

## ***Gravitational Lensing - Einstein's Unfinished Symphony***

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*(v3.0 released January 2010)*

Gravitational lensing - the deflection of light rays by gravitating matter - has become a major tool in the armoury of the modern cosmologist. Proposed nearly a hundred years ago as a key feature of Einstein's theory of General Relativity, we trace the historical development since its verification at a solar eclipse in 1919. Einstein was apparently cautious about its practical utility and the subject lay dormant observationally for nearly 60 years. Nonetheless there has been rapid progress over the past twenty years. The technique allows astronomers to chart the distribution of dark matter on large and small scales thereby testing predictions of the standard cosmological model which assumes dark matter comprises a massive weakly-interacting particle. By measuring distances and tracing the growth of dark matter structure over cosmic time, gravitational lensing also holds great promise in determining whether the dark energy, postulated to explain the accelerated cosmic expansion, is a vacuum energy density or a failure of General Relativity on large scales. We illustrate the wide range of applications which harness the power of gravitational lensing, from searches for the earliest galaxies magnified by massive clusters to those for extrasolar planets which temporarily brighten a background star. We summarise the future prospects with dedicated ground and space-based facilities designed to exploit this remarkable physical phenomenon.

### **1. Introduction**

One of the most intriguing aspects of the propagation of light rays in the cosmos is their deflection by massive objects. The phenomenon - termed *gravitational lensing* was predicted almost exactly a century ago by Albert Einstein as a feature of his theory of General Relativity and has now become one of the most powerful tools of the observational astronomer. We considered it appropriate, in this celebration of the 'Year of Light', to provide a non-specialist review of the progress that has been made utilising this phenomenon as well as introducing ambitious plans by the international community for future applications of gravitational lensing with upcoming facilities.

Einstein's prediction that light would be deflected by the Sun at the time of a solar eclipse was first verified by the famous 1919 expedition of the British astronomers Arthur Eddington and Frank Dyson. However, Einstein and Eddington were surprisingly skeptical of the long term utility of gravitational lensing. A renaissance began only in the 1970's when improved astronomical detectors and powerful large telescopes became available. The exquisite image quality of Hubble Space Telescope (HST), launched in 1990, provided a further major boost in progress.

Today gravitational lensing is being used to explore the distribution and nature of the poorly-understood dark matter that provides the dominant component of mass in the Universe. It is proposed that the phenomenon can yield unique insight into the mysterious *dark energy* - a property of space invoked to explain the accelerated expansion of the cosmos discovered in the late 1990's. Foreground mass structures such as nearby clusters of galaxies can also be used as natural 'telescopes' whereby distant objects are magnified as with a traditional optical lens; this

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provides unique information about galaxies seen at early cosmic times; without gravitational lensing such sources would be too faint to study.

Here we will briefly discuss the fascinating history of this phenomenon, explain in simple terms how gravitational lensing works, review the progress in areas of contemporary interest, and conclude with ambitious plans for future applications. For interested readers there are many excellent reviews of gravitational lensing [e.g., 1–4]. A comprehensive commented bibliography, including material suitable from high school to graduate students is given by [5].

### 1.1. A remarkable history

The earliest known mention of light being deflected by massive objects is the first query in Newton’s *Opticks* in 1704: ‘Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action strongest at the least distance?’ Unfortunately, the query does not distinguish between the action of gravity on a corpuscle and more conventional optical phenomena. Henry Cavendish is credited with the first (unpublished, 1784) calculation of the deflection angle  $\theta$  of a corpuscular light ray following a hyperbolic trajectory and the origin of the (Newtonian) equation  $\theta = 2GM/Rc^2$  where  $M$  is the mass of the deflector and  $R$  the radius at which the light ray arrives. Subsequently in 1804, Johann von Soldner published a similar calculation deriving a deflection of 0.84 arcsec for stars viewed close to the limb of the Sun. von Soldner additionally discussed the practicality of verifying this prediction but his work, as well as that of Cavendish, was largely forgotten as the corpuscular theory of radiation was increasingly discredited in favour of wave theories of light. Not only was there confusion as to whether a deflection was expected for a light wave but the small deflection was also considered unobservable.

In 1911, Einstein calculated a relativistic version of the solar deflection and derived a similar result to that achieved by von Soldner a hundred years earlier, 0.875 arcsec. However, the physical principles behind the two calculations are quite different. In the classical calculation, it is assumed that light can be accelerated and decelerated like a normal mass particle, whereas in Einstein’s calculation the deflection is based on gravitational time dilation. In 1915, Einstein considered the additional deflection arising from the curvature of space around the Sun in his newly-published *General Theory* from which he derived  $\theta = 4GM/Rc^2$  and a solar deflection of 1.75 arcsec. Beginning in 1912, Einstein sought observers who could verify his predicted deflection. The observational race to prove or disprove Einstein’s theory is a fascinating story well-documented in several recent books (e.g. [6][7][8][9]).

The Astronomer Royal, Frank Dyson, first proposed the May 29th 1919 eclipse expedition noting that the Sun would be in the rich field of the Hyades star cluster. Arthur Eddington had played a key role in promoting Einstein’s theory and took the lead in the organization. Eddington and his assistant Cottingham visited the island of Príncipe off the coast of West Africa (now part of the democratic republic of Sao Tomé and Príncipe); another team (Crommelin and Davidson) visited Sobral, Brazil. The results, confirming the full deflection, were presented in November 1919 ([10]). Although some have argued that Eddington was blinded by his enthusiasm for Einstein’s theory and biased in his analysis by discarding discrepant data [11], a recent re-analysis by Kennefick [12] shows this was not the case. The rejected Sobral astrograph plates were out of focus as a result of the rapid change in temperature during totality making it difficult to establish a proper plate scale. In 1979 the Sobral plates were more accurately re-measured yielding a deflection of  $1.55 \pm 0.32$  arcsec [13].

Eddington and Einstein were curiously reticent about possible applications of gravitational lensing. Chwolson [14] illustrated how lensing can produce multiple images of a distant source – a phenomenon now termed strong lensing (§2) but, as its occurrence depends on the precise alignment of a source and deflector, it was reasonable to conclude the probability of observing such phenomena would be very small. As a good illustration of thinking at the time, Einstein, urged by Mandl, discussed what Paczynsky later called microlensing – the temporary brightening

of a star due to the magnification induced by a foreground object that crosses the line of sight to the observer (§4). In this rare post-1919 article about lensing by its predictor [15], he states “of course there is no hope of observing this phenomenon.”

Fritz Zwicky emerges as the worthy prophet by arguing in 1937 that galaxies and galaxy clusters would be far more useful lenses given their greater mass and cross-section to background objects and, with great vision, foresaw many of the applications we review here [16]. In the 1960s, Barnothy & Barnothy [17] became tireless advocates of Zwicky’s position. The mathematics of multiply-imaged geometries was further developed independently by Klimov [18], Liebes [19] and Refsdal. Refsdal [20] demonstrated that if a background lensed source such as a quasar or supernova is variable in its light output, an absolute distance scale can be determined by measuring the time delay in the arrival of light observed in its multiple images; this offers a geometric route to measuring the rate of expansion of the Universe.

Why did it take so long before further observational progress was made in gravitational lensing? Firstly, gravitational lensing is a relative rare phenomenon in the celestial sky requiring fortuitous alignment of foreground and distant sources. Secondly, as in conventional optics, the background source must be substantially more distant than the lens. Until the 1960s, very few truly cosmologically-distant sources (i.e. at large redshift<sup>1</sup>) were known. Only as quasar surveys yielded many distant sources in the 1970s did it finally become likely one would be found behind a foreground galaxy. The first example, SBS 0957+561 A/B, was verified spectroscopically by Walsh, Carswell & Weymann in 1979 [21]. They discovered two images of the same distant (redshift  $z=1.413$ ) quasar gravitationally-lensed by a foreground galaxy with redshift  $z=0.355$ . Thirdly, surface brightness is conserved in the lensing process (as in conventional optics). However, as surface brightness dims with increased redshift  $z$  as  $(1+z)^{-4}$  due to relativistic effects associated with the expansion of the Universe, many lensed images viewed through foreground galaxy clusters lay undiscovered until the 1980s when efficient digital cameras became commonplace on large ground-based telescopes. The increased sensitivity led to the discovery in the mid 1980s of many ‘giant arcs’ - distorted images of background galaxies. For a few years there was some speculation as to the origin of these strange features. Eventually, Soucail and colleagues [22] confirmed, with a spectrum, that a giant arc in the rich cluster Abell 370 at a redshift  $z=0.37$  is the distorted image of a single background galaxy at redshift  $z=0.724$ . The improved angular resolution of the Hubble Space Telescope (HST), launched in 1990, was later critical in recognizing numerous distorted images of faint sources. HST features very prominently in science programmes exploiting gravitational lensing, for example via its role in conducting deep imaging surveys of foreground clusters for highly-magnified distant objects (Figure 1).

## 1.2. Gravitational lensing and Fermat’s principle

Figure 2 provides a useful qualitative overview of the lensing phenomenon. When the lens, for example a foreground galaxy or cluster of galaxies, is dense enough and well-aligned with a background source, multiple images of that source can be produced. More generally, when the intervening mass is less concentrated and/or poorly-aligned with the background source, only a small shape distortion occurs.

Quantitatively, gravitational lensing can be described in a similar manner to that of standard optics. The formulation of gravitational optics in terms of a generalized version of Fermat’s principle provides a very intuitive way to understand the underlying physics and the basic phenomenology. We give here a brief description of gravitational optics referring to [2] for a summary with equations and to [24] for a more thorough discussion.

The mathematic formalism of gravitational lensing can be conveniently described as a transformation between the positions and shapes of background sources as they would be observed

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<sup>1</sup>‘Redshift’ is defined as the factor by which the wavelength of the light of a source is stretched by the cosmic expansion and hence is a valuable measure of its distance, and ‘look-back’ time.

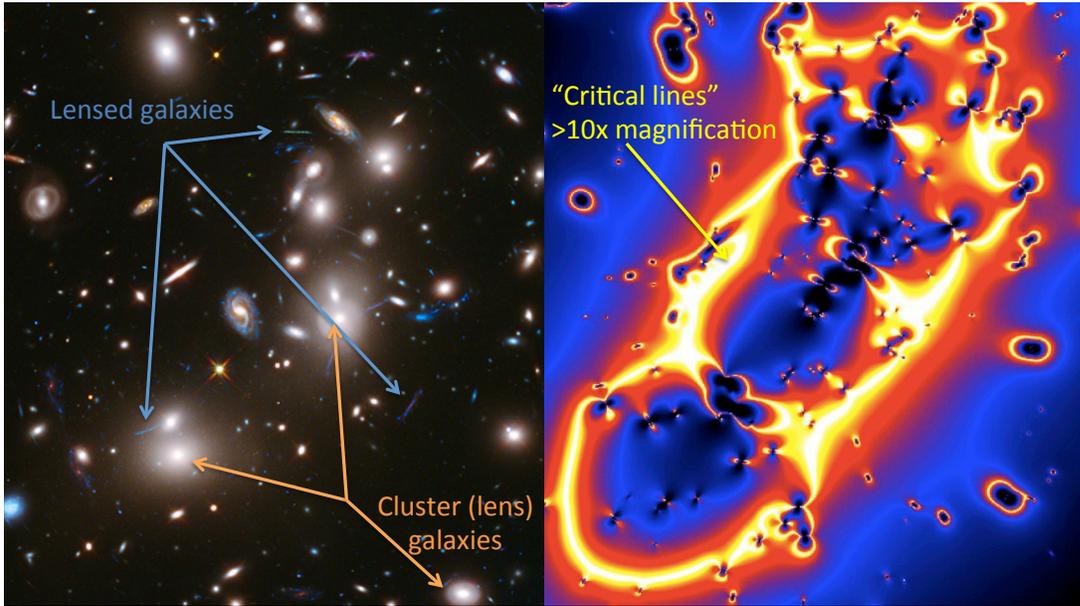


Figure 1. A striking example of gravitational lensing by a cluster of galaxies. Left: Hubble Space Telescope false colour image of Abell 2744 revealing many luminous members (white/yellow color) but also numerous background galaxies (typically blue) stretched and distorted by the gravitating mass in the cluster which is dominated by dark matter. The image is taken as part of the Frontier Fields programme . Right: Magnification map inferred via a gravitational lens model for the cluster [23]. The cluster acts as a natural telescope, so that a distant background source would appear magnified by a factor of  $\sim 10$  or more if it were to appear near the so-called critical lines (the regions surrounding the cluster centre colored in yellow/white, where magnification is very high, formally infinite for a point source). Sources in most of the solid angle behind the cluster are magnified by a factor of a few. Credits: NASA/ESA and [23].

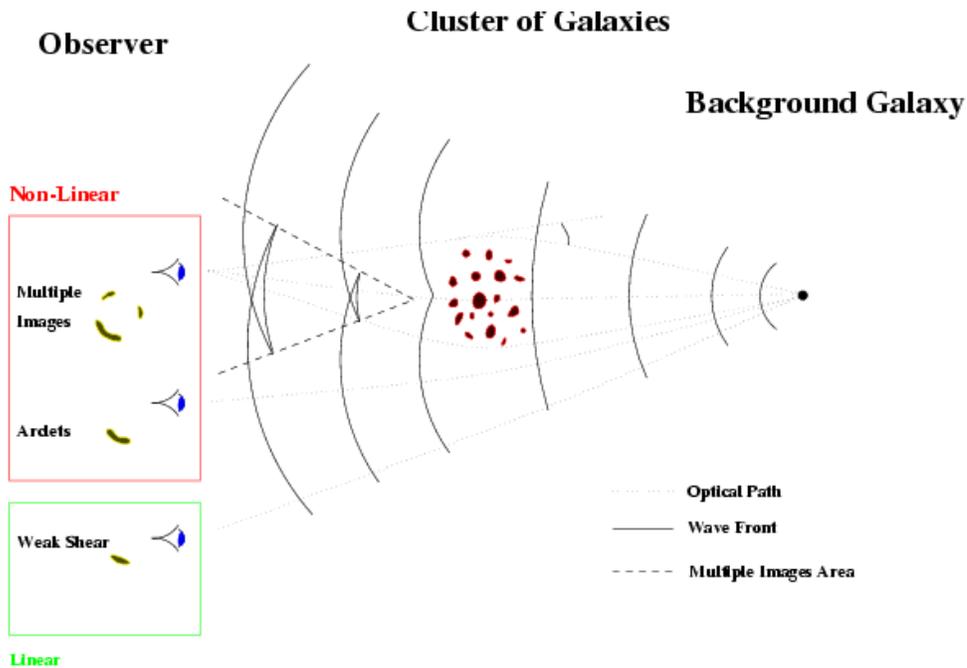


Figure 2. How gravitational lensing works: a foreground cluster of galaxies contains copious amounts of mass (mostly dark matter) and acts as a gravitational lens, distorting and magnifying the light of a background galaxy. The resulting image seen by the observer depends on the relative distances between the observer, lens and source, the concentration of mass (dark and visible) in the lens and the degree of alignment of the observer, lens and source. When the alignment and mass concentration is sufficient to create multiple-images of the background source, the phenomenon is called *strong lensing*. In this case the magnification and distortion can be very significant. When the alignment is not so fortuitous, only a modest stretching of a single image may occur ( *arclet* in the case shown). Quite generally dark matter in the cosmos induces a small distortion in the shapes of all background galaxies - a phenomenon called *weak lensing* or *cosmic shear*.

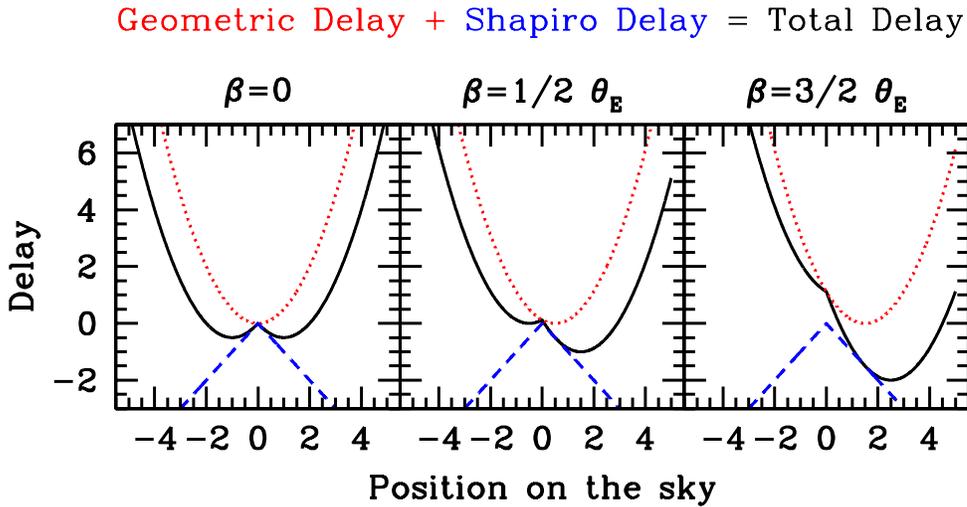


Figure 3. Illustration of gravitational lensing in terms of Fermat’s principle. For a given source position ( $\beta$ ), the time delay surface (black solid lines) is given by the sum of the geometric delay (red dotted lines) and the Shapiro delay (blue dashed lines) as a function of position in the image plane ( $\theta$ , in units of the Einstein Radius  $\theta_E$ , typically one arcsecond for galaxy-scale lenses). Images then form at the extrema of the time delay surface. The three panels show a section of the time delay surface in three different regimes of circularly symmetric deflector. Left panel: the source is perfectly aligned with the deflector ( $\beta = 0$ ); the time delay has a local maximum at the center and two minima at the same height. This configuration gives rise to a perfect Einstein Ring, with an infinitely demagnified image in the center (see centre panel of Figure 7 for example). Middle panel: the source is now offset to one side by half an Einstein Radius; the image forming on the outer minimum arrives first, then the central image corresponding to the local maximum, and last the image corresponding to the inner minimum. This configuration gives rise to the classic double configuration, with two bright images and an infinitely demagnified central image. Right panel: the source is now offset by more than the Einstein Radius; in this case there is only a minimum and thus only one image, i.e. no strong lensing.

in the absence of a deflector (the so-called *source plane*) and that received by observer (the so-called *image plane*). The transformation is achromatic and preserves surface brightness [3].

The transformation from source to image plane is given by Fermat’s principle. As in conventional optics, photons seem to “choose” special paths from the source to the observer, i.e. images form at the extrema of the arrival time surface (usually they pick the shorter time, i.e. minima, but also maxima or saddle points are allowed). However, whereas in standard optics the light travel time depends on the speed of light in the material as well as on the path length, in gravitational optics the role of the refractive material is played by the gravitational potential of the deflector via the so-called Shapiro delay [25], i.e. the delay measured by the observer for a light ray passing through a deep gravitational potential caused by general relativistic time dilation. The competition between the Shapiro delay, which increases with the gravitational potential, and the length of the light path gives rise to the variety of observed phenomena. The description of gravitational lensing in terms of Fermat’s principle is illustrated in Figure 3.

Under rare circumstances, if the Shapiro delay is strong enough, multiple images can appear to the observer, giving rise to the phenomenon of *strong lensing*. In this case the time delay between the arrival of light to the various images encodes information about the absolute path lengths traversed and hence the size of the Universe as function of cosmic time. As we describe in Section 2.2, this provides an opportunity for a direct measurement of various cosmological parameters. Conversely, if the Shapiro delay is not strong enough to counterbalance the geometric delay, only a single distorted image of the source appears to the observer. This phenomenon is called *weak lensing* and is a powerful tool to trace the distribution of dark matter outside of the confines of massive structures like cluster of galaxies (Section 3).

## 2. Strong Lensing

Strong lensing is perhaps gravitational lensing’s most visually impressive feature. A rich cluster of galaxies such as that in Figure 1 produces striking distorted images of background galaxies

which appear to swirl around the cluster core. Importantly the phenomenon is governed by the *total* mass in the cluster whether it is visible, as in the member galaxies, or dark. As we know the bulk of the gravitating matter in the Universe is in fact invisible, lensing offers us a remarkably powerful tool to study both the distribution and nature of dark matter.

### 2.1. *Lensing anomalies and the nature of dark matter*

The standard picture of dark matter is that it is comprised of a massive weakly-interacting or ‘cold’ particle. We know it cannot be baryonic (i.e. quarks) in form as this would violate measured abundances of the light elements synthesized in the Big Bang [26], and the observed power spectrum of cosmic microwave background [27]. Although physicists can attempt, with difficulty, to capture this weakly-interacting particle and constrain its mass and properties, astronomers have a unique ability to observe the dark matter using gravitational lensing.

A very robust prediction of the standard cold dark matter (CDM) cosmological model is that dark matter congregates in large ‘halos’ within which are numerous satellites or ‘subhalos’. The abundance of these subhalos should increase rapidly with decreasing mass ( $dN/dM_{\text{sub}} \propto M_{\text{sub}}^{-1.9}$ ) [28]. This behaviour stems directly from the ‘coldness’ (i.e. low thermal speed) of dark matter in the standard model. Alternate cosmological models with less massive (or ‘warmer’) dark matter particles [e.g. keV scale sterile neutrino 29] predict a lower mass cutoff to the distribution of subhalos [30–32].

The luminous satellites surrounding our Milky Way and external galaxies do not appear to be nearly as abundant as the predicted distribution of subhalos in CDM, a discrepancy dubbed the ‘missing satellite problem’ [33, 34] (see also [35] for an associated problem). The most favoured solution is that the lower mass subhalos cannot retain their hydrogen gas and are thus unable to form stars or be seen. This implies that there should be thousands of dark subhalos orbiting our own Milky Way.

Given the uncertainties in understanding star formation in low-mass galaxies, it is clear that only a direct census of these subhalos by mass can tell us conclusively whether these satellites do not exist or whether they are simply dark. This is a remarkably clean measurement in principle: if the dark subhalos do not exist, the standard cold dark matter model would be ruled out. Gravitational lensing provides a unique opportunity to perform this measurement, by means of so-called *strong lensing anomalies*. The presence of dark unseen satellites can be detected as small scale perturbations in the gravitational potential of a massive galaxy [36–38]. These perturbations with respect to an otherwise smooth mass distribution change the arrival time, apparent position, and observed flux of the lensed sources (hence they are named time-delay, astrometric, and flux ratio anomalies, respectively). An illustration of one of the methods is shown in Figure 4.

The results so far indicate that the abundance of dark subhalos is consistent with the expectations of CDM models, albeit with large uncertainties [39–42]. This is an area where great progress will be possible in the next decade, by studying large numbers of strong lenses at high angular resolution.

### 2.2. *Time delays as a probe of dark energy*

If the mystery of dark matter was not enough of a problem for the cosmologist, the discovery of the accelerated expansion of the Universe in 1999 from the study of distant supernovae [44, 45] raises a new conundrum. Over 70% of the energy density in the Universe is contained in the so-called *dark energy* - a label used to cover our ignorance of one of the most basic features of the Universe. Contemporary thinking suggests dark energy may be a natural property of empty space, a *vacuum energy density*, possibly the cosmological constant initially invoked by Einstein to retain a static Universe. A key question in considering the nature of dark energy is whether it is a constant property in cosmic time or whether it evolves. This is central to understanding

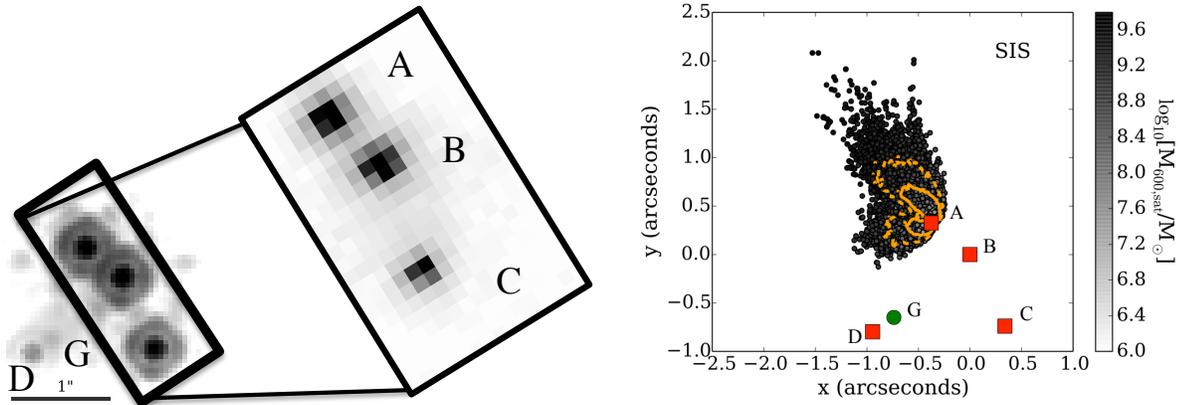


Figure 4. Detecting dark matter substructures to test the standard cold dark matter model. The left panels show four multiple images (A,B,C and D) of the gravitationally-lensed quasar B1422+231 obtained with the Hubble Space Telescope (leftmost panel) and with the imaging spectrograph OSIRIS behind the adaptive optics system on the W. M. Keck-I Telescope (zoomed in panel). The relative positions and narrow line spectroscopic fluxes of these images measured with OSIRIS contain clues as to the distribution of dark matter in the foreground lens (G). Accurate modelling indicates the requirement for a sub-halo, in addition to the primary one, whose relative position and mass are constrained as shown with the distribution of black points and orange contours in the right panel. Such positional and flux anomalies can be used to trace dark matter sub-halos with masses of  $\simeq 10^8$  solar masses, providing a critical test of the standard model (after [43]).

the fate of the Universe.

A powerful method to determine the nature of dark energy is to measure the time evolution of cosmic distances. In the same way as the strength of the Earth’s gravitational field can be inferred from the trajectory of a football, the evolution of physical scales in the universe provides information about its total energy density

Distance measurements are critical in cosmology. The supernovae observations that led to the surprising discovery of an accelerating Universe measured the relative distances between supernovae at different redshifts. Likewise, one of the fundamental quantities measured by cosmic microwave background satellites such as Planck [46] and WMAP [27] is the distance to the last scattering surface of the cosmic microwave background at redshifts  $\sim 1100$  (when the universe was 370,000 years old).

Strong gravitational lensing of a variable source provides a very elegant one step measurement of absolute distances in the Universe. The difference in arrival time induced by a lens is given by  $\Delta t \propto D_{\Delta t} \delta \phi$ . Here  $D_{\Delta t}$  is the so-called time delay distance and encompasses all of the cosmological dependence, and  $\delta \phi$  describes the geometry of the system and the gravitational potential of the main deflector. By measuring a time delay and determining a mass model for the main deflector, one obtains the time-delay distance  $D_{\Delta t}$  and, thus, a determination of the cosmological parameters [20, 47].

Refsdal’s idea [20] is fifty years old, but has only very recently been realized as a practical proposition. His original suggestion involved observing a multiply-imaged supernova, a rare phenomenon. In an exciting development, the first example was observed in 2014 [48, 49, (discovery images of the long awaited supernova aptly named ‘Refsdal’ are shown in Figure 5)]. A more productive endeavour, given the rarity of supernovae, has been the application of Refsdal’s method to lensed variable quasars. Earlier efforts were first stymied by the shortage of these sources, and later by the logistical challenges associated with the necessary long-term monitoring of them to measure accurate time delays. However, in the past few years, dedicated monitoring efforts [47, 50–55] and advances in time delay measurements [56, 57] and lens modeling [58–62] have led to substantial progress. Recent work has shown that a single lens is sufficient to measure absolute distances to 6% precision [63] and thus determine whether dark energy is the cosmological constant or a more exotic phenomenon (Figure 6). In the next decade with many planned wide field surveys and dedicated efforts, thousands of lensed quasars will be discovered and studied, yielding some of the most stringent constraints on the properties of dark energy [64, 65].

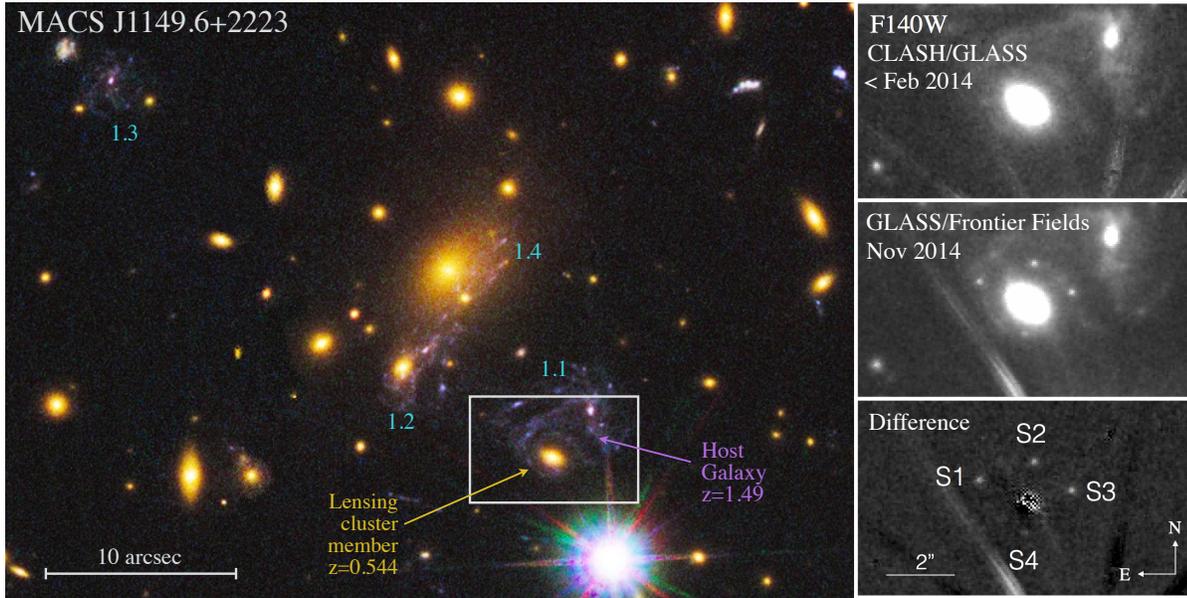


Figure 5. Discovery of the multiply-imaged supernova 'Refsdal'. The left panel shows an image of the cluster of galaxies MACS J1149.6+2223 taken prior to the explosion of the supernova. The host galaxy at  $z = 1.49$  is multiply imaged by the cluster, forming images 1.1, 1.2, 1.3, 1.4. A portion of the host galaxy is further multiply imaged by a galaxy in the lensing cluster at  $z = 0.544$ . The right panels zoom in on the multiply imaged supernova, (top) image prior to the supernova explosion from the CLASH program (PI: Postman), (middle) discovery image from the GLASS program (PI: Treu), (bottom) difference image revealing four images of the supernova in an 'Einstein Cross' configuration. Left panel image credit: NASA, ESA, W. Zheng (JHU), M. Postman (STScI), and the CLASH Team. The right panel image is taken from [48]. Montage and labels courtesy of S.A. Rodney.

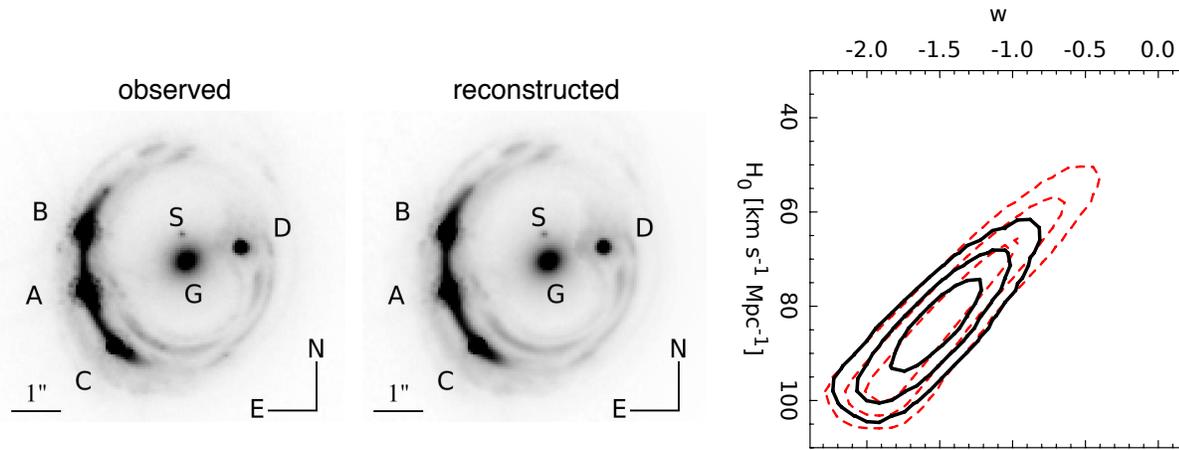


Figure 6. Constraining the nature of dark energy using gravitational time delays. The leftmost panel shows a Hubble Space Telescope image of the multiply-imaged variable quasar RXJ1131-1231 gravitationally-lensed by a foreground galaxy G and its associated satellite S. To its right is a reconstructed image based on a mass model for the lens. Accurate monitoring of the three images A, B and C provide an absolute measure of the different path lengths for a light ray through the lens and hence constraints on the present expansion rate of the Universe (Hubble's constant  $H_0$ ) and the equation of state parameter ( $w$ ; i.e. the ratio between pressure and energy density,  $w = 0$  for cold dark matter,  $w = 1/3$  for photons,  $w = -1$  for a cosmological constant) of dark energy as shown in the rightmost panel (after [63]). The dashed contours show the 68%, 95%, and 99 % posterior probability density contours based on the cosmic microwave data alone, and the solid contours show the improved precision with the inclusion of the time-delay distance measured from RXJ1131-1231.

### 2.3. Stellar and dark matter in massive galaxies

The idea that all galaxies are surrounded by halos of dark matter became commonplace by the early 1980s. But how can we quantify the distribution of dark matter around galaxies and verify its role in galaxy formation given it is invisible? Elliptical galaxies are compact and dense and thus serve as excellent gravitational lenses [1]. Using spectroscopic data from the Sloan Digital Sky Survey, the Sloan Lens Advanced Camera for Surveys (SLACS) team has so far

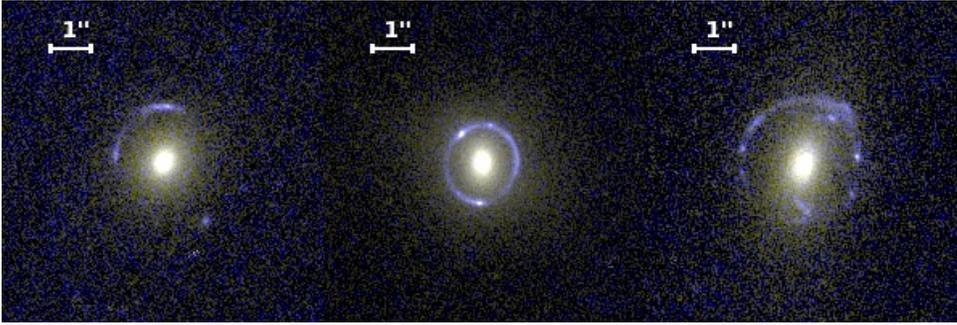


Figure 7. Examples of strong gravitational lensing by a galaxy. The background sources (blue) are lensed by the foreground massive galaxy (yellow) into an almost perfect ‘Einstein ring’. The radius of the ring gives the total mass enclosed with a precision of just a few percent. Combining this geometrical measure with the kinematics of the stars in the foreground galaxy, deduced spectroscopically, provides key data on the radial distribution of matter in the lens as well as the nature of the stellar population (after [79]).

isolated over 100 elliptical galaxies that strongly lens background blue star forming galaxies at  $z = 0.5 - 1$  [66, 67]. Since the redshifts of both the lens and background source are known, the lensing geometry, revealed by Hubble Space Telescope images (see examples of lenses in Figure 7 taken from the SL2S survey [68]), defines the total mass interior to the so-called Einstein radius irrespective of whether that material is shining. Together with a dynamically-based mass on a smaller physical scale derived from the dispersion of stellar velocities in the lensing galaxy itself, the total mass density in the lens as a function of radial distance within the galaxy,  $\rho(r)$ , can be determined.

Across a wide range in cosmic time and lens mass, the total mass distribution is remarkably uniform following an isothermal distribution,  $\rho(r) \propto r^{-2}$  [69, 70]. This distribution is spatially more extended than that of the visible baryons demonstrating clearly the existence of dark matter [71]. These important results confirm that the early formation of massive dark matter halos played the key role in encouraging a rapid formation of the cores of massive galaxies.

One surprising result from this line of inquiry is that the ratio between stellar mass and luminosity is much higher in massive elliptical galaxies than in the Milky Way [72–76]. This finding, in combination with detailed studies of the spectra of stars [77], suggest that mode of star formation in the most massive galaxies is different than in typical galaxies, and yields much many more low mass stars, down to the hydrogen burning limit [78].

#### 2.4. The inner regions of dark matter halos

A further area of tension between the standard cosmological model and observations is the inner regions of galaxies and clusters of galaxies. Dark matter only simulations predict that the density of dark matter should be ‘cuspy’, i.e. increase at small radii as  $\rho_{\text{DM}} \propto r^{-1}$  [80], whereas observations of many galaxies indicate a variety of density profiles including so-called ‘cores’ where the dark matter density is constant within a certain radius [81]. The origin of this discrepancy is not well-understood because of the poorly understood physics of gas, stars and black holes at the centres of galaxies. On the one hand, gas has the ability to cool and sink toward the centre of the dark matter potential wells; this makes the observational discrepancy even more of a problem. On the other hand, the energy released by explosive events such as supernovae and accretion onto black holes may potentially mitigate the problem by transforming the steep cusps into cores [82, 83].

Clusters of galaxies acting as a gravitational lens, provide a unique opportunity to shed light on dark matter in a particularly clean manner. When a source is very well aligned with the cluster centre, two of the multiple images can form a *radial arc*. The position of the radial arc is a direct measurement of the slope of the enclosed mass density profile close to the centre of the cluster. Incorporating other evidence, it is possible to derive the dark matter density profile of the cluster with unprecedented precision [84]. The results of this work also show that in

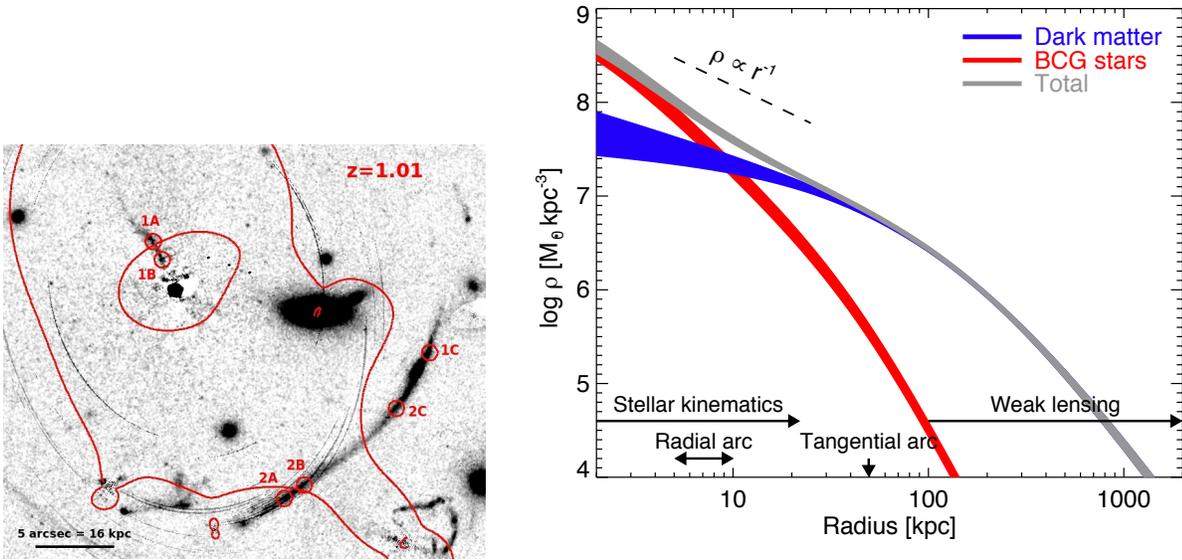


Figure 8. Probing how dark matter is distributed on small scale via strong gravitational lensing in the core of a massive cluster. The left panel shows a Hubble Space Telescope image of the centre of the rich cluster Abell 383. A radial arc (images 1A, 1B) is visible close to the massive central galaxy (whose image has been subtracted to improve the visibility of faint features). The position and redshift ( $z=1.01$ ) of this radial arc encodes the density profile of material close to the cluster centre, while the tangential arc (images 2A-C) measure the total enclosed mass. The red curve shows the critical line at this redshift. In the standard cold dark matter model, radial density profiles (right panel) are predicted to be very steep ( $\rho \propto r^{-1}$  - black dashed line), in conflict with the inferred dark matter density profile from observations (blue curve). Such differences may be explained either by physical processes occurring in such dense environments (e.g. supernovae explosions) which redistribute both the baryonic and dark matter or, more interestingly, by the suggestion that dark matter is ‘self-interacting’ (after [84]).

some cases the dark matter density profile is significantly flatter than predicted in simulations based on the standard cold dark matter model (Figure 8). Significant theoretical efforts [85–89] are currently underway to clarify whether this disagreement is due to a poor understanding of the physics of how baryons interact with dark matter or, more dramatically, whether dark matter has some property that is not yet understood. One appealing solution suggests that dark matter can interact with itself [90, 91]. In dense regions, such as the centre of a massive cluster, such *self-interacting dark matter* would naturally flatten the predicted cusps in agreement with observations. Interestingly, lensing observations of colliding clusters already set an upper limit to amount of self interaction [92, 93], so that the range of astrophysically interesting interaction strength allowed by lensing observation is limited.

### 2.5. ‘Natural’ telescopes

With extraordinary vision, in 1937 Fritz Zwicky [16] suggested that clusters of galaxies could be used as ‘natural telescopes to search for magnified images of very distant galaxies, thereby extending the reach of our existing telescopes’. In the past decade this has become a very effective way to locate and understand the properties of the earliest galaxies seen when the Universe was less than 10% of its current age. A few hundred millions years after the Big Bang, the first stellar systems emerged, as yet unpolluted by nuclear enrichment given only light elements were synthesised in the beginning. These systems were very hot and emitted copious amounts of ultraviolet radiation capable of photo-ionising the hydrogen in deep space. The arrival of the first galaxies, termed *cosmic dawn* is thought to be responsible for this *cosmic reionisation* [94].

The Hubble Space Telescope has given us our first glimpse of this last remaining frontier of cosmic history, but individual galaxies seen from this remote past are too faint for detailed study. All we can do at present is a census of their abundance and luminosity distribution. To understand whether, for example, the typical stars in these galaxies are pristine requires spectroscopic investigations which will be challenging, even with the next generation of ground

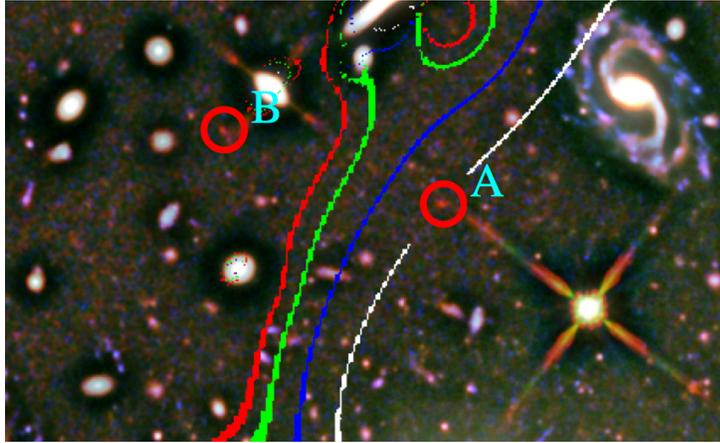


Figure 9. Identification of one of the most distant known galaxies via the magnification of the gravitational lensing phenomenon. The red colors of the multiply-imaged system (A and B) together with the symmetry of the position around the ‘critical line’ of high magnification for a distant source indicates that it is likely at  $z \sim 10$ , seen when the Universe was barely 5% of its present age. The dependence of the location of the critical line on source redshift is indicated by the red, green, blue and white lines which are for sources at redshifts 10, 3.6, 2.0 and 1.3, respectively (after [100]).

and space-based facilities. This is precisely where gravitational lensing can help. A foreground cluster of galaxies presents a large cross-section to this background population so the likelihood of magnified images is high (Figure 1). On the other hand, the distribution of mass in a cluster is less regular than in a single galaxy, so careful modeling is necessary to quantify the boost in signal. Some of the most distant galaxies known have been located by searching close to the so-called critical lines of massive clusters where magnifications of  $\times 20$ – $30$  are possible [95–99]; these systems would otherwise not have been detected. The Hubble Space Telescope is now undertaking a systematic deep survey of six such foreground clusters with its near-infrared camera (Figure 1). This programme, termed the Hubble Frontier Fields<sup>1</sup>, has already revealed the most distant galaxies known [100, Figure 9], and promises to deliver many examples of highly-magnified systems for subsequent study e.g. with the James Webb Space Telescope, a near-infrared large aperture space telescope due for launch in 2018.

Clusters not only magnify sources in their integrated brightness, rendering them more easily visible with our telescopes, but lensing also enlarges the angular size of a distant source making it easier to determine its internal properties. The most distant galaxies are physically very small – about 10 times less so than our Milky Way – and resolving them is a challenge for both Hubble Space Telescope and large ground-based telescopes equipped with adaptive optics (AO) – a technique that corrects for atmospheric blurring. However the combination of high resolution imaging from HST or AO *and* gravitational magnification offers spectacular opportunities. A distant galaxy at a redshift of 3 is typically only 0.2–0.3 arcsec across yet, when magnified by a factor of  $\times 30$ , it is possible to secure spectroscopic data point-by-point across the enlarged images of representative examples. Such studies of lensed  $z \simeq 2$ – $3$  galaxies observed during a time of peak activity in galaxy assembly demonstrate that many have primitive rotating disks [101], [102], and that there is a range of chemical gradients in their gaseous composition [103, 104].

### 3. Weak lensing and Cosmic Shear

Strong lensing signatures are usually straightforward to recognise in astronomical images via the presence of multiply-imaged sources, often with highly distorted shapes. Weak lensing, which affects all photons travelling in the universe is much harder to recognize. The principal signal is a small distortion in the shape of a background galaxy which depends on the curvature (or second derivative) of the foreground gravitational potential. In an idealised case, the observer

<sup>1</sup><http://www.stsci.edu/hst/campaigns/frontier-fields/>

sees the background source slightly stretched (or sheared) tangentially around a circle whose center is the lensing structure (Figure 2). Unfortunately, this weak lensing signal is too feeble to be inferred for a single background source and, of course, we would need to assume the true shape and orientation of a source to measure it quantitatively. However, the presence of foreground structures can still be inferred by statistically averaging the distorted shapes of many background galaxies in a given direction assuming the sources are, overall, randomly oriented. Since it is so pervasive, weak lensing offers the prospect of making maps of the otherwise invisible dark matter everywhere in the sky, on scales much larger than galaxies or clusters of galaxies. Consider the following remarkable fact: the night sky does not present us with a faithful picture of the Universe! In all directions light rays from distant galaxies are being subtly deflected as they traverse the cosmos by low density dark matter structures – a phenomenon we call *cosmic shear*.

Weak gravitational lensing holds enormous promise in observational cosmology as the technique, properly employed, can reveal the distribution of dark matter independently of any assumptions about its nature. However, the technical challenges are formidable. Foremost the signal arising from large scale structure is small – amounting to a change in the ellipticity of a faint distant galaxy of only a few percent. Secondly, as a statistical technique, a high surface density of measurable galaxies must be secured in order to gather sufficient signal, so deep imaging is essential. Finally, as the Earth’s atmosphere smears the shapes of faint galaxies, painstaking corrections must be made to recover the cosmological signal.

The first claimed detection of a weak lensing signal was by Tyson and collaborators [105] in the field of a rich cluster. But the techniques for robustly analyzing the pattern of distortions of background galaxies and inverting these to map the foreground dark matter were developed by Kaiser and others soon after [106, 107]. Detections of weak lensing from the large scale distribution of dark matter along random sightlines, that is “cosmic shear”, were not announced until 2000 ([108][109][110]). These early papers analyzed the strength of the signal to constrain the amount of dark matter per unit volume, confirming independently values from other methods. Later papers used the techniques to produce maps of the projected dark matter distribution [Figure 10 111] [see also 112]. These maps can be compared to that of the light in the same direction as revealed by visible galaxies and X-ray emitting clusters. To first order there is a reassuring similarity, indicative of the fact that dark matter acts as the gravitational framework (or scaffolding) for the normal baryonic material.

Ambitious ground-based surveys are now being undertaken with telescopes equipped with panoramic camera with a goal not only of charting the distribution of dark matter in two dimensions but also tracing its clustering with time. Structure formation develops according to two basic forces - the attractive force of gravity and the repulsive effect of dark energy. The 3.6m Canada France Hawaii Telescope has undertaken a survey of 154 deg<sup>2</sup> in five photometric bands with the MegaCam 1 deg<sup>2</sup> CCD imager [113]. The European Southern Observatory’s 2.6m VST is undertaking the Kilo Degree Survey (KIDS) - a 1500 deg<sup>2</sup> survey in four photometric bands with a similar camera called OmegaCam [114]. The Pan-STARRS project at the University of Hawaii plans a series of telescope upgrades in order to chart the distribution of dark matter. The Dark Energy Survey has just begun and in the next 5 years it will image 5000 deg<sup>2</sup> of sky in 5 photometric bands using a 520 megapixel camera on the 4m Blanco Telescope in Chile<sup>1</sup>. Finally, the Subaru Telescope is now equipped with the most impressive HyperSuprimeCam - a 870 megapixel CCD camera providing a 1.5 deg<sup>2</sup> field. This instrument is likewise being used to undertake a 1200 deg<sup>2</sup> survey<sup>2</sup>. By virtue of the larger aperture of the Subaru 8.2m, this will be the deepest weak lensing survey in the next few years.

These surveys, collectively, aim to pinpoint the nature of dark energy in two respects. Firstly, by comparing the rate at which structure grows to the rate of cosmic expansion (for example from

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<sup>1</sup> <http://www.darkenergysurvey.org/index.shtml>

<sup>2</sup><http://www.naoj.org/Projects/HSC/surveyplan.html>

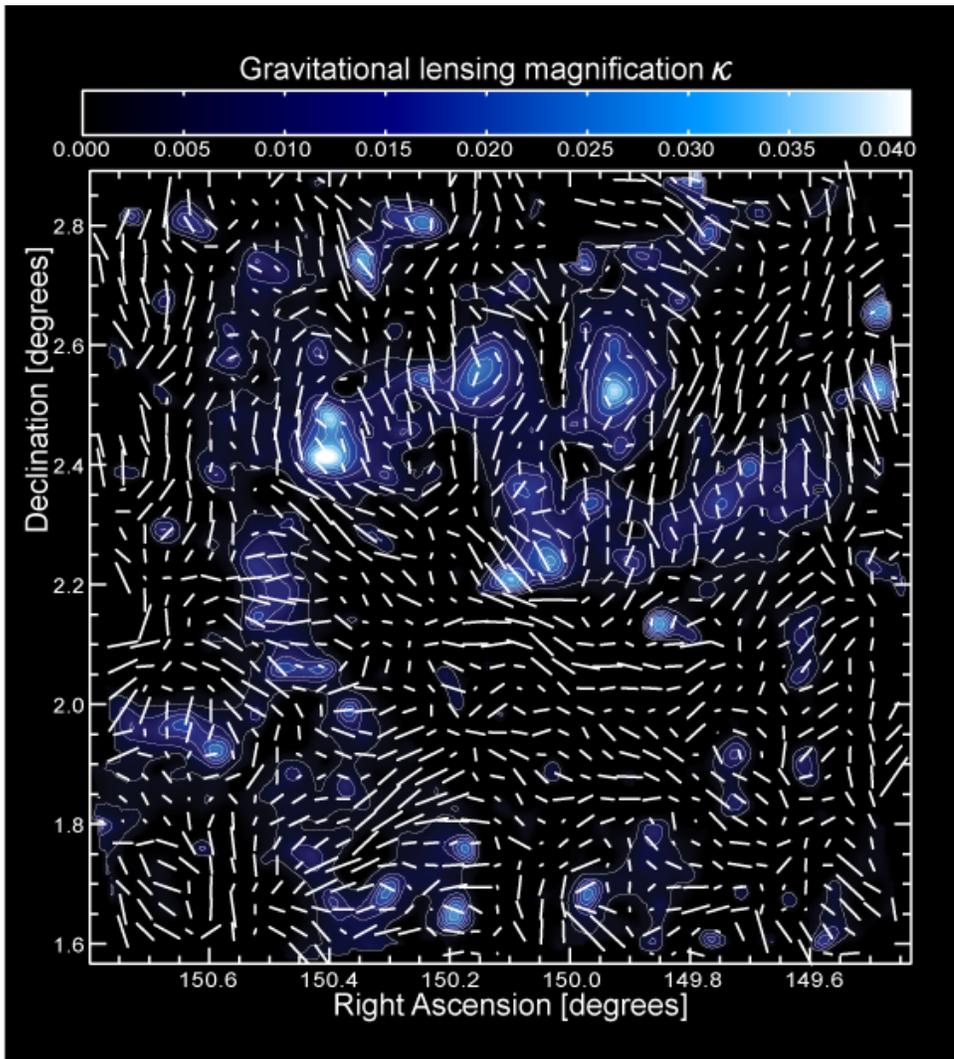


Figure 10. Mapping dark matter using weak gravitational lensing. The vector tick marks represent the mean alignments of faint galaxies across a 2 square degree field imaged by the Hubble Space Telescope. The pattern can be used to make a projected 2-D map of the dark matter along the line of sight. Where the pattern swirls tangentially around a region, it indicates a mass concentration in that direction; where the pattern is radially outward, an under density. By slicing such day matter maps according to look-back time, the growth of structure in the Universe can be traced. This is a powerful measure of the competition between the positive gravitational influence of dark matter and the repulsive effect of dark energy (after [111]).

studies of luminous supernovae or gravitational time delays), we gain insight into whether dark energy is an illusion caused by a failure of Einstein's theory of gravity on large scales. Surprising though it may seem to challenge Einstein, who founded the very topic of this article, one must remember that skeptics challenged Einstein when he dared to insist that Newtonian gravity needed amending! Secondly, the data will assist in determining whether dark energy is a constant or evolving property of space. Right now the limited data imply that dark energy is consistent with a constant energy density (sometimes called the 'cosmological constant', corresponding to  $w = -1$  in the language of the equation of state of dark energy (see Figure 6) to a precision of about 5% over the past 8 billion years. However, these upcoming surveys will significantly improve this constraint. Whatever the outcome – a new theory of gravity on large scales, verifying a 'cosmological constant' or discovering an evolving component of dark energy – new physical insight is guaranteed!

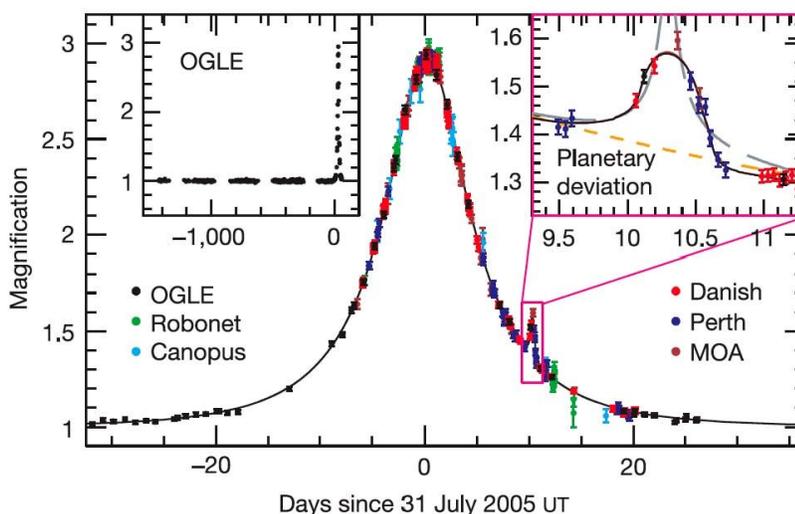


Figure 11. Detection of a low mass exoplanet (OGLE 2005 BLG 390LB) via a small perturbation to the microlensing light curve. The light curve represents the time-dependent brightness of a background star in magnitudes (a logarithmic scale). As a foreground star crosses the path, it magnifies the background source. An associated planet in the foreground provides an additional boost over a shorter timescale. Microlensing can detect low mass planets further away from their host stars than other methods and thus promises a more complete inventory of their abundance in the Milky Way (after [115]).

#### 4. Microlensing finds extrasolar planets

In 1936 Einstein was not convinced gravitational lensing would yield observable returns because the probability that two stars are sufficiently well-aligned so that one magnifies the other is very low. But with panoramic imaging cameras, many tens of millions of stars can be monitored efficiently. Together with the fact that there can be relative motion between the source and lens, there is still the likelihood of observing an effect. Microlensing - a term introduced by Bohdan Paczynski - generally refers to the case where either the source or both the source and lens are unresolved. Consequently, the deflection and distortion of light from the background source cannot be seen. The key signature is a temporal brightening of the combined signal from source plus lens as the one passes in front of the other. The timescale of the brightening can be anything from seconds to years and the observed light curve gives information on the lens mass, the relative distances and the motion of the lensing object (assuming the background object is stationary). As microlensing is a transient phenomenon, an effective survey strategy is to monitor a dense stellar field repeatedly, searching for that rare occasion when an individual star increases its brightness. Of course, complicating such searches is the fact that many stars are genuinely variable in their output. Once a likely event has been triggered, it can be monitored more intensively to see if the light curve is of the form expected for microlensing.

Microlensing has had a major impact in astronomy in two areas, both involving monitoring of tens of millions of stars in the Milky Way or nearby Magellanic Clouds: (i) the search for dark matter in the Galactic halo in the form of compact objects of moderate mass ( 0.1 solar masses or less) and (ii) locating and assessing the abundance of extrasolar planets down to as low as the mass of the Earth. In the case where the lens comprises a star with an orbiting planet, the light curve deviates from that expected for a single lens. Such a signal was first observed in 2004 by Bond and colleagues [116] leading to the detection of a 1.5 Jupiter mass planet. Figure 11 shows one example where a 5.5 Earth mass planet was detected via a perturbation to the light curve of a microlensing event. More recently a planet of two Earth masses has been detected in an orbit similar in radius to that of the Earth around the Sun [117].

Because it is a geometric technique, microlensing probes planet in the cold outer regions, far from their host stars. By contrast, other techniques for finding exoplanets, such as monitoring the radial velocity of the host star or searching for transits, are sensitive to planets much closer to their host stars. Moreover, in sharp contrast to other methods, the signal does not necessarily

decline as the planet mass decreases. Microlensing has, like the other aspects of lensing reviewed above, not yet been thoroughly exploited. It offers the exciting prospect of a more complete inventory of planets around stars in the Milky Way.

## 5. The future: gravitational lensing in the next decade

It is remarkable to consider that, as recently as 1970, gravitational lensing was largely considered as a mathematical curiosity with no practical purpose, neglected by the observational community. The only observational progress was improved precision in measurements of the solar deflection initiated in 1919 and in the Shapiro time delay [118].

Today, there is a thriving community of scientists working on the subject, so much that it has become impossible to do justice to the entire field in a single review, but we had to focus on a few highlights. This young and energetic community is not only making great strides in answering profound scientific questions, but it is also pushing the boundaries with new computational, statistical, and mathematical tools [119], as well as novel ways of conducting collective science. Teams of postdoctoral astronomers compare their mass models for the clusters being imaged by HST for the Frontier Field program at dedicated science workshops, while others participate in blind tests of their weak lensing analysis codes by applying them to simulated datasets [120], or in blind tests of the recovery of gravitational time delays [121].

Significant observational surveys are underway or planned, like the Large Synoptic Survey Telescope (LSST<sup>1</sup>). With dedicated state-of-the-art imaging cameras to chart large areas of the sky in depth and through a range of colour filters these surveys trace weak lensing signals at various cosmic distances in order to map the evolving dark matter distribution (see §4) and learn about dark energy [122]. The same surveys will be used to locate large numbers of strong lenses for detailed studies using methods such as those described in §2, when combined with follow-up high resolution information from the James Webb Space Telescope [123] or the next generation of adaptive optics systems on large and extremely large telescopes. The commissioning of the Atacama Large Millimeter Array (ALMA) and the planned Square Kilometer Array<sup>2</sup> guarantee that radio wavelengths will also play an important role in the scientific exploitation of the gravitational lensing effect [124–127].

The successful progress with these ground-based surveys, together with evident advantage of high angular resolution data revealed by HST, has given the international community the necessary impetus to plan two ambitious satellite missions whose science case builds on the cosmological opportunities afforded by gravitational lensing. The European Space Agency's *Euclid* mission<sup>3</sup> is a 1.2 metre telescope with a 0.7 degree diameter field of view which will undertake a survey of 15,000 square degrees when it is launched in 2020. A high priority in NASA's future programme is an even more ambitious mission currently called WFIRST-AFTA<sup>4</sup> which will be based on a 2.4 metre telescope 'inherited' from the US National Reconnaissance Office. The detailed science payload for this mission is still being determined but panoramic imaging to limits even deeper than achievable with *Euclid* are likely.

Of course, as in any relatively new field, with the giant leaps in technology, there is also the possibility of discovering new hitherto exotic phenomena, like lensing by cosmic strings [128] or lensing in the strong gravity field of a black hole [129, 130].

Nearly one hundred years after Einstein's remarkable realization that he could solve Newton's dilemma of how the gravitational force acts a distance, one wonders what he would make of both the mysterious composition of the Universe, dominated by dark matter and dark energy, as well as how the phenomenon he predicted has now taken a central role in making progress. Einstein

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<sup>1</sup><http://www.lsst.org/lsst/>

<sup>2</sup><https://www.skatelescope.org/>

<sup>3</sup><http://www.euclid-ec.org/>

<sup>4</sup><http://wfirst.gsfc.nasa.gov/about/>

was not afraid to admit his previous errors of judgement on several occasions, for example in initially disputing the expanding Universe. We suspect therefore that he would be an enthusiastic promoter of gravitational lensing, as are we!

## Acknowledgements

TT acknowledges support by National Aeronautics and Space Administration (NASA), the National Science Foundation (NSF), and the Packard Foundation through a Packard Research Fellowship. TT thanks the American Academy in Rome and the Observatory of Monteporzio Catone for their kind hospitality during the writing of this manuscript.

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