ASTR 3830: Problem Set 5

(due in class Friday May 4th)

1. The Navarro-Frenk-White (NFW) profile is a theoretical model for the distribution of dark matter within a galaxy cluster. The profile can be written as:

$$\rho = \frac{C}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}.$$

Here, ρ is the density of dark matter at radius r from the cluster center. C and r_s are both constants.

(a) Plot a graph of log ρ as a function of log (r/r_s).

(b) The mass M(r) interior to radius r is given by the usual integral $M(r) = \int_0^r 4\pi r^2 \rho dr$. Carry out this integral to find an expression for M(r) [Hint: one way to do this is to first simplify things by making the substitution $x \equiv r/r_s$, and then substitute $t = 1 + x$ to put the integral into tractable form…]

2. For a source with a luminosity L, and measured flux (in some waveband) f, the luminosity distance d_L is defined as:

$$d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2}.$$

(a) Type Ia supernovae are good standard candles, with an intrinsic luminosity that corresponds to an absolute magnitude (in the blue) M = -19.6 mag. By using the above definition of the luminosity distance, together with the definitions of the apparent magnitude m [$m = -2.5 \log f + \text{constant}$] and the absolute magnitude [the absolute magnitude M is the apparent magnitude a source would have at a distance of 10 pc], show that a supernova at luminosity distance d_L has an apparent magnitude given by:

$$m = 5 \log\left(\frac{d_L}{1 \text{ Mpc}}\right) + 5.4.$$

[Note: because you know the absolute magnitude, you *don't* need to know the value of the constant entering into the magnitude definition to do this.]

(b) Consider two different universes with parameters:

- $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$ (model A)
- $\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$ (model B).

Make a plot (using any plotting software you like) showing the apparent magnitude of Type Ia supernovae as a function of redshift in the two different cases. To do this, use the calculator available on the web at:

http://www.astro.ucla.edu/~wright/CosmoCalc.html

to obtain the luminosity distance corresponding to different redshifts in the two models (note: both are `flat' models). Then convert to apparent magnitude using the equation above. Assume that the Hubble constant is 71 km s^{-1} Mpc^{-1} (the default value in the calculator). I obtained a nice graph, using points at $z = 0.1, 0.2, \ldots 1.0$.

(c) What is the difference in the apparent magnitude of the supernovae in the two models between redshift $z = 0.5$ and $z = 1$? The two models I've chosen represent roughly the actual uncertainty in the cosmological parameters derived from current data, so this is an estimate of how well we need to do to make further progress (of course, we can in practice average many supernovae to improve the accuracy of the mean magnitude).