Queen Mary
University of London

M.Sc./M.Sci. EXAMINATION BY COURSE UNITS

ASTM002 / MAS430   The Galaxy

Tuesday, 22 May, 2007
6:15 – 9:15 p.m.

Time Allowed: 3 hours

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 5 questions answered will be counted.

Calculators ARE permitted in this examination. The unauthorised use of material stored in a pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Useful information

In this paper π and e represent the conventional mathematical constants.
G represents the gravitational constant with $G = 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$.
c is the velocity of light, with $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$.
1 pc $= 3.09 \times 10^{16}$ m.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} .$$

The Jeans equations in a steady-state (i.e. constant over time), spherically-symmetric galaxy give the result for a (r, θ, ϕ) spherical coordinate system

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(n\,\langle v_r^2\rangle\Big) + \frac{n}{r}\Big[\,2\langle v_r^2\rangle - \langle v_\theta^2\rangle - \langle v_\phi^2\rangle\Big] = -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r} ,$$

where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

The gravitational lensing optical depth through a distribution of microlenses of mass M_L along a path length to a source at a distance D_S is given by

$$\tau = \frac{4\pi G}{c^2 D_S}\int_0^{D_S} D_L\, D_{LS}\, \rho(D_L)\, \mathrm{d}D_L ,$$

where ρ is the local mass density of the lenses in space, D_L is the distance of the lens from the observer, and D_{LS} is the distance between the lens and source.

1. (a) How does the gas content of galaxies vary with galaxy type? How does this explain the difference in the optical spectra of elliptical and irregular galaxies? [3]

(b) A spherical elliptical galaxy is found to have a surface brightness at a distance R from its centre that is given by $I(R) = I_0\,\mathrm{e}^{-(R/R_0)^{1/4}}$, where I_0 and R_0 are constants. What is the total apparent flux from the galaxy in terms of I_0 and R_0? (You may find useful the standard result $\int_0^\infty x \exp(-(x/a)^{1/4})\,\mathrm{d}x = 4\,.\,7!\,a^2 = 20160\,a^2$.) [3]

(c) Explain how the Tully-Fisher relation can be used to determine the distances of spiral galaxies.
Observations show that two spiral galaxies have rotation velocities $v_{rot} = 200\mathrm{km\,s^{-1}}$ and $300\ \mathrm{km\,s^{-1}}$. What is the ratio of their luminosities? [4]

(d) Explain the difference between dissipational and dissipationless collapse in galaxy formation.
Explain why gas is likely to settle to a rotating disc during galaxy formation or during mergers. [4]

(e) Briefly explain how in the monolithic collapse model, through the collapse of a single protogalactic cloud with some net angular momentum, the main components of the Galaxy could have been formed. Include an explanation of how the kinematics, ages and metallicities of these components arise in this model. [5]
An alternative model for the formation of galaxies proposes that clumps of dark matter containing embedded baryonic matter merged to form the galaxies we observe today. Which of these two models is supported by modern computer simulations? [1]

[Total 20 marks for question]

2. (a) A researcher carries out N-body numerical modelling of an elliptical galaxy using 10^5 particles in a computer simulation. However, the galaxy actually contains 10^{11} stars. The relaxation time T_{relax} and crossing time T_{cross} of a uniform system of N stars are related approximately by

$$\frac{T_{relax}}{T_{cross}} \simeq \frac{N}{12\ln N} \ .$$

Explain why the numerical modelling will be inaccurate if the gravitational field of each particle is represented with the standard $\Phi = -Gm/r$ Newtonian potential. Explain why a softening parameter is added to the gravitational potentials of particles for N-body modelling. [4]

(b) The continuity equation for the distribution function f of stars in the six-parameter phase space $(x_1, x_2, x_3, v_1, v_2, v_3)$ of position $\mathbf{x}$ and velocity $\mathbf{v}$ states that

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(\frac{\partial}{\partial x_i}\left(f\frac{\mathrm{d}x_i}{\mathrm{d}t}\right) + \frac{\partial}{\partial v_i}\left(f\frac{\mathrm{d}v_i}{\mathrm{d}t}\right)\right) = 0 \ ,$$

[This question continues overleaf . . .]

where t is time.
Derive the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\frac{\partial f}{\partial x_i} + \frac{\mathrm{d}v_i}{\mathrm{d}t}\frac{\partial f}{\partial v_i}\right) = 0$$

from the continuity equation, showing your working. [6]

(c) Derive the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{3}\frac{\partial n\langle v_i\rangle}{\partial x_i} = 0\,,$$

from the collisionless Boltzmann equation, where n is the number density of stars and $\langle v_i\rangle$ is the mean value of the v_i velocity component at a point. Explain your working. [6]

(d) The velocity dispersion tensor is defined by

$$\sigma_{ij}^2 = \frac{1}{n}\int\Big(v_i - \langle v_i\rangle\Big)\Big(v_j - \langle v_j\rangle\Big)\, f\, \mathrm{d}^3\mathbf{v}$$

where v_1, v_2 and v_3 are the components of the velocity vector $\mathbf{v}$, n is the number density of stars in space, f is the distribution function, and i and $j = 1, 2$ and 3. Prove that

$$\sigma_{ij}^2 = \langle v_i v_j\rangle - \langle v_i\rangle\langle v_j\rangle$$

where $\langle p\rangle = \frac{1}{n}\int p f \mathrm{d}^3\mathbf{v}$ denotes the mean value of some parameter p at a point in space. [4]

[Total 20 marks for question]

3. (a) Assume that the Galactic halo is spherical, that it has no net rotation, that its velocity dispersion is isotropic and constant, that it has a potential $\Phi(r) = v_0^2 \ln r$ where v_0 is a constant, and that it has a stellar density profile of the form $n(r) \propto r^{-l}$ where l is a constant. Hence determine an expression for the velocity dispersion σ of halo stars in the solar neighbourhood in terms of v_0 and l.
If $v_0 = 220\ \mathrm{km\,s^{-1}}$, calculate σ for a plausible value of l. How does this figure match observations? [5]

(b) The Jeans equations in a cylindrical coordinate system (R, θ, z) centred on the Galaxy, with $z = 0$ in the plane, give

$$\frac{\partial(n\langle v_z\rangle)}{\partial t} + \frac{\partial(n\langle v_R v_z\rangle)}{\partial R} + \frac{\partial(n\langle v_z^2\rangle)}{\partial z} + \frac{n\langle v_R v_z\rangle}{R} = -\,n\frac{\partial\Phi}{\partial z}\,,$$

where n is the star number density, v_R and v_z are the velocity components in the R and z directions, $\Phi(R, z)$ is the Galactic gravitational potential and t is time.

[This question continues overleaf . . .]

Assuming that the Galaxy is in a steady state, derive an expression for the surface mass density $\Sigma(z, R_0)$ within a distance z of the mid-plane of the Galactic disc at the solar radius R_0 in terms of n and v_z for stars lying towards the Galactic poles. State any other assumptions you make. [8]

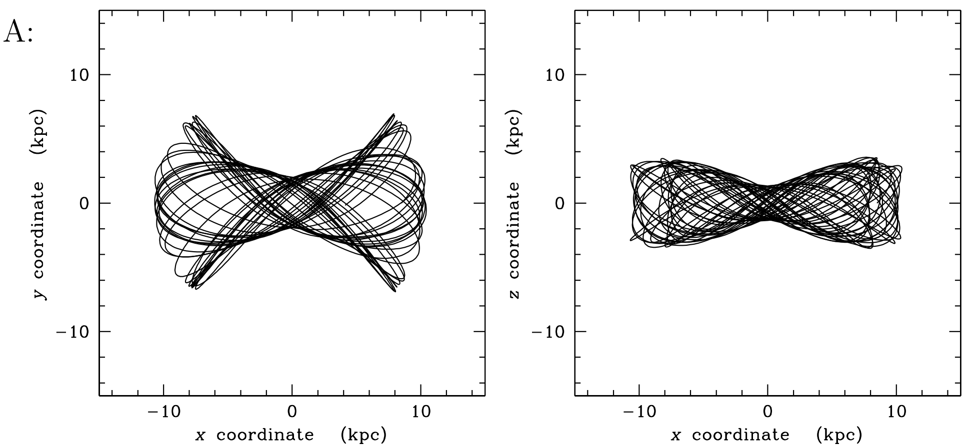
(c) Explain how this enables the surface mass density of the Galactic disc to be found from observations of stars. [5]

(d) Does any significant quantity of dark matter exist within the Galactic disc? [2]

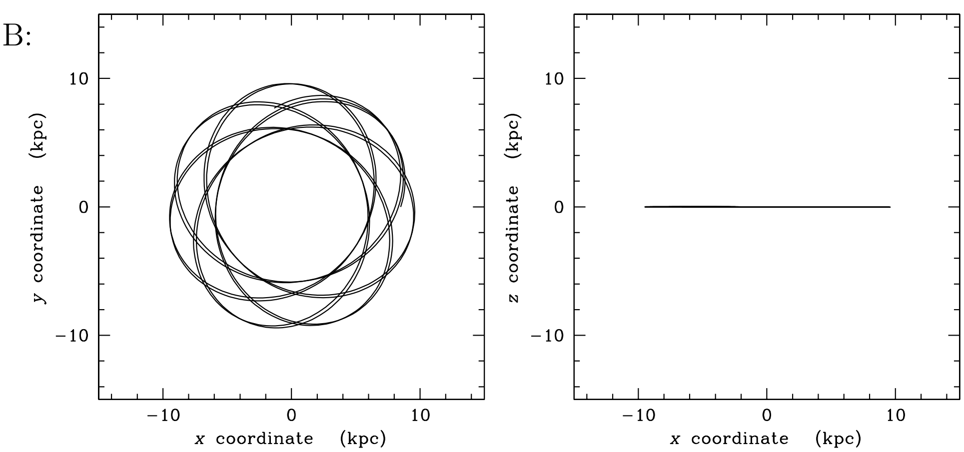
[Total 20 marks for question]

4. (a) The diagrams below show the orbit of a star in two gravitational potentials, shown projected in the $x - y$ and the $x - z$ planes.

Potential A:



Potential B:



What do you conclude about each of the potentials A and B: are they (i) spherical, (ii) flattened (oblate), or (iii) triaxial? Justify your answer on the basis of the character of the orbit. [6]

[This question continues overleaf . . .]

(b) The distribution function f of stars in the six-parameter phase space is related to the density ρ at a point in space by

$$\rho(r) \;=\; 4\pi\,\overline{m}\int f v^2\,\mathrm{d}v \;=\; (4\pi)^2\sqrt{2}\,G\,\overline{m}\int\sqrt{E_m-\Phi(r)}\;f(E_m)\,\mathrm{d}E_m$$

for a spherically symmetric potential, where r is the radial distance from the centre of the potential, Φ is the gravitational potential, $\mathbf{v}$ is the velocity of a star, E_m is the energy per unit mass, and $\overline{m}$ is the mean mass of the stars.
Derive the limits on these two integrals. [4]

(c) What is meant by an integral of the motion? [2]

(d) Which of the following are integrals of the motion for stars moving in a spherical potential: velocity; momentum; energy per unit mass; potential energy; angular momentum per unit mass? [2]

(e) The distribution of stars in a spherically symmetric galaxy is modelled using a distribution function

$$f(v) \;=\; \frac{n_0}{(2\pi\sigma^2)^{\frac{3}{2}}}\,\exp\left(-\,\frac{\frac{1}{2}v^2+\Phi}{\sigma^2}\right)\;,$$

where E_m is the energy of a star per unit mass, v is the speed of a star, $\Phi(r)$ is the gravitational potential at a distance r from the centre, and σ and n_0 are constants.
Determine the number density $n(r)$ at a radius r in terms of the potential $\Phi(r)$ implied by this profile assuming that there is no dark matter present. Comment on the upper limit of the integration implied by this $f(v)$. You may find the standard integral $\int_0^\infty x^2 \mathrm{e}^{-ax^2}\mathrm{d}x = \sqrt{\pi/a^3}\,/4$ useful. Hence obtain an expression relating the density $\rho(r)$ and potential $\Phi(r)$ at a radius r.
Without derivation state one possible solution to the density profile for this $f(v)$. [6]

[Total 20 marks for question]

5. (a) Explain the difference between pressure support and rotational support for a galaxy. [2 marks]

(b) A spherically-symmetric galaxy is modelled using a density profile

$$\rho(r) \;=\; \frac{k}{(r+a)(r^2+a^2)}\;,$$

where r is the distance from the centre, and k and a are constants.
Show that the mass interior to a radius r in this model is given by

$$M(r) \;=\; 2\pi k\left(\ln\left(\left(1+\frac{r}{a}\right)\sqrt{1+\frac{r^2}{a^2}}\;\right) \;-\; \tan^{-1}\left(\frac{r}{a}\right)\right)\;.$$

[4]

[This question continues overleaf . . .]

What is the circular velocity v_{circ} as a function of radius r? How does v_{circ} scale with r for $r \gg a$? [3]

You may find helpful the standard integral

$$\int \frac{x^2 \, \mathrm{d}x}{(x+a)(x^2+a^2)} = \frac{1}{2} \ln\left((x+a)\sqrt{x^2+a^2}\right) - \frac{1}{2} \tan^{-1}\left(\frac{r}{a}\right) + \text{constant}\,.$$

(c) A spherically-symmetric galaxy is dark matter dominated and has a gravitational potential

$$\Phi(r) = -\frac{k}{r^2+a^2}$$

at a radial distance r from its centre, where k and a are constants. Show that the density profile implied by this potential is

$$\rho(r) = \frac{k}{2\pi G} \frac{3a^2 - r^2}{(r^2+a^2)^3}\,.$$

[6]

A population of stars is distributed within this potential. The stars contribute negligibly to the total density. The system of stars has an isotropic velocity distribution with a velocity dispersion σ that is constant across the galaxy, and has zero net rotation. Assuming that the potential is constant over time, derive an expression for the number density n of stars as a function of radius r and in terms of the number density n_0 of stars at the centre and of k. [5]

[Total 20 marks for question]

6. (a) In which region of the electromagnetic spectrum is the CO molecule most easily observed? What type of energy level transitions are responsible for this emission? Why are observations of CO emission used to trace the distribution of cold molecular gas in the Galaxy, rather than observations of H_2 molecules directly? [3]

(b) Observations of an isolated H II region show that the total luminosity through the Hα emission line is 1.4×10^{48} photons s^{-1}, while it is 9×10^{47} photons s^{-1} in the Hβ line, $7 \times 10^{47}\,\mathrm{s}^{-1}$ in Hγ, $5 \times 10^{47}\,\mathrm{s}^{-1}$ in Hδ, and $8 \times 10^{47}\,\mathrm{s}^{-1}$ in all other Balmer emission (lines and continuum). What is the total flux of ultraviolet photons with wavelengths shorter than 912 Å from stars inside the H II region? Why is this wavelength of 912 Å significant? [4]

(c) Explain under what circumstances absorption lines from interstellar gas can be observed. Summarise the properties of these lines and the likely mechanisms that cause them. [2]

(d) Explain the concept of the components of the interstellar medium. [3]

(e) Observations of the extinction of starlight by dust show that the ratio of colour excesses, $E_{\mathrm{U-B}}/E_{\mathrm{B-V}}$, for the (U–B) and (B–V) colour indices is nearly constant

[This question continues overleaf . . .]

across the Galaxy, regardless of the strength of the extinction. Prove that the parameter

$$Q \equiv (\mathrm{U}-\mathrm{B}) - \frac{E_{\mathrm{U-B}}}{E_{\mathrm{B-V}}}(\mathrm{B}-\mathrm{V})$$

is independent of interstellar extinction.

A hot main sequence star in the Galactic plane is observed to have magnitudes U = 11.22, B = 11.65 and V = 11.00. What is the Q parameter for this star given that a standard value for $E_{\mathrm{U-B}}/E_{\mathrm{B-V}}$ is 0.72 in the Galaxy?

The table below lists intrinsic colour indices and Q parameters for different types of main sequence stars. Using the table, determine the (B–V) colour excess of the star, estimate the V-band interstellar extinction A_V, and estimate the intrinsic V-band magnitude. [5]

Spectral type	O5V	B0V	B5V	A0V
$(\mathrm{B-V})_0$	−0.35	−0.31	−0.16	0.00
Q	−0.90	−0.84	−0.43	0.00

(f) A small dust particle in the interstellar medium has a mass of 1.0×10^{-24} kg and an initial temperature of 10 K. It is struck by a single ultraviolet photon with a wavelength of 1.0×10^{-7} m. If the grain has a thermal heat capacity of 2.0×10^{-21} J K^{-1}, estimate the increase in the temperature of the grain. What effect does this have on the wavelengths of photons emitted by the grain after the heating as compared to before. (The Planck constant is $h = 6.63 \times 10^{-34}$ J s. The velocity of light is $c = 3.00 \times 10^{8}$ m s^{-1}.) [3]

[Total 20 marks for question]

7. (a) A star has an oxygen-to-iron abundance ratio 1/10th that in the Sun by number. What is the [O/Fe] parameter? [1]

(b) Explain how the different properties of type Ia and type II supernovae can explain the observed trend of [O/Fe] as a function of [Fe/H] for G and K dwarf stars in the solar neighbourhood. [4]

(c) What is the basic difference between the s-process and r-process for neutron capture in nucleosynthesis? [2]

(d) The change δZ in the heavy element mass fraction Z when the mass of gas M_{gas} in a volume of space changes by δM_{gas} is given in the Simple Model by

$$\delta Z = -p\,\frac{\delta M_{\mathrm{gas}}}{M_{\mathrm{gas}}} ,$$

where p is the yield of heavy elements. Show that the mass $M_{stars}(t)$ of stars at time t is related to the heavy element fraction $Z(t)$ and the initial gas mass $M_{gas}(0)$ by

$$Z = -p \ln\left(1 - \frac{M_{\mathrm{stars}}(t)}{M_{\mathrm{gas}}(0)}\right) ,$$

[This question continues overleaf . . .]

[5]

(e) Show that in the Simple Model, the mean metallicity of a population of long-lived stars is given by

$$\langle Z \rangle \;=\; p\left(\frac{M_{\rm gas}(0)}{M_{\rm stars}} - 1\right)\ln\left(1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)}\right) \;+\; p\;.$$

Show that this can be written as $\langle Z \rangle \;=\; p\left(1 + \dfrac{\mu \ln \mu}{1-\mu}\right)$ in terms of the gas fraction μ.

What is the mean metallicity $\langle Z \rangle$ when the gas fraction $\mu \to 0$ as gas is used up entirely in star formation? [6]

You may find helpful the standard integral

$$\int \ln(1 - x/a)\,\mathrm{d}x \;=\; (x-a)\ln(1-x/a) - x + \text{constant}.$$

(f) How do the chemical compositions of stars in globular clusters, field stars in the Galactic halo, and stars in the thin disc of the Galaxy compare with the Sun respectively? [2]

[Total 20 marks for question]

8. (a) Assuming that the dark matter halo of the Galaxy can be represented by an isothermal sphere of compact microlensing objects with a density distribution $\rho(r) = \sigma^2/2\pi G r^2$ where r is the radial distance from the Galactic centre, G is the constant of gravitation and σ is a constant, show that the optical depth for gravitational microlensing towards a globular cluster at the north galactic pole is

$$\tau \;=\; \frac{\sigma^2}{2c^2}\left(2\ln 2 \;+\; \pi \;-\; 4\right)\;,$$

if the distance to the cluster is the same as the distance R_0 of the Sun from the Galactic centre. You may assume that

$$\int \frac{x\,\mathrm{d}x}{a^2+x^2} \;=\; \tfrac{1}{2}\,\ln\left(a^2+x^2\right) \;+\; \text{constant}\,,$$

and

$$\int \frac{x^2\,\mathrm{d}x}{a^2+x^2} \;=\; x \;-\; a\,\tan^{-1}\left(\frac{x}{a}\right) \;+\; \text{constant}\,.$$

If $\sigma = 200\ \mathrm{km\,s^{-1}}$ and $c = 3.00\times10^5\ \mathrm{km\,s^{-1}}$, estimate the optical depth τ. If there are 5×10^5 stars in the globular cluster, how good are the chances of observing a microlensing event? [9]

[This question continues overleaf . . .]

(b) Why in practice are star fields in the Large Magellanic Cloud and the Galactic bulge used for microlensing studies, rather than stars in random fields? [2]

(c) Does gravitational lensing affect the colour and spectrum of a source? How does this help in microlensing surveys? [2]

(d) The separation l between the Galaxy and the Andromeda Galaxy M31 satisfies the equation

$$\frac{\mathrm{d}^2 l}{\mathrm{d}t^2} = -\frac{GM}{l^2} ,$$

where M is the combined total mass of the two galaxies and t is time.
Verify that $l = k\,t^n$ is a solution to this equation, where k and n are constants, and determine the required values of k and n.
How well does this solution describe the actual dynamics of M31 and the Galaxy? What dynamical situation does the solution represent? [7]

[Total 20 marks for question]

[End of examination paper]