Lecture 27 : The big summary

FINAL EXAM

- *Friday, 15 May, 8:00-10:00*
- Exam is in this room
- Cumulative, but with emphasis on material after the midterm
- No notes or books allowed
- **Bring calculator**
- Q&A review session in class, May 12
- As for the mid-term all relevant (and some not relevant) formulas and constants will be given

Please fill in your course evaluation!

- www.CourseEvalUM.umd.edu- closes Friday evening May 13!

- We are at only 24% (10 people!)-
- What did we do that you liked-disliked
- How can I improve?
- Help your fellow students and me.
- Identification numbers are not linked to your feedback in the evaluation reporting system

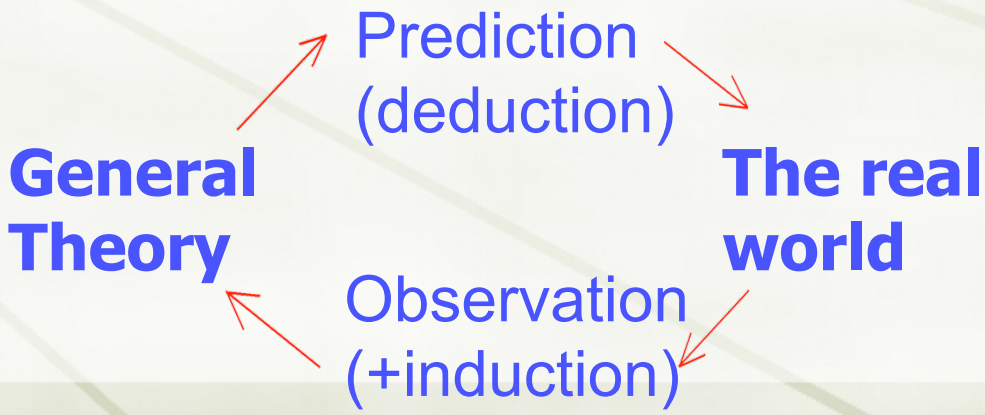
Class Summary

- We have covered an enormous amount of material and a vast range of subjects (more than 600 slides !)
- I am proud that so few have dropped this, perhaps harder than expected,class.

Kudos- praise given for achievement

Scientific cosmology

- Non-anthropocentric narrative
- Based on concept of causality, but not purpose
- Derives from and is checked by data = objective (reproducible), quantitative observations of the physical world
- Models/theories are continually re-evaluated based on the *scientific method*
- To be scientific, *a theory must be falsifiable* : whole or part may be rejected based on new data
- New data can support an existing theory, but cannot prove it

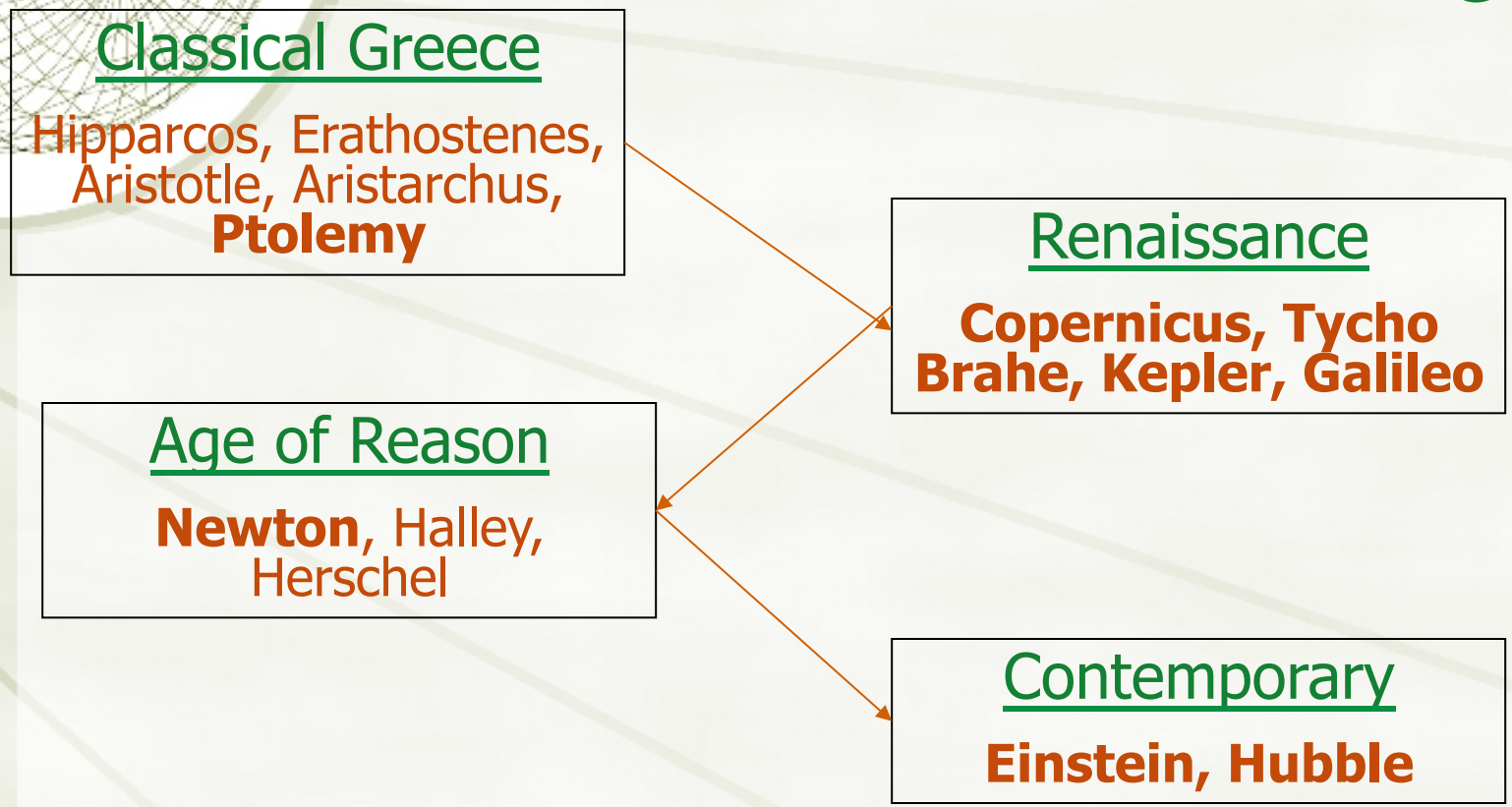


The scientific method

- Relevant (explanatory power)
- Consistent (within and without)
- Predictive (qualitative and quantitative)
- Testable (falsifiable)
- 'Simple' (Occam's razor)

A hypothesis that survives significant tests of many of its predictions can become a *theory*, and perhaps even a *law*.

Progression of Cosmological Thought



Aristotle's celestial physics

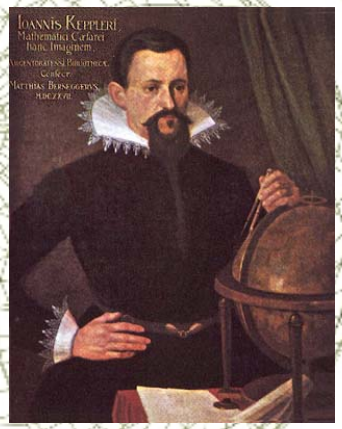
- Heavens are governed by *different* laws from Earth
- Celestial bodies are composed of "ether," a fifth element not present on Earth
- "Natural motions" of celestial spheres are different from terrestrial motions:
  - circular, constant, and eternal
  - Aristotle needed 55 spheres to explain observed motions of Sun, Moon, planets, stars (not simple !)
- Space is finite, bounded by outer sphere
  - But the edge is unreachable: motions become circular in the ethereal domain
- Time is infinite
- (But why is such a perfect universe centered on such an imperfect Earth?)

Copernicus

- Copernicus - heliocentric (Sun centered) model for the solar system
  - Rejected Ptolemy's geocentric model because it was *too complicated*
  - Preferred heliocentric model with perfect circular motions

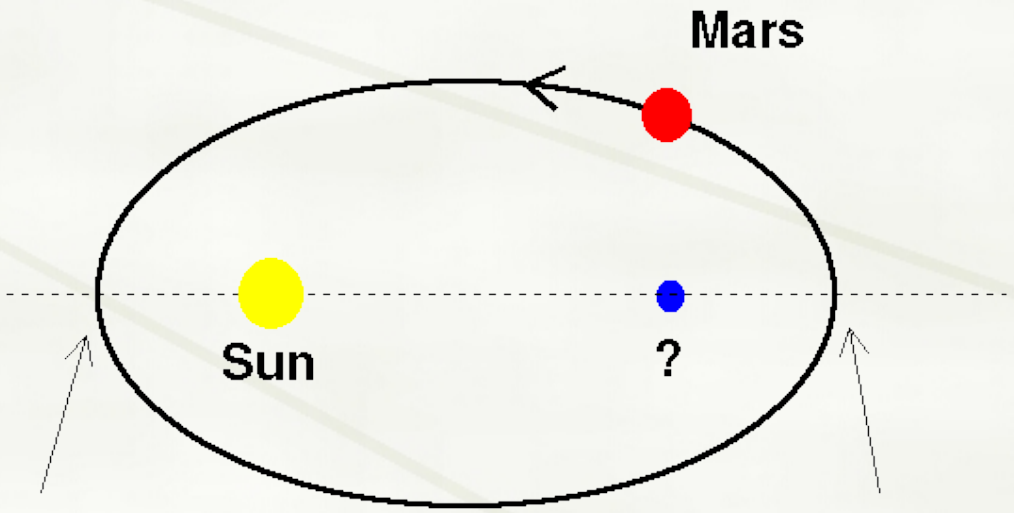
- **The Copernican Principle : The Earth is not at a special location in the Universe.**
- The **Generalized Copernican Principle: There is no special place in the universe, i.e., the universe has no center.**

- **Test of Copernican idea with phases of Venus (Tycho Brahe)**



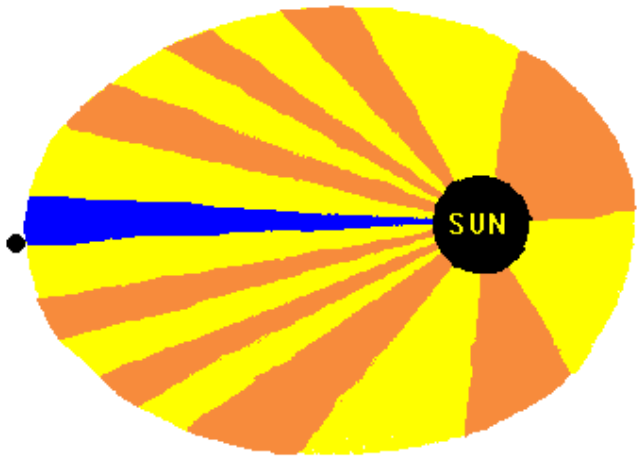
Kepler's first law

- **Planets move around the Sun in ellipses, with the Sun at one focus.**



Kepler's second law

- **The line connecting the Sun and a given planet sweeps out equal areas in equal times.**
  - Therefore, planets move faster when they are nearer the Sun
  - Consequence of angular momentum conservation (Newton).



http://home.cvc.org/science/kepler.gif

Kepler's third law

- **The square of the period *P* of the orbit is proportional to the cube of the semi-major axis *R***
- Period (*P*) = time it takes for planet to complete one orbit
- Semi-major axis (*R*) = half of the length of the "long" (i.e. major) axis of the ellipse- for a circle it is the radius.

$$P^2 = constant \times R^3$$

$$constant = 4\pi^2/G(M+m) \quad \text{(Newton)}$$

Newton

Newton's first law

Newton's first law (N1) - If a body is not acted upon by any forces, then its velocity, *v*, remains constant

- N1 sweeps away the idea of "being at rest" as a *natural* state
- N1 includes special case with v = 0, i.e. a body at rest remains at rest if F = 0, as part of a more general law

Newton's second law

Newton's 2nd law (N2) - If a body of mass *M* is acted upon by a force *F*, then its acceleration *a* is given by

$$F = Ma$$

- N2 defines "inertial mass" as the degree by which a body resists being accelerated by a force.
- Since momentum $p = mv$ and acceleration a = rate of change in v,

  ma = rate of change in $(m\ v)$
- Thus, another way of saying N2 is that force = rate of change of momentum
- **a** and **v** are vectors

Newton's third law

Newton's 3rd law (N3) - If body A exerts force $F_{A\to B} = f$ on body B, then body B exerts a force $F_{B\to A} = -f$ on body A.

- N3 is often phrased in terms of "equal" (in magnitude) and "opposite" (in direction) forces
- From N3, the *total force* on a closed system is *0*, i.e.

  $F_{tot} = F_{A\to B} + F_{B\to A} = f + (-f) = 0$
- Combining with N2, this implies that the total momentum of a closed system is *conserved* [does not change] **if there are no external forces, i.e.**

  Any momentum change of one part of a closed system is compensated for by a momentum change in another part

Newton Law of Gravity and frames of reference

- For Newton all velocities are relative
- To find a velocity in a new frame of reference use the Galilean velocity addition law,
- Distinction between inertial and accelerated frames, real and fictitious forces
- **Newton's law of Gravitation: A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by**

$$F = \frac{Gm_1m_2}{r^2}$$

Weak equivalence principle- mass defined from force and from gravity are the same

Subsequent developments: electromagnetic waves

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations)
  - Unify the electric and magnetic forces in a single theory

You do NOT have to know these equations- but should recognize that the speed of light is special in them

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

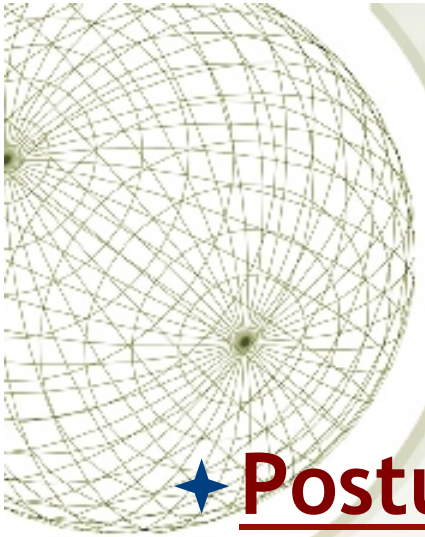
$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / c + (1/c)\partial \mathbf{E} / \partial t$$

A special velocity!

Ether and light waves

- Luminiferous Ether (19th century)
  - Hypothetical substance that fills space - provides a "medium" through which light can travel.
    - Maxwell's equations, would apply only in frame of ether
  - "explains" why the speed of wave propagation "c" is a constant in the equations
  - If speed of light in ether is "c" , and *if Galilean relativity holds*, then speed of light measured in other frames would be different from "c"
  - Albert Michelson & Edward Morley attempted (1887) to measure motion of Earth through ether… and found *no* evidence for its existence!



Einstein's postulates of Relativity

- **Postulate 1** – The laws of nature are the same in all *inertial* frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all *inertial* frames of reference.

- Inertial frames are frames which are not accelerating (constant velocity or not moving)

Time dilation- Muon Experiment

- Clock always ticks most rapidly when measured by observer in its own rest frame (proper time)
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at velocity **V** with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

- This slowing factor is called the **Lorentz factor, γ**

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Length contraction

- Consider two "markers" in space.
- Suppose spacecraft flies between two markers at velocity V.
- A flash goes off when front of spacecraft passes each marker, so that anyone can record it
- Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft…
- Observer at rest w.r.t. markers says:
  - **Time interval is t_R**; distance is $L_R = V \times t_R$
- Observer in spacecraft says:
  - Time interval is t_S; distance is $L_S = V \times t_S$
- We know from before that $t_R = t_S \gamma$
- Therefore, $L_S = V \times t_S = V \times t_R \times (t_S / t_R) = L_R / \gamma$
- *The length of any object is **contracted** in any frame moving with respect to the rest frame of that object, by a factor* γ (same γ as for time dilation)

Relativistic velocity addition law

- Einstein's theory of special relativity was partly motivated by Galilean velocity transformation (simple adding/subtracting frame velocity) gives **incorrect results** for electromagnetism
- Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities
- For a particle measured to have velocity V_p by an observer in a spaceship moving at velocity V_s with respect to Earth, the particle's velocity as measured by observer on Earth is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator <<1 and therefore $V \sim V_p + V_s$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed of light
- This is consistent with everyday experience
- Also notice that if the particle has $V_p = c$ in the spaceship frame, then it has $V_p = c$ in the Earth frame. **The speed of light is frame-independent!**

Mass and Energy

- Einstein reworked Newton's laws of mechanics using his new relativistic formulae.
- Formula for the energy of a moving object with mass *m* and speed *V* –

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - V^2/c^2}}$$

- Thus, energy increases as the speed increases, and energy would become infinite if *V* approaches *c*
- *At rest*, $E=mc^2$

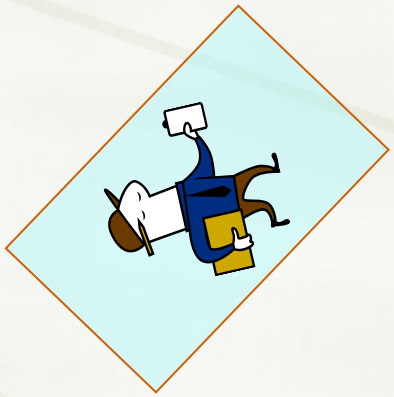
EQUIVALENCE PRINCIPLES

- Recall the "**weak" equivalence principle:**
  - All objects are observed to accelerate at the same rate in a given gravitational field.
  - Therefore, the inertial and gravitational masses must be the same for any object.
  - This has been verified experimentally, with fractional difference in masses $<10^{-11}$
- As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot locally be distinguished

THE STRONG EQUIVALENCE PRINCIPLE

- Einstein introduced the **strong equivalence principle** – when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- More generally, all inertial and freely-falling reference frames are equivalent, and there is no (local) experiment that can distinguish them

a=9.8 m/s^2



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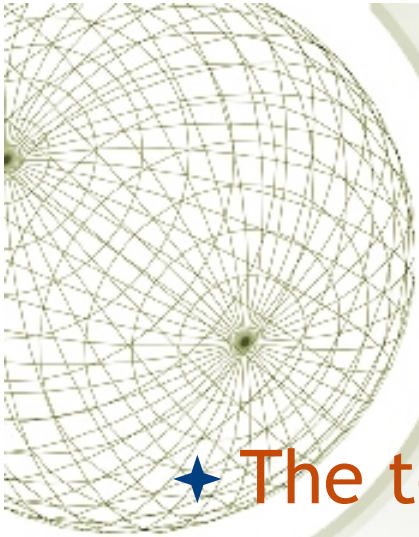


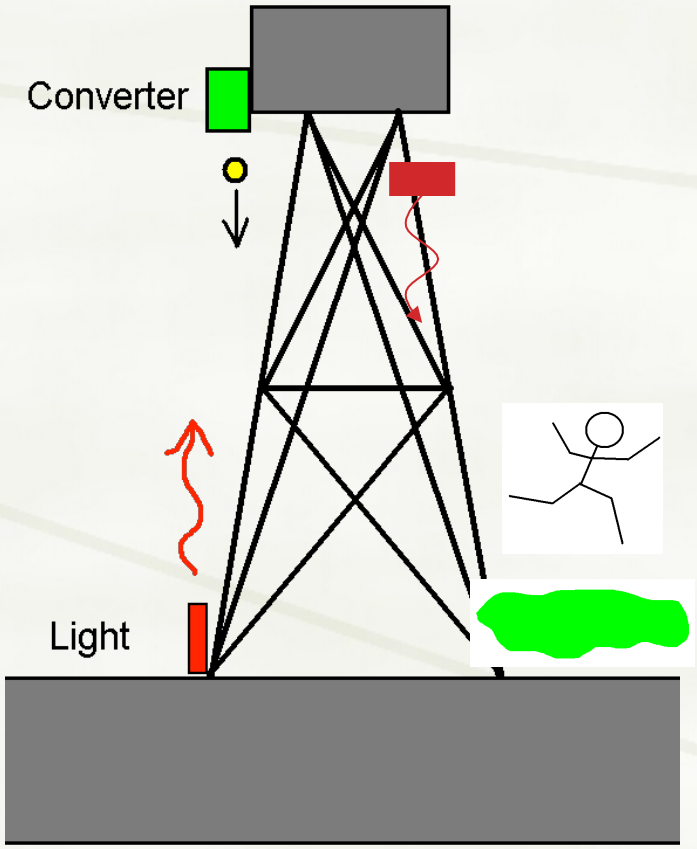
General Relativity

- The tower experiment
- Gravitational redshift and time dilation
- Black holes
- Curved space time

Resolving the tower problem

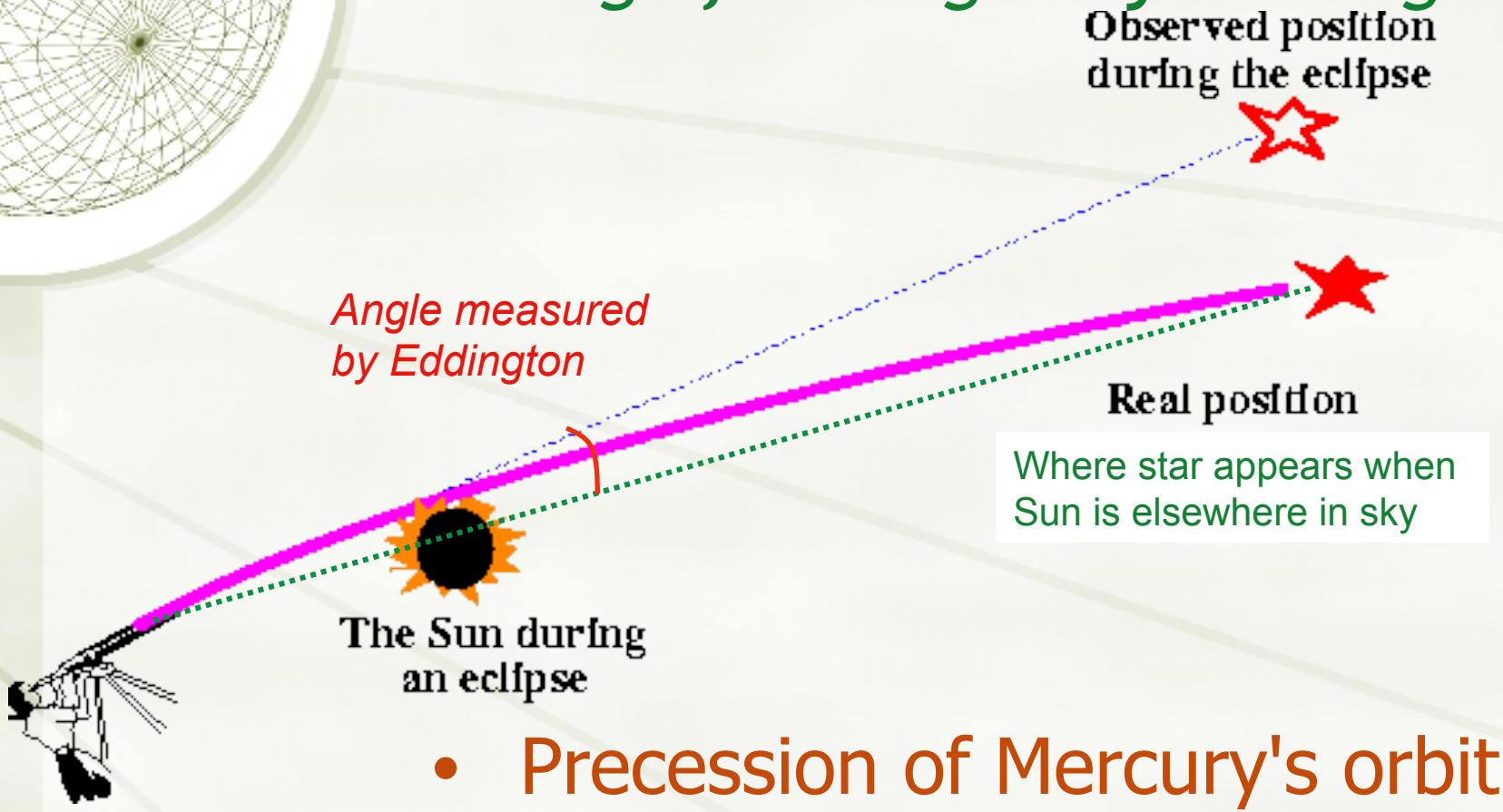
- Consider a light ray aimed from top to bottom of tower
- Free-falling(FF) observer sees light ray travel **unaffected** by gravity,since freefall is an inertial frame
- From"Earth's"frame…
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity
  - If FF observer is **supposed to** see a constant frequency light beam, then light must get relatively *blueshifted* as it falls in a gravitational field, to compensate
  - Light beam aimed upward must conversely be increasingly *redshifted* with height
  - ***Gravitational redshifting removes just the right amount of energy to solve the tower paradox!***

(remember that the energy of a photon is $E=h\nu=hc/\lambda$)



Testing of General Relativity

Bending of starlight by Sun's gravity



- Precession of Mercury's orbit

Binary pulsar

- Gravitational lensing by clusters

General Relativity- Curved Space Time

- 4-dimensional space-time is "curved," not flat
  - Example: surface of sphere is curved 2D space; surface of football field is flat 2D space
- Free-falling objects and light move on geodesics through curved space-time- geodesic is shortest distance between 2 points in this space-time
- The curvature (bending) of space-time is produced by matter and energy

"Space-time curvature tells matter/energy how to move. Matter/energy tells space-time how to curve."

Black Holes

- Features of Schwarzschild's solution:First solution of Einstein's equations of GR
- Describes gravitational field in (empty) space around a point mass
  - Consistent with Newton's law of gravity, with flat space, at large R
  - Space-time curvature becomes infinite at center (R=0; this is called a **space-time singularity**) Laws of GR breakdown
  - Gravitational time-dilation effect becomes *infinite* on a spherical surface known as the **event horizon**
    - gravitational time-dilation is infinite as observed from large distance.
    - Any light emitted at R_s would be infinitely redshifted - hence could not be observed from outside
  - Radius of the sphere representing the event horizon is called the **Schwarzschild radius,** $R_s = 2GM/c^2$
- Black holes are real-
  - best explanation of astronomical objects (quasars, x-ray binaries)

The Distance Scale and Redshift

- Standard candles
  - Cepheids
  - Type Ia SN
- Parallax

- If galaxy is moving towards us, wavelengths are shortened ⇒ **spectrum blueshifted**
- If galaxy is receding from us, wavelengths are lengthened ⇒ **spectrum redshifted**

redshift

- Redshift

$z = (\lambda_{observed} - \lambda_{emited}) / \lambda_{emited}$

- $1+z = \sqrt{\{(1+v/c)/(1-v/c)\}}$

or

- $1+z = \gamma\{1+(v/c)\}$

or

$$z = \frac{(\lambda_{Galaxy} - \lambda_{rest})}{\lambda_{rest}}$$

Hubble Expansion

- Hubble found in the 1920's that almost all galaxies were moving away from us by measuring the Doppler shift of the light from the galaxies and getting distance using 'standard' candles (Cepheid stars)
- The further the galaxy the faster it is moving with respect to us

Hubble's Law

- Hubble interpreted redshift-distance relationship as a linear increase of the recession velocity of external galaxies with their distance
- Mathematically, the Hubble law is

$$v = H \times d$$

where v=velocity and d=distance

- Modern measurement gives the Hubble constant as $H = 69$ km/s/Mpc
- In fact, Hubble's interpretation is only "sort of" correct
- What **really** increases linearly with distance from the MW is simply wavelength of light observed, and this redshift is due to the *cosmological expansion of space itself over the time since the light left the distant galaxy and arrived at the Milky Way!*

Cosmological redshift, z

- If galaxies move apart, z describes a Doppler shift from the expansion velocity
- More fundamentally, it comes from the change in metric scaling, ***R*(*t*)**
- galaxies are carried apart by the **expansion of space itself**, not by the forces of an explosion! Since it's relativistic, it affects time as well as length
- Because this is NOT a velocity effect galaxies can move apart from each other faster than the speed of light- this is **not** a violation of Einstein

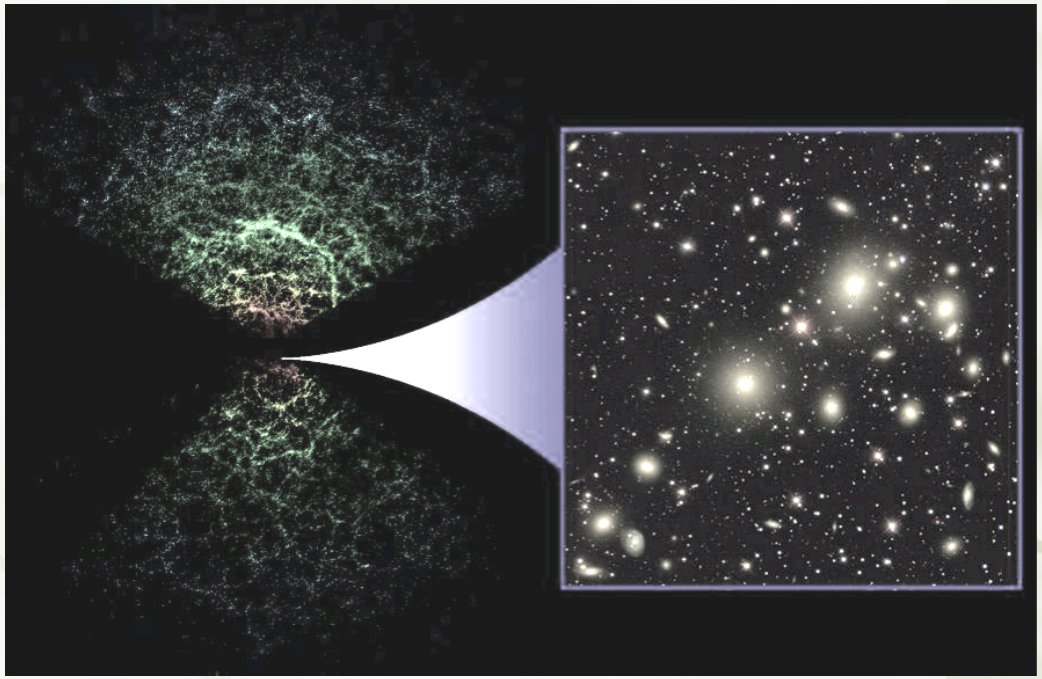
How to Use Einstein's GR Equations

- make the following assumptions
  - Universe is **homogeneous** – every place in the universe has the same conditions as every other place, on average.
  - Universe is **isotropic** – there is no preferred direction in the universe, on average.

Observational evidence for homogenity and isotropy

galaxy distribution

cosmic microwave background



Distribution Of Galaxies From The Sloan Digital Sky Survey

Friedmann Eq

Exact solution of Einstein's field equations of general relativity; it describes a homogeneous, isotropic expanding or contracting universe

The general form of the solution follows from homogeneity and isotropy

Einstein's field equations are only needed to derive the scale factor of the universe as a function of time (R(t)).

There are 3 general solutions described by 'k'

Friedmann Equation

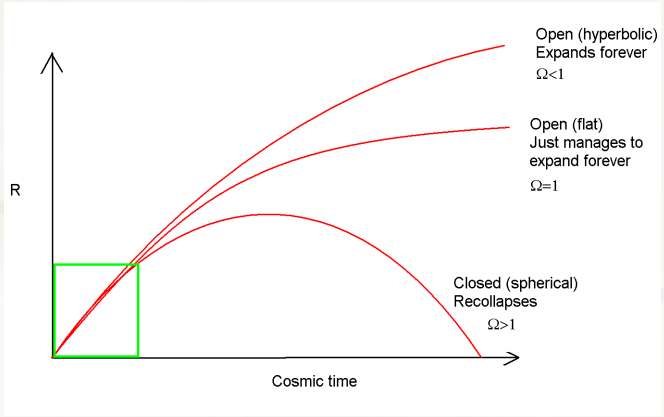
- **Where do the three types of evolutionary solutions come from?**
- Back to Einstein's eq....
- When we put the FRW metric in Einstein's equation and go though the GR, we get the **Friedmann Equation**... this is what determines the dynamics of the Universe

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- What are the terms involved?
  - G is Newton's universal constant of gravitation
  - *dR/dt* is the rate of change of the cosmic scale factor
    This is same as $\Delta R/\Delta t$ for small changes in time
    In textbook, symbol for *dR/dt* is $\dot{R}$ *(pronounced "R-dot")*
  - ρ is the total energy density $\div c^2$; this equals mass/volume for "matter-dominated" Universe
  - *k* is the geometric curvature constant *(= +1,0,-1)*

Omega in standard models

$$\Omega = 1 + \frac{kc^2}{H^2 R^2}$$

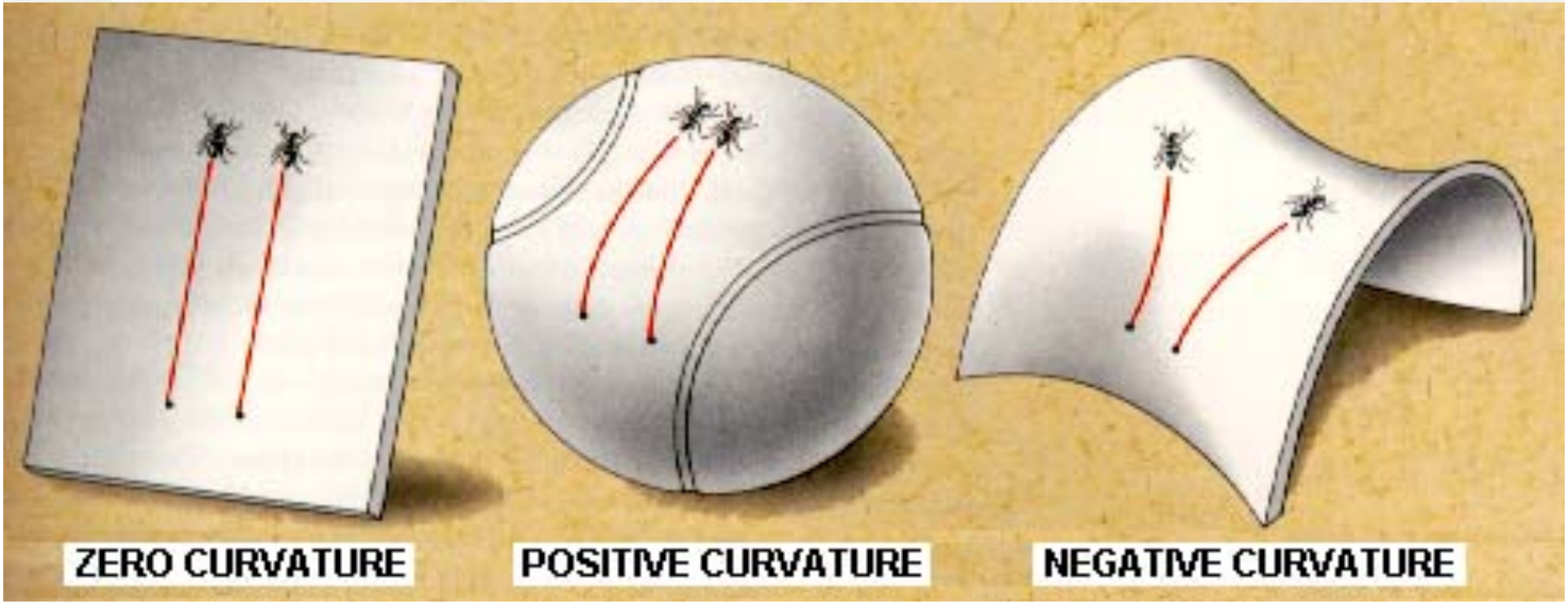


- Thus, within context of the standard model:
  - $\Omega<1$ if k=-1; then universe is hyperbolic and will expand forever
  - $\Omega=1$ if k=0; then universe is flat and will (just manage to) expand forever
  - $\Omega>1$ if k=+1; then universe is spherical and will recollapse
- Physical interpretation:

  If there is more than a certain amount of matter in the universe ($\rho>\rho_{critical}$), the attractive nature of gravity will ensure that the Universe recollapses!

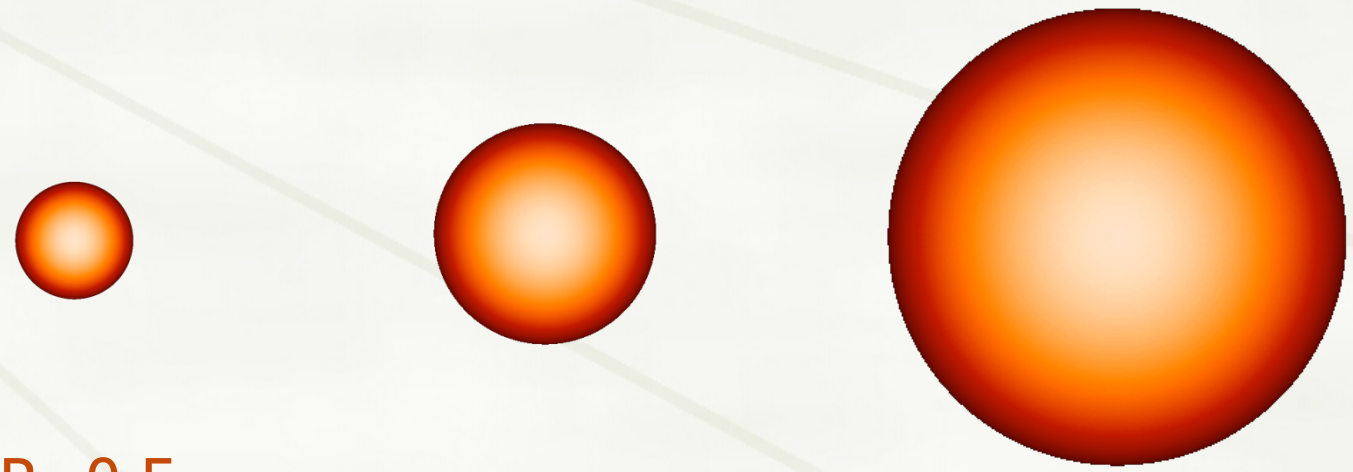
Curvature of Universe

- 3 types of general shapes: flat surface at the left :zero curvature, the spherical surface : positive curvature, and the saddle-shaped surface : negative curvature.
- Each of these possibilities is tied to the amount of mass (and thus to the total strength of gravitation) in the universe, and each implies a different past and future for the universe.
- In GR space itself is 'curved'



Meaning of the scale factor, R(t)

- Scale factor, *R(t)*, is a central concept!
  - *R(t)* tells you how "big" the space is…
  - Allows you to talk about changing the size of the space (expansion and contraction of the Universe - even if the Universe is infinite).
- Simplest example is spherical case
  - Scale factor is just the radius of the sphere



Important features of standard models…

- All models begin with R→0 at a finite time in the past
  - This time is known as the **BIG BANG**
  - Space and time come into existence at this moment… **there is no time or space** before the big bang!
  - The big bang happens everywhere in space… not at a point!
  - Concept of the Hubble time-t_H=1/H gives an estimate of the age of the Universe
  - Hubble 'constant'

$$H = \frac{1}{R}\frac{\Delta R}{\Delta t} = \frac{1}{R}\frac{dR}{dt}$$

There is a connection between the geometry and the dynamics

- *Closed* solutions for universe expand to maximum size then re-collapse
- *Open* solutions for universe expand forever
- *Flat* solution for universe expands forever (but only just barely…).
- Definition of the density parameter $\Omega=\rho/\rho_{crit}$

$$\Omega_B = \frac{\rho_B}{\rho_{crit}} = \frac{\rho_B}{3H_0^{\,2}/(8\pi G)}$$

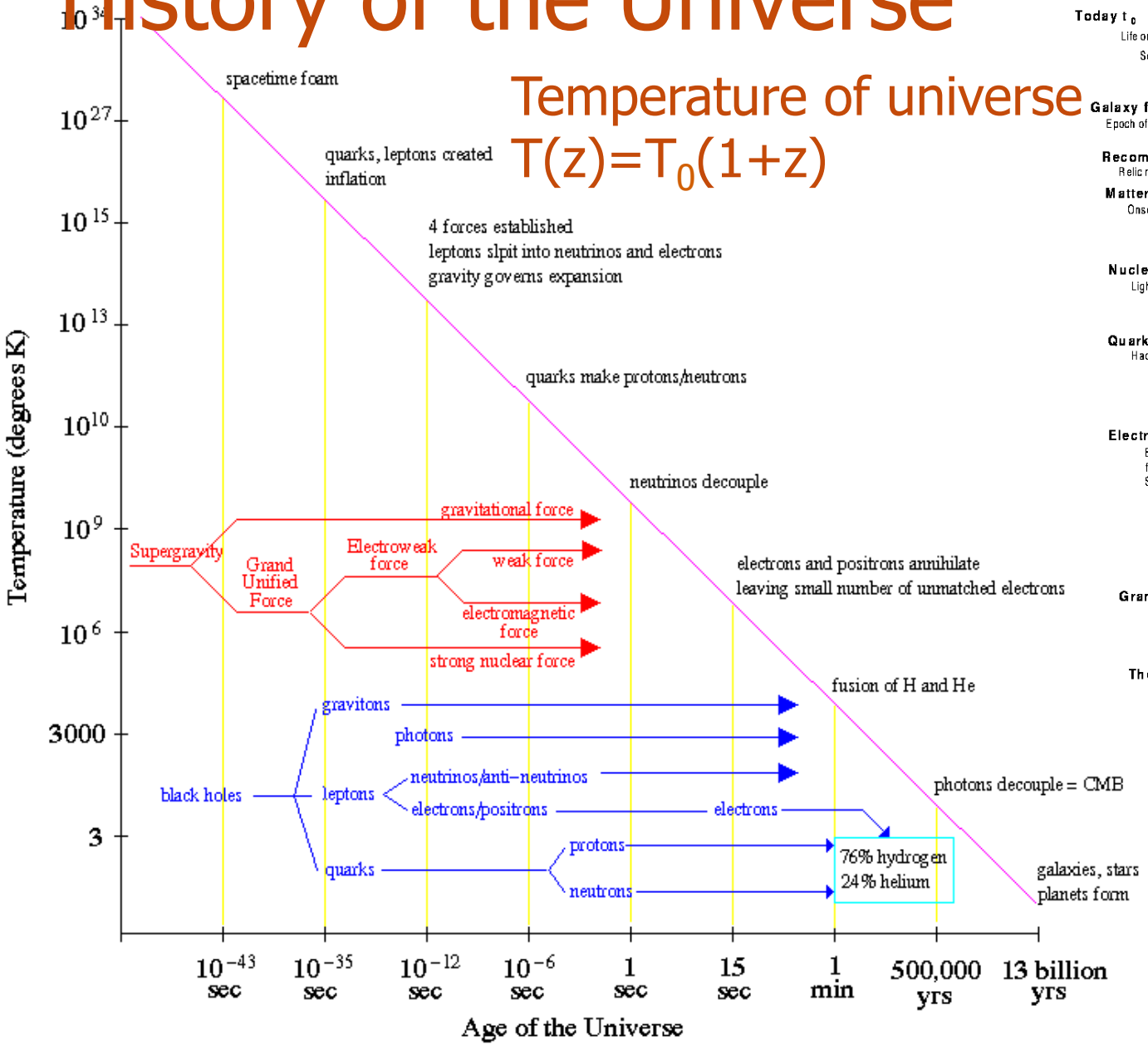
- With no cosmological constant if $\rho=\rho_{crit}$

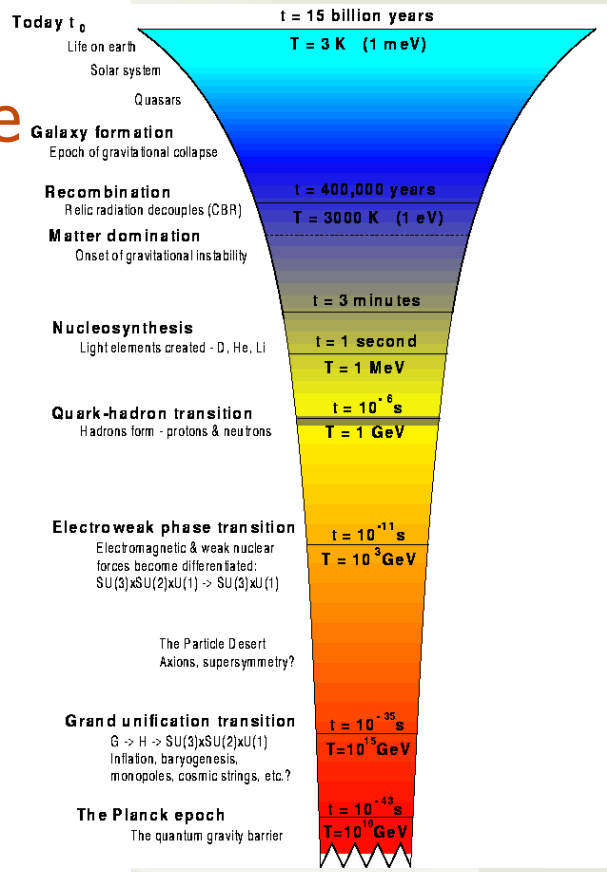
(amount of matter in the universe), the Universe is flat

History of the Universe

Temperature of universe

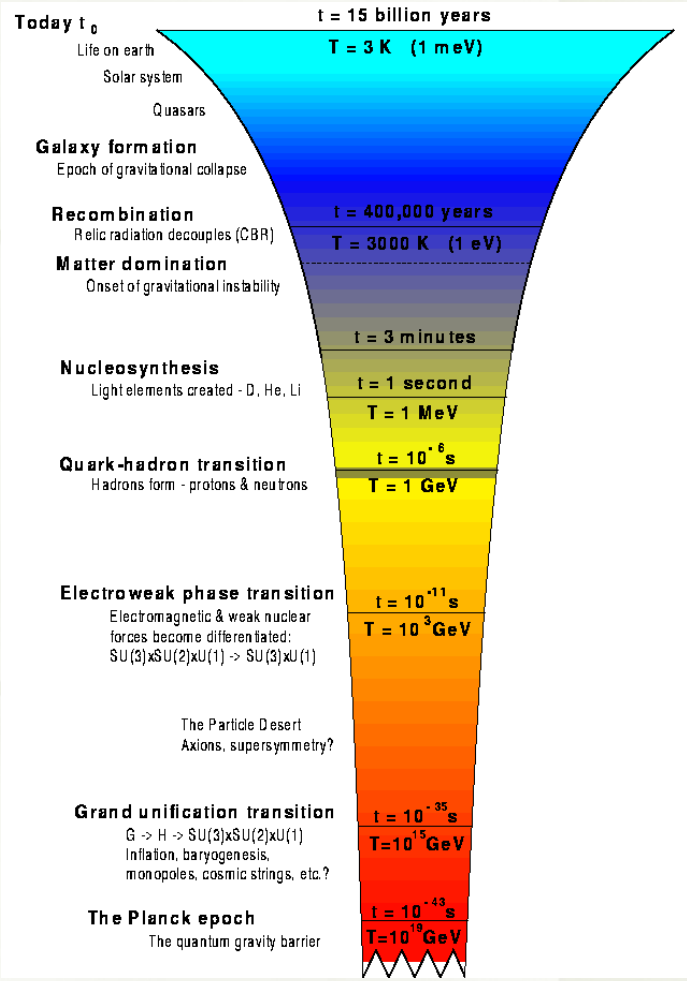
$T(z)=T_0(1+z)$





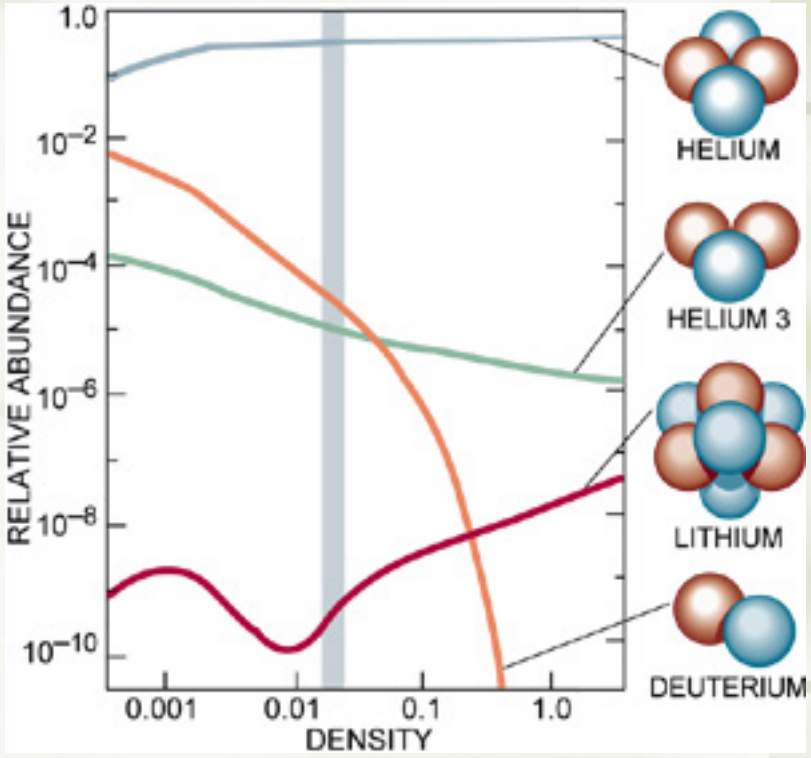
A brief look at the stages of the Universe's life…

- Crude overview:
  - t=0: The Big Bang
  - For first 400,000 yrs, an expanding "soup" of tightly coupled radiation and matter
    - Earliest epochs were "extreme" physics
    - Then more "normal" physics: protons & neutrons form
    - Then came nucleosynthesis
  - After 400,000 yrs, atoms form ("recombination") and radiation and matter "decouple"
  - Following decoupling, matter and radiation evolve independently
  - Galaxies, stars, planets, etc can then form and evolve



Primordial nucleosynthesis

- All things considered, we have $\Omega_B h^2 \approx 0.019$.
- If H_0=72km/s/Mpc,
  - h=0.72
  - $\Omega_B \approx 0.04$
- This is far below Ω=1!
- Baryons alone would give open universe



$\Omega_B h^2$

Weighing the Universe, and the need for dark matter

Constraints on the baryon density parameter Ω_B

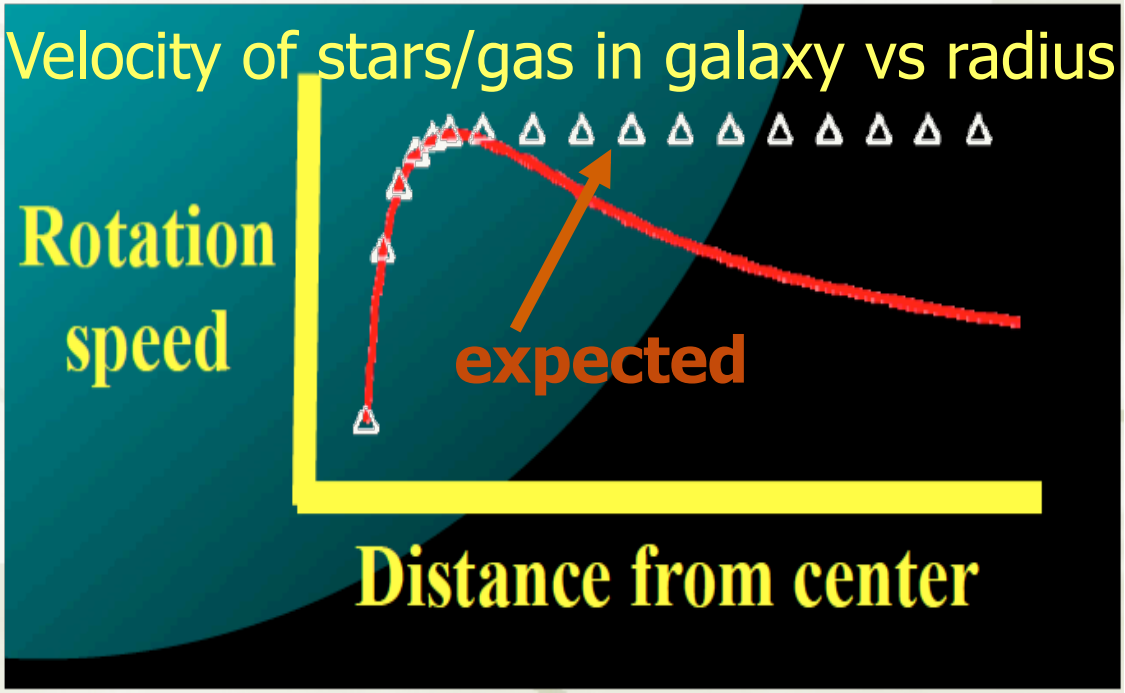
- The importance of measuring the **total** density parameter Ω

- Measuring the mass of the Universe
  - Mass to light ratio
  - Mass of luminous stars
  - Masses of galaxies and galaxy clusters
- the accounting of all forms of mass/energy in the Universe…
  - Baryonic matter- stars, gas, dust
  - Non-baryonic dark matter
  - Radiation

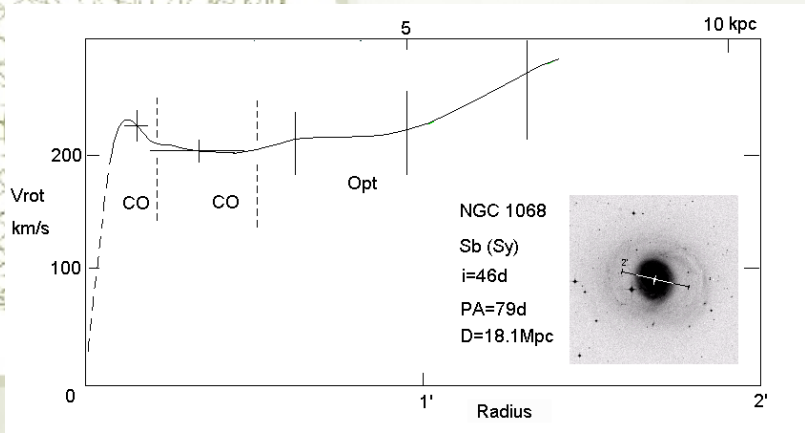
Dark Matter Dominates the Universe and the Outer Parts of Galaxies

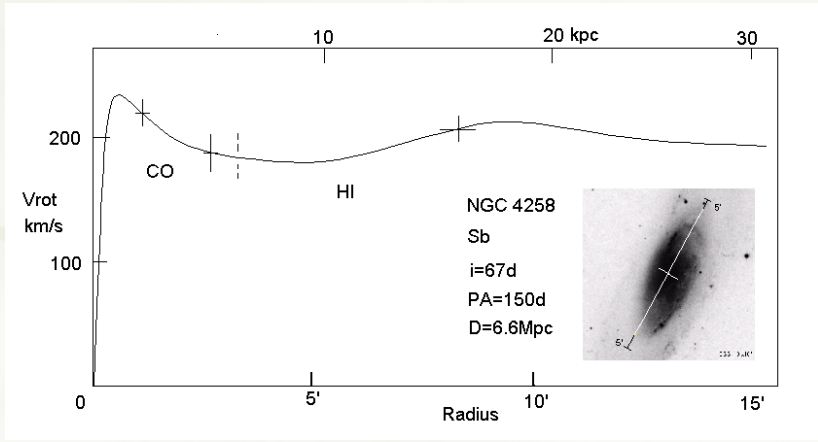
- The further out one moves from the center of the galaxy the more 'mass is missing'- lots of mass and no light from stars or indication of gas
- The material that accounts for the 'extra' velocity is ***DARK***
- *Dark matter is a way of expressing our ignorance- the stars and gas do not move like we 'expect'*
- gravitational lensing dynamics of stars and gas confirm the existence of dark matter

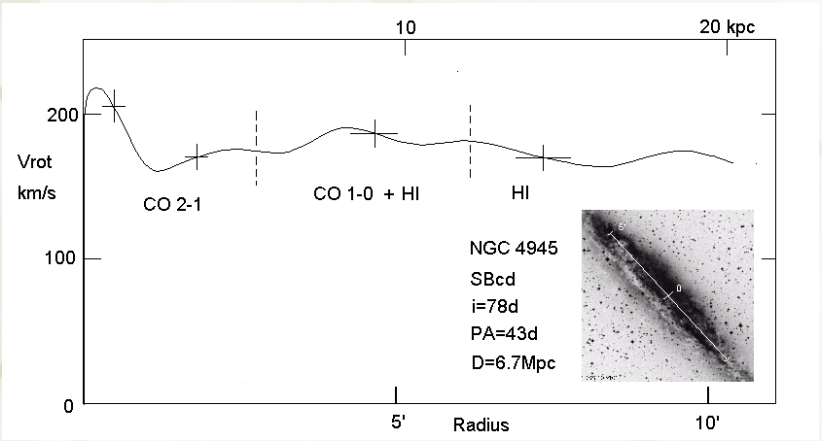
- Nucleosynthesis arguments constrain the density of baryons ($\Omega_B \approx 0.037$)
- But there seems to be much more mass in galaxy and cluster halos ($\Omega \sim 0.1\text{-}0.3$)
- So, most of the matter in the Universe is not baryonic- its dark matter

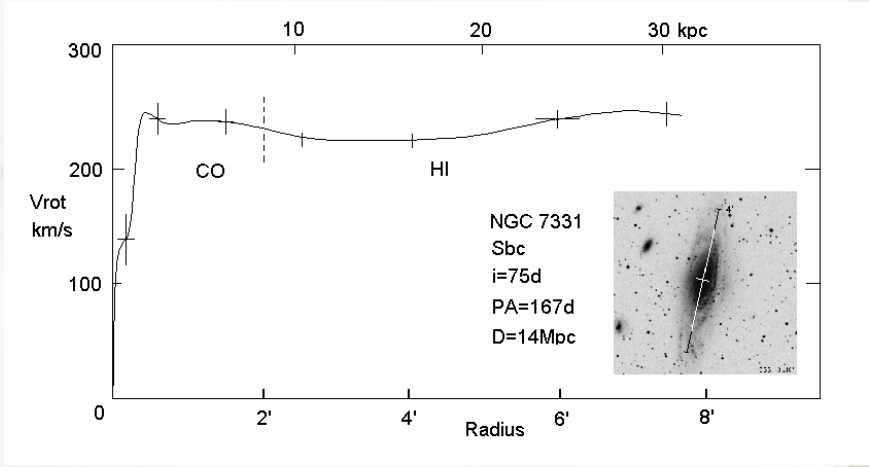


Dark matter?





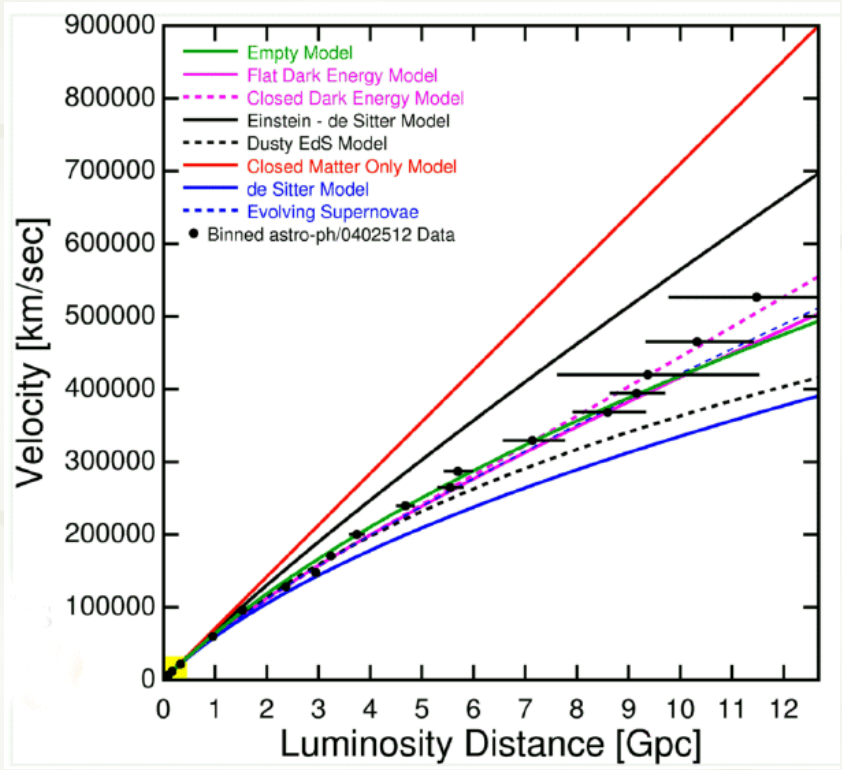




In the outermost parts of galaxies, *V* and *R* are based on measurements of hydrogen gas atoms that orbit the galaxy, rather than stars

Supernovae verify acceleration

- Large effort to measure distant SNIa give accurate value for Hubble's constant
  - H = 72 km/s/Mpc
- Find acceleration, not deceleration, at large distance!
  - Very subtle, but really is there in the data!
  - Profound result!



Purple curve: Ω_M =0.3, Ω_Λ=0.7

Black curve: Ω_M =1, Ω_Λ=0

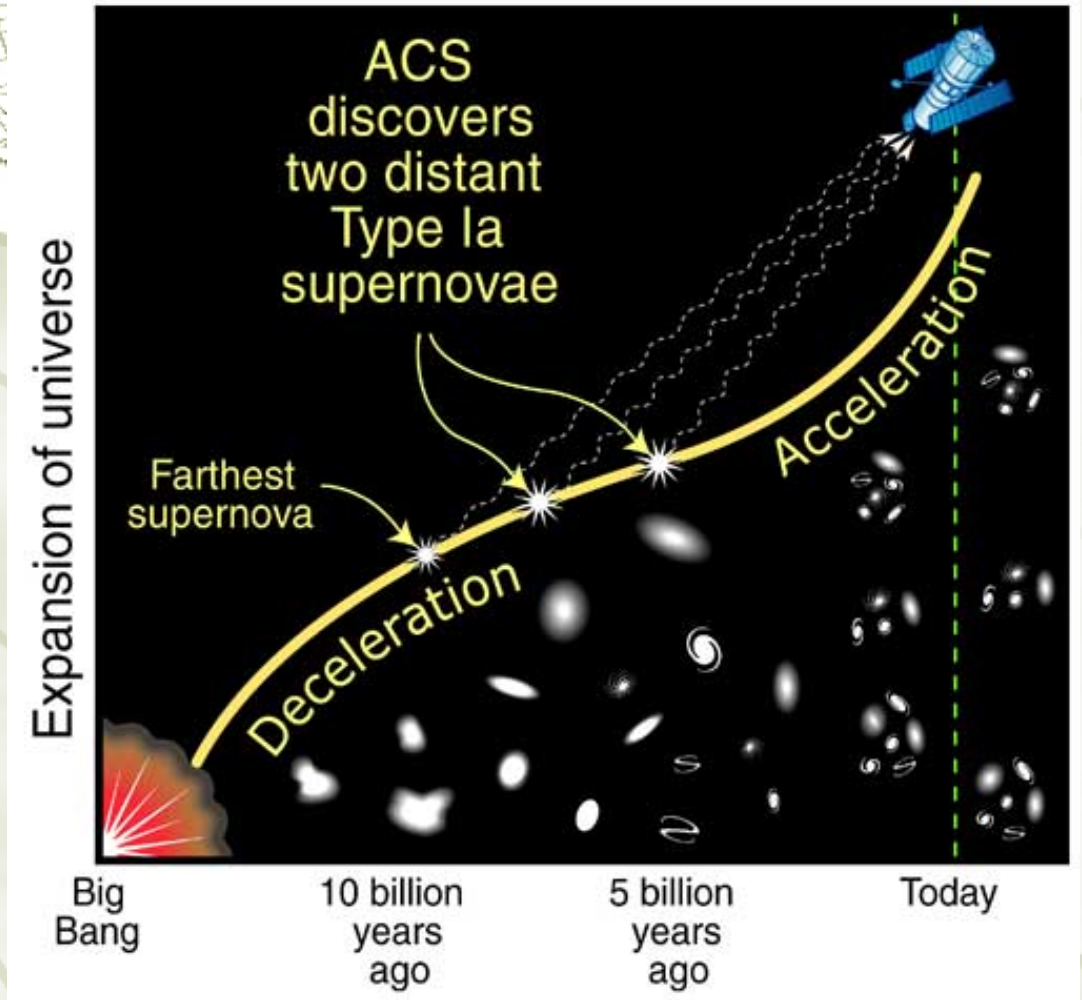
Green curve: Ω_M =0, Ω_Λ=0

Graphics: Ned Wright, UCLA

The accelerating universe

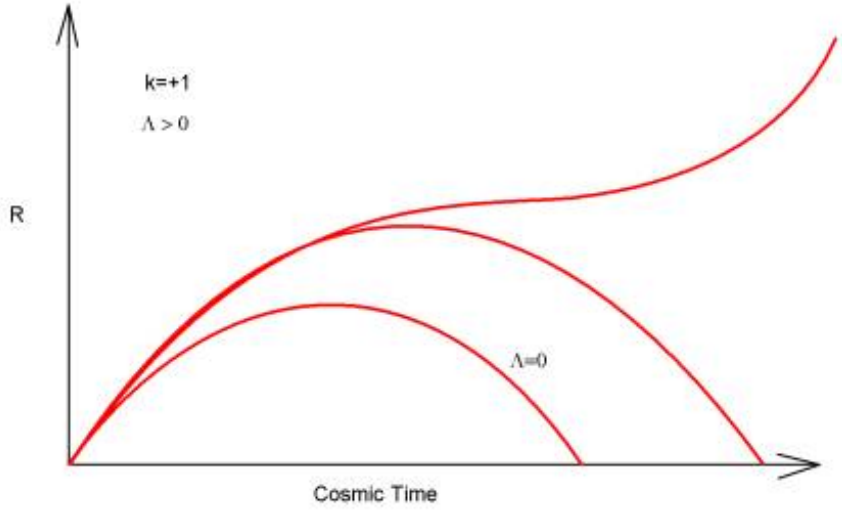
Need for a cosmological constant/dark energy

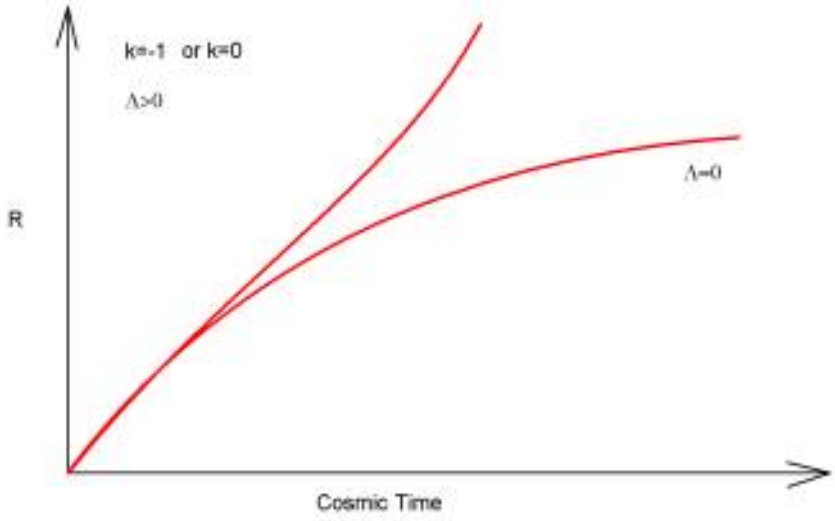
ACS= camera on Hubble



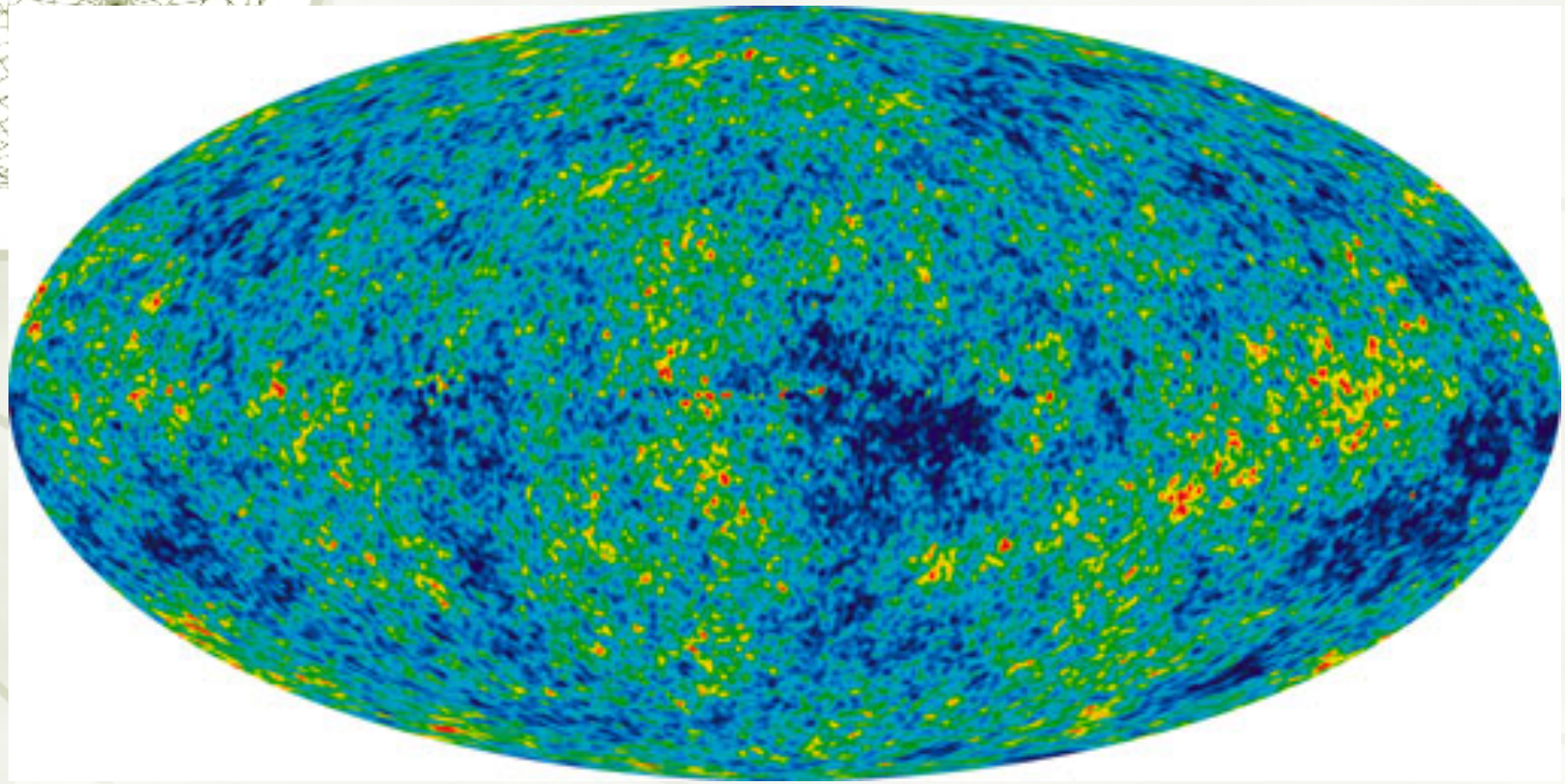
Generalized Friedmann Equation

- Because Λ term appears with *positive* power of *R* in Friedmann equation, effects of Λ increase with time if *R* keeps increasing
- Positive Λ can create accelerating expansion!



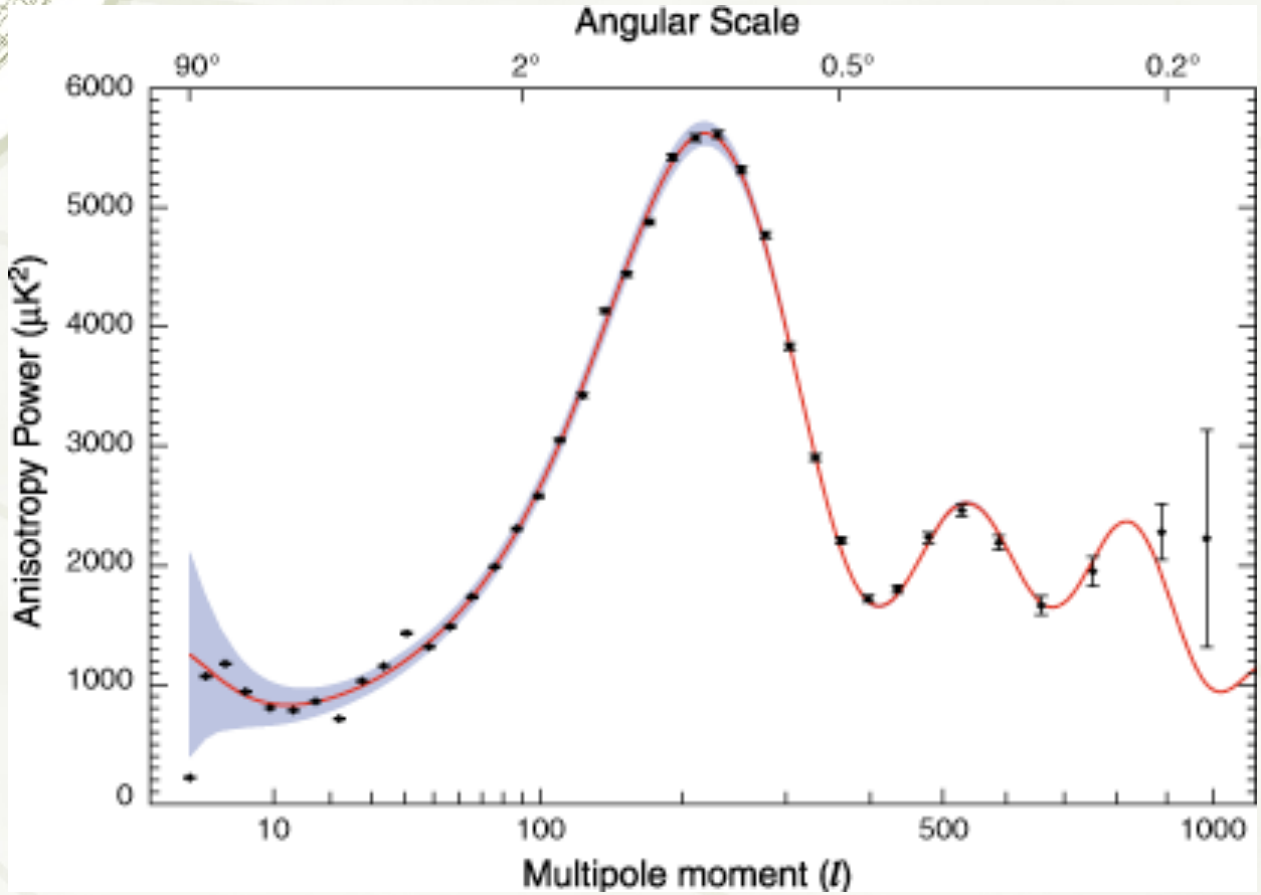


WMAP's map of the CMB



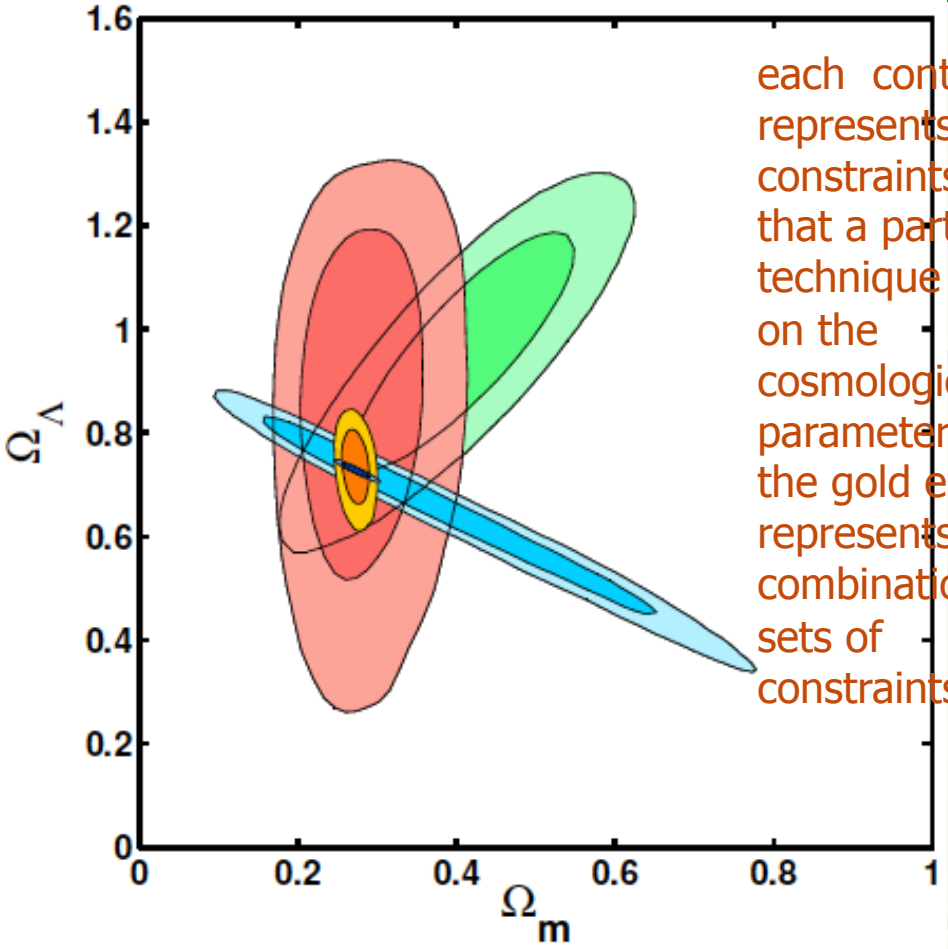
The microwave light captured in this picture is from 379,000 years after the Big Bang, over 13 billion years ago!

Cosmic lumps: the power spectrum of the CMB



Concordance Cosmology

- No one technique definitely 'proves' the existence of dark energy
- Physics of clusters (pink), CMB(blue) type Ia Sn (green)
- Hubble constant: H_0 = 72 km/s/Mpc
- Geometry: Flat!
- Baryon density: Ω_B = 0.04
- Dark matter density: Ω_{DM} = 0.22
- Cosmological constant: Ω_Λ = 0.74
- Age: t_0 = 13.7 billion years

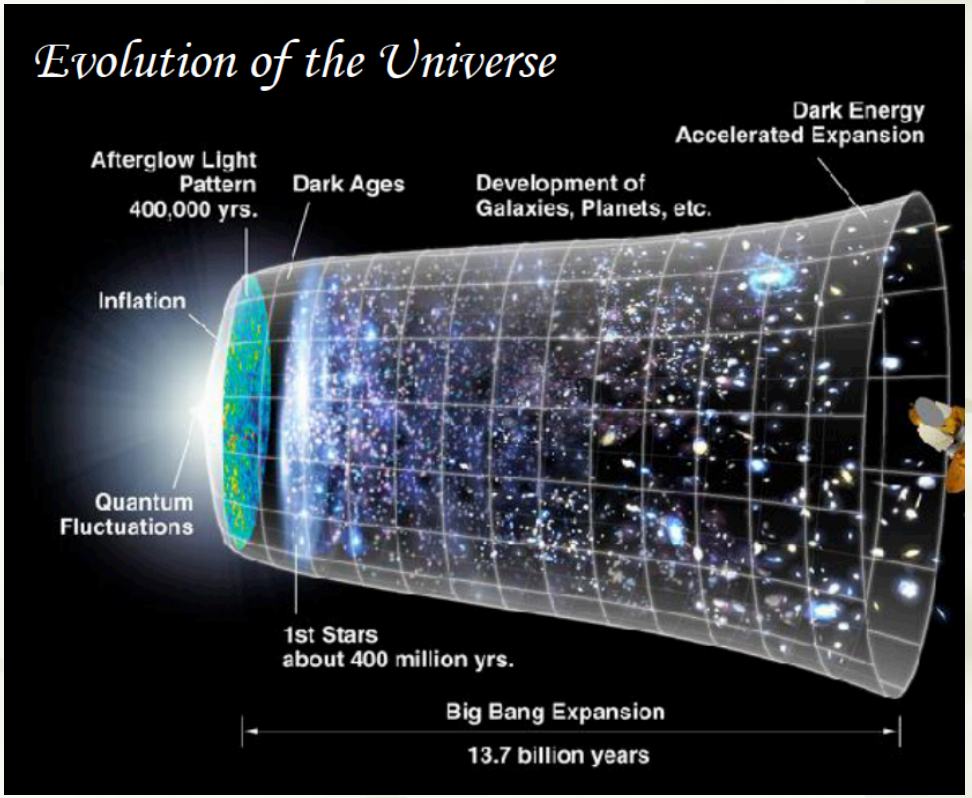


each contour represents the constraints that a particular technique puts on the cosmological parameters- the gold ellipse represents the combination of 3 sets of constraints

Wide variety of techniques agree on cosmological parameters

Where did the galaxies come from

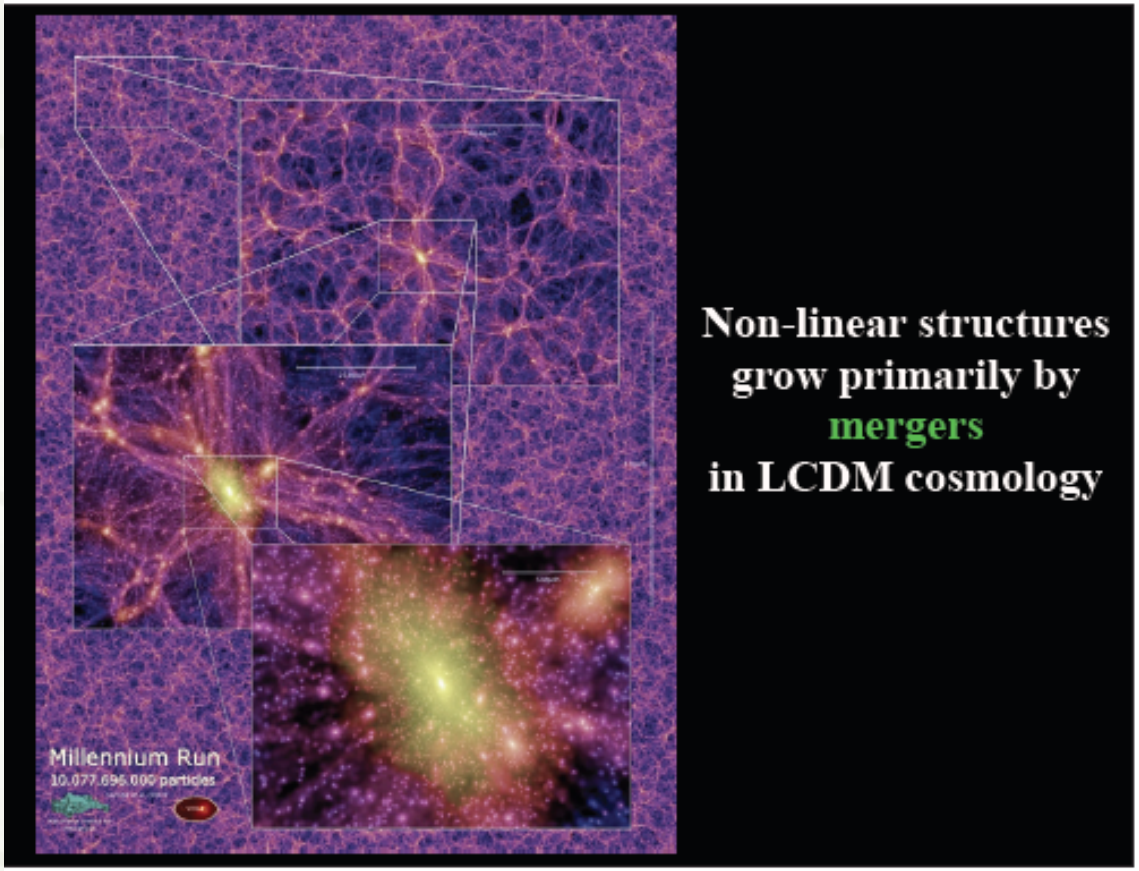
- From homogeneity to structure…
  - Gravitational evolution of dark matter
  - Formation of dark matter halos
- Galaxy formation



- Denser parts of the universe collapse under their own gravity (dark matter)
- Normal matter falls into halo, cools, settles to center
- Once cool dense clouds form, can get star formation
- Through this process, a galaxy is built up

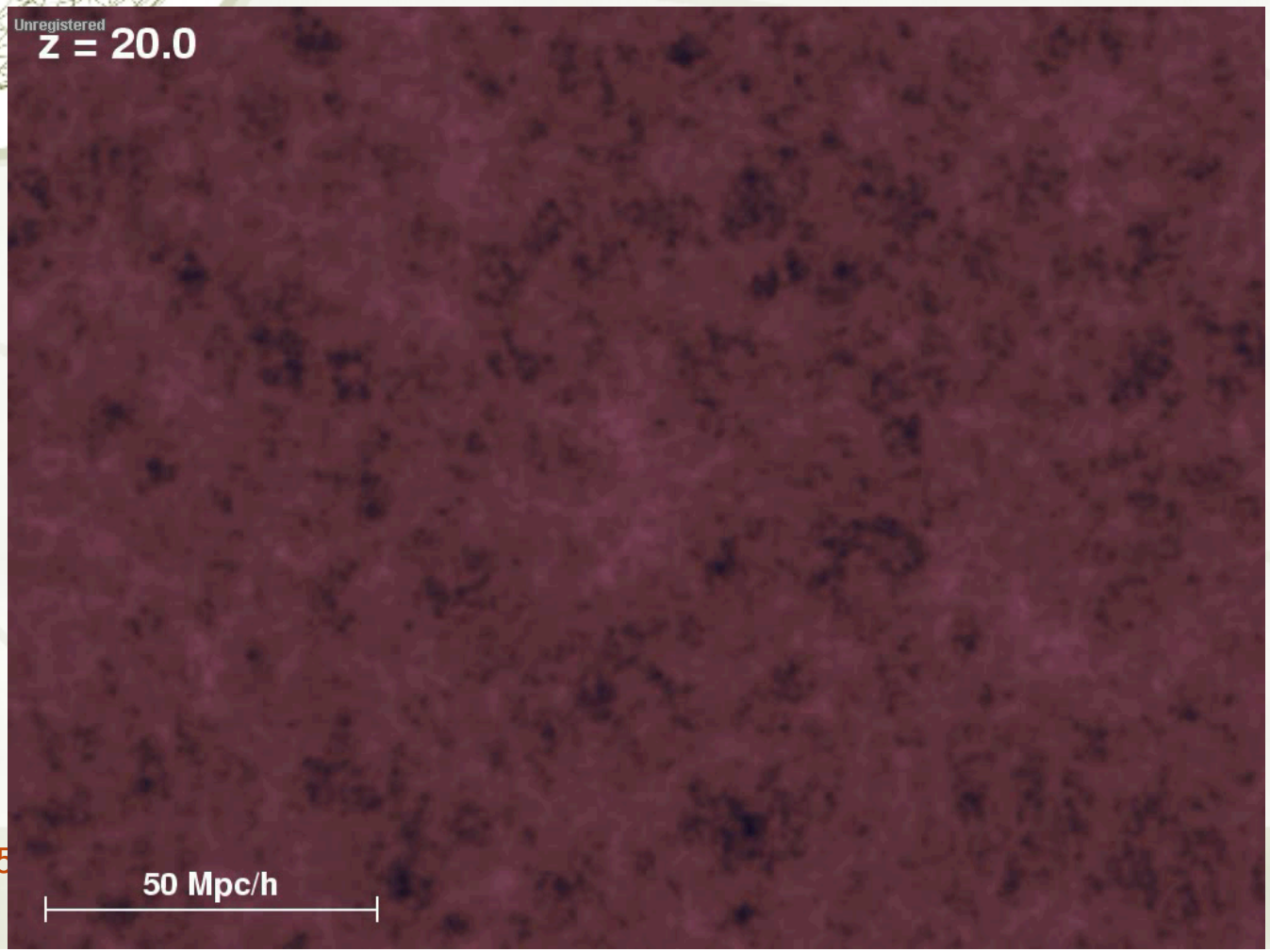
Galaxy Formation

- Numerical Simulations of Structure formation
- Solve complex equations- have inputs of
  - Friedmann eq
  - concordance universe parameters



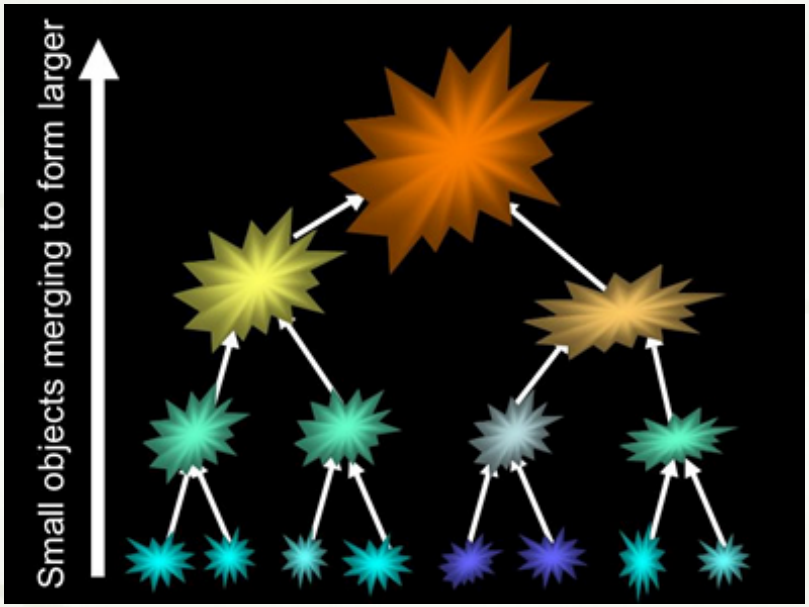
Filamentary structure of the universe (brighter colors- denser regions

Formation of structure in the Millenium simulation



How Things Form

- Gravity acts on overdensities in the early universe making them collapse.
- As time goes on these collapsed regions grow and merge with others to make bigger things

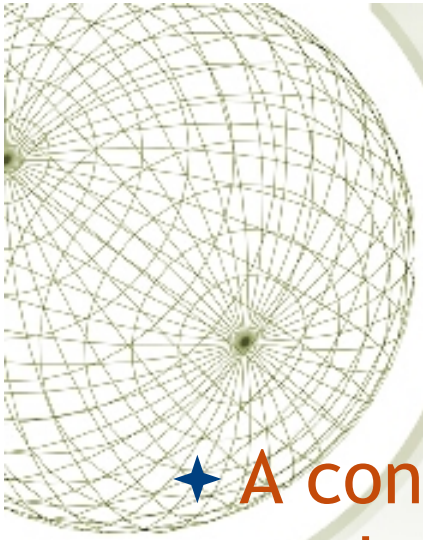


•Hierarchical clustering (or hierarchical merging) is the process by which larger structures are formed through the continuous merging of smaller structures.

•The structures we see in the Universe today (galaxies, clusters, filaments, sheets and voids) are predicted to have formed by the combination of collapse and mergers according to Cold Dark Matter cosmology (the current concordance model).

So, success! Except… Some striking cosmological puzzles and a solution

- Puzzles:
  - flatness problem
  - horizon problem
  - structure problem
  - relic problem
- Inflation
  - Basic idea
  - How it solves the cosmological puzzles
  - Problems with inflation



Here we are

- A consistent picture of the history and future evolution of the Universe
- Supported by observations and experimentation
- Profoundly surprising: most of the Universe is made of something else (dark matter, dark energy) !
- Many open avenues for new observations and theory: this will get even better and more interesting SOON !