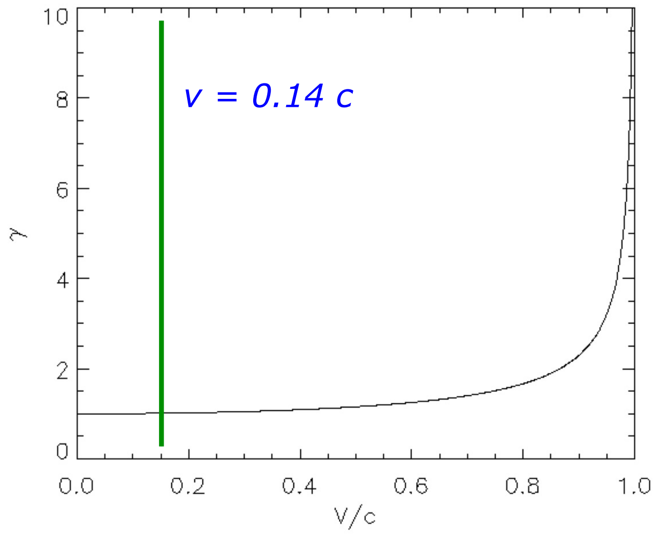
ASTR350 Black Holes (Spring 2020)
Prof. Richard Mushotzky

RECAP

- Maxwell's equations say that speed of light is the same in any (inertial) frame of reference… experimentally verified by Michelson-Morley experiment
- Einstein's postulates
  - Laws of physics same in any inertial frame
  - Speed of light same in any inertial frame
- Time dilation
  - Moving clocks run more slowly as speed increases

Lorentz factor



A 1% effect at v = 0.14 c, or 42,000 km/s

Lorentz factor goes to infinity when V→c!

But it is very close to 1 for V/c small

Optical Clocks and Relativity

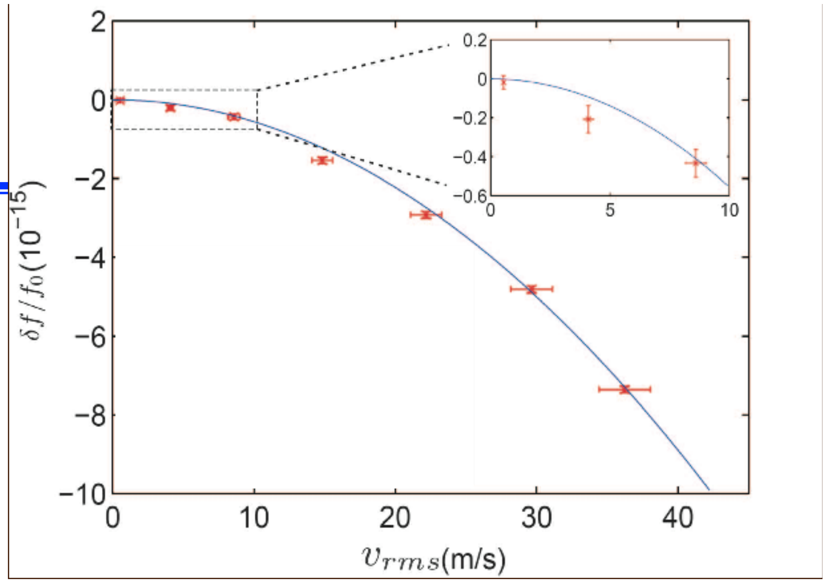
C. W. Chou,* D. B. Hume, T. Rosenband, D. J. Wineland

ı. We observed time dilation from relative speeds of less than 10 meters per second by comparing two optical atomic clocks connected by a 75-meter length of optical fiber. We can now also detect time dilation due to a change in height near Earth's surface of less than 1 meter. This technique may be extended to the field of geodesy, with applications in geophysics and hydrology as well as in space-based tests of fundamental physics.

The moving clock ticks more slowly
*This effect is called **time-dilation**, and the quantity "γ" is the **Lorentz factor**.*

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Time Dilation at Normal Speeds



Using an ultra-precise atomic clock one can measure even tiny amounts of time difference.
Comparison of observed (red) vs. predicted time dilation at normal speeds. *predictions of special relativity confirmed (Science V329 pg 1631-2010*
10m/s=22 mph

Other Experimental Tests of Time Dilation

- Hafele and Keating, in 1971, flew cesium atomic clocks east and west around the Earth in commercial airliners, to compare the elapsed time against that of a stationary clock. **Results were within 4% of the predictions of relativity.**
- in 1996 on the 25th anniversary of the original experiment, using more precise atomic clocks during a flight from London to Washington, D.C. and back again. **A time gain of 39 ± 2 ns was observed, compared to a relativistic prediction of 39.8 ns.**[5] In June 2010, the National Physical Laboratory again repeated the experiment, this time around the globe (London - Los Angeles - Auckland - Hongkong - London). The predicted value was 246 ± 3 ns, the measured value 230 ± 20 ns.[6]
- More than 20 more experiments with decaying particles (pion,kaon,muons) in accelerators

If you want more info…

- A good youtube to watch is
http://www.youtube.com/watch?v=xWST2gpbnvw-
"Physics-X" lecture series,15 minutes long
- taught at Michigan Technological University by Dr. Robert Nemiroff

- Relativity In 5 Minutes
- http://www.youtube.com/watch?v=KYWM2oZgi4E

This class

- More time dilation
- Length contraction
- Relativity of simultaneity
- Causality
- New velocity addition law
- Mass and Energy

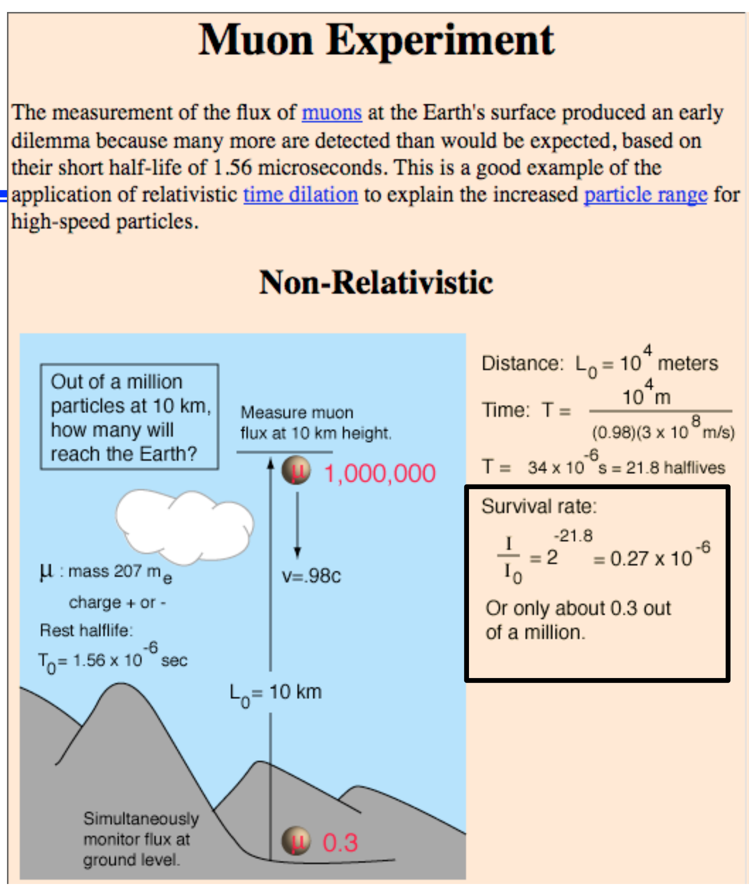
Length (Fitzgerald) contraction

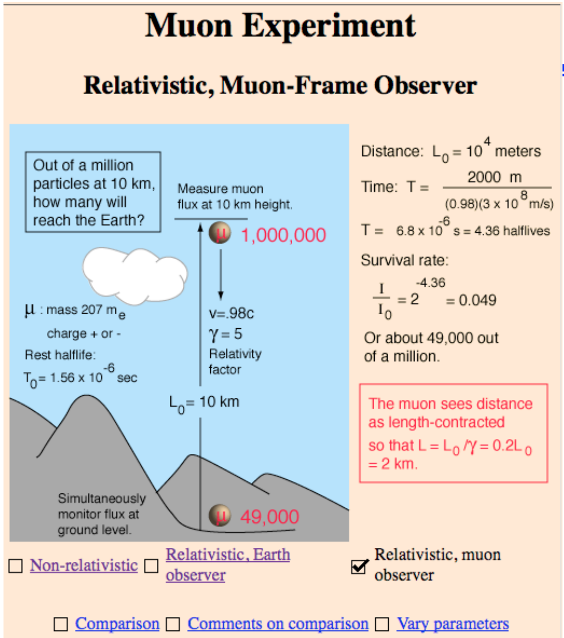
Think again about the muon experiment… <u>but now from the muon's perspective!</u>

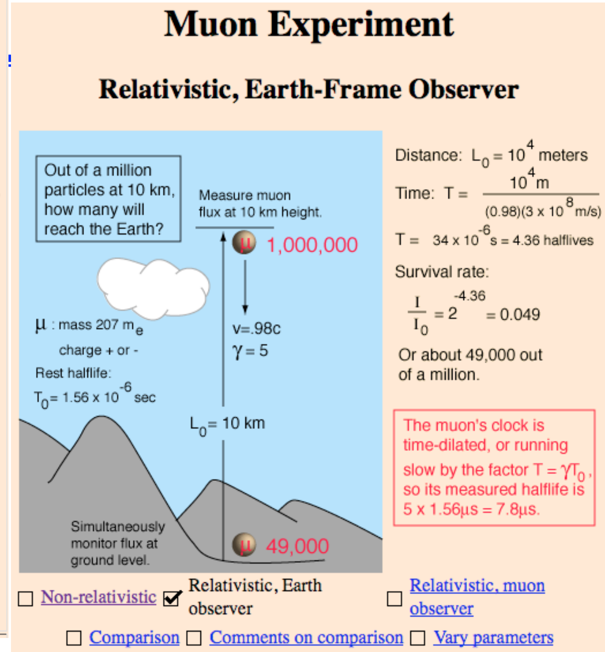
- *Fitzgerald contraction…*
  - *A moving object **contracts** by a factor γ (the same Lorentz factor) in the direction of motion*

This is really a <u>contraction of space itself</u>… the object does not experience forces or stresses that make it contract

Again, everything is relative… if someone watches you travel past them at high speed, you will appear to be contracted in the direction of motion







Muons ... again!

- Consider atmospheric muons again, *this time from point of view of the muons* i.e. think in frame of reference in which muon is at rest
- Decay time in this frame is 2 μs (2/1,000,000 s)

How do they get from top of the atmosphere to sea level before decaying?

- From point of view of muon, the atmosphere's height *contracts by factor of* γ
- Muons can then travel the reduced distance (at almost speed of light) before decaying.

I Length (Fitzgerald) contraction

Consider two "markers" in space.

- *Suppose spacecraft flies between two markers at velocity V.*

A flash goes off when front of spacecraft passes each marker, so that anyone can record it

- *Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft…*

Length (Fitzgerald) contraction

Observer at rest w.r.t. markers says:

Time interval is t_R; distance is $L_R=Vxt_R$

Observer in spacecraft says:

Time interval is $t_{S;}$ distance is $L_S=Vxt_S$

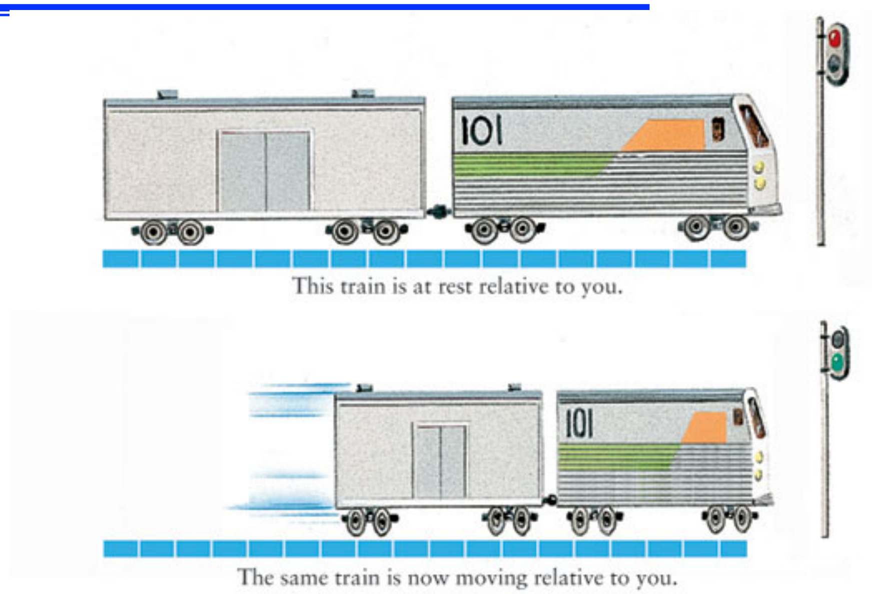
We know from before that $t_R = t_S \gamma$

Therefore, $L_S=Vxt_S= Vxt_{Rx}(t_S/t_R)=L_R/\gamma$

The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor γ

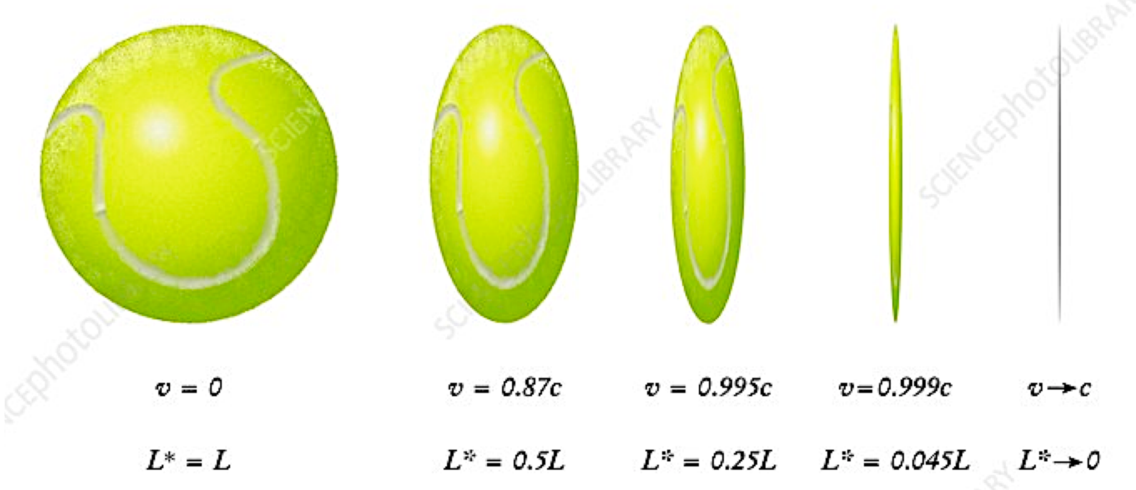
In addition to time, length depends on your frame of reference !

FITZGERALD CONTRACTION



(a) Length contraction

FITZGERALD CONTRACTION



https://www.sciencephoto.com/media/146860/view/lorentz-fitzgerald-contraction-artwork

Length contraction

- So, moving observers see that objects contract *along the direction of motion*.
- **Length contraction**… also called

  Lorentz contraction

  FitzGerald contraction

  Note that there is *no contraction of lengths* that are **perpendicular** to the direction of motion

- *Recall M-M experiment: results consistent with one arm contracting*

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

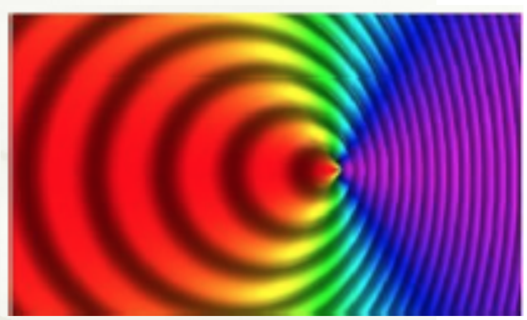
Other Experimental Tests of Time/Length Dilation

- frequency of wave (sound) due to motion towards or away from the observer
- If ν_{obs} is the frequency seen by the observer and ν_{emit} is the wavelength emitted by the object moving at velocity ,v, then
- The Doppler shift is $\nu_{obs}=\nu_{emit}(1+v/c)$ when v<<c

when viewed from in front the pitch, (frequency of sound), gets higher)

$\nu_{obs}=\nu_{emit}$*sqrt((1+v/c)/(1-v/c))
for the observer in front of the source
and

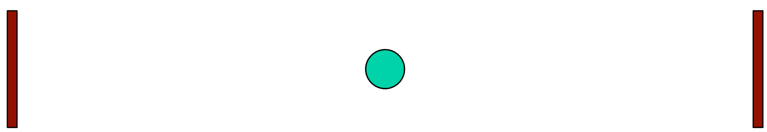
$\nu_{obs}=\nu_{emit}$*sqrt((1-v/c)/(1+v/c))
for the observer in back



result is due to contraction of length (change in wavelength) or time dilation (change in frequency) of the wave
Observer in front see contraction one in back expansion

II: Relativity of Simultaneity

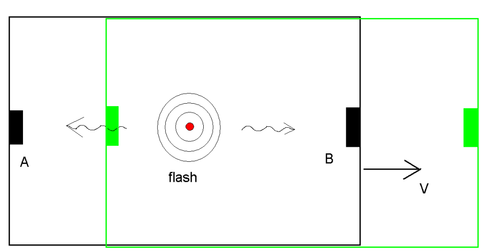
- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



- **Since speed of light is constant**, light rays will hit opposite walls at precisely the same time. Call these events A and B.

Change frames…

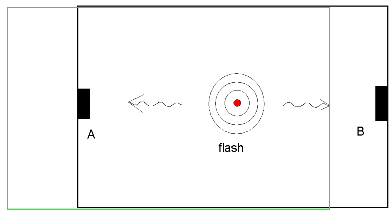
- Imagine performing same experiment aboard a moving spacecraft, and **observing** it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (*since the walls are moving!*). Event A will happen before event B.



- *But astronaut in spacecraft thinks events are simultaneous*.
- **Concept of "events being simultaneous" (i.e. simultaneity) is different for different observer**

Change frames again!

- What about perception of a 3rd observer who is moving faster than spacecraft?



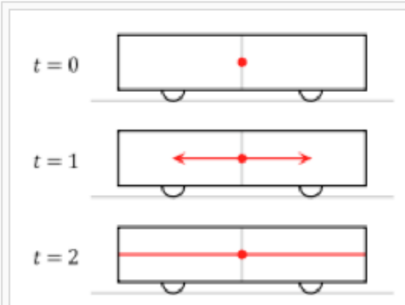
- 3rd observer sees event B before event A
- So, **order** in which events happen can depend on the frame of reference.

Simultaneity

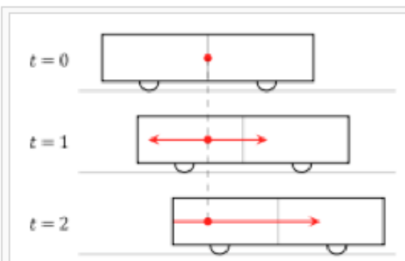
- Time can be different for two observers
- whether two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame.
- reference frame that is moving relative to the first will generally assign different times to the two events

- In the moving train analogy - we shine a light to the front and the back of the train car- *the speed of the train is NOT added to the speed of light*

  light ALWAYS moves at the same speed.!
- Since the train is moving the stationary observer will observe the light to strike the rear of the train before it hits the front
- For the passenger the light strikes the front and the back at the same time, for the stationary observer they occur at different times!
- **Time itself is different for the two observers !**



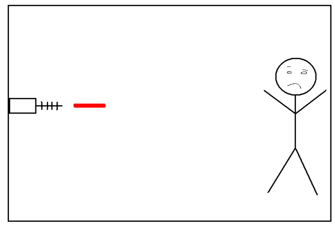
The train-and-platform experiment from the reference frame of an observer onboard the train



Reference frame of an observer standing on the platform (length contraction not depicted)

The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.



- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

- This is a question of **causality**.
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- In fact, it is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.

- This is true provided that
  - The laser blast does not travel faster than the speed of light
  - We do not change to a frame of reference that is going faster than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information**.

- **Question:**
- Two events that are simultaneous in one frame of reference will be:

A) simultaneous in all frames of reference

B) simultaneous in another frame that is moving in the opposite direction

C) simultaneous in another frame of reference that is moving in the same direction

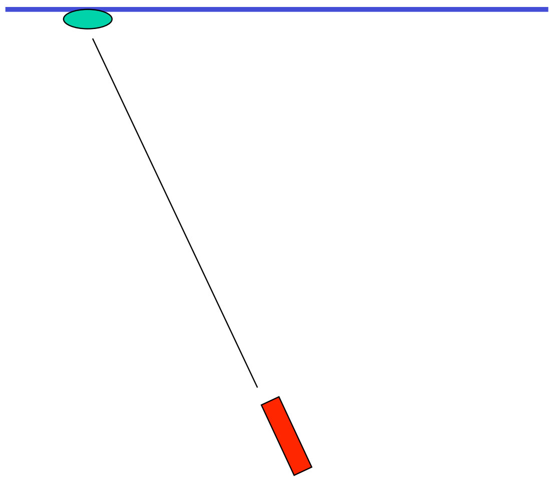
D) simultaneous in the same frame of reference

E) simultaneous in another non-inertial frame of reference.

From the special theory of relativity two events which are observed simultaneous in one inertial frame of reference may not be simultaneous in other frame of reference. For example time spent in a frame of reference varies in one inertial frame relative to another inertial frame which moves relative to first.

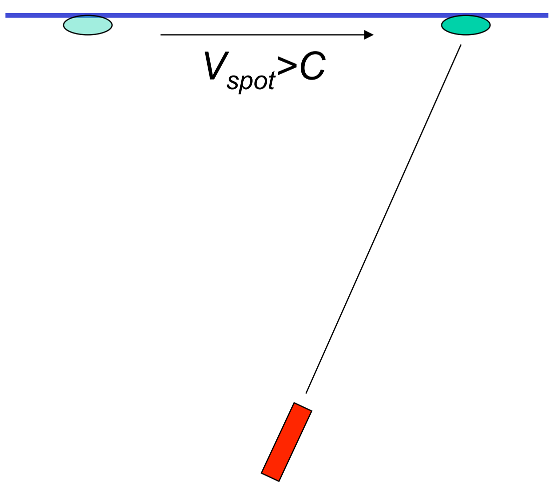
Therefore the two events will be simultaneous only in the same frame of reference*.*

Some things move faster than light ?



- But they transmit no information
- E.g., light spot on a distant screen

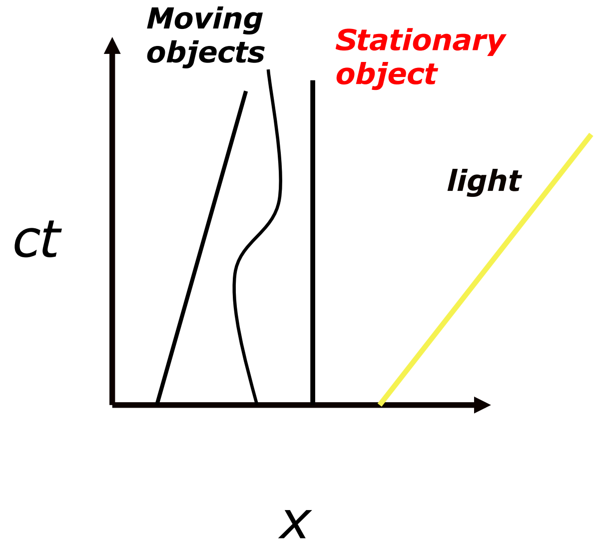
Some things move faster than light



- But they transmit no information
- E.g., light spot on a distant screen

III Space-time diagrams

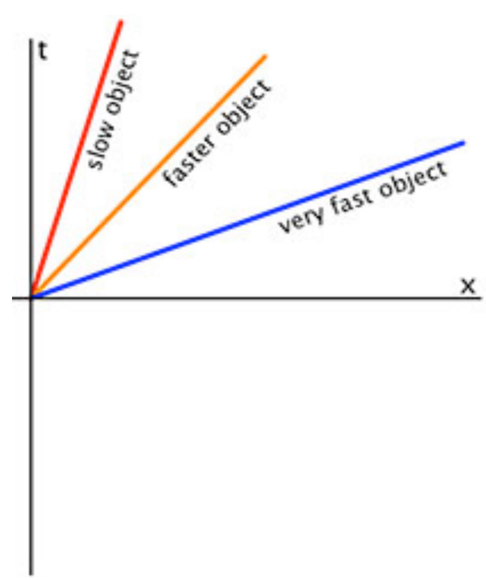
- Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- This is simplest to do for events that take place along a line in space (*one-dimensional space*)
  - Plot as a 2D graph
  - use two coordinates: ***x* and *ct***



World lines of events

Space-time diagrams

- If moving at constant velocity slope of line is how fast you are moving (maximum is speed of light)



Space Time Diagrams

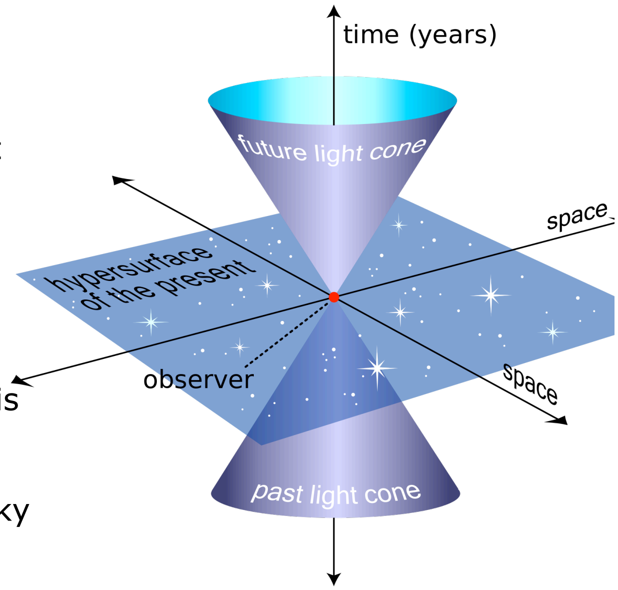
Can't get there from here

the light cone

The **future** of the given event is formed by all events that can be reached traveling slower than light

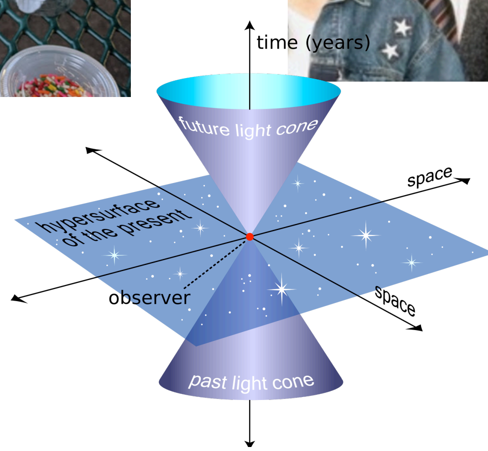
The **past** of the given event is formed by all events that can influence the event

The **lightcone** at the given event is formed by all events that can be connected through light rays with the event. When we observe the sky at night, we basically see only the past light cone.

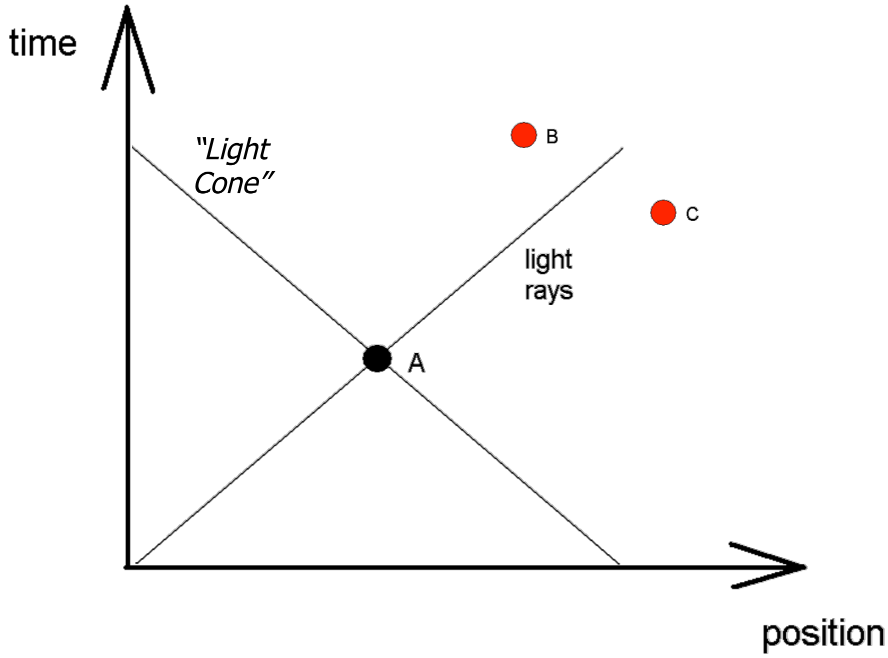


https://en.wikipedia.org/wiki/World_line#/media/File:World_lir

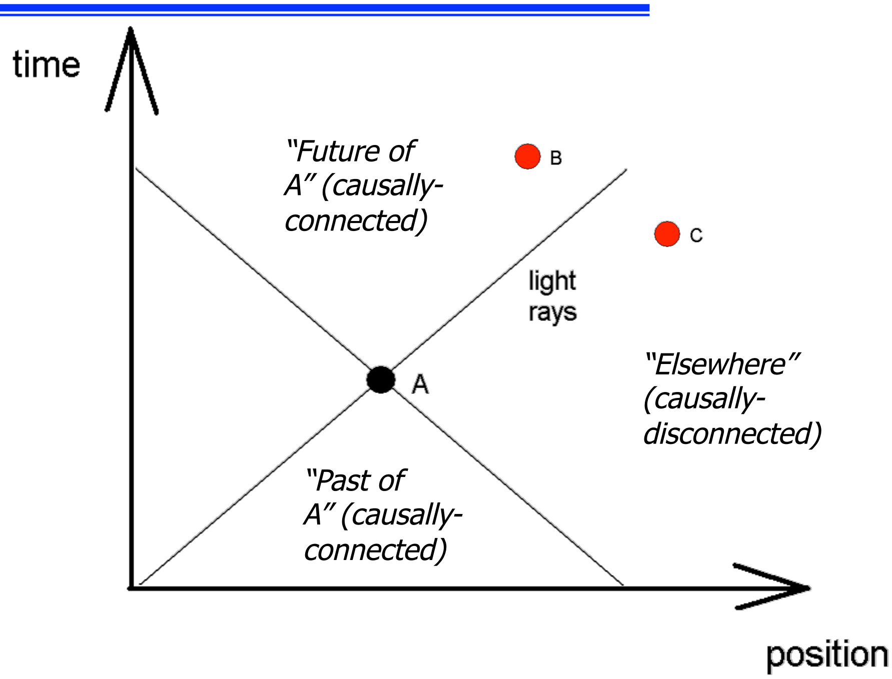
Light Cones



Light cone for event “A”

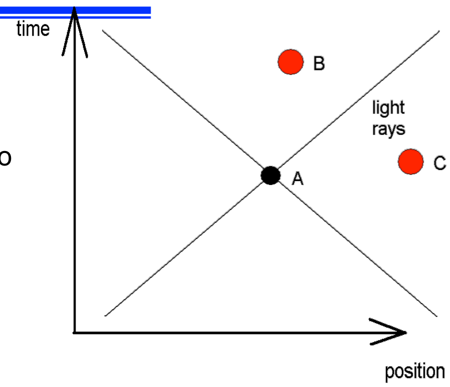


Past, future and “elsewhere”.



IV Causality

- Events A and B…
  - Cannot change order of **A** and **B** by changing frames of reference.
  - **A** can also communicate information to **B** by sending a signal at, or less than, the speed of light.
  - This means that **A** and **B** are causally-connected.
- Events A and C…
  - Can change the order of **A** and **C** by changing frame of reference.
  - If there were any communication between **A** and **C**, it would have to happen at a speed faster than the speed of light.
- If idea of **cause and effect** is to have any meaning, we must conclude that no communication can occur at a speed faster than the speed of light.



IV : Causality

- Can causality be proved?
  - No, it is an axiom of physics
- What if causality doesn't hold?
  - Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - This would be a true "end of science."
- So we will *insist* on causality as we continue to explore relativity.

Distances in time and space

- Two events A and B separated by distance Δs in space (x, y, z):

  $\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$

  (Thanks, Pythagoras!)

where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

Distances in time and space

- Two events A and B separated by distance Δs in time (Δt):

  $\Delta s = [(c\Delta t)^2]^{1/2}$

  where $\Delta t = t_A - t_B$, and we've multiplied by c to make the units of Δs come out as a distance

- Two events A and B separated in x and t:

  $\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$

  one space dimension+time

- For a light ray since $\Delta x= c\Delta t$

Δs^2=sqrt($(c\Delta t)^2-(\Delta x)^2$))=0

- Not like Euclidean space

if $(\Delta x)^2 > (c\Delta t)^2$ the events are separated by a 'spacelike' interval - can't get from here to there or more

formally

not enough time passes between their occurrence for there to exist a causal relationship crossing the spatial distance between the two events at the speed of light or slower

V: NEW VELOCITY ADDITION LAW

- Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simply adding/ subtracting frame velocity) give incorrect results for electromagnetism

- Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities

- For a particle measured to have velocity V_p by a observer moving at velocity V_s to a stationary observer ,the particle's velocity as measured by the observer is

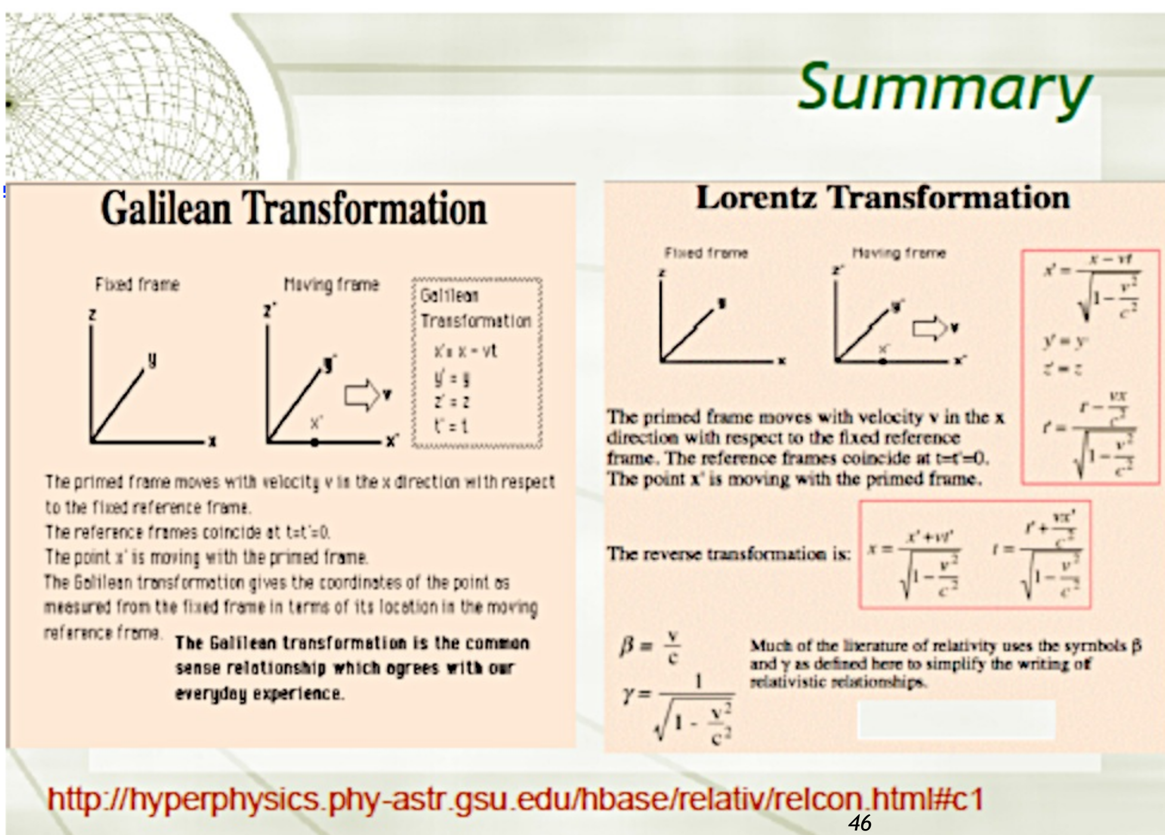
$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

NEW VELOCITY ADDITION LAW

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator is very small and therefore $V \sim (V_p + V_s)$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed o flight
- This is consistent with everyday experience
- Also notice that if the particle has V_s=c in the moving frame, then it has V_p=c in the stationary frame
- the speed of light is frame-independent!

(algebra e.g $V_p+V_s=2c$; $V_p{*}V_s=c^2$; $2c/(1+1)=c$)

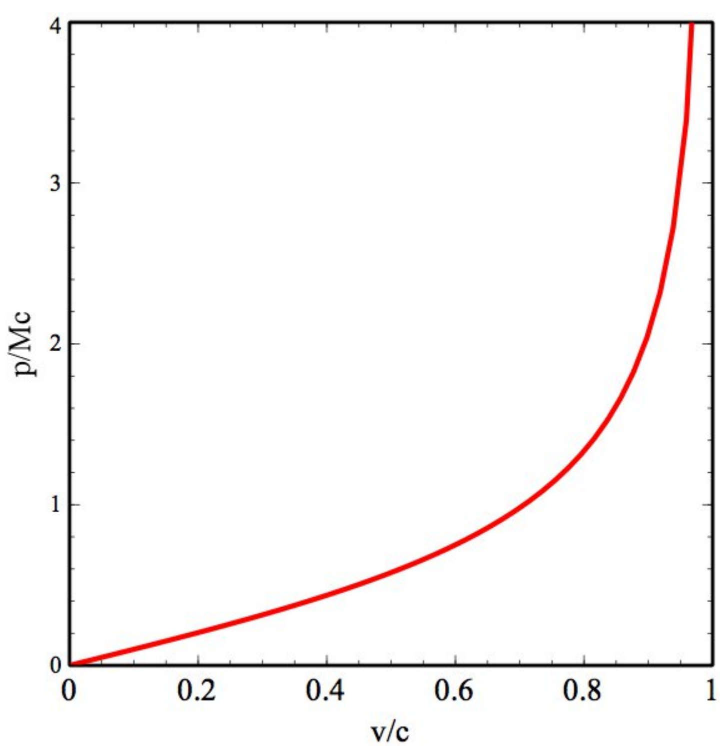


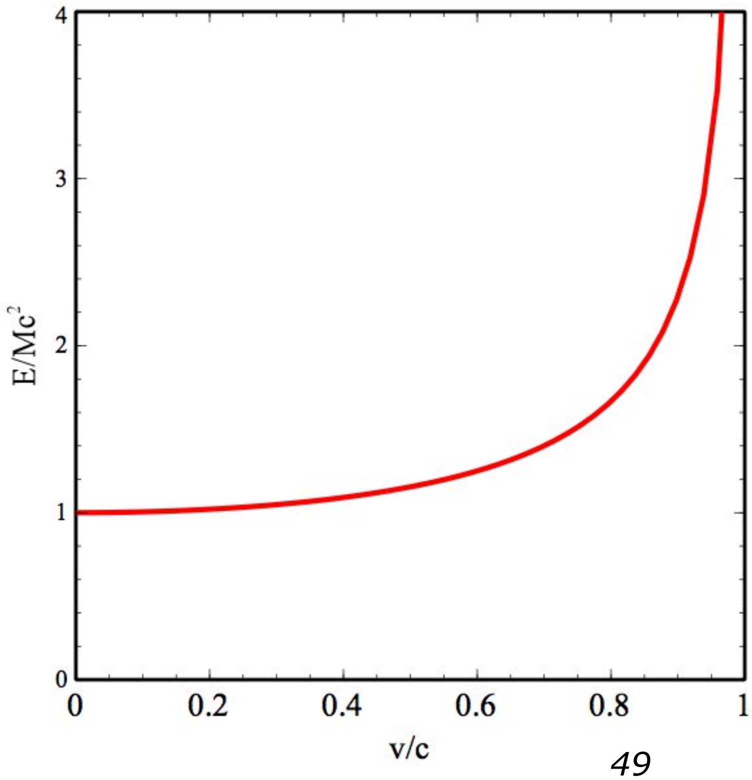
VI : Mass and Energy

- Einstein reworked Newton's laws of mechanics using his new relativistic formulae
- The relationships between mass (M), velocity (v), momentum (p) and energy (E) **are different than those found in Newtonian mechanics**

$$\mathbf{p} = \gamma M \mathbf{v}$$
$$E = \gamma M c^2$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Thus energy increases as the speed increases, and energy would become infinite if V = c; γ is the Lorentz factor





Newton was only a little bit wrong

- What about objects moving at "small velocity"?
- It can be shown that: (binomial expansion)

$$E \approx mc^2 + \frac{1}{2}mV^2$$

  - The $\frac{1}{2}mv^2$ is the Newtonian expression for the **kinetic energy** of a moving object.
- What counts as "small velocity"?
  - For car going at 30mph, approximate formula is wrong by 1 part in 10^{30}
  - For rocket going at 30,000mph, this approximate formula is wrong by 1 part in 10^{18}
  - So, Newtonian approximation is fine for all velocities experienced in everyday life.

VII Rest mass energy

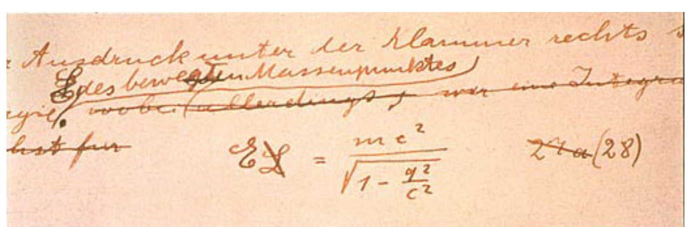
- Put V=0 in Einstein's energy formula, we get…

$$E = mc^2$$

- What does this mean?
  - Maybe it is some fundamental "irreducible" (i.e., inaccessible) energy that every object possesses?
  - Or, perhaps this energy can be accessed? In other words, maybe mass can be turned into "usable" energy? **It turns out that this is correct!**
  - Also, this can go the other way – energy can be turned into mass!

Einstein's Insight

- Einstein speculated $E=mc^2$ was not simply an academic exercise; he believed that it might explain how an ounce of radium could emit 4,000 calories of heat per hour indefinitely (Marie Curie), seemingly violating the first law of thermodynamics-



Once again, relativity forced a major revision in classical physics. Before, the first law of thermodynamics, which states that the total amount of energy can never be created or destroyed.

Now the total combined amount of ***matter and energy*** *is the conserved quantity.*

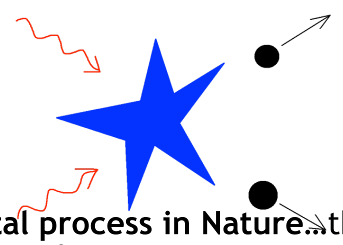
adapted from text by Michio Kaku

Einstein and the Atomic Bomb

- When Einstein learned that the Germans might figure out how to split the atom, he wrote to President Franklin Roosevelt with his concerns. Einstein's 1939 letter helped initiate the U.S. effort to build an atomic bomb
- http://www.amnh.org/exhibitions/past-exhibitions/einstein/peace-and-war/the-manhattan-project

EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy(e.g.,gamma-rays) can produce particle/anti-particle pairs



  - **Very fundamental process in Nature**…this process, operating in the early universe, is responsible for all of the mass that exists today!

conservation of energy sets a minimum photon energy required for particle creation : this threshold energy must be greater than the total rest energy of the fermions created.

To create an electron-positron pair the total energy of the photons must be at least $2m_ec^2$=2x0.511KeV=1.022MeV

Some Consequences of Special Relativity

There is no absolute time or absolute space.

It is impossible for two events to be simultaneous for all possible observers.

There are pairs of events which will happen in one order for some observers and in the other order for other observers.

The kinetic energy of massive moving bodies increases without bound as the velocity of the body approaches the speed of light.

The same holds for the momentum of massive moving bodies: it increases without bounds as the velocity approaches the speed of light

No massive object can travel at or faster than the speed of light. All massless objects can only travel at the speed of light.