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HIGH-ENERGY GAMMA-RAY EMISSION FROM SOLAR FLARES: CONSTRAINING THE ACCELERATED PROTON SPECTRUM

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Abstract. Using a multi-component model to describe the γ-ray emission, we investigate the flares of December 16, 1988 and March 6, 1989 which exhibited unambiguous evidence of neutral pion decay. The observations are then combined with theoretical calculations of pion production to constrain the accelerated proton spectra.

The detection of π^0 emisson alone can indicate much about the energy distribution and spectral variation of the protons accelerated to pion producing energies. Here both the intensity and detailed spectral shape of the Doppler-broadened π^0 decay feature are used to determine the spectral form of the accelerated proton energy distribution. The Doppler width of this γ-ray emission provides a unique diagnostic of the spectral shape at high energies, independent of any normalisation. To our knowledge, this is the first time that this diagnostic has been used to constrain the proton spectra. The form of the energetic proton distribution is found to be severely limited by the observed intensity and Doppler width of the π^0 decay emission, demonstrating effectively the diagnostic capabilities of the π^0 decay γ-rays.

The spectral index derived from the γ-ray intensity is found to be much harder than that derived from the Doppler width. To reconcile this apparent discrepancy we investigate the effects of introducing a high-energy cut-off in the accelerated proton distribution. With cut-off energies of around 0.5–0.8 GeV and relatively hard spectra, the observed intensities and broadening can be reproduced with a single energetic proton distribution above the pion production threshold.

1. Introduction

High-energy γ-ray emission from solar flares can be used as a diagnostic for the number and energy spectrum of the accelerated protons and heavier ions in the flare environment. The Solar Maximum Mission Gamma-Ray Spectrometer (SMM/GRS) (Forrest *et al.*, 1980) demonstrated that many flares exhibited the characteristic signature of high-energy proton interactions with the ambient solar atmosphere (Rieger, 1989; Chupp, 1990). In particular, the observations of γ-ray spectra rich in nuclear de-excitation lines have been comprehensively investigated and used to determine the form of the accelerated proton energy spectrum under the assumption of a specified acceleration process (e.g., Ramaty, Kozlovsky, and Lingenfelter, 1979; Murphy, Dermer, and Ramaty, 1987, hereafter referred to as MDR). Detection of nuclear de-excitation lines in the range 4–7 MeV signifies

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the presence of energetic protons with energies exceeding 10 MeV. In addition, the strong neutron capture line at 0.511 MeV, recorded in several flare events (Chupp, 1984), imply the presence of fast neutrons and charged pions, respectively, which, in turn, indicates accelerated proton energies in excess of several hundred MeV (Ramaty, 1986). Gamma-ray continuum and broad-band emission at photon energies $>$10 MeV have also been detected in several flares and are characteristic of interactions involving relativistic electrons and/or GeV protons. The γ-ray continuum emission from many of these high-energy events is dominated by bremsstrahlung radiation from relativistic electrons (cf. Vestrand *et al.*, 1987; MacKinnon and Brown, 1989). However, in a few events a distinct hardening of the photon spectrum is observed around 100 MeV, characteristic of the decay of neutral pions (Forrest *et al.*, 1985; Dunphy and Chupp, 1991). The presence of this radiation suggests interacting protons with energies in excess of several hundred MeV.

In analysing such events, very careful consideration must be given to the relative contributions of primary and secondary electron bremsstrahlung in addition to the neutral pion-decay γ-rays. The production of charged pions in conjunction with the π^0s leads, via the various decay channels, to high-energy secondary electrons which result in bremsstrahlung radiation in the highest energy bins of the SMM/GRS. In any detailed analysis, the form of this bremsstrahlung emission, together with that due to the primary electrons, is important, as it may alter the shape and intensity of any observed broad-band feature associated with neutral pion decay. However, in many events the degree of spectral hardening is severe enough to imply unequivocally the presence of large numbers of neutral pions. This single observational feature alone can indicate much about the energy distribution of protons at energies above the pion production threshold. Thus, the observed γ-ray fluences resulting from the decay of neutral pions provides an important diagnostic for the accelerated proton spectrum at these energies irrespective of any assumed acceleration process.

Another component of the neutral pion production is that due to the interaction of energetic alpha particles, accelerated in the flare, with the ambient solar atmosphere. At present, however, there is no unambiguous way of differentiating between the emission caused by energetic protons and energetic alphas over the whole energy range from MeV to GeV. Nuclear de-excitation lines due entirely to $\alpha - \alpha$ interactions do exist and provide a potential direct diagnostic of the accelerated α-particle spectra and abundances (Murphy *et al.*, 1991; Ramaty *et al.*, 1993). In particular, the interaction of energetic α-particles with ambient solar ^{4}He results in excited nuclides of ^{7}Be and ^{7}Li which decay to produce γ-ray lines at 0.429 and 0.478 MeV, respectively (Ramaty *et al.*, 1975). It should be noted, however, that with the exception of the April 27, 1981 event studied by Murphy *et al.* (1991), these lines are not well resolved in most flares observed by SMM/GRS and therefore do not provide adequate information about the energetic α-particles. In addition, the analysis of such lines also depends very strongly upon the uncertain

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abundances of both the ambient ^{4}He and the accelerated αs, on the presence of nearby lines and, also, on the assumed model of the continuum at these energies.

Even for events where the $\alpha - \alpha$ lines are well resolved and the ambient abundances and the form of the continuum spectrum are known, relevant information about the energetic αs can only be obtained for the low-energy ($<$100 MeV) component of the accelerated α spectrum; the lines discussed above have a production threshold of around 10 MeV. No unique diagnostic for accelerated α particles exists for the determination of the spectral form at, say, pion production energies. Consequently any inferences made from these γ-ray lines about the energetic α spectrum at high energies cannot be fully justified. In addition, we shall demonstrate below that the accelerated proton spectral form inferred from the π^0 decay γ-ray intensity is not significantly altered by the inclusion of an accelerated α particle distribution. We will also show that the observations under consideration in this paper preclude the suggestion that the α particle distribution could significantly alter the shape of the π^0 decay feature and consequently we do not expect the exclusion of energetic α particles to affect strongly the inferences made from this shape. It is the view of the present authors, then, that the least ambiguous approach is to neglect the contribution from the unknown α particle spectrum and treat the data-imposed conditions on the proton spectra as strict upper or lower limits. It is, of course, possible to compare the γ-ray emission resulting from an energetic α spectrum produced in any specified acceleration process with the observations at high energies (cf. MDR), but such an approach has no more *a priori* justification than the one adopted here.

It is the aim of the work discussed here to utilise the observations to infer the properties of the accelerated proton spectrum, responsible for the neutral pion production, for specific flare events without making restrictive assumptions about the acceleration mechanisms or interaction models. This is done by assuming a power-law form for the accelerated proton spectrum at energies above the pion-production threshold. The normalisation of the proton spectrum is achieved by coupling this power-law to a Bessel function spectrum at lower energies. This latter form for the proton spectrum at low energies, while representative of a specific model of particle acceleration, is found to provide good fits to the nuclear de-excitation line emission (MDR) and is here used solely to provide the correct number of γ-ray producing protons and to provide the correct nuclear γ-ray line yields. The details of our procedure will be discussed in Section 3.

In this paper we will concentrate on the event of December 16, 1988, which displayed evidence for a neutral pion decay component (Dunphy and Chupp, 1991, 1992). We will also discuss briefly the flare of March 6, 1989. These observations and their analysis are discussed in the following section. In Section 3 we discuss the results of our spectral fitting procedure and the interpretation of our results. The effect of high-energy cut-offs in the proton spectra are considered in Section 4 and we conclude with a summary in Section 5.

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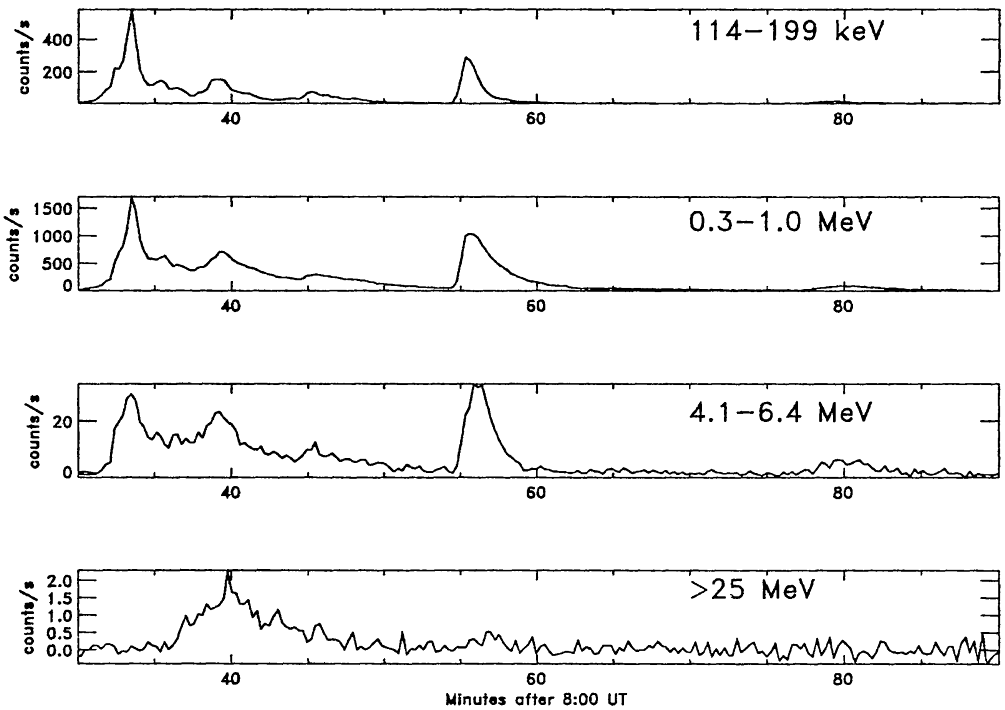

Fig. 1. Time history of the December 16, 1988 flare in various SMM/GRS energy-loss channels.

2. Observations

The SMM/GRS was sensitive to hard X-rays and γ-rays from solar flares over an energy range of $\sim$14 keV to $\sim$200 MeV. Two X-ray detectors covered the range 14–199 keV. An array of seven 7.6 cm$\times$7.6 cm Na I(Tl) 'main channel' detectors measured energy losses between 0.3–9 MeV and the GRS High-Energy Matrix (HEM) covered an energy loss range of about 10–140 MeV. Details of the GRS detectors can be found in Forrest *et al.* (1980).

2.1. Flare of December 16, 1988

To illustrate our approach we first present the SMM/GRS observation of the December 16, 1988 flare. The X4.7/1B flare of December 16, 1988 began at $\sim$08:30 UT. It was produced by active region 5278 located at N27 E33 on the solar disk, corresponding to a heliocentric angle of 43°. This flare was one of the most intense above 10 MeV seen by the SMM/GRS, with $>$10 MeV γ-ray emission lasting some 24 min. The time histories of several GRS energy loss channels are shown in Figure 1.

It has been demonstrated (Murphy and Ramaty, 1985; Hua and Lingenfelter, 1987a, hereafter referred to as HL) that the spectrum of solar flare protons can be evaluated by using the ratio of various γ-ray emissions, namely: (1) the line at 0.511 MeV from positron annihilation, (2) the line at 2.223 MeV from neutron capture, (3) the emission between 4 and 7 MeV from nuclear de-excitation, and (4) the broad peak from π^0 decay centred near 70 MeV. In the present analysis,

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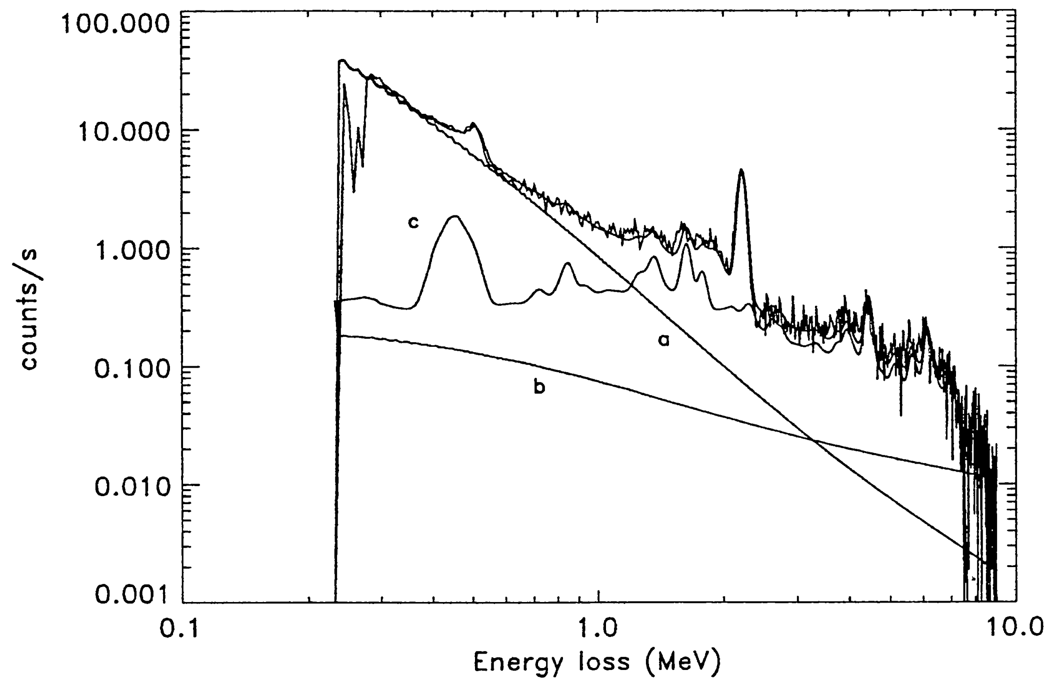

Fig. 2. GRS main channel energy-loss spectrum (background subtracted) for 08:38:39–08:41:40 UT on December 16, 1988. Also shown is a fit with a photon spectrum combining (a) power-law continuum, (b) an exponential continuum, and (c) a theoretical nuclear line spectrum lines at 0.511 and 2.223 MeV.

we use the intensities of the 2.223 MeV line and the de-excitation γ-rays from the GRS main channel, and the π^0 decay feature from the HEM data. The main channel spectrum is fitted by a model that combines a power-law continuum, a nuclear line spectrum, and an exponential continuum. The power-law continuum is associated with primary electron bremsstrahlung. The nuclear line spectrum is produced by prompt de-excitation of nuclear levels, by positron annihilation, and by neutron capture by H. The prompt de-excitation spectrum is a theoretical spectrum calculated to fit the nuclear lines from the flare of April 27, 1981 (Murphy *et al.*, 1985). The exponential continuum can be caused by bremsstrahlung from pion-decay electrons (see below) and unresolved nuclear lines (cf. 'model 1' of Murphy *et al.*, 1990). Figure 2 shows a main channel energy loss spectrum accumulated over a 180 s time interval during the December 16 flare as well as the components used to fit the spectrum.

The high-energy (>10 MeV) photon spectrum is found from the HEM data using a model-independent iterative fitting technique. A test spectrum (including both γ-rays and high-energy neutrons) is folded through the detector response and varied to minimize the sum of the weighted squared residuals using the Marquardt method (cf. Bevington, 1969). Once the high-energy γ-ray spectrum has been determined, we fit it using a model which is the sum of a logarithmic Gaussian (a Gaussian with a logarithmic energy scale) and a continuum. The continuum shape, a power law plus an exponential, is constrained by requiring it to be consistent

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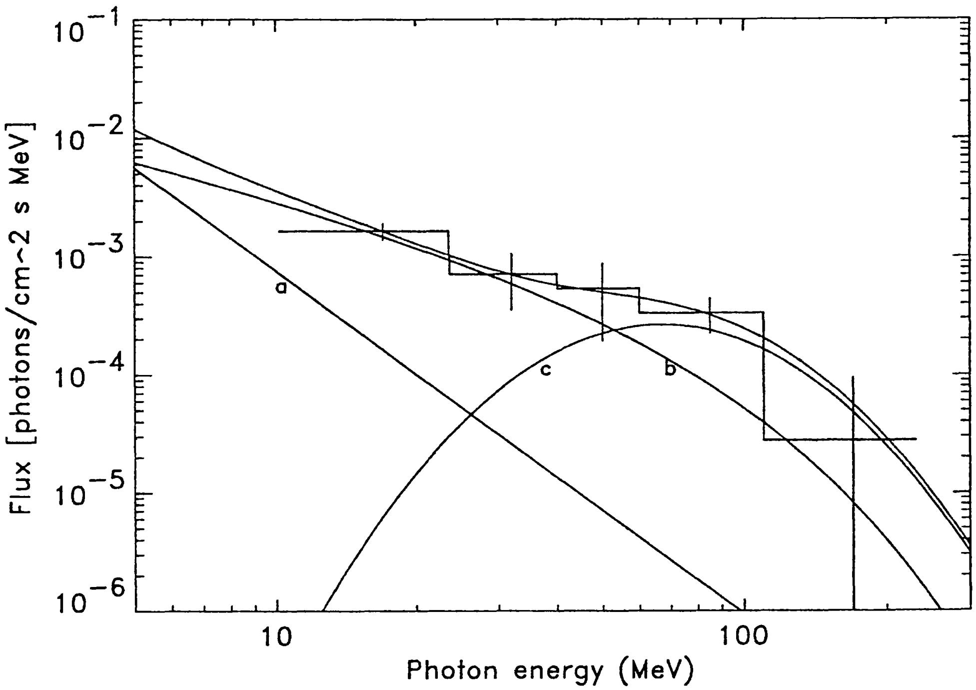

Fig. 3. Photon spectrum derived from the GRS HEM channel for the same time interval as Figure 2. The smooth curve is a fit using a logarithmic Gaussian peak (c) plus the power law (a) and exponential continuum (b) used in Figure 2.

with the main channel spectrum <10 MeV. Thus, we describe the photon spectrum above 10 MeV as

$$F_{pH}(\epsilon_\gamma) = \epsilon_\gamma^{-a} + \frac{b\exp(\epsilon_\gamma/\epsilon_{\gamma 0})}{\epsilon_\gamma} + $$
$$+c\exp\left(-\frac{1}{2}\left(\frac{\log(\epsilon_\gamma)-\log(0.07)}{\Delta w}\right)^2\right) \quad (\text{photons cm}^{-2}\,\text{GeV}^{-1})\,, \tag{1}$$

where a, b, c, $\epsilon_{\gamma 0}$, and Δw are adjustable parameters. The power-law continuum (at low energies, <1 MeV) and the nuclear de-excitation lines (mainly >1 MeV) are clearly required to describe the data (cf. Ramaty, Kozlovsky, and Suri, 1977). The exponential continuum has been used previously (Murphy *et al.*, 1990) as one of the components to model the spectrum of the flare of April 27, 1981. This functional form can also be justified because an exponential continuum plus a logarithmic Gaussian gives a good description of the theoretically calculated spectrum of γ-rays expected from pion decay (e.g., MDR). However, it should be recognized that the π^0-decay peak and the underlying continuum are not clearly separable, so that the net intensity of the π^0-peak fluence is somewhat dependent upon the form chosen for this continuum. Figure 3 shows the high-energy (>10 MeV) γ-ray spectrum for the same time interval as Figure 2.

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The neutron-capture line at 2.223 MeV is delayed with respect to the prompt nuclear lines by the time required for neutron thermalisation and capture. This time is $\sim$100 s (Prince *et al.*, 1983; Hua and Lingenfelter, 1987b). Both of the flares which we study here consist of multiple bursts. As a result, the time behaviour of the 2.223 MeV line is modeled using a single time constant (Prince *et al.*, 1983) in order to find the fluence of 2.223 MeV γ-rays that is to be associated with a given burst of nuclear de-excitation radiation. The line ratios given below, therefore, properly account for the neutron capture time delay.

Our analysis of the December 16, 1988 flare concentrates on the 721 s time interval from 08:35:39 to 08:47:40 UT, when there was significant emission of π^0-decay γ-rays. This burst is well defined in the GRS HEM at energies >25 MeV (Figure 1). The fluence of the nuclear de-excitation line radiation in the 4–7 MeV energy band for this time interval was found to be (102 ± 15) cm^{-2}. The corresponding fluence in the 2.223 MeV line was (211 ± 25) cm^{-2}. The ratio of the fluences $(F_{2.2}/F_{4-7})$ can be used to characterize the proton spectral shape at energies >10 MeV (Murphy and Ramaty, 1985; HL; Ramaty *et al.*, 1993). The spectral shape determined from the ratios is dependent upon the model of particle acceleration assumed by these authors, and so for consistency we will adopt the calculations of HL throughout this analysis. Assuming a Bessel-function shape for the injected proton spectrum in the energy range 10–300 MeV, and an isotropic velocity distribution, the ratio, $F_{2.2}/F_{4-7}$, implies a spectral shape parameter, αT, of 0.029 ± 0.003 (cf. HL: Figure 14a). For a horizontal (fan-beam) distribution, $\alpha T = 0.050 \pm 0.008$. The energetic proton intensity $N_p(>30\,\text{MeV})$, inferred from the γ-ray yield (2.223 MeV) is then $(1.1 \pm 0.2) \times 10^{33}$ in both cases. In addition, we determine a fluence in the π^0-decay emission of 10.5 ± 2.7 cm^{-2}.

2.2. Flare of March 6, 1989

Active region 5395 was visible on the solar disk between March 6 and March 19, 1989 during which time it produced 11 GOES X-class flares and numerous smaller flares. The first of these X-class events was an X15/3B flare on March 6 at $\sim$13:05 UT located at N35 E69 on the solar disk (heliocentric angle 77°). The time history of this event as observed by the GRS is illustrated in Figure 4.

The γ-ray emission, >1 MeV, of this event was produced in several distinct bursts lasting a total of some 56 min (cf. Rieger and Marschhäuser, 1990; Rieger and Kanbach, 1991). In the present analysis we concentrate on a 721 s time interval from 14:05:56 to 14:17:57 UT, which displayed spectral hardening in the HEM indicating the presence of π^0-decay γ-rays. The fluence of nuclear de-excitation line radiation in the 4–7 MeV energy band for this time interval was found to be (42.7 ± 2.2) cm^{-2} with a corresponding fluence in the neutron capture line at 2.223 MeV of (24.7 ± 8.2) cm^{-2}. The latter figure is lower than that reported by Dunphy and Chupp (1991) because here the difference in the time of production of the 2.223 MeV line relative to the prompt 4–7 MeV emission is properly taken into account. These fluences yield a ratio, $F_{2.223}/F_{4-7} = 0.58 \pm 0.20$ which,

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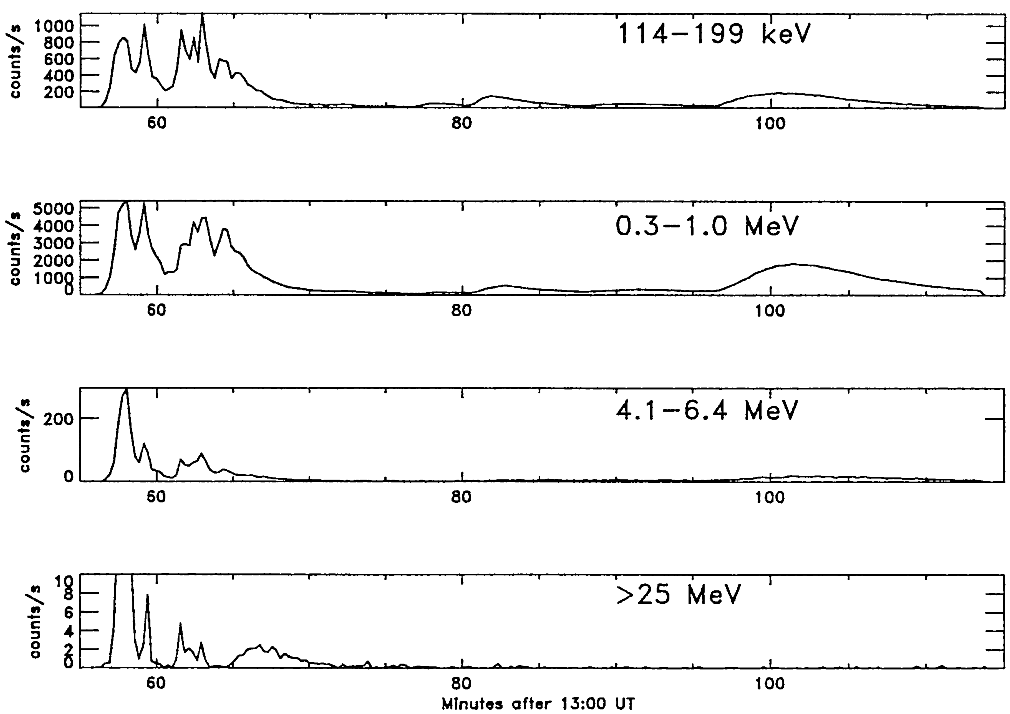

Fig. 4. Time history of the March 6, 1989 flare in various SMM/GRS energy-loss channels.

when we use the calculations of HL, corresponds to spectral shape parameters of $\alpha T = 0.020 \pm 0.005$ for isotropic injection and $\alpha T = 0.028 \pm 0.008$ for horizontal injection. The energetic proton intensity, $N_p(>30\,\mathrm{MeV})$, for this event is found to be $(3.6 \pm 0.9) \times 10^{32}$.

3. Results of Spectral Fitting

In this section we carry out the detailed comparison of the numerical calculations of π^0 decay emission, given an injected proton energy distribution, with the deconvolved results from the analysis of the observations of the event of December 16, 1988. This comparison enables us to infer the general properties of the accelerated proton spectra as implied by the observed γ-ray emission.

3.1. π^0 γ-RAY PRODUCTION

We only consider the production of secondary neutral pions from interactions of high-energy protons with the ambient solar H and He. We neglect the contribution from accelerated α particles entirely and our reasons for doing so were discussed fully in the Introduction. The charged pion contribution to the γ-ray emission at energies exceeding 10 MeV are accounted for by the fitting procedure detailed in the preceding section. We calculate only the total (thick-target) yield of protons, thus assuming that all protons stop in the source during the interval of the observations.

The production spectrum of secondary π^0 produced in $p - H$ collisions in solar flares is given by

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$$F_{pH}(T_\pi) = \frac{1}{4\pi R^2} \int_{T_{p,\,\mathrm{min}}}^{\infty} \mathrm{d}T_p j_p(T_p) \times$$
$$\times \int_{T_{p,\,\mathrm{min}}}^{T_p} \frac{\mathrm{d}\sigma(T_p',\, T_\pi)}{\mathrm{d}T_\pi} \frac{P(T_p,\, T_p')}{m_p[\mathrm{d}T_p'/\mathrm{d}N]_p} \mathrm{d}T_p' \;(\mathrm{pions\,cm^{-2}\,GeV^{-1}})\,, \quad (2)$$

where $j_p(T_p)$ (protons GeV^{-1}) is the incident proton spectrum; R is the Earth –Sun distance, and $T_{p,\,\mathrm{min}}$ is the minimum kinetic energy required by a proton in the lab. system to produce a pion of kinetic energy T_π. The integral over T_p' expresses the pion production rate per unit proton kinetic energy, T_p, allowing for the modification of the proton spectrum by thick-target losses and nuclear collisions. $P(T_p,\, T_p')$ is the probability for an energetic proton to survive from its initial energy T_p to the interaction energy, T_p'. Expressions for $\mathrm{d}T_p'/\mathrm{d}N$ and $P(T_p,\, T_p')$ can be found in Murphy (1985) and Alexander and MacKinnon (1993). The differential cross-section for the production of a pion with energy T_π due to a collision of accelerated proton energy T_p is expressed by $\mathrm{d}\sigma(T_p,\, T_\pi)/\mathrm{d}T_\pi$ and is taken from Dermer (1986).

The inclusion of contributions from collisions of the energetic protons with the ambient atmospheric He is calculated by multiplying Equation (2) by the ambient He abundance which we take to be 7% by number (Cameron, 1982) and replacing the $p - p$ inclusive cross-section with that appropriate for $p - \alpha$ interactions, i.e., we multiply Equation (2) by the energy-dependent factor $A_{\mathrm{He}}[\sigma_{p+\alpha\to\pi+X}/\sigma_{p+p\to\pi+X}]$ (see MDR). More recent work (e.g., Murphy *et al.*, 1990) has shown that a He abundance of 10% may be mroe appropriate for the solar atmosphere. However, in what follows we infer values of αT (the proton spectral shape parameter) from the earlier published results and maintain the Cameron abundances for consistency.

The proton spectrum incident upon the target region, $j_p(T_p)$, is chosen such that it has the form of a Bessel function spectrum, characteristic of some stochastic acceleration processes (cf., Ramaty, 1979), up to a proton kinetic energy, T_{p0}. Since we are mainly concerned with energies greater than ~300 MeV, we choose the Bessel function form to represent the proton spectrum at lower energies as a convenience and because it has been shown to provide good fits to nuclear line data (cf., Ramaty and Murphy, 1987). The spectrum we use to describe our population of accelerated protons is then given by

$$j_p(T_p) = \begin{cases} A_N K_2[2(3p_p/m_p c\alpha T)^{1/2}]\,, & T_p < T_{p0}\,, \\ A_N K_2[2(3p_{p0}/m_p c\alpha T)^{1/2}]\,(T_p/T_{p0})^{-\delta}\,, & T_p > T_{p0}\,, \end{cases} \quad (3)$$
$$(\mathrm{protons\ GeV^{-1}})\,,$$

where A_N is a normalisation factor to be determined from comparison with observations; K_2 is the modified Bessel function; p_p is the total proton momentum;

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αT measures the hardness of the Bessel function spectrum. It is important to emphasize that the value of αT is crucial in not only providing us with the correct normalisation, when the spectra are compared with data, but also in making our inferred proton distribution compatible with the γ-ray line emission observed at lower energies.

Having determined the pion distribution, the γ-ray spectrum resulting from the decay of these pions is simply

$$F_{pH}(\epsilon_\gamma) = 2 \int\limits_{E_{\pi,\,\mathrm{min}}}^{\infty} \mathrm{d}E_\pi \frac{F_{pH}(E_\pi)}{(E_\pi^2 - m_\pi^2)^{1/2}} \ (\text{photons cm}^{-2}\ \text{GeV}^{-1}) , \tag{4}$$

where $E_{\pi,\,\mathrm{min}} = \epsilon_\gamma + m_\pi^2/4\epsilon_\gamma$ from relativistic considerations (cf., Stecker, 1971). This, then, is the photon energy spectrum expected from a distribution of energetic protons with spectral form (3), accelerated during the initial energy release in a flare and interacting with the ambient solar atmosphere.

3.2. Spectral Fitting

The spectral fitting technique discussed in the previous section enables us to derive the observed γ-ray fluence corresponding to the neutral pion decay feature. The net π^0 emission is compared with the theoretically calculated Doppler-broadened π^0 decay line resulting from Equation (4) given an injected proton energy distribution of the form (3) and normalized to the number of protons required to produce the observed γ-ray line fluences.

The injected proton distribution chosen in this work is characterized by three independent parameters, namely, αT, the spectral hardness associated with the Bessel function spectrum, T_{p0}, the transition energy from Bessel function to power law, and δ, the power-law spectral index. The normalisation constant, A_N, is also required. The Bessel function form chosen to represent the accelerated proton spectrum at γ-ray line producing energies is only strictly valid up to about 100 MeV (Ramaty, 1979). We will restrict T_{p0} to be one of two values; $T_{p0} = 0.1$ GeV or $T_{p0} = 0.3$ GeV. This maintains agreement with the proton spectra used by MDR to explain the nuclear de-excitation line production without allowing this model-dependent spectral shape to affect the neutral pion production.

The use of the γ-ray line ratios to infer a value of αT, when combined with the γ-ray line fluences, enables us to determine the normalisation constant, A_N. This is possible because of the steepness of the proton spectra involved in which less than 1% of the protons have energies greater than the pion production threshold. Having determined αT and A_N from the de-excitation lines and chosen T_{p0} to be one of our two preferred values, the π^0 emission is then sufficient to determine the spectral index, δ.

In Figures 5 and 6 we illustrate the π^0 decay emission resulting from the calculations discussed above, for a range of power-law spectral indices, δ, and for the values of αT adopted from HL for the event of December 16, 1988. Both the

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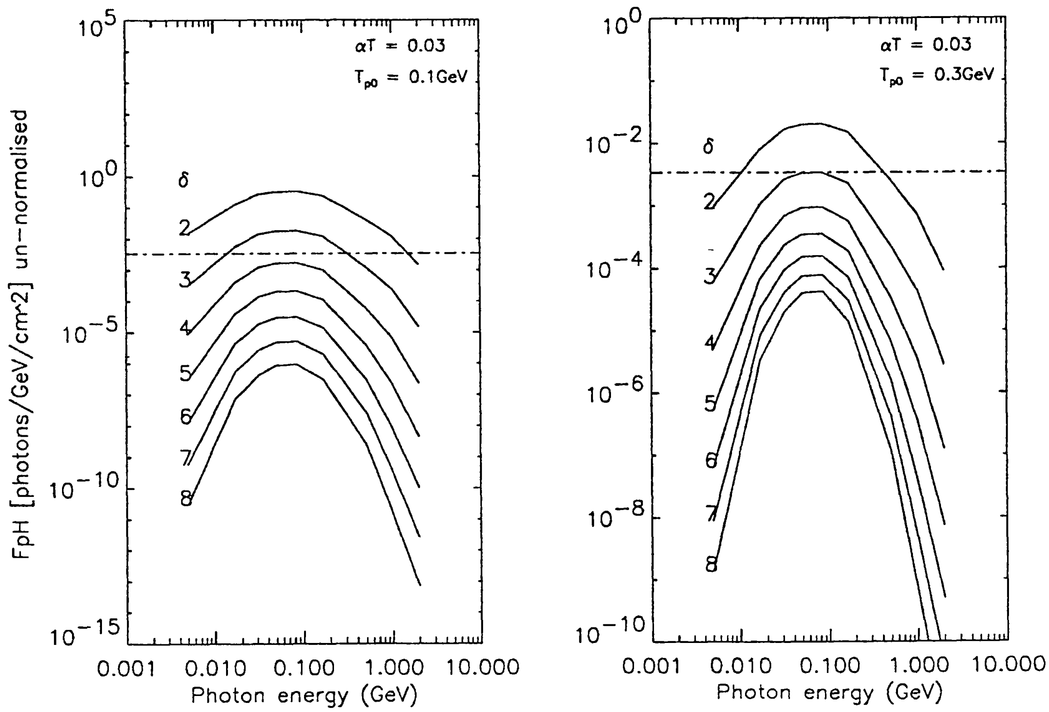

Fig. 5. Theoretical π^0 decay emission for a range of the power-law spectral index, δ, assuming $\alpha T = 0.03$ (isotropic model of Hua and Lingenfelter, 1987a). The fluences have been calculated for $A_N = 10^{35}$. The dot-dash line signifies the appropriately scaled, observed fluence at the π^0 decay peak.

intensity and Doppler width of the resulting emission is clearly very sensitive to the assumed spectral parameters and thus we note that the observations are able to provide useful constraints on the form of the injected proton spectra.

Before considering specific observations it should be noted that the deconvolved π^0 emission is characterized by two parameters; the peak intensity at ~70 MeV and the Doppler width. The width provides a direct measure of the spectral shape (independent of any normalisation), whereas the intensity requires to be scaled to the number of pion-producing protons. However, since the width is the less reliably determined quantity (see below) we will use the γ-ray line yields and the inferred αT to deduce the normalisation and use the π^0 decay γ-ray intensity to infer the power-law spectral index. We will also use the width of the π^0 feature to determine a spectral index which may be compared with that obtained from the intensity.

The calculated π^0 emission appropriate to the event being considered is determined by performing a χ^2 fit on the derived π^0 data and minimising with respect to δ,

$$\chi^2_{\min} = \min \sum_i (I_{ji} - I_{0i})^2/\sigma_i^2 \,, \tag{5}$$

where j runs over all the calculated spectra with specified αT, T_{p0} and varying δ, $i = 1, 5$ corresponds to the energy bins for the γ-ray data, I_{ji} are the calculated

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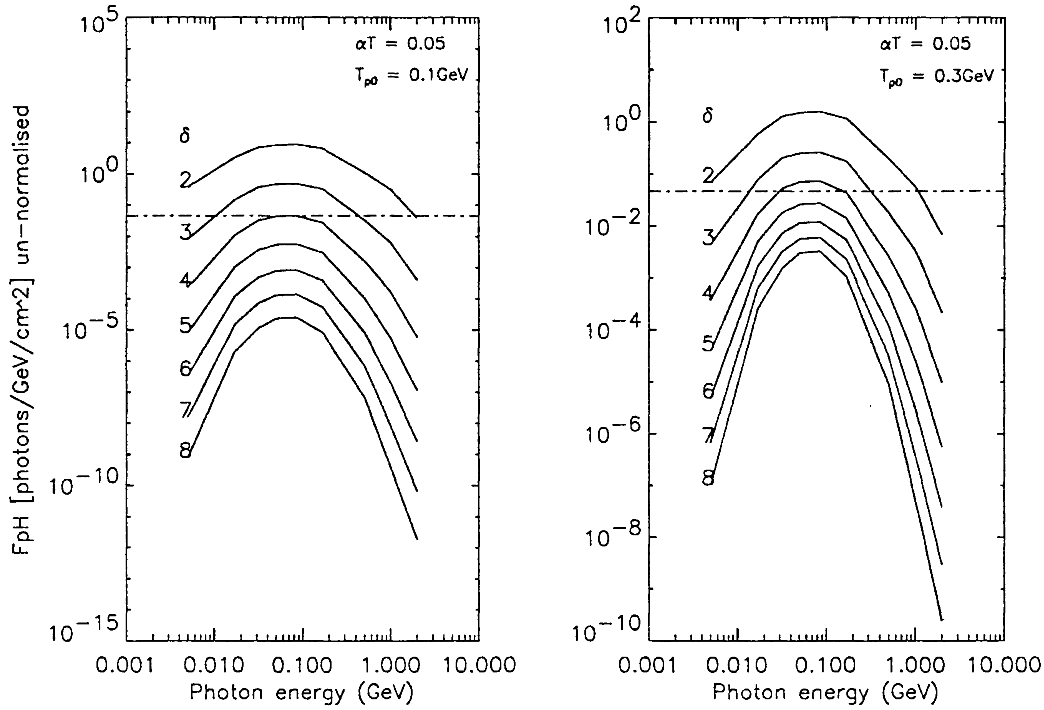

Fig. 6. As Figure 5 with $\alpha T = 0.05$ corresponding to the horizontal injection model of Hua and Lingenfelter (1987a).

spectra, I_{0i} are the net π^0 decay spectra data points and σ_i represents the derived errors at these points.

3.2.1. Flare of December 16, 1988

Recent yield calculations of HL considered the angular distribution of the energetic protons and discussed three specific cases: downward, with pitch angle $\theta = 0°$, horizontal, with pitch angle $\theta = 89°$, and isotropic. Given the observed γ-ray line fluences and ratios for the December 16, 1988 flare (see Section 2), we have used the results of HL to determine the best fit value of αT and the corresponding normalisation $N_p(>30\text{ MeV})$ which can then be used to deduce A_N. For the event of December 16, 1988 we find that $[\alpha T,\ N_p(>30\text{ MeV})]_I = [0.029 \pm 0.003,\ 9.5 \pm 1.1 \times 10^{32}]$ and $[\alpha T,\ N_p(> 30\text{ MeV}]_H = [0.050 \pm 0.008,\ 11 \pm 2 \times 10^{32}]$ where the subscript 'I' denotes isotropic and 'H' denotes horizontal injection. (The results for the downward distribution are not significantly different for our purposes from the isotropic injection.) In obtaining these results we use the total emission from this burst, i.e., the data from the 721 s time interval specified in Section 2. The temporal behavior during this time interval will be discussed below.

We have

$$N_p(> 30\text{ MeV}) = \int_{30}^{\infty} j_p(T_p) \simeq A_N \int_{30}^{100} K_2[2(3p_p/m_p c\alpha T)^{1/2}]\, dT_p\ , \qquad (6)$$

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TABLE I
Best fit spectral indices, δ_I, for π^0 decay emission of December 16, 1988 derived from the peak intensity at ~70 MeV

	αT \ T_{p0}	0.1	0.3
Isotropic	0.03	3.7 ± 0.1	3.0 ± 0.1
Horizontal	0.05	4.0 ± 0.1	4.4 ± 0.1

where the parameters are as defined in Equation (3). Substituting for the values of αT and $N_p(> 30$ MeV) given above we find $[A_N]_I = (3.39 \pm 0.56) \times 10^{39}$, $[A_N]_H = (2.53 \pm 0.65) \times 10^{38}$. Note that we are using the nuclear de-excitation line yields to derive the normalisation constant, A_N, and have consequently truncated the integral in Equation (6) to include only contributions from protons with energies below 100 MeV. This is easily extended to higher energies, taking account of the values of T_{p0} and δ. However, this only varies A_N by $<5\%$ for $T_{p0} = 0.1$ GeV and $\ll 1\%$ for $T_{p0} = 0.3$ GeV. The values of A_N quoted above have therefore been averaged over δ and T_{p0} with the range being incorporated into the uncertainties which are dominated by the errors in $N_p(>30$ MeV).

Having determined the normalisation constant we can then use the observed flux at the π^0 decay peak to determine the spectral index, δ_I, using Equation (4). The peak γ-ray flux at ~70 MeV for the event of December 16, 1988 was 116 ± 31 photons cm^{-2} GeV^{-1}. Using Figures 5 and 6 we see that the values corresponding to the isotropic proton injection ($\alpha T = 0.03$) as discussed by HL yield a proton spectral index above the pion production threshold of $\delta_I = 3.7 \pm 0.1$ for $T_{p0} = 0.1$ GeV and $\delta_I = 3.0 \pm 0.1$ for $T_{p0} = 0.3$ GeV. When applied to the case of horizontal injection ($\alpha T = 0.05$) we obtain $\delta_I = 4.0 \pm 0.05$ for $T_{p0} = 0.1$ GeV and $\delta_I = 4.4 \pm 0.1$ for $T_{p0} = 0.3$ GeV, where we have removed the averaging over T_{p0} and δ in the exact determination of A_N. These results are summarized in Table I.

For a fixed T_{p0} we see that the harder the spectrum required to fit the γ-ray line observations, the softer is the distribution required to give an agreement with the observed higher energy emission. This is a direct result of the use of the γ-ray line observations to provide the normalisation for our theoretical spectra and the assumption that the protons responsible for producing the pions belong to the same population as those responsible for the line emission.

As T_{p0} increases we find that there are two possible behaviours. On the one hand, if the αT inferred from the γ-ray lines is relatively soft (e.g., $\alpha T = 0.03$) then a harder spectrum at higher energies is required to provide the necessary photon fluence at 70 MeV. On the other hand, a hard spectrum at low energies results in softer power laws being required as T_{p0} increases. This is to be expected because the typical power-law spectral index inferred from the π^0 decay data, $\delta_I \sim 4$, has

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a slope which is flatter than a Bessel function with a parameter $\alpha T \leq 0.04$ and steeper than $\alpha T \geq 0.04$ in the energy range 0.1–0.3 GeV.

We also investigated the time dependence of the F_{π^0}/F_{4-7} flux ratio in each of four 180 s time bins displaying significant >10 MeV emission for the event of December 16, 1988. Both the 4–7 MeV and π^0 decay emissions are prompt and consequently the ratios of the fluences in these energy ranges can be used to probe the time variation of the spectral shape of the energetic protons (cf. Murphy and Ramaty, 1985). However, it was found that, while there is a considerable spread in F_{π^0}/F_{4-7}, each of the four time bins yielded the same value for this ratio to within the observational uncertainties; the average ratio for the total time interval 08:35:39 to 08:47:40 UT was 0.102 ± 0.023. Thus, for this time period there was no apparent shift in the relative contributions from the γ-ray line and π^0 decay emissions and, therefore, no significant evolution of the proton spectral shape. A time-independent F_{π^0}/F_{4-7} ratio does not, of course, preclude a temporal variation in the overall γ-ray flux but does indicate a consistent set of spectral parameters. Our normalisation is governed by the γ-ray line fluxes and any time dependence in these fluxes would result in a variation in this normalisation. However, this is only important if the value of αT for the lines differs between different time bins. This behaviour is difficult to determine since in order to isolate the parameters inferred from the line emission (i.e., αT) we need to restrict ourselves to $F_{0.511}/F_{4-7}$ and $F_{2.223}/F_{4-7}$ ratios, both of which are delayed with respect to the prompt de-excitation lines, making any time dependence of these ratios difficult to determine. We, therefore, use the constancy of the F_{π^0}/F_{4-7} ratio to indicate the time independence of the relative spectral parameters, αT and δ. It is interesting to note that this constancy would also appear to be inconsistent with a picture involving prolonged acceleration involving long-lived trapping of protons (cf. Ryan and Lee, 1991).

If we now consider the spectral shape of the π^0 decay feature we find that the Gaussian width consistent with the observations is $\Delta w = 0.22 \pm 0.10$ in appropriate units where the 68% confidence level is given. Using our calculated π^0-decay spectral shapes we determine the best fit spectral index, δ_w, which yields a photon spectrum with a corresponding line width. We obtain $\delta_w = 9.5 \pm 3.1$ (or $\delta_\omega = 9.5 \pm 5.1$ for 90% confidence level). The range of viable δ_ω is extremely large and this reduces the reliability of the inferred spectral values. This value of δ_ω is unaffected by the different values of αT and T_{p0} adopted, since there is no assumed Bessel function component in the pion production range. Consequently, αT and T_{p0} serve only to normalise the emission. There is a significant difference between the derived value of δ_w and that inferred from the intensity, δ_I, although both are just about consistent when we consider the 90% confidence level, i.e., there is ~10% chance of δ_I being consistent with δ_w. The proton spectrum inferred from the observed π^0-decay fluence results in a Doppler-broadened feature which is much wider than that observed. Observationally, the π^0 decay feature for the flare of December 16, 1988 is an intense narrow feature which is difficult to explain with the form of the energetic proton spectrum adopted. There are several

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TABLE II
Best fit spectral indices, δ_I, for π^0 decay emission of March 6, 1989 derived from the peak intensity at ~70 MeV

	T_{p0} / αT	0.1	0.3
Isotropic	0.020	2.82 ± 0.16	1.41 ± 0.16
Horizontal	0.028	2.99 ± 0.16	2.00 ± 0.16

possible reasons for this discrepancy and we will deal with these directly in Section 4.

3.2.2. *Flare of March 6, 1989*

The observed γ-ray line fluences for the March 6, 1989 flare yield best fit αT values of $[\alpha T]_I = 0.020 \pm 0.005$ and $[\alpha T]_H = 0.028 \pm 0.08$ where the subscripts 'I' and 'H' again denote the isotropic and horizontal injection models of HL. In both cases the number of energetic protons was $N_p(> 30 \text{ MeV}) = (3.6 \pm 0.9) \times 10^{32}$. These values are obtained for the 721 s time interval specified in Section 2.

Following the procedure outlined above for the December 16, 1988 flare we obtain normalisation constant of $[A_N]_I = (1.75 \pm 0.44) \times 10^{40}$ and $[A_N]_H = (1.87 \pm 0.44) \times 10^{39}$. The observed flux of the π^0 decay emission at the peak photon energy of 70 MeV was 210 ± 90 photons cm^{-2} GeV^{-1}. Given the normalisation constants above we determine that for the case of isotropic injection a spectral index of $\delta_I = 2.82 \pm 0.16$ ($\delta_I = 1.41 \pm 0.16$) is required to reproduce the observed pion decay intensity when $T_{p0} = 0.1$ GeV ($T_{p0} = 0.3$ GeV) while for the case of horizontal proton injection model we obtain $\delta_I = 2.99 \pm 0.16$ for $T_{p0} = 0.1$ GeV and $\delta_I = 2.00 \pm 0.16$ for $T_{p0} = 0.3$ GeV (see Table II).

An inspection of Table II shows that much harder spectra at large energies are required to produce the observed π^0 decay emission in this event than in the December 16, 1988 flare. In addition, the line ratios calculated from the March 6, 1989 data yield Bessel function spectra which are much softer than the corresponding December 16, 1988 spectra and consequently relatively more high-energy protons are necessary to produce the 70 MeV γ-ray emission.

The equivalent Gaussian width, describing the spectral shape of the March 6 π^0 decay feature, $\Delta w = 0.15 \pm 0.07$ (68% confidence level), is much narrower than that obtained for the December 16 flare. This reflects the fact that for the March 6, 1989 event there were very few counts detected in the highest energy bins of the High-Energy Matrix. The corresponding power-law spectral index is $\delta_w \simeq 18 \pm 7$. Thus the discrepancy between the form of the high-energy proton distribution inferred from the π^0 decay feature intensity and its width is even more apparent in this flare. In what follows we will concentrate on the interpretation of

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the December 16, 1988 results although it is clear that many of the arguments will also apply to the results of March 6, 1989.

4. Interpretation of Results

It was demonstrated above that the energetic proton spectrum inferred from the intensity of the π^0 decay emission at 70 MeV was significantly different from that obtained using the shape of the Doppler-broadened π^0 line. The large errors in the observed width resulted in a difference of $\sim$3–5 in the power-law spectral indices. In addition to the uncertainty in the width due to the observational errors, such a large discrepancy could also have several other causes: (1) the possible contribution of energetic α particle interactions, neglected in this work, while expected to contribute only $\sim$20% to the intensity, may significantly affect the shape of the π^0 feature, particularly if the α spectrum is quite distinct from the simple power law chosen for the protons; (2) the bremsstrahlung component due to primary electrons and secondary electrons and positrons may differ from the functional form used in this work; (3) the assumed accelerated proton spectrum requires a degree of modification.

Any errors associated with the exclusion of energetic α particles may be neglected for the following reasons. We demonstrated in the previous section that the proton spectrum inferred from the width of the π^0 decay feature was significantly softer than that inferred from the intensity. Equivalently, we may say that the π^0 decay feature is much narrower than expected from the observed intensity. Consequently, if the energetic α particle contribution was to be the source of this error then it must not only result in a narrow π^0 decay line but must also be the dominant source of neutral pions in these events. (The wide shape inferred from the intensity can only be significantly narrowed by a much stronger source of neutral pions.) This is clearly not the case in solar flares since the energetic α's can only contribute at most $\sim$20–30% of the γ-ray fluence. This strengthens our arguments for the neglect of these particles.

The assumption that the normalisation constant inferred from the γ-ray line data is also appropriate for the high-energy emission may be invalid if the two different energy regimes result from distinct acceleration processes. A combination of the shape and intensity of the π^0 feature would yield a normalisation constant, which may be compared with that obtained from the γ-ray lines. The discrepancy between δ_w and δ_I for the event of December 16, 1988 (for either the isotropic or horizontal injection model) may be characterized by considering the number of protons accelerated to pion-producing energies, $N_p(\pi^0)$. Allowing the normalisation to be obtained from the γ-ray nuclear de-excitation lines we find that $[N_p(\pi^0)]_I = 3.2 \pm 0.5 \times 10^{30}$ and $[N_p(\pi^0)]_H = 4.1 \pm 0.6 \times 10^{30}$, where we have used a power-law spectral index appropriate for these parameters and chosen $T_{p0} = 0.1$ GeV. With the normalisation obtained from the observed width of π^0 decay feature we find $[N_p(\pi^0)] = 1.4^{+2.4}_{-1.1} \times 10^{29}$, where the large error bars are

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due entirely to the large spread in the inferred spectral index.

Fewer pion-producing protons are required to produce the observed emission around 70 MeV than would be suggested by a single population of accelerated protons of the form (3). In other words, the γ-ray emission at high energies is softer than would be implied from the low-energy emission. This may indicate that there are two distinct distributions of energetic protons, or that the simple power-law extension of the low-energy Bessel function requires modification. One such modification suggested by ground-based observations of energetic neutrons originating at the Sun (Heristchi, Trottet, and Perez-Peraza, 1976) would be the inclusion of a high-energy cut-off in the accelerated proton spectrum. Such a curtailed spectrum would produce narrow π^0 decay features since the 'average' Doppler broadening would be reduced.

4.1. Effect of High-Energy Cut-Offs in the Proton Spectra

Evidence for a maximum in the energy distribution of particles accelerated at the Sun was discussed by Heristchi, Trottet, and Perez-Peraza (1976). By studying data from the worldwide neutron monitor network, these authors determined energetic proton spectra which displayed upper cut-offs at energies of a few GeV nucl^{-1}. Because the existence of any upper cut-off is inferred from neutron measurements it is argued that it cannot be the result of transport effects in interplanetary space and that, in fact, such an energy distribution is a direct consequence of the acceleration process responsible for the production of the energetic particles. Such a maximum must exist since the attainment of any particular particle energy requires a finite time, the magnitude of this time increasing with attained energy. Since any real acceleration can only have functioned for a finite length of time, the power laws, Bessel functions, etc., characteristic of steady-state acceleration, will inevitably be truncated at high energies (as can be seen, for example, in the time-dependent acceleration discussed by Ramaty, 1979). In what follows we consider the effects of introducing a maximum injected proton energy on the π^0 decay γ-ray emission. We will restrict our attention to the event of December 16, 1988 and assume a transition energy of $T_{p0} = 0.1$ GeV. The need for a high-energy cut-off is even more apparent in the event of March 6, 1989. Our aim here is to obtain an exact correspondence between the spectral index inferred from the intensity of the π^0 decay emission and that inferred from its Doppler width. Naturally, the introduction of a sharp cut-off at high energies will result in a narrower and less intense emission for any given spectral index. Consequently, as the cut-off energy decreases, both δ_I and δ_ω will decrease, i.e., a harder proton spectrum will be required.

Figure 7 demonstrates the variation of the Doppler width with cut-off energy for $\alpha T = 0.03$, one of the values appropriate for the December 16, 1988 flare, $T_{p0} = 0.1$ GeV and $\delta = 2.0$. The introduction of a cut-off energy results in the removal of a significant number of high-energy protons which strongly affects both the intensity (due to the much smaller number of pion-producing protons) and the Doppler width (due to the preferential removal of high-velocity protons). The

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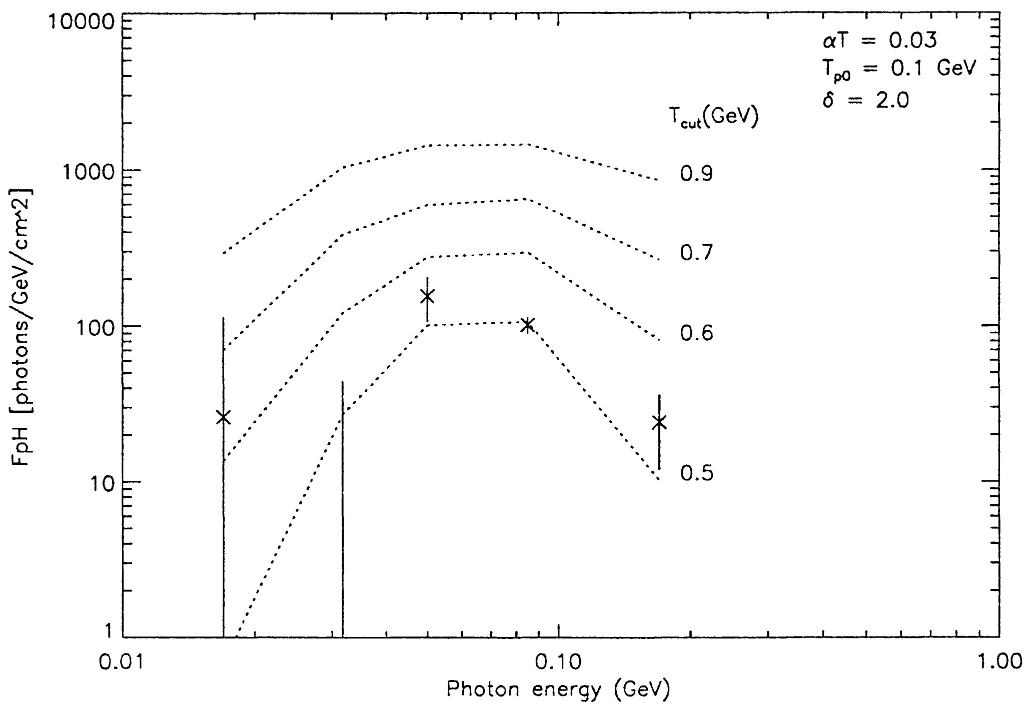

Fig. 7. Comparison of SMM/GRS data for the event of December 16, 1988 with theoretical π^0 decay spectra for a variety of upper cut-off energies. The spectral parameters adopted are shown in the figure.

most important diagnostic here is the Doppler width; the intensity can always be fitted by a suitably hard spectrum. If we require a line shape corresponding to the optimum Gaussian width of $\Delta w = 0.22$, it is found that for all spectra harder than $\delta = 6$ a cut-off energy between 0.5 and 0.6 GeV is necessary. For $\alpha T = 0.03$, corresponding to the isotropic injection model of HL, we find that the optimum logarithmic Gaussian width and the observed γ-ray intensity is achieved with a cut-off of $T_{\text{cut}} = 0.53$ GeV and a spectral index of $\delta = 2.0 \pm 0.1$ (see Figure 7). These alter to $T_{\text{cut}} = 0.53$ GeV and $\delta = 2.5 \pm 0.1$, for the horizontal injection model with $\alpha T = 0.05$. Thus, with the aid of a high-energy cut-off in the injected proton spectrum we are able to obtain a single power-law spectral index from the two independent observational parameters of the π^0 decay emission, namely the intensity and the Doppler broadening.

If we now consider the 68% confidence interval, $\Delta w = 0.22 \pm 0.07$, we find values of $\delta_{\text{max}} = 3.0$ for $\alpha T = 0.03$ and $\delta_{\text{max}} = 3.6$ for $\alpha T = 0.05$ with cut-off energies of 0.7 and 0.8 GeV, respectively. Here δ_{max} corresponds to the softest spectrum compatible with both the 68% confidence interval of the Doppler width and the γ-ray intensity. Thus, with the inclusion of a high-energy cut-off in the accelerated proton spectrum, we are able to obtain a single spectrum which can reproduce the detailed spectral shape of the π^0 decay γ-ray emission (intensity and broadening) observed in the solar flare of December 16, 1988. It should also be noted that our approach of using the γ-ray nuclear de-excitation lines to nor-

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malize our proton spectrum means that our results for this event are automatically compatible with the γ-ray emission in the 4–7 MeV range.

The introduction of a high-energy cut-off is necessary in order to explain the very intense but relatively narrow π^0 decay features in this event and we have demonstrated that cut-off energies of around 700 MeV are sufficient. This finding has implications for models of flare particle acceleration of the kind alluded to above. This will be investigated in more detail elsewhere. The existence of these cut-offs and their magnitudes can also be determined by the detailed analysis of neutron spectra in flare events (cf. Heristchi, Trottet, and Perez-Peraza, 1976). For these flares, however, there have been no reports of positive measurements or upper limits from ground level neutron monitors, which can respond to solar neutrons above a few hundred MeV. Also, we should note that the neutron spectra observed by SMM/GRS for the December 16 and March 9 flares (Dunphy and Chupp, 1991, 1992) cannot rule out a cut-off around 700 MeV.

5. Summary

We have investigated in detail the SMM/GRS high-energy spectra from bursts during the solar flares of December 16, 1988 and March 6, 1989. These bursts exhibited significant >10 MeV emission with evidence for the production and decay of neutral pi-mesons. Theoretical calculations of neutral pion production and decay, used in conjunction with these observations, constrain the accelerated proton spectra responsible for these emissions.

The γ-ray data are analysed using a spectral fitting routine which combines a power-law continuum at low energies (<1 MeV), a nuclear line spectrum (1–10 MeV) and an exponential continuum that connects smoothly to the >10 MeV emission. This spectral fitting procedure enables us to isolate the emission associated with the decay of neutron pions. The intensity and width of this feature prove to be ideal diagnostics fot the study of these high-energy processes in solar flares. In particular, the Doppler broadening is only influenced by the spectral shape of the proton energy distribution above the pion production threshold and the intensity can yield further information regarding the total number of pion producing protons. Thus, by comparing the derived π^0 emission with detailed calculations of π^0 production, we are able to discuss the spectral characteristics of the accelerated protons responsible.

The injected proton distribution is assumed to have the form of a Bessel function up to kinetic energies of a few hundred MeV, coupled to a power law at higher energies. These assumptions assure consistency with the de-excitation line data, while allowing parametric freedom in the distribution at π^0 producing energies. The normalisation of the proton spectrum is achieved by utilising the γ-ray line fluences observed in these events. The spectral form at high energies is then constrained by the π^0 decay radiation. In this work, we have completely ignored any possible contribution from interactions involving accelerated α-particles, since there is no

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unambiguous method of isolating the effects of these ions at the photon energies we consider.

It was found from the theoretical calculations that the π^0 decay emission was extremely sensitive to the choice of spectral parameters. However, the observations indicated that the spectral index derived from the γ-ray intensity was much harder than that derived from the Doppler width. It was argued that the most probable source of this discrepancy was the choice of a continuous power-law representation of the injected proton energy distribution at high energies and that a modification of this spectral form was required to explain the observations of the December 16, 1988 flare. We investigated the effect of introducing a sharp cut-off in the accelerated proton spectra. Such cut-offs have been detected in neutron observations of solar flares (Heristchi, Trottet, and Perez-Peraza, 1976) and are expected on simple theoretical grounds.

The introduction of a sharp cut-off naturally resulted in narrower and less intense emission and it was demonstrated that a single power-law spectral index, which produced photon spectra consistent with both the width and intensity of the observed π^0 emission, could be obtained. The production of an intense, narrow π^0 decay feature required cut-off energies of around 700 MeV, a significant result for particle acceleration models in solar flares.

The sensitivity of the π^0 decay Doppler width to the form of the accelerated proton energy distribution above $\sim$300 MeV makes it an ideal diagnostic of the acceleration of particles to these energies. We have shown here that the limitations of the SMM/GRS are such that the Doppler width determined is subject to a great deal of uncertainty. This is mostly due to contamination of π^0 emission by primary and secondary bremsstrahlung radiation. Instruments with better sensitivity and with spectral capability to much higher energies (e.g., EGRET experiment on the Compton Gamma-Ray Observatory) may realise more fully the potential of the π^0 decay Doppler width as a diagnostic of the highest energy processes occurring in solar flares.

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