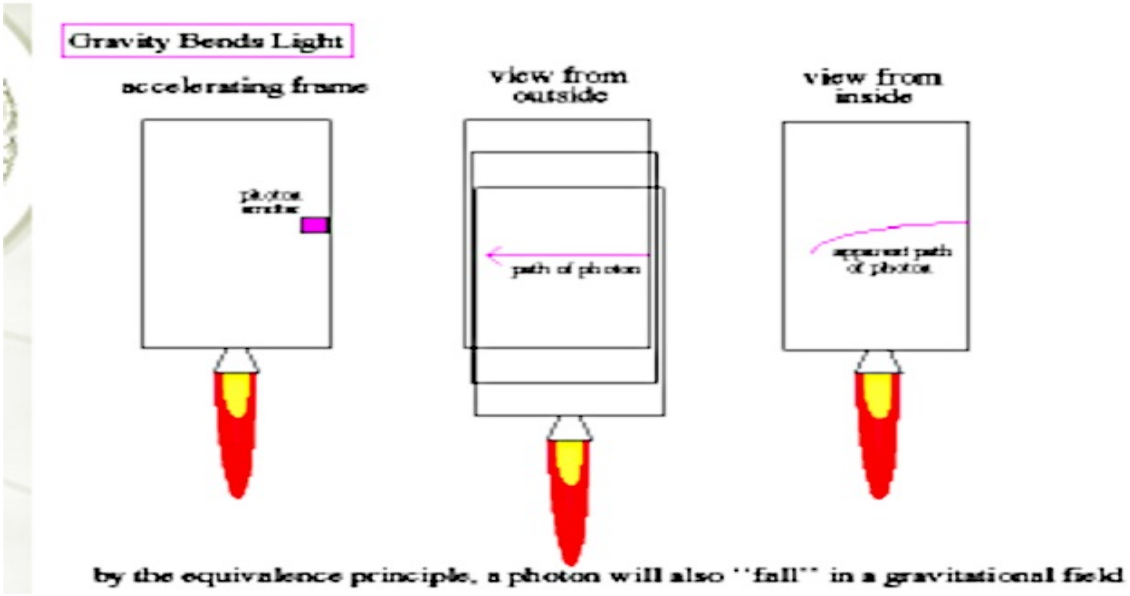
So the 'trick' is: The energy received must equal the energy emitted minus the energy due to the potential energy

Finally: another way of looking at this From the principle of equivalence between gravity and force due to acceleration, the shift in frequency in a gravitational field can be related to the relativistic Doppler shift due to an accelerating light source.

- Derivation:

for velocities v<<c the Doppler shift formula is $\nu=\nu_0[1+V/c]$
For an observer accelerated to velocity V in a time t=L/c (L is some length)
$V=\mathbf{a}t=\mathbf{a}L/c$ and then $\nu=\nu_0[1+aL/c^2]$ and replacing the arbitrary acceleration '**a**' by gravitational acceleration 'g'
$\nu=\nu_0[1+gL/c^2]$

Bending of light by Gravity



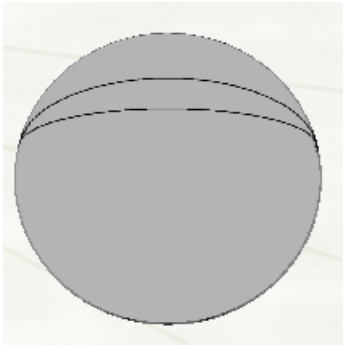
The principle of equivalence renders the gravitational field fundamentally different from all other force fields encountered in nature.

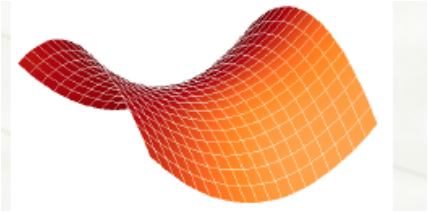
http://abyss.uoregon.edu/~js/cosmo/lectures/lec06.html

- Einstein's suggestion
  - 4-dimensional space-time is curved
  - Free-falling objects move on "geodesics" (generalizations of straight lines) through curved space-time.
  - Matter and energy causes space-time to bend.
- What is a geodesic?
  - Shortest path between two points on a surface
  - E.g. path flown by aircraft
  - Geodesics that start parallel can converge or diverge (or even cross).

Einstein's proposal

- 4-dimensional space-time is "curved," not flat
- Examples: surface of sphere is curved (2D space);
  surface of football field is flat (2D space)
- Free-falling objects move on geodesics* through curved space-time
- The curvature (bending) of space-time is produced by matter and energy
- "Space-time curvature tells matter/energy how to move. Matter/energy tells space-time how to curve."





a geodesic is the generalization of a straight line in flat space to curved space
It is the shortest path between two points on a surface; for instance, the path flown by an aircraft between cities on the globe
Unlike straight lines in flat space, geodesics that start as parallel can converge or diverge (or even cross on a curved surface

Structure of GR

- **Gravity is geometry**, not a normal force!

- Free-fall particles follow geodesics "straight-lines" through the 4-d spacetime (Strong Equivalent Principle)…
  I.e., **spacetime curvature tells matter/light how to move**
- Distribution of matter and energy determines curvature of spacetime (Einstein Field Equations).

I.e. **matter/energy tells spacetime how to curve**

Einstein's Master Equation

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

The Einstein curvature tensor "**G**" is a mathematical object describing curvature of 4-D space-time.
The Stress-Energy tensor "**T**" is mathematical object describing distribution of mass/energy.

Newton's constant of gravitation "G" and the speed of light "c" appear as fundamental constants in this equation.

This is actually a horrendous set of 10 coupled non-linear partial differential equations!!
For weak gravitational fields, this is equivalent to Newton's law of gravitation

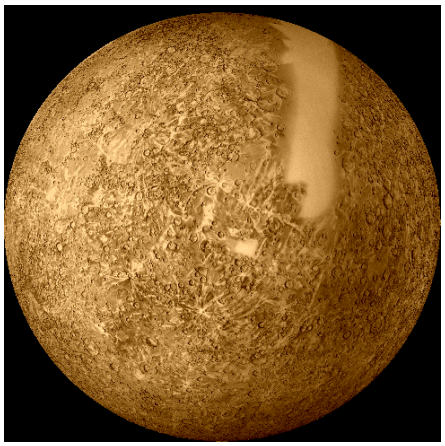
Tests of General Relativity

- Tests in the solar system and the laboratory have shown that at 'small scales' and low M/R General Relativity is accurate to better than 1 part in 10^5

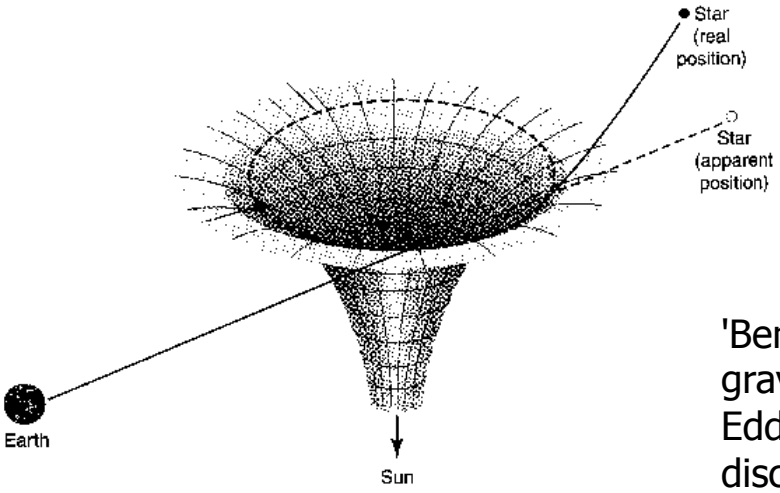
- The most famous test is the precession of the perihelion of Mercury.

GR EFFECTS IN THE SOLAR SYSTEM

- Have already heard about bending of star light by the Sun (detected by Eddington).
- **Orbit of Mercury:**



Warping of space by Sun's gravity

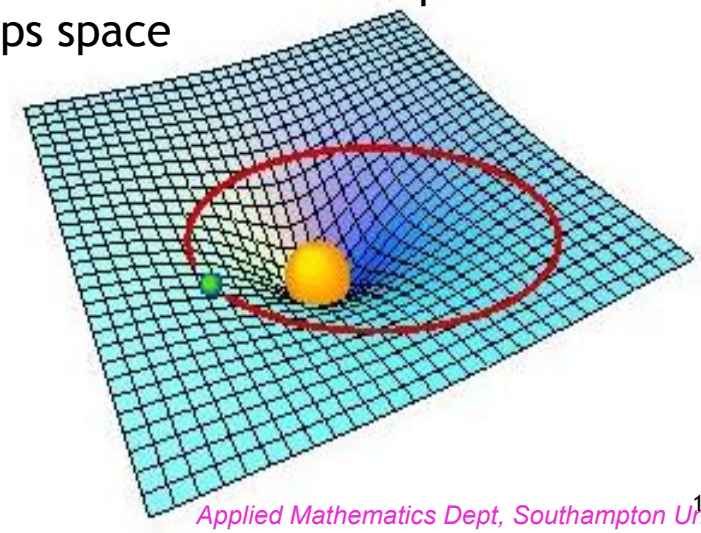


'Bending of light by gravity of sun- the Eddington test we discussed last time

- Light rays follow geodesics in warped space

How are orbits of planets affected?

- Green marble would follow straight line if yellow weight were not there
- Marble's orbit becomes curved path because weight warps space

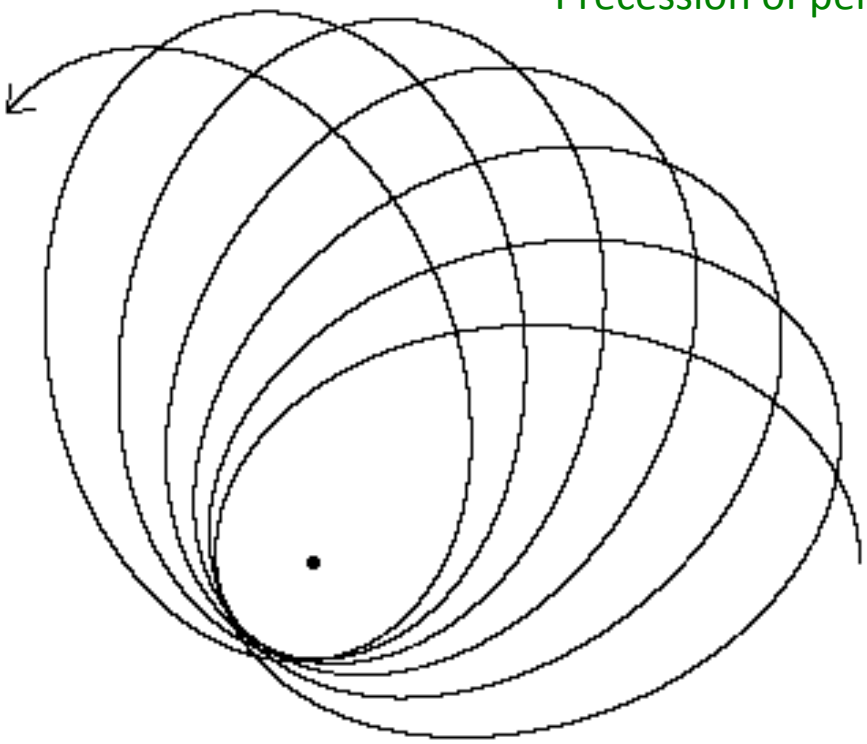


Applied Mathematics Dept, Southampton University

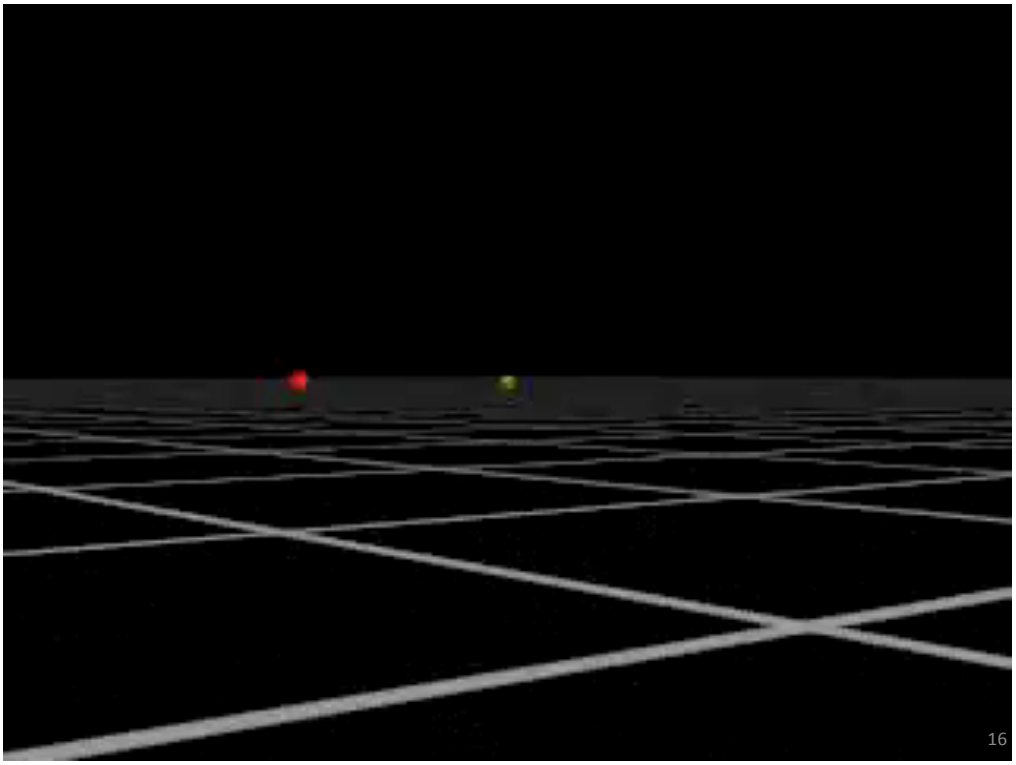
Mercury's Precession

- Under Newtonian physics, a (small) object orbiting a spherical mass would trace out an ellipse with the spherical mass at a focus (Kepler's laws) but ...
  - Gravitational effect of other planets
  - Oblate deformation of the Sun
- Leads to "precession of perihelion" even for Newtonian physics…
  - But, Newton wouldn't have worried about non-inertial nature of Earth's frame!

Precession of perihelion



Precession of the orbit of Mercury



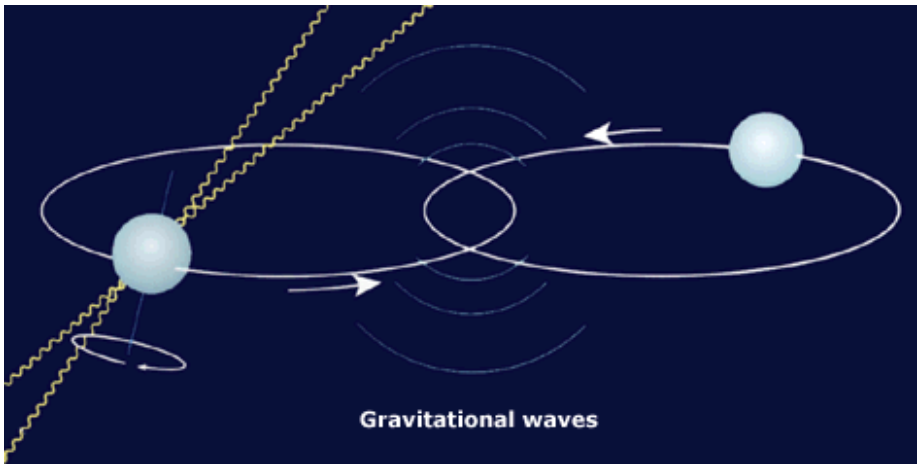
Precession of Mercury's orbit

- Mercury orbit about Sun is rather elliptical (e=0.206)
- Orbit precesses around Sun by 5600 arcsec/century… most of this is due to perturbations from other planets
- 43 arcsec/century ***unaccounted for*** by Newtonian effects of known planets
  - This was a discovered in 1859 and was certainly known to Einstein
  - Various Newtonian effects to explain this (planet Vulcan, ring of planetoids, breakdown of inverse-square law) all unsuccessful

GR provided very natural and accurate explanation for precisely this difference.

The binary pulsar (PSR1913+16)

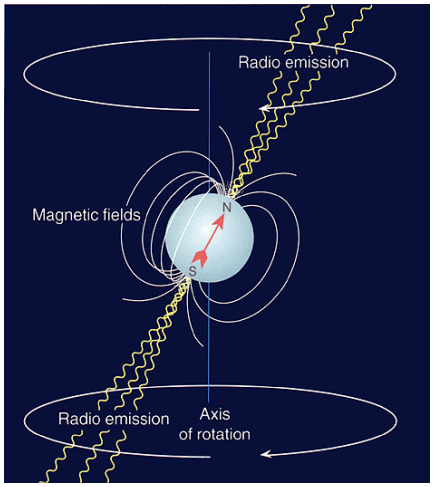
- Russell Hulse & Joseph Taylor (1974)
  - Discovered remarkable double star system
  - Nobel prize in 1993



From Nobel Prize website

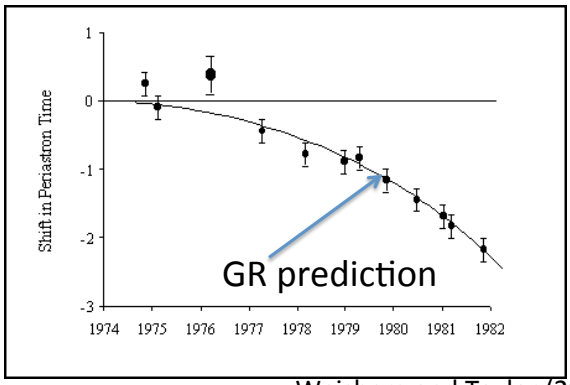
Hulse-Taylor system

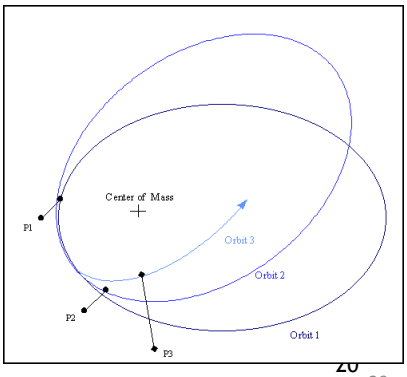
- Two **neutron stars** orbiting each other
- One neutron star is a pulsar –
  - Neutron star is spinning on its axis (period of 59ms)
  - Emits pulse of radio towards Earth with each revolution
  - Acts as a very accurate clock!
- Strong gravity- good place to test GR
  - Orbit precesses 4 deg/year!
  - Orbit is shrinking due to gravitational waves
  - Why?



Precise test of certain aspects of GR

- When pulsar is approaching Earth, pulse frequency increases (Doppler shift); when pulsar is receding, pulse frequency decreases -- orbit of pulsar can therefore be "mapped"
- Orbit seen to be precessing (same physics as for Mercury) and shrinking (loss of energy due to gravitational waves) at exactly the rate predicted by Einstein's theory



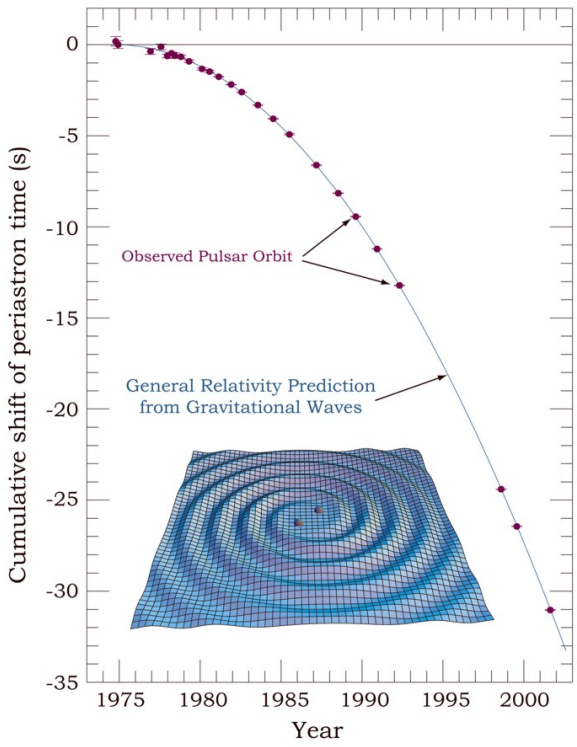


Weisberg and Taylor (2004)

As data have gotten better the agreement with General Relativity is more and more accurate

A strong theory survives stronger and stronger tests

Binary pulsars are the only unambiguous detection of the effects of gravitational radiation



GRAVITATIONAL WAVES-Another Prediction of GR and a Test

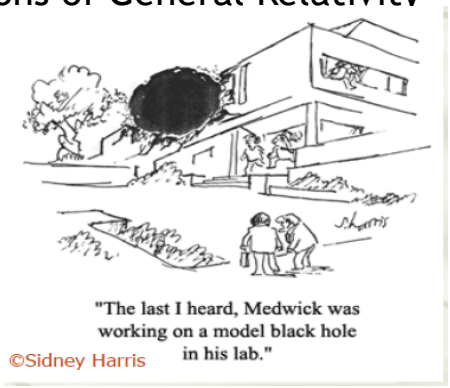
- Accelerating masses produce continual changes in space-time geometry
- Periodically-moving bodies (e.g. orbiting stars) create ripples in space-time curvature
- Ripples travel at speed of light through space-time
- These are called **gravitational waves**
- Usually **VERY** weak unless LARGE masses are experiencing LARGE accelerations- merger of black holes
- Yet to be detected- stay tuned for Advanced LIGO in 3-4 years

Black Holes

– One of the most unusual predictions of General Relativitv

Back to Newtonian Physics
What goes up must come down"… or must it?
Escape velocity, V_{esc} Critical velocity object must have to just escape the gravitational field of an object (Earth)
$V<V_{esc}$: object falls back to Earth
$V>V_{esc}$: object never falls back to Earth
Escape velocity given in general by

$V_{esc}=\text{sqrt}(\mathbf{2GM/R})$



where the mass of an object is M and the distance from the center is R-
G is the gravitational constant
Starting from **Earth's** surface, V_{esc}=11 km/s
Starting from **Sun's** surface, V_{esc}= 616 km/s

Black Holes

By making M larger and R smaller, V_{esc} increases- in Newtonian physics to arbitrary values
Idea of an object with gravity so strong that light cannot escape first suggested by Rev. John Mitchell in 1783

General Relativity

Karl Schwarzschild (1916) First solution of Einstein's equations of GR
Describes gravitational field in (empty) space around a non-rotating mass
Space-time interval in Schwarzschild's solution (radial displacements only) is rather complex- see text book

- Features of Schwarzschild's solution:
  Yields Newton's law of gravity, with flat space, at large distances (Large R)
  Space-time curvature becomes infinite at center (R=0; this is called a **space-time singularity**)
- Gravitational time-dilation effect becomes infinite on a spherical surface known as the **event horizon**, where coefficient of Δt is zero
- Radius of the sphere representing the event horizon is called the **Schwarzschild radius,** $R_s = 2GM/c^2$

Event horizon

Point of no-return… the location where the **escape velocity equals the speed of light**

- The **gravitational redshift becomes infinite** here (as seen by an outside static observer)
- Nothing occurring inside can be seen from outside (or have any causal effect on the external Universe!)
- So… as a practical matter, astrophysicts never need concern themselves with the Universe *interior* to the event horizon

$$R_{Sch} = \frac{2GM}{c^2} \approx 3\left(\frac{M}{M_\odot}\right) \text{ km}$$

symbol for sun, $M_\odot$ =solar mass

- Radius corresponding to event horizon for a non-spinning black hole is known as the **Schwarzschild radius R_{Sch}**

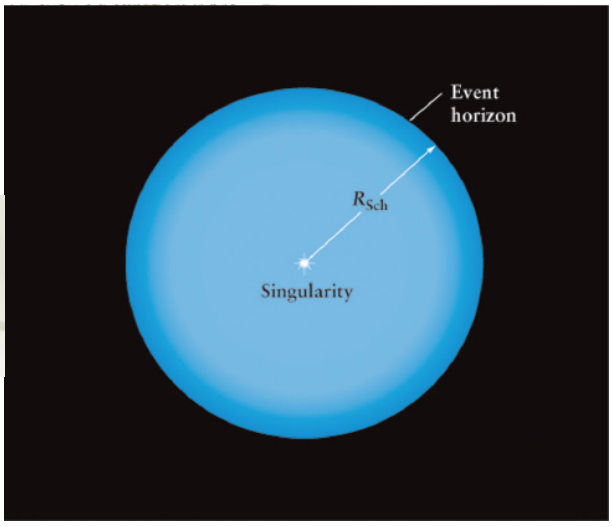
GR black holes

Gravitational redshift outside of a spherical object with mass M is

$$\nu_{obs} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \nu_{emit}$$

ν is frequency of the light

As r $\longrightarrow$ R_s
ν_{obs} goes to zero
wavelength of emitted radiation goes to ∞
infinite redshift



- Radius corresponding to event horizon for a non-spinning black hole is known as the **Schwarzschild radius R_S**

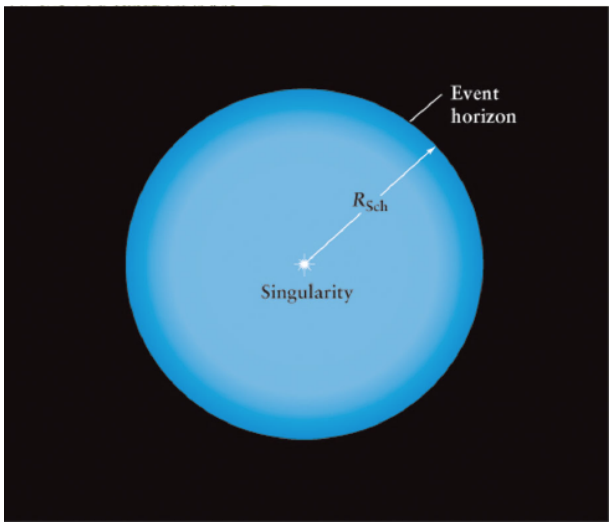
GR black holes

Gravitational length contraction
$L'=L\,(1-R_s/R)^{1/2}$

Gravitational time dilation
$t_0=t_f(1-R_s/R)^{1/2}$; t_0 is the time that the observer near the BH measures
t_f is the time that a distant observer measures – **as you get closer to the black hole time slows down**

$R_s= 2GM/c^2$

As R ⟶ R_s time goes to ∞
length goes to zero

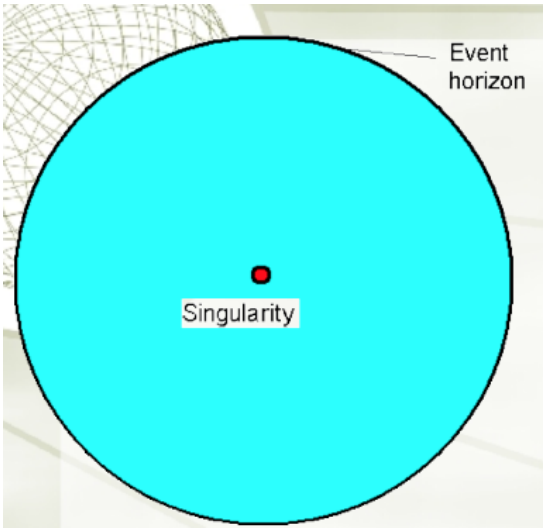


- Radius corresponding to event horizon for a non-spinning black hole is known as the **Schwarzschild radius R_S**

GR black holes

For a body of the Sun's mass,
Schwarzschild radius
$R_S = 2GM/c^2$ →3km

- **Singularity** – spacetime curvature is infinite. Everything destroyed. Laws of GR break down.
- **Event horizon (R_s)** – gravitational time-dilation is infinite as observed from large distance.
- Any light emitted at Rs would be infinitely redshifted - hence could not be observed from outside



Schwarschild radius is NOT the singularity

At the Schwarzschild radius gravitational time dilation goes to infinity and lengths are contracted to zero

More features of Schwarzschild black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)- remember the Minkowski diagrams !
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)

- Observer who enters event horizon would only "feel" "strange" gravitational effects if the black hole mass is small, so that R_s is comparable to observer's size

- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
- Thus black hole would produce "shadow" on sky

Black holes!

$$r_{Sch} = \frac{2GM}{c^2}$$

for the Sun :

$$r_{Sun-Sch} \approx \frac{2\times(7\times10^{-11})\times(2\times10^{30})}{9\times10^{16}}$$

$$\approx 3\times10^{-11+30-16} = 3{,}000\,(\text{units?})$$

- How about the Earth? Or say, you?

Falling Into a Black Hole

The Singularity

The center of the blackhole… the place into which all of the matter making the black hole has been crushed

General Relativity gives nonsense answers here (infinite density, infinite spacetime curvature)… **so GR must break down here**
Some new theory - **quantum gravity** is needed

Real black holes

So much for theory – what about reality

Thought to be two classes of black hole in nature

- **"Stellar mass black holes"** – left over from the collapse/ implosion of a massive star ($M>8M_{\odot}$)
- **"Supermassive black holes"** – giants that currently sit at the centers of galaxies (range of mass of BH from 10^6-10^9 $M_{\odot}$)

Why Do We Think Black Holes are Real

In binary stars can determine the mass of the stars by measuring their orbits- *Kepler's laws*

In x-ray binaries one of the stars is 'normal'- the other has strange properties (emits lots of x-rays and is very small and other things)

Mass of the the 'strange' star is larger than a neutron star can be (maximum mass of a neutron star is set by quantum mechanics (!))

So lots of mass and very small size leads to the idea of a black hole

Gravity overcomes all other forces and the object is squished as small as GR lets it be

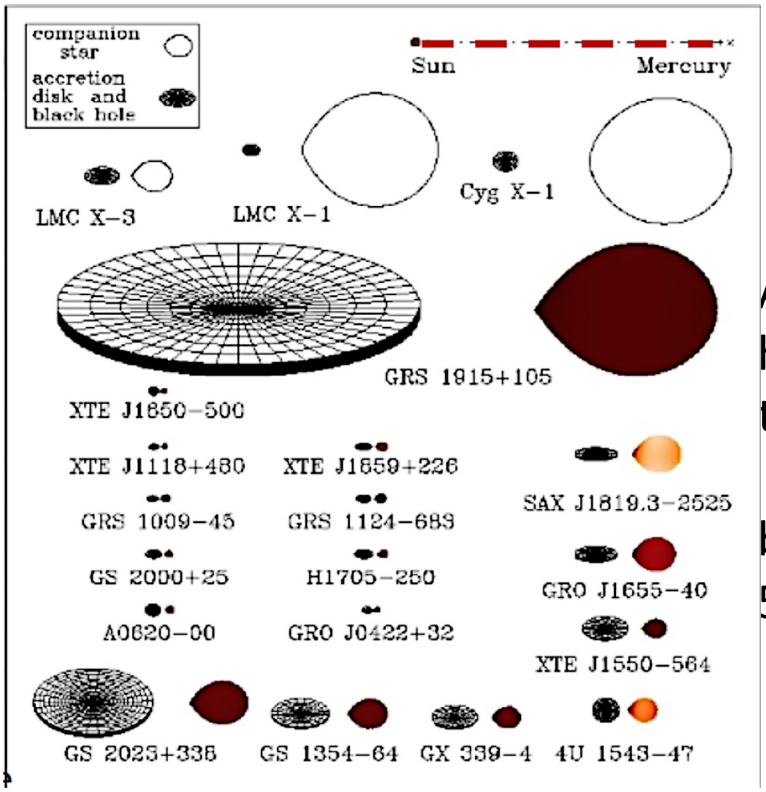


Figure is to scale- From Jerome Orosz

The Known Galactic Black holes

About 20 black holes known in the Milky Way

BH Mass between 5-16$M_\odot$

All *known* galactic black holes are in binary star systems.

Tidal forces can rip matter off the normal star

Matter goes into orbit around black hole – forms an **accretion disk**

As matter flows in towards the black hole, it gives up huge amount of energy

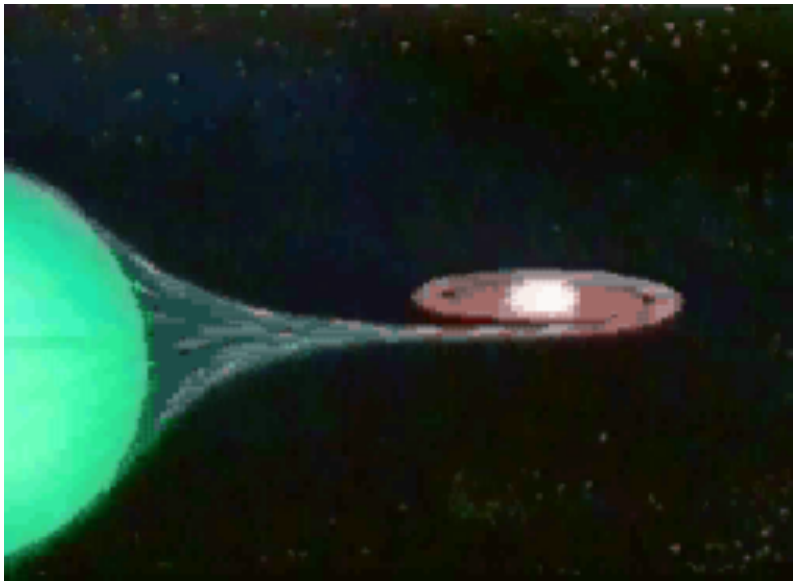
analogy to hydroelectric power derived when water falls over a dam

Energy is first converted to heat, raising gas temperature in accretion disk to millions of degrees

Hot accretion disk radiates away energy, emitted mainly as X-rays

Black holes in binary systems

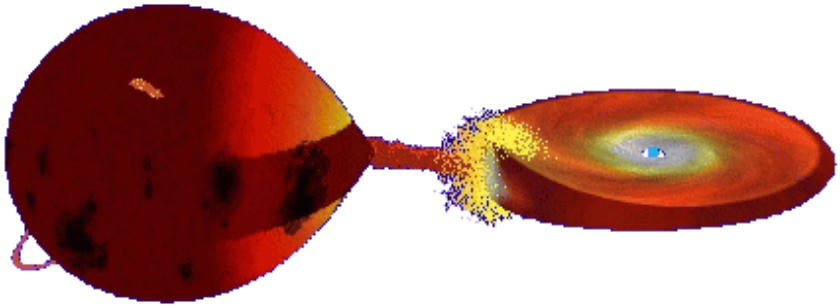
These systems are called **X-ray binaries**



"X-ray" binary

Outer layers of a nearby star are pulled off by tides, crash into accretion disk around BH

Accretion Onto A Compact Object

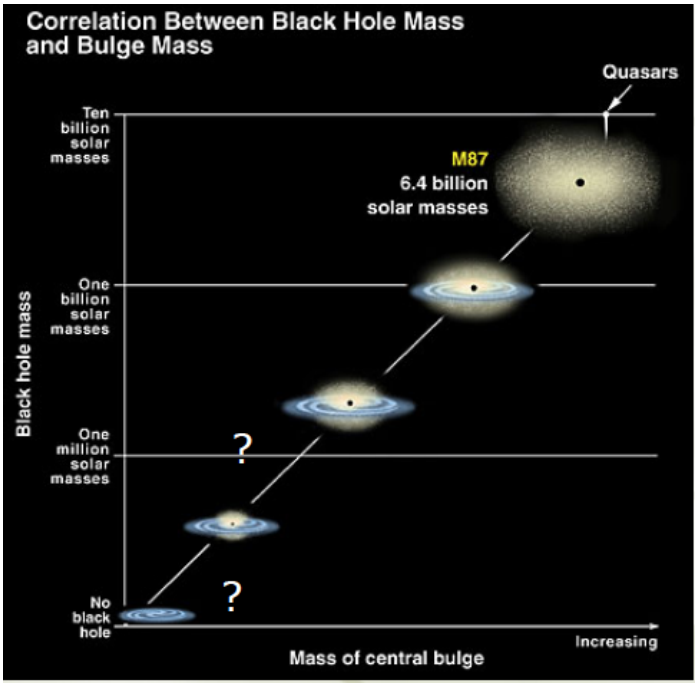


http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif

Supermassive black holes (SMBHs)

Found in the centers of most big galaxies

Mass of black hole and galaxy are correlated

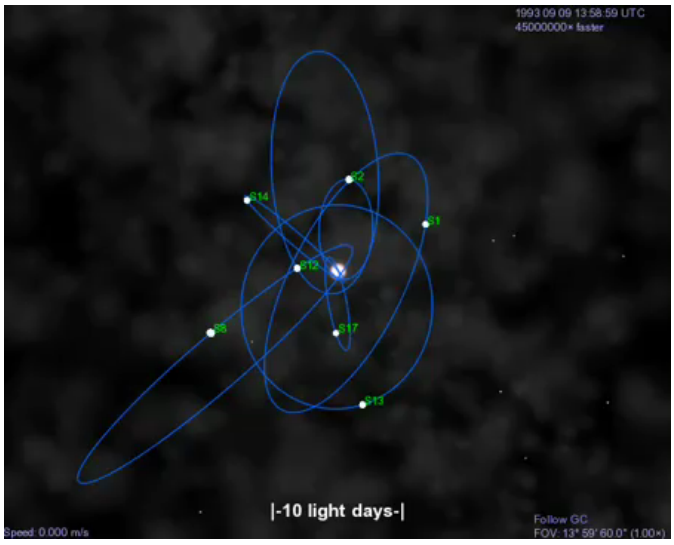


*Center of the Milky Way: Sgr A**

- The center of our own Galaxy
- Can directly observe stars orbiting an unseen object

Best fit requires a black hole with $M \sim 3.7 \times 10^6 M_\odot$ to explain stellar orbits

- Best case yet of a black hole.

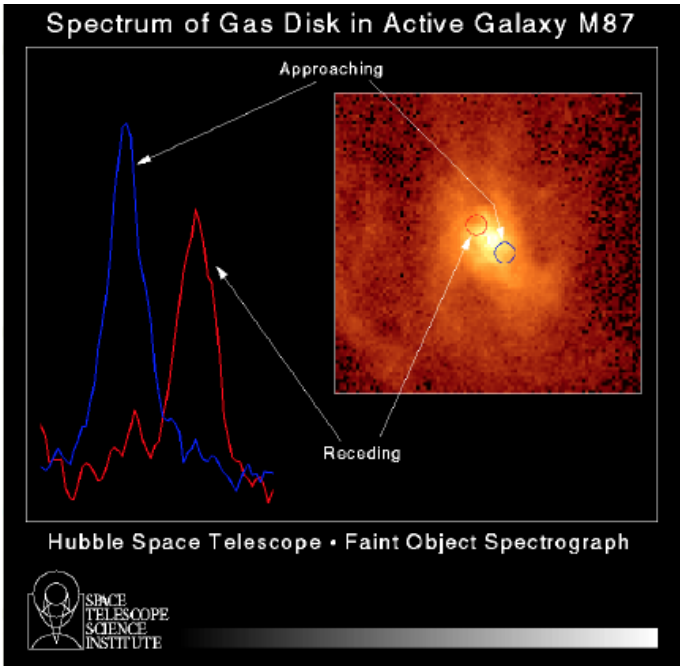


Nearby Galaxy M87

Another example – the SMBH in the galaxy M87
HST detects a gas disk orbiting galaxy's center

- Measure velocities of the gas using the *Doppler effect* (red and blue shift of light from gas)

- Need a 3 **b**illion solar mass SMBH (Kepler's Laws)to explain gas diskvelocities



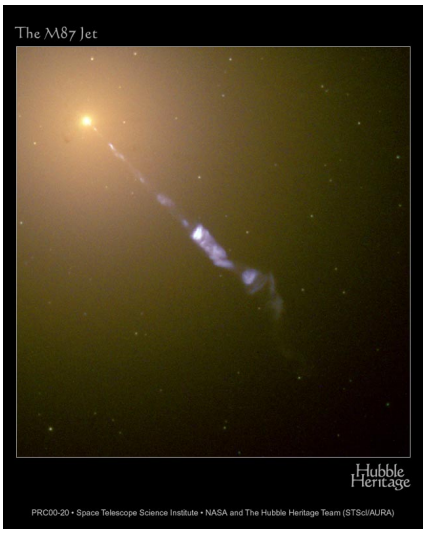
Active Galaxies

M87 is example of an "active galactic nucleus"

- Material flows (accretes) into black hole
- Energy released by accretion of matter powers

very energetic phenomena

Particularly powerful active galactic nuclei are sometimes called **Quasars-can be up to 10^{14} x more luminous than the sun**



HST Image of M87

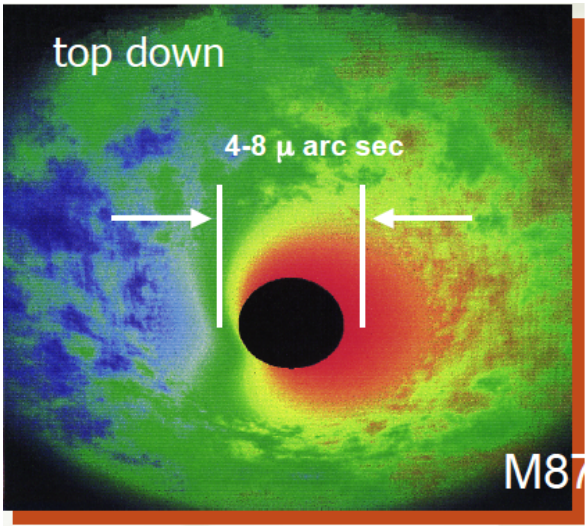
1966 Time Magazine Cover, discovery of QSOs

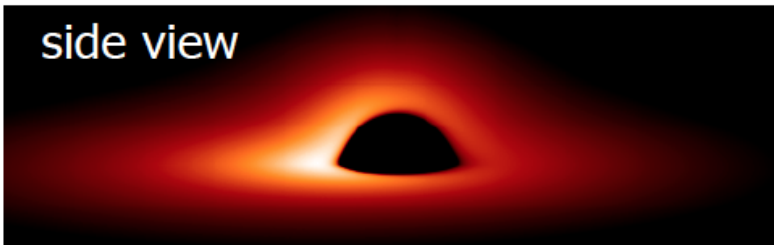
quasars can be seen at redshifts >6, universe 10^9 yrs old

'Images' of a Black Hole

A theoretical calculation of what the region near a black hole might look like

- The red/blue represent the Doppler shift and the asymmetries due the effect of GR(bending of light)





Active galaxies (e.g. radiating supermassive black holes) emit radio, IR, optical, UV, x-ray and γ-ray radiation !

Broad band spectrum very different than stars

In the x-ray band there is a signature of the physics very close to the event horizon

time variability of an active galaxy with total luminosity ~10^{10} Sun's intensity changing on timescales of minutes!