STRUCTURE OF GALAXIES

1. Structure, kinematics and dynamics of the Galaxy

Piet van der Kruit
Kapteyn Astronomical Institute
University of Groningen
the Netherlands

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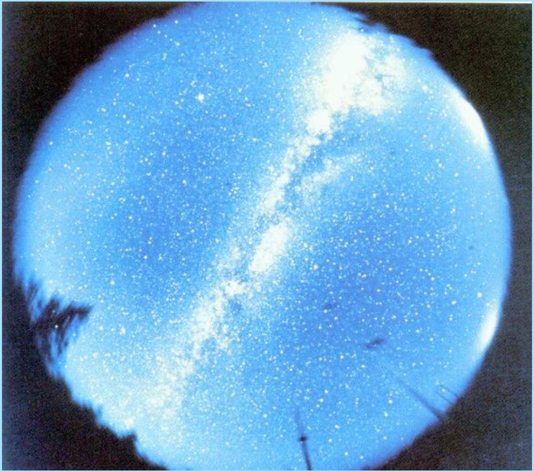
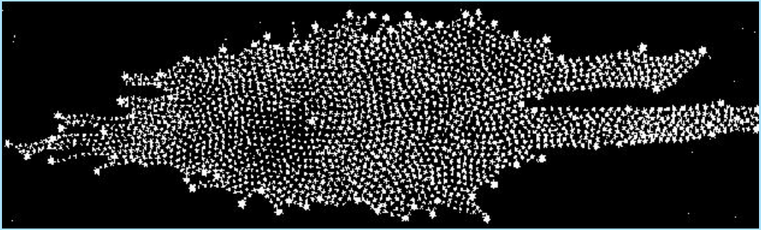
Outline

Structure of the Galaxy

History

Our **Galaxy** can be seen on the sky as the Milky Way, a band of faint light.



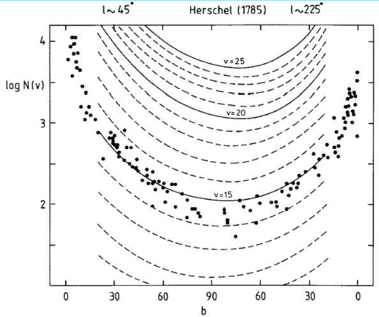


The earliest attempts to study the structure of the Milky Way Galaxy (the Sidereal System; really the whole universe) on a global scale were based on star counts.

William Herschel (1738 – 1822) performed such "star gauges" and assumed that (1) all stars have equal intrinsic luminostities and (2) he could see stars out ot the edges of the system.

Then the distance to the edge of the system in any direction is proportional to the square-root of the number of stars per square degree.

It can be shown by comparing to current star counts that Herschel counted stars down to about visual magnitude 14.5[1].



[1] P.C. van der Kruit, A.&A. 157, 244 (1986)

Jacobus C. Kapteyn (1851 – 1922) improved upon this by determining locally the luminosity function $\Phi(M)$, that is the frequency distribution of stars as a function of their absolute magnitudes.

The observed distribution of stars $N_{\rm m}$ in a given direction as a function of apparent magnitude m relates to the space density of stars $\Delta(\rho)$ at distance ρ as

$$\frac{dN_{\rm m}}{dm} = 0.9696 \int_0^\infty \rho^2 \Delta(\rho) \Phi(m - 5 \log \rho) d\rho$$

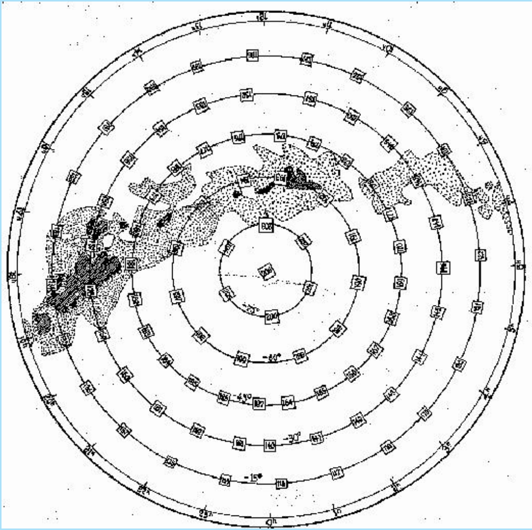
Kapteyn proceeded to investigate (numerical) methods to invert this integral equation in order to solve it.

Kapteyn suspected that interstellar absorption was present and even predicted that it would give rise to reddening[2].

But he found that the reddening was small (0.031 ± 0.006 mag per kpc in modern units) and chose to ignore it.

Under Kapteyn's leadership an international project on Selected Areas over the whole sky to determine star counts (and eventually spectral types and velocities) in a systematic way was started.

[2] J.C. Kapteyn, Ap.J. 29, 46 & 30, 284/398 (1909)


-15°
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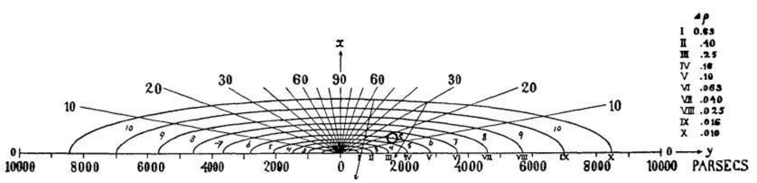
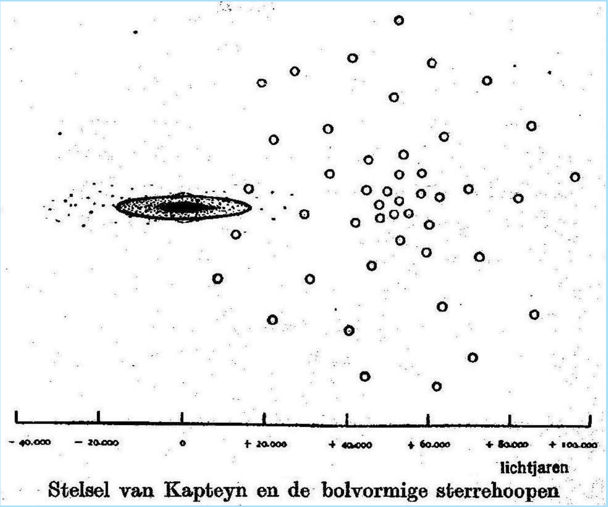
Towards the end of his life he used star counts to construct what became known as the Kapteyn Universe[3]:



The Sun is near the center. That was suspicious.

Indeed the work of Harlow Shapley (1885 – 1972) on the distances of Globular Clusters showed that the Sidereal System really was much larger.

[3] J.C Kapteyn & P.J. van Rhijn, Ap.J. 52, 23 (1920); J.C. Kapteyn, o.J. 55, 302 (1922)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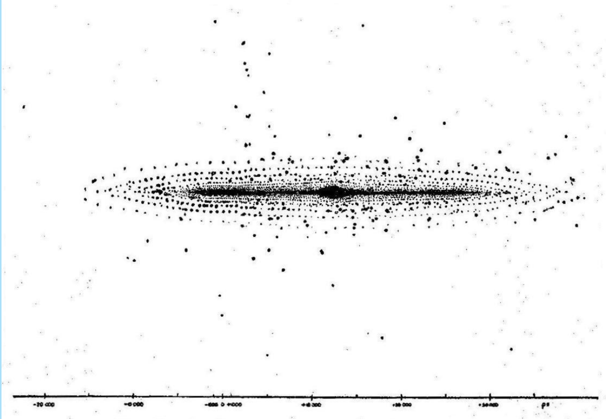
lichtjaren
Stelsel van Kapteyn en de bolvormige sterrehoopen

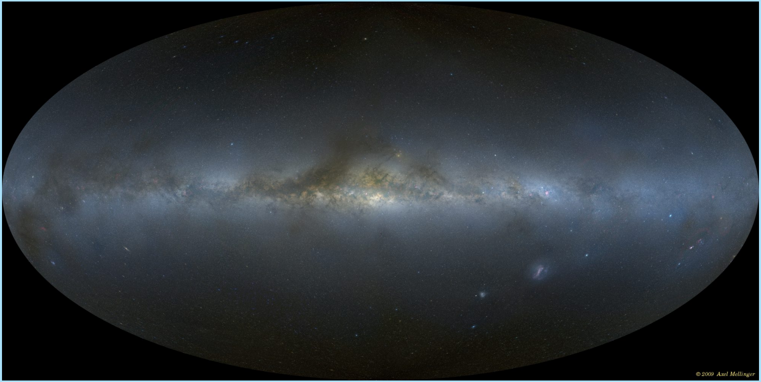
Astronomers like Jan H. Oort (1900 – 1992) found that absorption reconciled the two models.



Dr. Oort's diagram van het melkwegstelsel

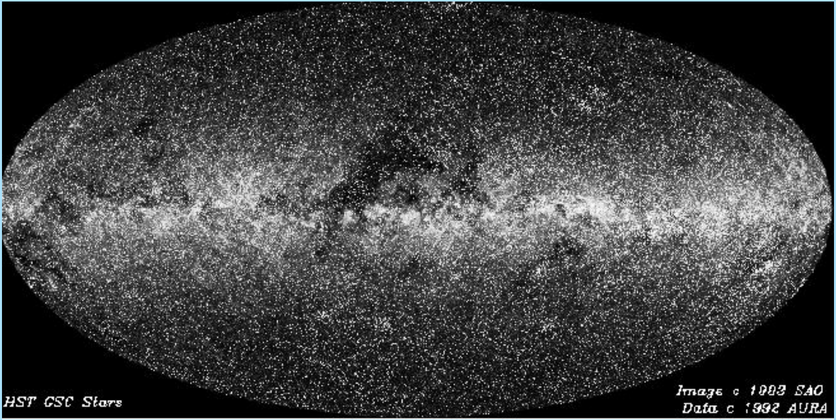
All-sky pictures

Here is a composite picture[4] covering the full sky at $36''\text{pixel}^{-1}$.

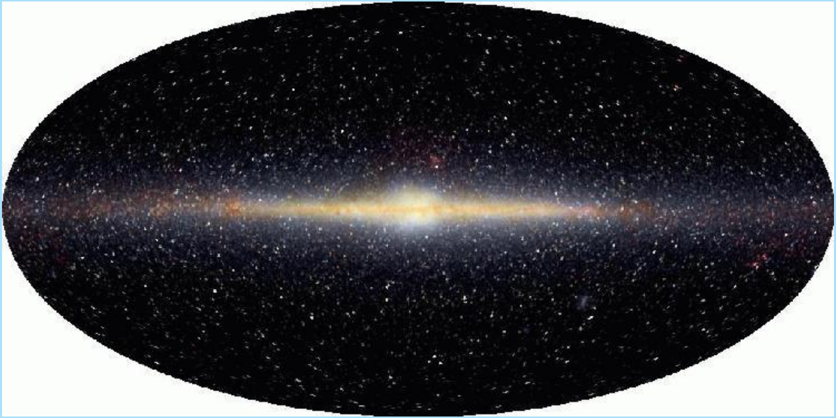


[4] A. Mellinger, P.A.S.P. 121, 1180 (2009); also Astronomy Picture of the Day for 2009 November 25: antwrp.gsfc.nasa.gov/apod/ap091125.html

Here is a plot of all stars in the Guide Star Catalogue of the Hubble Space Telescope down to about magnitude 16.



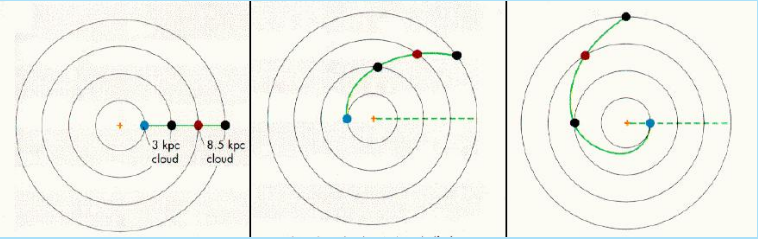
The Cosmic Background Explorer (COBE) satellite did see the Milky Way in the near-infrared as follows:



Kinematics of the Galaxy

Differential rotation

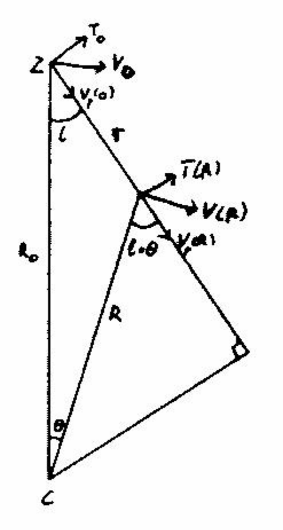
The Galaxy does not rotate like a solid wheel. The period of revolution varies with distance from the center. This is called differential rotation.

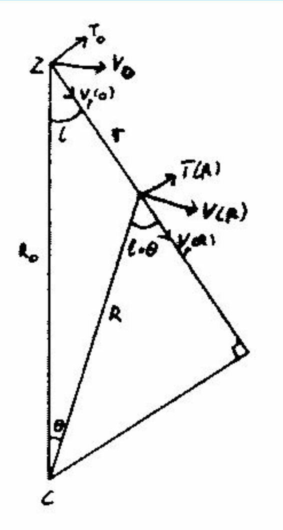


Each part moves with respect to those parts that do not happen to be at the same galactocentric distance.

Say, the rotation speed is $V(R)$ and in the solar neighborhood it is $V_\circ$.

If the Sun Z is at a distance $R_\circ$ from the center C, then an object at distance r from the Sun at Galactic longitude l has a radial velocity w.r.t. the Sun $V_{\rm rad}$ and a tangential velocity T.





$$V_{\rm rad} = V_{\rm r}(R) - V_{\rm r}(0) = V(R)\sin(l+\theta) - V_\circ \sin l$$

$$T = T(R) - T(0) = V(R)\cos(l+\theta) - V_\circ \cos l$$

$$R\sin(l+\theta) = R_\circ \sin l$$

$$R\cos(l+\theta) = R_\circ \cos l - r$$

Substitute this and we get

$$V_{\rm rad} = R_\circ \left(\frac{V(R)}{R} - \frac{V_\circ}{R_\circ} \right) \sin l \qquad (1)$$

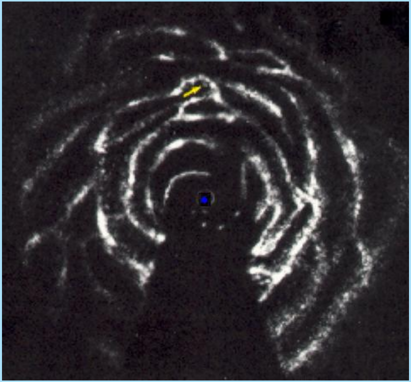
$$T = R_\circ \left(\frac{V(R)}{R} - \frac{V_\circ}{R_\circ} \right) \cos l - \frac{r}{R} V(R) \qquad (2)$$

So, if we would know the rotation curve $V(R)$ we can calculate the distance R from observations of $V_{\rm rad}$. From this follows r with an ambiguity symmetric with the sub-central point.

The latter is that point along the line-of-sight that is closest to the Galactic Center.

$V(R)$ can be deduced in each direction l by taking the largest observed radial velocity. This will be the rotation velocity at the sub-central point.

With the 21-cm line of HI, the distribution of hydrogen in the Galaxy has been mapped[5]. This was the first indication that the Galaxy is a spiral galaxy.



[5] K.K. Kwee, C.A. Muller & G. Westerhout, Bull. Astron. Inst. Neth. 12, 211 (1954); J.H. Oort, F.J. Kerr & G. Westerhout, Mon.Not.R.A.S. 118, 379 (1958) and J.H. Oort, I.A.U. Symp. 8, 409 (1959)

Local approximations and Oort constants

We now make local approximations; that is $r \ll R_\circ$.

Change to angular velocities $\omega(R) = V(R)/R$ and $\omega_\circ = V_\circ/R_\circ$ and make a Tayler expansion

$$f(a+x) = f(a) + x\frac{df(a)}{da} + \frac{1}{2}x^2\frac{d^2f(a)}{d^2a} + \ldots.$$

for the angular rotation velocity

$$\omega(R) = \omega_\circ + (R - R_\circ)\left(\frac{d\omega}{dR}\right)_{R_\circ} + \tfrac{1}{2}(R - R_\circ)^2\left(\frac{d^2\omega}{dR^2}\right)_{R_\circ}$$

The cosine-rule gives

$$R = R_\circ \left[1 + \left(\frac{r}{R_\circ} \right)^2 - \frac{2r}{R_\circ} \cos l \right]^{1/2}$$

Make a Tayler expansion for this expression and ignore terms of higher order than $(r/R_\circ)^3$.

$$R = R_\circ \left[1 - \frac{r}{R_\circ} \cos l + \frac{1}{2} \left(\frac{r}{R_\circ} \right)^2 (1 - \cos^2 l) \right]$$

$$R - R_\circ = -r \cos l + \frac{1}{2} \frac{r^2}{R_\circ} (1 - \cos^2 l)$$

$$(R - R_\circ)^2 = r^2 \cos^2 l$$

Substitute this in the equation for ω

$$\omega(R) = \omega_\circ + \left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ \left[-\frac{r}{R_\circ}\cos l + \frac{1}{2}\left(\frac{r}{R_\circ}\right)^2 (1-\cos^2 l)\right] + \frac{1}{2}\left(\frac{d^2\omega}{dR^2}\right)_{R_\circ} R_\circ^2 \left(\frac{r}{R_\circ}\right)^2 \cos^2 l$$

or in linear velocity

$$V_{\rm rad} = \left(\frac{r}{R_\circ}\right)^2 \left(\frac{d\omega}{dR}\right)_{R_\circ} \frac{R_\circ^2}{2} \sin l - \frac{r}{R_\circ}\left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ^2 \sin l \cos l + \frac{1}{2}\left(\frac{r}{R_\circ}\right)^2 \left[-\left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ^2 + \left(\frac{d^2\omega}{dR^2}\right)_{R_\circ} R_\circ^3\right] \sin l \cos^2 l$$

Use $2 \sin l \cos l = \sin 2l$ and ignore terms with $(r/R_\circ)^2$ and higher orders. Then

$$V_{\rm rad} = -\tfrac{1}{2} R_\circ \left(\frac{d\omega}{dR} \right)_{R_\circ} r \sin 2l \equiv A r \sin 2l$$

So, stars at the same distance r will show a systematic pattern in the magnitude of their radial velocities accross the sky with Galactic longitude.

For stars at Galactic latitude b we have to use the projection of the velocities onto the Galactic plane:

$$V_{\rm rad} = A r \sin 2l \cos b$$

For the tangential velocities we make a change to proper motions μ. In equivalent way we then find

$$\begin{aligned}\frac{T}{r} = 4.74\mu &= -\omega_\circ + \frac{3}{2}\left(\frac{d\omega}{dR}\right)_{R_\circ} r\cos l - \left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ \cos^2 l \\ &+ \frac{r}{2R}\left[-\left(\frac{d\omega}{dR}\right)_{R_\circ} + \left(\frac{d^2\omega}{dR^2}\right)_{R_\circ} R_\circ^2\right]\cos^3 l\end{aligned}$$

Now use $\cos^2 l = \frac{1}{2} + \frac{1}{2}\cos 2l$ and ignore all terms $(r/R_\circ)$ and higher order.

$$\begin{aligned}4.74\mu &= -\omega_\circ - \frac{1}{2}\left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ - \frac{1}{2}R_\circ\left(\frac{d\omega}{dR}\right)_{R_\circ} \cos 2l \\ &\equiv B + A\cos 2l\end{aligned}$$

Now the distance dependence has of course disappeared. Agian for higher Galactic latitude the right-hand side will have to be multiplied by $\cos b$.

The constants A and B are the Oort constants. Oort first made the derivation above (in 1927) and used this to deduce the rotation of the Galaxy from observations of the proper motions of stars.

The Oort constanten can also be written as

$$A = \frac{1}{2}\left[\frac{V_\circ}{R_\circ} - \left(\frac{dV}{dR}\right)_{R_\circ}\right]$$

$$B = -\frac{1}{2}\left[\frac{V_\circ}{R_\circ} + \left(\frac{dV}{dR}\right)_{R_\circ}\right]$$

Furthermore

$$A + B = -\left(\frac{dV}{dR}\right)_{R_\circ} \quad ; \quad A - B = \frac{V_\circ}{R_\circ}$$

Current best values are

$R_\circ \sim$8.5 kpc　　$A \sim$13 km s^{-1} kpc^{-1}
$V_\circ \sim$ 220 km s^{-1}　　$B \sim$-13 km s^{-1} kpc^{-1}

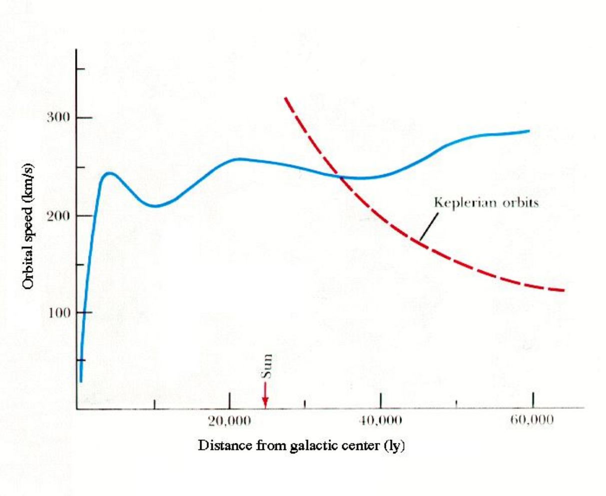
Rotation curves and mass distributions

The rotation curve $V(R)$ is difficult to derive beyond $R_\circ$ and this can only be done with objects of known distance such as HII regions).

In a circular orbit around a point mass M we have $M = V^2 R/G$ (as in the Solar System). This is called a Keplerian rotation curve.

One expects that the rotation curve of the Galaxy tends to such a behavior as one moves beyond the boundaries of the disk. However, we do see a flat rotation curve.

One determination of the Galactic rotation curve:



We see that up to large distances from the center the rotation velocity does not drop.

We also see this in other galaxies. It shows that more matter must be present than what we observe in stars, gas and dust and this is called dark matter.

With the formula estimate the mass within $R_\circ$ as $\sim 9.6 \times 10^{10} M_\odot$.

At the end of the measured rotation curve this enclosed mass becomes $\sim 10^{12} M_\odot$.

Galactic dynamics

Fundamental equations

There are two fundamental equations.

The first is the continuity equation, also called the Liouville or collisionless Boltzman equation.

It states that in any element of phase space the time derivative of the distribution function equals the number of stars entering it minus that leaving it, if no stars are created or destroyed.

Write the distribution function in phase space as $f(x, y, z, u, v, w, t)$ and the potential as $\Phi(x, y, z, t)$.

Now look first for the one-dimensional case at a position x, u. After a time interval dt the stars at $x - dx$ have taken the place of the stars at x, where $dx = udt$.

So the change in the distribution function is

$$df(x,u) = f(x - udt, u) - f(x,u)$$

$$\frac{df}{dt} = \frac{f(x - udt, u) - f(x,u)}{dt} = \frac{f(x - dx, u) - f(x,u)}{dx}u = -\frac{df(x,u)}{dx}u$$

For the velocity replace the positional coordinate with the velocity x with u and the velocity u with the acceleration du/dt. But according to Newton's law we can relate that to the force or the potential.

So we get

$$\frac{df}{dt} = -\frac{df(x,u)}{du}\frac{du}{dt} = \frac{df(u,x)}{du}\frac{d\Phi}{dx}$$

The total derivative of the distribution function then is

$$\frac{\partial f(x,u)}{\partial t} + \frac{\partial f(x,u)}{\partial x} u - \frac{\partial f(x,u)}{\partial u}\frac{\partial \Phi}{\partial x} = 0$$

In three dimensions this becomes

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} - \frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial u} - \frac{\partial \Phi}{\partial y}\frac{\partial f}{\partial v} - \frac{\partial \Phi}{\partial z}\frac{\partial f}{\partial w} = 0$$

If the system is in equilibrium, $f(x, y, z, u, v, w)$ is independent of time and

$$\frac{\partial f}{\partial t} = 0.$$

In cylindrical coordinates the distribution function is $f(R, \theta, z, V_{\mathrm{R}}, V_\theta, V_{\mathrm{z}})$ and the Liouville equation becomes

$$V_{\mathrm{R}} \frac{\partial f}{\partial R} + \frac{V_\theta}{R} \frac{\partial f}{\partial \theta} + V_{\mathrm{z}} \frac{\partial f}{\partial z} + \left(\frac{V_\theta^2}{R} - \frac{\partial \Phi}{\partial R} \right) \frac{\partial f}{\partial V_{\mathrm{R}}} -$$
$$\left(\frac{V_{\mathrm{R}} V_\theta}{R} + \frac{1}{R} \frac{\partial \Phi}{\partial \theta} \right) \frac{\partial f}{\partial V_\theta} - \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial V_{\mathrm{z}}} = 0.$$

The second fundamental equation is Poisson's equation, which says that the gravitational potential derives from the combined gravitational forces of all the matter. It can be written as

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \equiv \nabla^2 \Phi = 4\pi G \rho(x, y, z),$$

or in cylindrical coordinates

$$\frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R}\frac{\partial \Phi}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho(R, \theta, z).$$

For an axisymmetric case this reduces to

$$\frac{\partial K_{\mathrm{R}}}{\partial R} + \frac{K_{\mathrm{R}}}{R} + \frac{\partial K_{\mathrm{z}}}{\partial z} = -4\pi G \rho(R, z).$$

Here

$$K_{\rm R} = -\frac{\partial \Phi}{\partial R} \qquad K_{\rm z} = -\frac{\partial \Phi}{\partial z}$$

From the collisionless Boltzman equation follow the moment or hydrodynamic equations.

These are obtained by multiplying the Liouville equation by a velocity-component (e.g. $V_{\rm R}$) and then integrating over all velocities.

For the radial direction we then find:

$$\frac{\partial}{\partial R}(\nu \langle V_{\rm R}^2 \rangle) + \frac{\nu}{R}\{\langle V_{\rm R}^2 \rangle - V_{\rm t}^2 - \langle (V_\theta - V_{\rm t})^2 \rangle\} +$$

$$\frac{\partial}{\partial z}(\nu \langle V_{\rm R} V_{\rm z} \rangle) = \nu K_{\rm R}.$$

By assumption we have taken here $V_{\rm t} = \langle V_\theta \rangle$ and $\langle V_{\rm R} \rangle = \langle V_{\rm z} \rangle = 0$.
This can be rewritten as:

$$-K_{\rm R} = \frac{V_{\rm t}^2}{R} - \langle V_{\rm R}^2 \rangle \left[\frac{\partial}{\partial R} (\ln \nu \langle V_{\rm R}^2 \rangle) + \right.$$

$$\left. \frac{1}{R} \left\{ 1 - \frac{\langle (V_\theta - V_{\rm t})^2 \rangle}{\langle V_{\rm R}^2 \rangle} \right\} \right] + \langle V_{\rm R} V_{\rm z} \rangle \frac{\partial}{\partial z} (\ln \nu \langle V_{\rm R} V_{\rm z} \rangle).$$

The last term reduces in the symmetry plane to

$$\langle V_{\rm R} V_{\rm z} \rangle \frac{\partial}{\partial z} (\ln \nu \langle V_{\rm R} V_{\rm z} \rangle) = \frac{\partial}{\partial z} \langle V_{\rm R} V_{\rm z} \rangle$$

and may then be assumed zero.

In the vertical direction the hydrodynamic equation becomes

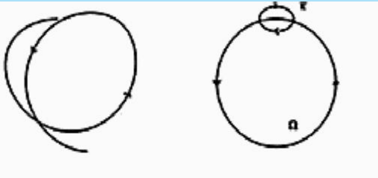
$$\frac{\partial}{\partial z}(\nu \langle V_{\mathrm{z}}^2 \rangle) + \frac{\nu \langle V_{\mathrm{R}} V_{\mathrm{z}} \rangle}{R} + \frac{\partial}{\partial R}(\nu \langle V_{\mathrm{R}} V_{\mathrm{z}} \rangle) = \nu K_{\mathrm{z}}.$$

If the radial and vertical motions are not coupled (as in a plane-parallel potential) the cross-terms with $\langle UW \rangle$ vanish and we are left with

$$\frac{\partial}{\partial z}(\nu \langle V_{\mathrm{z}}^2 \rangle) = \nu K_{\mathrm{z}}.$$

Epicycle orbits

For small deviation from the circular rotation, the orbits of stars can be described as epicyclic orbits.



If $R_\circ$ is a fudicial distance from the center and if the deviation $R - R_\circ$ is small compared to $R_\circ$, then we have in the radial direction

$$\frac{d^2}{dt^2}(R - R_\circ) = \frac{V^2(R)}{R} - \frac{V_\circ^2}{R_\circ} = 4B(A - B)(R - R_\circ),$$

where the last approximation results from making a Taylor expansion of $V(R)$ at $R_\circ$ and ignoring higher order terms.

Similarly we get for the tangential direction

$$\frac{d\theta}{dt} = \frac{V(R)}{R} - \frac{V_\circ}{R_\circ} = -2\frac{A-B}{R_\circ}(R - R_\circ),$$

where θ is the angular tangential deviation seen from the Galactic center.

These equations are easily integrated and it is then found that the orbit is described by

$$R - R_{\rm circ} = \frac{V_{\rm R,\circ}}{\kappa}\sin\kappa t,$$

$$\theta R_\circ = -\frac{V_{\rm R,\circ}}{2B}\cos\kappa t$$

and the orbital velocities by

$$V_{\mathrm{R}} = V_{\mathrm{R},\circ} \cos \kappa t,$$

$$V_{\theta} - V_{\circ} = \frac{V_{\mathrm{R},\circ} \kappa}{-2B} \sin \kappa t.$$

The period in the epicycle equals $2\pi/\kappa$ and the epicyclic frequency κ is

$$\kappa = 2\{-B(A-B)\}^{1/2}.$$

In the solar neighborhood κ ~36 km s^{-1} kpc^{-1}.

For a flat rotation curve we have

$$\kappa = \sqrt{2}\frac{V_{\circ}(R)}{R}.$$

Through the Oort constants and the epicyclic frequency, the parameters of the epicycle depend on the local forcefield, because these are all derived from the rotation velocity and its radial derivative.

The direction of motion in the epicycle is opposite to that of galactic rotation.

The ratio of the velocity dispersions or the axis ratio of the velocity ellipsoid in the plane for the stars can be calculated as

$$\frac{\langle V_\theta^2 \rangle^{1/2}}{\langle V_{\rm R}^2 \rangle^{1/2}} = \sqrt{\frac{-B}{A-B}}.$$

For a flat rotation curve this equals 0.71.

With this result the hydrodynamic equation can then be reduced to the so-called asymmetric drift equation:

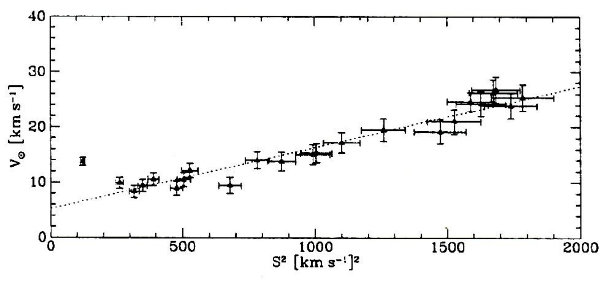
$$V_{\rm rot}^2 - V_{\rm t}^2 =$$

$$-\langle V_{\rm R}^2 \rangle \left\{ R\frac{\partial}{\partial R} \ln \nu + R\frac{\partial}{\partial R} \ln \langle V_{\rm R}^2 \rangle + \left[1 - \frac{B}{B-A} \right] \right\}.$$

If the asymmetric drift $(V_{\rm rot} - V_{\rm t})$ is small, the left-hand term can be approximated by

$$V_{\rm rot}^2 - V_{\rm t}^2 \sim 2V_{\rm rot}(V_{\rm rot} - V_{\rm t}).$$

The term asymmetric drift comes from the observation that objects in the Galaxy with larger and larger velocity dispersion lag more and more behind in the direction of Galactic rotation.



Vertical motion

For the vertical motion the equivalent approximation is also that of a harmonic oscillator.

For a constant density the hydrodynamic equation reduces to

$$K_{\mathrm{z}} = \frac{d^2 z}{dt^2} = -4\pi G \rho_\circ z.$$

Integration gives

$$z = \frac{V_{\mathrm{z},\circ}}{\lambda} \sin \lambda t \quad ; \quad V_{\mathrm{z}} = V_{\mathrm{z},\circ} \cos \lambda t.$$

The period equals $2\pi/\lambda$ and the vertical frequency λ is

$$\lambda = (4\pi G \rho_\circ)^{1/2}.$$

For the solar neighbourhood we have $\rho_\circ \sim 0.1\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$.

With the values above for $R_\circ$, $V_\circ$, A and B, the epicyclic period $\kappa^{-1} \sim 1.7 \times 10^8$ yrs and the vertical period $\lambda^{-1} \sim 8 \times 10^7$ yrs. The period of rotation is 2.4×10^8 yrs.

The Sun moves with $\sim$20 km s^{-1} towards the Solar Apex at Galactic longitude $\sim 57^\circ$ and latitude $\sim +27^\circ$.

From the curvature of the ridge of the Milky Way the distance of the Sun from the Galactic Plane is estimated as 12 pc.

The axes of the solar epicycle are about $\sim$0.34 kpc in the radial direction and $\sim$0.48 kpc in the tangential direction.

The amplitude of the vertical motion is $\sim$85 pc.