Solar Observations with a Millimeter-Wavelength Array[1]

S. M. White and M. R. Kundu

Dept. of Astronomy, Univ. of Maryland, College Park MD 20742

submitted to **Solar Phys.**, *1991 December*
revised, 1992 May

[1] Contributed paper for the 1991 CESRA meeting, Ouranopoulis, Greece

Abstract

Rapid developments in the techniques of interferometry at millimeter wavelengths now permit the use of telescope arrays similar to the Very Large Array at microwave wavelengths. These new arrays represent improvements of orders of magnitude in the spatial resolution and sensitivity of millimeter observations of the Sun, and will allow us to map the solar chromosphere at high spatial resolution and to study solar radio burst sources at millimeter wavelengths with high spatial and temporal resolution. Here we discuss the emission mechanisms at millimeter wavelengths and the phenomena which we expect will be the focus of such studies. We show that the flare observations study the most energetic electrons produced in solar flares, and can be used to constrain models for electron acceleration. We discuss the advantages and disadvantages of millimeter interferometry, and in particular focus on the use of and techniques for arrays of small numbers of telescopes.

Subject headings: Sun: flares; Sun: radio radiation

1. Introduction

The purpose of this article is effectively to introduce the field of solar millimeter interferometry. In recent years solar observations at millimeter wavelengths have been relatively few and underemphasized, particularly in the West, when compared with solar observations at microwave wavelengths. For the last fifteen years microwave observations have been dominated by large multielement sidereal arrays (the Westerbork Synthesis Radio Telescope and the Very Large Array). Observations at millimeter wavelengths have not kept pace with the microwave observations, because they have been limited to single-dish radiotelescopes which cannot match the spatial resolution, sensitivity and ability to make maps on very short timescales which these microwave arrays offer. However, recently there have been rapid developments in arrays operating at millimeter wavelengths, and there are now such arrays operating in Japan (the 5–element interferometer at Nobeyama), the U.S. (3–element arrays at Hat Creek and Owens Valley) and Europe (the IRAM array). Most of these arrays are in the process of adding new elements, and in the US there are plans to build a 40–element Millimeter Array. Given appropriate encouragement these new facilities will be an important tool in solar radiophysics for the next decade. We have been using the Berkeley-Illinois-Maryland Array (BIMA) at Hat Creek to study the Sun since 1989. This paper is a report on these observations and on solar millimeter observing in general, as a guide for future use. We will discuss the radiation mechanisms relevant for solar millimeter astronomy, compare observations in the millimeter and microwave wavelength ranges, describe the use of closure phases which is important for arrays containing a small number of elements, and give some examples of observations so far. This article is not written solely for solar radio astronomers, and so we will describe some basic concepts for the general reader where relevant. We start with a review of recent work in millimeter solar radio astronomy.

The early work in solar millimeter–wavelength astronomy has been reviewed by Kundu (1965) and Kundu (1982), and we will not cover it in detail here. All of the early observations (late 1950's) involved single-dish telescopes, whose sensitivity is greatly limited by atmospheric conditions and fluctuations (see below, section 4). Generally they were confined to measurements of the solar brightness temperature at millimeter wavelengths. Modelling of the chromosphere was also possible using observations of the brightness distribution at the limb during eclipses, and anomalous results so obtained were explained in terms of fine structure (Hagen 1957). Even from the early observations it was known that the solar disk at millimeter wavelengths was relatively uniform (i.e., little contrast across the disk; Coates 1958), while limb brightening was seen in the eclipse observations. Single-dish mapping in the late 1960's and early 1970's showed that active regions can have peak excess brightness temperatures of 700 K at 3.5 mm (Kundu 1970); they tend to be bipolar when observed in circularly polarized emission, with low degrees of polarization (Kundu and Gergely 1973).

Most of the work in solar millimeter astronomy within the last 10 years has been carried out in the USSR, Japan, Brasil and Finland. In the Soviet Union the RT-25$\times$2 transit radiotelescope has been used at 6.3 and 8.6 mm to scan across the solar disk with a $1.1' \times 11'$ beam; by this technique the solar radius can be measured, and solar cycle variations in brightness studied (Pelyushenko and Chernyshev 1983; Pelyushenko 1985). Mapping has been carried out with the RT-7.5 millimeter radio telescope at 3 mm wavelength, with $2'$ resolution (Nagnibeda and Piotrovich 1990). These observations found that the active-latitude belts showed enhanced emission even in the absence of active regions on the disk. Urpo, Hildebrand and Krüger (1987) have also mapped the Sun at several millimeter wavelengths using the 14–m Metsähovi telescope in Finland, and have compared the results with models to conclude that fine structure does play a role in the observed active-region brightness temperatures.

In Japan the Nobeyama 46m radiotelescope has been used to map the Sun with high spatial resolution: the beam sizes are $46''$ at 36 GHz and $17''$ at 98 GHz (Kosugi *et al.* 1986; Shibasaki 1992). These are probably the highest spatial-resolution maps of the Sun at 3 mm made so far. They confirm the existence of a thin brightening at 36 GHz (but not at 98 GHz) associated with the polar-cap coronal holes, which was first seen in observations by Kundu and McCullough (1972), but first noticed by Babin *et al.* (1976).

Another large single-dish millimeter telescope used for solar work is the 14m Itapetinga antenna in Brasil (Kaufmann *et al.* 1982). This has a beam size of $2'$ at 44 GHz and $80''$ at 90 GHz. Maps made with drift-scan observations have been used to measure the height of the solar limb at 44 GHz (Costa *et al.* 1986).

There have also been a number of theoretical studies investigating the likely properties of millimeter emission. Zlotnik (1987) has investigated the likely spectra of active region emission in the mm/short cm regime and the conditions under which the radio spectrum will turn up again at higher frequencies. Krüger and Hildebrandt (1988) have calculated the expected circular polarization of active region emission at mm wavelengths.

The literature on observations of solar flares at millimeter wavelengths is sparse by comparison with the literature on microwave observations. This is easily understood. Most solar flares exhibit a radio spectrum which falls towards higher frequencies, and thus there is less flux available at millimeter wavelengths. However, the solar thermal flux rises with frequency, so that the background level against which any enhancement due to a flare must be seen is higher (typically 10^4 sfu at 3 mm wavelength). The intrinsic sensitivity of the receivers at the higher frequencies is often poorer than at longer wavelengths, and any real solar variations must be distinguished from fluctuations due to short-term variations in the opacity of the sky at millimeter wavelengths (discussed further in section 4). All of these effects make it difficult to observe solar flares with a single-dish millimeter telescope: effectively only the largest solar flares could be seen at millimeter wavelengths. Croom (1970) reported minimum detectable fluxes of 370 sfu at 70 GHz with a 1 m dish in clear sky conditions, degrading to 750 – 1500 sfu in more usual weather conditions. Few bursts ever reach such fluxes at millimeter wavelengths: in over two years of monitoring during a solar maximum Croom (1970) saw only seven bursts. Early observations of millimeter radio bursts are presented by Croom and Powell (1969), Croom (1970), Feix (1970), Shimabukuro (1970, 1972), Cogdell (1972) and Akabane *et al.* (1973).

An important conclusion of these observations was that most of the observed millimeter flux was due to thermal bremsstrahlung from hot dense plasma produced in the corona after the impulsive phase of the flare, which was also responsible for strong enhancements seen in soft X-ray emission (Shimabukuro 1970; Shimabukuro 1972; Hudson and Ohki 1972).

For single-dish observations of flares a larger telescope is better, not only because of the larger collecting area but also because the smaller beam admits less of the solar thermal flux and thus the background level is lower. Kaufmann *et al.* (1985) have used the Itapetinga telescope at several millimeter wavelengths to study a solar flare, with surprising results. They can reach an effective sensitivity of about 1 sfu and operate with high time resolution (1 millisecond). The flare studied showed two remarkable features: a radio spectrum which was microwave-poor and whose peak frequency was above 90 GHz, in contrast to peak frequencies near 10 GHz which are normal; and the presence of rapid subsecond oscillations at 90 GHz which correlated well with similar variations seen in simultaneous hard X-ray measurements. de Jager *et al.* (1987) interpreted this event as due to hot (5×10^8 K), dense (10^{11} cm^{-3}) plasma in a strong magnetic field region (1400 – 2000 G) in the corona. The very high turnover frequency can also be attributed to synchrotron emission by ultrarelativistic electrons (Kaufmann *et al.* 1986); for this reason, gamma-ray-emitting flares are expected to be strong sources of millimeter emission. This point may have been first made by Kawabata *et al.* (1982), who gave a number of examples at 35 GHz. More recent single-dish (RT-25$\times$2) observations of a solar flare at 3, 4, 6 and 8 mm have been reported by Mal'tsev *et al.* (1990), and RT-7.5 telescope observations of two flares by Nagnibeda *et al.* (1990). Long-duration off-limb mm burst sources have been studied by Urpo *et al.* (1989); similar emission associated with an eruptive prominence has been studied by Zodi *et al.* (1988).

One of the first uses of an interferometer at (long) millimeter wavelengths was by the group at Nagoya, who have used a fan-beam interferometer operating at 35 GHz (8.6 mm) since 1970 (Kawabata *et al.* 1974). This telescope consists of 8 antennas used as an adding interferometer, and produces one-dimensional fanbeam scans with a spatial resolution of about $30''$. The group finds a good association between strong 35 GHz bursts and γ–ray emission (Kawabata *et al.* 1982), although this result may be influenced by the "Big Flare Syndrome" (Kahler 1982). They also note that, at least in the stage when the flare emission is at its peak, there often seem to be two spatially-separated components in the millimeter emission; motions have also been seen. Recent flare observations with this instrument have been reported by Wang *et al.* (1987) and Kawabata and Ogawa (1989).

The first use of an interferometer to study the Sun at short millimeter wavelengths (here 3 mm) that we know of was carried out by an Australian group in 1976, in order to study the brightness distribution at the limb during a solar eclipse. This observation was all the more remarkable because the interferometer was portable (Archer 1977; Labrum 1978; Labrum *et al.* 1978). The first flare observation with a millimeter interferometer in the 3 mm window may have been carried out with the (then) Berkeley two-element interferometer at Hat Creek in 1980, during the balloon flight of a hard X-ray spectrometer (Lin *et al.* 1981). A flare was actually detected, but it was outside the time of the hard X-ray observations, and the data were never pursued (J. Bieging, private communication). Patrol observations on a regular basis at 3 mm using an interferometer are carried out by the group at Nobeyama, who correlate the signals from two small dishes placed at a separation such that the correlated amplitude is not sensitive to the flux from the solar disk. In this way they are able to greatly improve the sensitivity of solar flare measurements at 3 mm, and can routinely achieve sensitivities down to 10 sfu over the whole disk (Nakajima *et al.* 1985).

The present 3–element BIMA array affords much greater sensitivity and spatial resolution than the observations reviewed above. We have reported BIMA interferometer observations at 3.5 mm in several publications (Kundu *et al.* 1990; Kundu *et al.* 1991; White *et al.* 1992; Lim *et al.* 1992). In the following sections we first describe the processes that are expected to produce solar millimeter emission, then go on to describe some of the aspects of observations peculiar to solar millimeter interferometry based on our experience at BIMA.

2. Emission Mechanisms at Millimeter Wavelengths

In this section we will discuss the likely emission mechanisms which will be relevant at millimeter wavelengths, and their properties. A general review of the relevant emission mechanisms may be found in Dulk (1985). Details of gyrosynchrotron emission specifically relevant to solar observations were discussed by Ramaty (1969), Holt and Ramaty (1969), Ramaty and Petrosian (1972), and Takakura (1972).

2.1. Gyroemission

Electrons moving in a magnetic field exhibit a gyration about the direction of the magnetic field, and the acceleration associated with this gyration leads to radiation by the gyromagnetic process. The electron gyrofrequency associated with the gyration of electrons in a magnetic field B is $f_B = 2.8 \times 10^6 \ B_{gauss}$. In the solar photosphere the magnetic field rarely exceeds 3000 G; in the corona it does not usually exceed 2000 G (Abramov-Maksimov and Gelfreikh 1983; White *et al.* 1991), corresponding to a gyrofrequency of 6 GHz. Thus emission at 86 GHz (in the 3 mm atmospheric window; atmospheric effects discussed below only permit short millimeter wavelength observations in "windows" at 1 and 3 mm) must be at a harmonic of at least 14. Because the typical frequency of gyroemission of a particle with Lorentz factror γ is $\gamma^3 f_B$, nonrelativistic particles will not emit at such high harmonics; in particular, a thermal plasma, even at a temperature of 10^8 K, has negligible opacity at 86 GHz, and thus only nonthermal gyroemission by relativistic electrons can be seen at millimeter wavelengths. This is only likely to occur during solar flares. Except in rare cases the emission will be optically thin, and the radio spectral index α will be related to the electron energy spectral index δ by the expression

$$\alpha = 1.22 - 0.90\delta \tag{1}$$

(Dulk and Marsh 1982; this assumes that the typical harmonic of emission does not exceed about 100).

Based on this argument, in solar flares the radio emission at millimeter wavelengths should be most sensitive to electrons with energies in excess of 1 MeV, i.e., those electrons which emit gamma rays rather than those which emit hard X-rays in the range 25 – 100 keV. To quantify this statement somewhat, Figure 1 shows what happens to the gyrosynchrotron spectrum of radio emission from nonthermal electrons in a homogeneous magnetic field as one removes the lower energy electrons. The energy spectral index is 4.00, and the magnetic field is 300 G. The electron energy distribution has an upper cutoff at 10 MeV, and low energy cutoffs of 10, 100, 300, 1000 and 2000 keV. As the low-energy electrons are removed, two things happen: the low-frequency optically-thick flux increases; and the flux in the microwave range above the peak in the spectrum drops once we start removing the electrons with energies above 300 keV. The first effect occurs because the flux level in the optically thick limit is determined by the average energy of the electrons, and as we remove the low-energy electrons the mean energy must rise: this demonstrates the important point that the optically thick low-frequency microwave emission can be quite sensitive to the electrons which emit hard X-rays (energy range 10 – 100 keV). At the same time the frequency of the spectral peak drops and the optically-thin emission above the peak decreases because there are fewer electrons present. It is clear that most of the optically-thin microwave flux is due to electrons with energies above 300 keV, and that the flux at 86 GHz is not affected until one starts removing MeV electrons, confirming that millimeter emission is largely due to the MeV energy electrons.

This is demonstrated further in Figure 2, where we have plotted contours of the ratio of the 86 GHz flux from a distribution with a lower cutoff at 1 MeV to the flux from a distribution with a lower cutoff at 10 keV, as a function of energy spectral index and of magnetic field, for two values of the angle θ between the line-of-sight and the magnetic field, 30° and 70°. For θ = 30° and magnetic field strengths below 1000 G essentially all the 86 GHz flux comes from the MeV electrons. At θ = 70° the lower-energy electrons contribute relatively more at 86 GHz. As one goes to steeper and steeper energy distributions the ratio decreases for a given magnetic field, because there are relatively fewer MeV electrons present in the first place.

Thus we expect that flares which have no MeV electrons will produce little or no detectable millimeter emission in the impulsive phase. If the energetic electrons in solar flares typically have energy spectra of the form deduced for one flare by Lin and Schwartz (1987), which falls off steeply above several hundred keV, and if the gamma-ray producing flares really are a different class of flares and are all large (Bai and Sturrock 1989), then a typical flare will not produce much millimeter emission in the impulsive phase. It has not really been possible to test this until now, because millimeter observations could only see the large flares, not the small flares. Observations with a millimeter interferometer are sensitive enough to observe millimeter emission from "typical" small flares and can thus test this idea. The observations we have so far indicate that most small impulsive flares do produce MeV–energy electrons (Kundu *et al.* 1994; Lim *et al.* 1992).

Finally we add a cautionary note. The results of this section suggest that millimeter emission in the impulsive phase of solar flares should come predominantly from high-magnetic-field regions, i.e., at the footpoints of magnetic loops, because gyrosynchrotron emissivity is a strong function of magnetic field strength. However, the same prediction can be made for high–microwave–frequency burst sources (e.g., see the discussion by Petrosian 1982), and this prediction is often found to be wrong (Alissandrakis and Kundu 1978; Marsh and Hurford 1980; Hoyng *et al.* 1983; Willson 1983; Kundu *et al.* 1987). We may well find that millimeter impulsive burst sources also occur over photospheric neutral lines, where simple theory leads one to expect that the field strength could be relatively low (this has been claimed for 35 GHz sources by Kawabata and Ogawa 1989). If this is the case, it may indicate that extreme cases of trapping of energetic electrons at the tops of magnetic loops are common.

2.2. Plasma emission

The electron plasma frequency, associated with oscillations of plasma electrons with respect to the background distribution of ions, $f_p = 9 \times 10^3 \sqrt{n_e}$, reaches values of only 10 GHz in a chromospheric density of 10^{12} cm^{-3}. Even if emission at high harmonics of the plasma frequency were possible in the chromosphere, it would be rapidly absorbed by collisional absorption in the high density plasma. Thus plasma emission, which is responsible for much of the metric and decimetric solar radio emission, is unlikely to play any role at millimeter wavelengths. The highest frequency at which it may have been seen on the Sun is 8 GHz (Bruggman *et al.* 1990).

2.3. Thermal bremsstrahlung

The mechanism which will dominate non-flare emission from the Sun at millimeter wavelengths is thermal bremsstrahlung (or collisional absorption). The opacity of this mechanism has the form

$$\kappa = 0.01 \begin{Bmatrix} 18.2 + 1.5 \ln T - \ln f \\ 24.5 + \ln T - \ln f \end{Bmatrix} \frac{n_e^2}{f^2 T^{1.5}}\ \mathrm{cm}^{-1} \qquad \begin{cases} T < 2 \times 10^5\ \mathrm{K} \\ T > 2 \times 10^5\ \mathrm{K} \end{cases} \tag{2}$$

where n_e is the electron density and f the frequency (Dulk 1985). Because thermal opacity favours cool dense plasmas and low frequencies, the corona is optically thin to thermal bremsstrahlung at millimeter wavelengths. This may be seen from the typical peak brightness temperature of an active region at 15 GHz of around 30,000 K (White *et al.* 1991; White *et al.* 1992); given the f^{-2} law of eqn. (2), this corresponds to a maximum brightness temperature contribution of 1000 K at 86 GHz. Outside active regions the coronal contribution will be much less than 100 K at 86 GHz, and can be considered negligible compared with the expected contrast provided by density structure in the chromosphere (discussed below). At 300 GHz (the 1 mm atmospheric window) the coronal contribution will less than 100 K everywhere.

The chromosphere will be optically thick to thermal bremsstrahlung at some height. The chromospheric models of Vernazza, Avrett and Loeser (VAL; 1981), based predominantly on EUV observations, predict that at 3 mm the chromosphere becomes optically thick at heights between 1300 and 2000 km, where the temperature is around 7000 K and the gradient of temperature with height is relatively small. Quiet-Sun measurements (summarized by Vernazza *et al.* 1981) at 3 mm show temperatures of 6500 – 7500 K, in agreement with their models. The models do not include the effects of fine structure, notably spicules: these are small-scale jets which reach from the photosphere up to heights of about 6000 km, and they seem to dominate the brightness distribution at the solar limb where there are many spicules along any line of sight. When looking down on disk center, the filling factor of spicules within the viewing area is presumably small and they should not dominate the emission. However, this will be one of the questions to be addressed by high-spatial-resolution observations: what role do spicules play in millimeter emission seen on the disk? Spicules on the disk cannot be seen optically because of their low optical depth.

As we discuss below, an interferometer is only sensitive to flux on spatial scales smaller than the corresponding fringe spacing, and hence does not see the uniform background solar flux. The fact that temperature increases with height in the chromosphere in the altitude range where radiation at 3 mm is optically thick is of great importance. What the interferometer will see are the temperature fluctuations across the optically thick (τ = 1) layer; these temperature fluctuations will be largely due to local density enhancements which boost the height of the τ = 1 layer up to a height where the temperature is greater. Thus we expect relative contrasts typically of several hundred K at 3 mm due to chromospheric structure in quiet regions. More so than at microwave wavelengths, we expect that there will be both positive and negative contrasts with respect to the "uniform" background, since low-density regions will drop the optically-thick level to lower temperatures. Where the density is very high, 3 mm emission may become optically thick above 2000 km in the narrow ($\sim$ 100 km) jump region from the chromospheric temperature plateau at around 7000 K to the Lyman step at about 25,000 K; in this region the temperature gradient is very steep with height, and we could see contrasts of several thousand K (however, it should also be noted that the latest atmospheric models apparently differ somewhat from VAL in the region of the Lyman step; Avrett 1992).

Because the temperature gradient in the chromospheric plateau region is small, the models of Vernazza *et al.* (1981) also predict that there will be a quite significant contribution to the brightness temperature from the optically thin region above the τ = 1 layer at 3 mm, and this contribution will be sensitive to local density gradients in the upper part of the chromospheric plateau. However, the contribution ceases at the steep temperature rise to the Lyman step, because of the $T^{-1.5}$ dependence of the opacity.

Observations at 3 mm up to now have been limited by spatial resolution in measuring the true brightness temperatures of fine-scale features. During 82 days of observations in 1967 Mayfield, Higman and Samson (1970) measured enhancements of up to 1000 K at 3.3 mm with a 2.8′ beam. Using a beamwidth of 1.2′ Kundu (1970) found enhancements of up to 700 K at 3.5 mm associated with active regions. However, at these spatial resolutions these observations represent an average over an active region, and we expect that the true peak enhancements would have been somewhat larger. Emission features associated with prominences above the limb have also been seen (Kundu 1972); however, filaments on the disk appear as density depressions of up to 400 K at this resolution (Kundu 1970). Higher resolution observations such as those of Kosugi *et al.* (1986; 17″) have enough resolution to measure the enhancements associated with features such as the supergranular network (Shibasaki 1992).

At 1 mm wavelength thermal radiation arises much lower in the chromosphere in general, and probably by a different mechanism. Vernazza *et al.*'s (1981) models predict that 1 mm will become optically thick in the range 600 – 1400 km above the photosphere. In these levels the temperature is below 6000 K, and the ionization fraction of hydrogen is low. There are more H^- ions present than there are protons, and electron–H^- collisions provide the bulk of the opacity at 1 mm (Vernazza *et al.* 1976). There will also probably be less optically thin contribution to the observed brightness from the chromosphere above the τ = 1 layer at 1 mm. High-resolution observations probing these layers at 1 mm and shorter wavelengths have been carried out recently with the 15m James Clerk Maxwell Telescope (Lindsey *et al.* 1990; Naylor *et al.* 1991; Lindsey and Roellig 1991; Lindsey and Jefferies 1991). They indeed find that the typical temperature of the quiet Sun at 0.85 mm wavelength is about 5400 K, but also that temperatures up to 7000 K may be seen in regions of plage. They conclude that their observations are consistent with the range of the VAL models.

2.4. Thermal bremsstrahlung in flares

Solar flares can produce much hot, dense plasma in the corona which emits strongly at soft X-ray wavelengths. This same plasma will also radiate strongly at millimeter wavelengths due to optically-thin thermal bremsstrahlung. In this subsection we present some simple formulae for predicting the flux at 3 mm from the soft X-ray measurements of the GOES satellites, using the results of Thomas *et al.* (1985).

The procedure is to model the soft-X-ray emitting plasma as a single–temperature plasma. The GOES detectors produce broadband measurements of the soft X-ray flux in two wavelength ranges, 0.5 – 4 Å (3 – 25 keV) and 1 – 8 Å (1.5 – 10 keV). We label the instantaneous fluxes (in W m^{-2} above the background) in these two wavelength ranges as B_4 and B_8, respectively. Following Thomas *et al.* (1985) we define the ratio of these fluxes, $R(T) = B_4/B_8$, which is a function of temperature alone. The temperature is then determined by the following formula:

$$T(R) = 3.15 + 77.2R - 164R^2 + 205R^3, \tag{3}$$

where T is in millions of degrees K. To determine the emission measure we must first calculate the temperature-dependent part of the expression for the flux from the temperature T:

$$10^{55} b_8(T) = -3.86 + 1.17T - 0.0131T^2 + 1.78 \times 10^{-4} T^3 \tag{4}$$

Then the emission measure is

$$EM = \frac{B_8}{b_8} \tag{5}$$

in units of cm^{-3}.

The radio flux may now be derived by assuming that the plasma (typically 10^7 K) is optically thin at high frequencies, which should be satisfied (e.g., with a line-of-sight depth of 10^{10} cm and a temperature of 10^7 K, one requires a density of 3×10^{12} cm^{-3} to make the soft-X-ray plasma optically thick at 86 GHz; the corresponding GOES X-ray class of such a flare would be X10^4 for a volume of dimension 10^{10} cm). With T and *EM* known, the optically-thin radio flux is

$$S = 1.4 \times 10^{-45} \frac{EM}{\sqrt{10^6 T}} \text{ sfu.} \tag{6}$$

(here T is still measured in units of 10^6 K). A similar expression was derived by Hudson and Ohki (1972). The flux is independent of frequency as long as the emission is optically thin. Typically a GOES-class M flare has a peak emission measure of 10^{49} cm^{-3}; at a temperature of 10^7 K the radio flux due to the soft-X-ray emitting material will be 4 sfu.

We note two things, however. The soft-X-ray emitting plasma is generally distributed over large spatial scales, and if the fringe spacing on a given baseline is short, it will not see the emission from the smoothly-distributed soft-X-ray plasma. Secondly, the asumption of a single-temperature plasma is almost certainly incorrect for the material in the corona during a flare. The radio emission falls as temperature rises, whereas the soft X-ray emission rises as temperature rises. Thus the radio observations are more sensitive to cool material, whereas the GOES measurements are more sensitive to hot material. Any significant amounts of plasma in the temperature range 10^5 – 10^6 K could easily produce a stronger response at millimeter wavelengths than the plasma seen by GOES.

4. Atmospheric Effects at Millimeter Wavelengths

The atmosphere plays a much more important role at millimeter wavelengths than it does at microwave wavelengths. This is due to two, not exactly unrelated, effects (e.g., see Chap. 13 of Thomson *et al.* 1986).

One of these effects is atmospheric opacity due to the large number of strong rotational transitions which common atmospheric species such as H_2O and O_2 display in the millimeter range. Opacity is very large from 50 to 70 GHz, at 120 GHz and from 170 to 200 GHz, which leaves suitable observing ranges known as "atmospheric windows" at 9 mm, 3mm and 1 mm. This opacity is usually not too much of a problem for solar observations with an interferometer, since we are generally doing continuum observations and can choose our observing frequency to lie in one of the "atmospheric windows" where opacity is small. It does become a problem when heavy wet clouds pass in front of the Sun; these can easily render the sky effectively opaque even at frequencies in an atmospheric window. Opacity is also a more serious problem for single-dish millimeter telescopes, since small variations in the brightness temperature of the sky are difficult to distinguish from real variations on the Sun. Interferometer amplitudes are not sensitive to such small variations in the sky, since the lateral scale lengths of these variations in the atmosphere are usually larger than a typical baseline and their effects are thus the same at both ends of the baseline.

The second effect is much more severe for an interferometer. This is the variable phase shift in the signal introduced by varying refractive index distributions along the atmospheric paths to the two telescopes which form an interferometer pair. The molecular components in the atmosphere affect the refractive index of the atmosphere, and the refractive index along the line of sight to a telescope determines the actual path length measured in wavelengths, which is the important quantity when correlating the signals from the two telescopes. Atmospheric molecules typically contribute 10^{-8} to the index of refraction. At 3 mm wavelength the atmosphere is at least 3×10^7 wavelengths thick, and a difference of only 10^{-9} between the mean indices of refraction along the two paths is enough to change the correlated phase by about 10°. Clearly this effect will be worse when the atmosphere is disturbed and sharp inhomogeneities of the atmospheric water vapour distribution on length scales comparable to a typical interferometer baseline are common (e.g. late on summer afternoons when thermal updrafts carry water vapour from the surface up into the atmosphere); similarly, it is always worse on long rather than short baselines because variations on two widely separated lines of sight are less likely to be correlated than they would be on nearby lines of sight. These effects are readily seen directly in solar millimeter observations as a wandering of the phase on short timescales (e.g., see Fig. 4 below), and acts as a source of phase noise which degrades the dynamic range of the maps. Fortunately, the Sun is such a strong source that the technique of self-calibration (e.g., see the discussion in Thompson *et al.* 1986, Chap. 11) should be able to largely remove this phase noise for multi-element arrays (although not for three-element arrays). For most (weak) cosmic sources this is not possible.

5. The Sun as a Target for a Millimeter Interferometer

The Sun is a very useful diagnostic target for a millimeter interferometer, because it is one of the few objects in the sky which is brighter than the sky at millimeter wavelengths! This means that some effects, which may be important for all observations but are inconspicuous in observations of (relatively weak) celestial sources, show up strongly in solar observations. We cite two examples from our own experience at BIMA. One of the antennas had a problem tracking the correct sky position in even light winds for a time due to backlash in the telescope drives. This showed up prominently in solar observations because the effect was bad enough to blow the dish off the Sun and the total power dropped sharply; in observations of other sources where the sky dominates the signal this could not be seen directly in the data because one part of the sky is as bright as another. The fact that this tracking error occurred for a large fraction of daylight observing was not appreciated by all telescope users. A second effect is an oscillation in the total power from the receiver in one antenna which appears to be associated with a water chiller cycle on the telescope, and which produces a corresponding sporadic oscillation (at around the solar oscillation period of 300 seconds, unfortunately) in the correlated amplitudes and phases on baselines including this telescope. The cause of this is not yet fully understood.

The Sun is often also a dangerous target for millimeter telescopes. In order to work at short wavelengths telescopes must have very smooth surfaces, and if the surface is *too* smooth then a large fraction of the Sun's enormous thermal flux at infrared wavelengths can be sent into the receiver system. Even a small fraction of the Sun's infrared flux, concentrated on a very small area by a 10m-class telescope, is more than enough to melt the electronics. This happened most recently to the Swedish SEST telescope by accident, when the Sun drifted through the beam of the telescope while it was stowed.

This is not an insurmountable problem as long as it is anticipated. Two approaches are usually used. Telescopes operating in the submillimeter range need to have dishes so smooth that they must specularly reflect a large fraction of the solar infrared: these can be shielded by use of a telescope awning which is opaque in the infrared but transparent in the millimeter range (e.g., the Goretex cover used at the James Clerk Maxwell Telescope, or the plastic "tent" used recently at the Caltech Submillimeter telescope). At millimeter telescopes the surface can be "roughened" to avoid specular reflection of the infrared: at BIMA this is achieved by "scalloping" the metal surface of the dish when it is machined, while at the Owens Valley millimeter array it is now achieved by the simple expedient of painting the dish surfaces white (before this precaution was taken some damage to the telescopes resulted from an unshielded solar observation).

Another difficulty with observing the Sun at high frequencies is that the field of view tends to be very restricted, and if one is studying activity there is a good chance that any flares will occur outside the field of view. This will continue to be a problem for solar radioastronomers, since the drive in the non-solar community is usually to use larger dishes with better collecting area and thus better sensitivity. A 6–m diameter dish is excellent for 3 mm solar observations, since it has a primary beam of $2'$ and most active regions fit nicely into its field of view. With 10–m dishes some large active regions will not fit within the field of view. At 1 mm the field of view of even a 6–m dish is so small that an active region cannot be mapped with only one pointing position; however, there are mosaicking techniques which allow us to build up maps of a large region from a number of smaller maps.

The science issues to be addressed by observations with a millimeter array have been recently summarized by Dulk *et al.* (1990).

6. Observing with a 3–Element Array

Observations with the BIMA telescope are described by Kundu *et al.* (1990) and White *et al.* (1992). In order to achieve the highest possible time resolution we use the analog correlator operating in the 3 mm window. The time resolution is presently computer-limited to about 0.4 seconds. Two sidebands separated by 3.2 GHz are observed; our standard frequencies are the SiO frequency at 86 GHz plus the sideband at 89 GHz. This is merely for convenience, and because one can then also use strong SiO maser sources as calibrators.

The present observations at the BIMA interferometer can only use 3 elements. This provides only 3 baselines (each interferometer pair forms one baseline) and is not adequate to map either steady features using earth-rotation synthesis, or flares using snapshot mapping. We must obtain all our information from the three baselines, each of which provides us with one amplitude and one phase (described below) at any instant. Thus observing is considerably different from using an instrument such as the VLA, where typically one proceeds by making maps without ever inspecting individual baselines (of which there are up to 351 at the VLA, i.e., too many to inspect).

In interferometry the signals from each pair of antennas are correlated, resulting in a one-dimensional spatial Fourier transform (a complex number) of the brightness distribution within the primary beam of the telescope. In the case of BIMA, the 6m dishes have a primary beam width at the half-power points of 2.3 arcminutes at 86 GHz; they would have a beamwidth of only 0.6 arcmin in the 1 mm window. The length D of the projection of the baseline joining the two antennas onto the plane orthogonal to the line–of–sight to the source determines the fringe spacing (or wavelength) F of the spatial Fourier transform according to the formula $F = 2.06 \times 10^5\ \lambda/D$ arcsec, and the line of nodes of the Fourier pattern is parallel to the projected baseline on the sky. For example, at BIMA a typical short projected baseline of 10 meters at λ = 3.3 mm gives $F = 70''$; a typical long projected baseline of 100 meters gives $F = 7''$.

If the radio brightness distribution on the sky is a point source, then the amplitude of the complex number resulting from the spatial Fourier transform is the flux of the source, and the phase represents the distance of the source from the pointing center along the direction of the Fourier transform, modulo 2π. For this reason burst location is difficult with 3–baseline observations: each baseline provides a set of parallel straight lines separated by a fringe spacing on the sky as equally likely locations, and the burst could be at any intersection of the three sets of lines. Given the effective width of the lines due to noise and source width, a large number of possible positions results (which is smaller when the baselines are shorter), and in the absence of other information we have no way of deciding which intersection is "correct".

It is straightforward to derive the response of an interferometer to a gaussian-shaped brightness distribution. Suppose that the source is located at right ascension α_1 and declination δ_1, and has a total flux S distributed on the sky as

$$\exp\left[-\frac{(\alpha-\alpha_1)^2\cos^2\delta_1}{2\Delta\alpha^2}-\frac{(\delta-\delta_1)^2}{2\Delta\delta^2}\right] \tag{7}$$

(i.e., angular widths $\Delta\alpha$ and $\Delta\delta$ in right ascension and declination, measured here in radians). Let u and v be the east-west and north-south components of the projected baseline, respectively, measured in units of length, and the telescope pointing center be (α_0,δ_0). Then the (calibrated) correlated amplitude measured by the interferometer is

$$|V| \;=\; S\exp\left[-2\pi^2\,\frac{u^2\Delta\alpha^2+v^2\Delta\delta^2}{\lambda^2}\right] \tag{8}$$

while the (calibrated) phase is

$$2\pi\frac{u(\alpha_1-\alpha_0)\cos\delta_0+v(\delta_1-\delta_0)}{\lambda} \tag{9}$$

Effectively the visibility amplitude varies as

$$|V| \;=\; S\exp\left[-\frac{\pi^2}{2}\,\frac{\Delta\theta^2}{F^2}\right] \tag{10}$$

where $\Delta\theta$ is the angular width of the source on the sky (approximately the full-width-half-maximum): thus unless the angular width is less than about one-tenth of the fringe spacing the flux measured by the interferometer will be significantly attenuated. If the gaussian width of the source on the sky is of the order of the fringe spacing then very little of the flux will be seen, and this is the basis for the statement that an interferometer does not respond to structures larger than a fringe spacing.

Thus with three baselines we can obtain information on the flux on the spatial scales corresponding to each of the fringe spacings. In principal there is enough information to fit a simple gaussian of the form (1) and to determine S, $\Delta\alpha$ and $\Delta\delta$. In practice, however, it is rare that the actual brightness distribution on the sky can be modelled by such a simple function. Most sources consist of several components on different spatial scales. In particular, for active regions there is invariably a lot of structure within the primary beam and simple models cannot be useful. For radio bursts simple models may be applicable in some circumstances, but usually the best we can do is determine the minimum flux of the burst at millimeter wavelengths, and estimate the spatial scale of the source size by comparison of the fluxes on the different baselines. When the burst is larger than the largest available fringe spacing we expect that the time profiles on the three baselines will not match each other well. We also obtain information from closure phase, which we discuss further below.

In principle we can also use the total power information from the telescope: an output voltage from each receiver, which is equivalent to a total power measurement, is recorded at each integration. The larger flares recorded by BIMA are often seen in these total power data. In practice, two problems have presented us from using this information effectively. The most serious is the effect of delay line jumps. In an interferometer the "pointing" is achieved by combining the signal from each telescope with the appropriate phasing of the signals so that the path lengths of the signals from a point source exactly at the pointing center to the correlator (where the signals are combined) are the same for each antenna. Phase changes are achieved by sending the signals through different lengths of delay line which produce different time delays and thus control the phase. As the source moves across the sky the required phase delay changes for each antenna. Since only the relative phases of the signals are important, in practice phasing is carried out with reference to the center of the array so that an antenna there can maintain a constant delay while an antenna at any other station must have its delay varied. Unfortunately, to change the delay as a source moves on the sky different lengths of delay line are moved in and out of the path, and each time this happens there is a small (purely instrumental) jump in the total power voltage. This voltage jump is often larger than a small flare appears in total power.

For an antenna close to the array center the jumps may occur, e.g., every 20 seconds, but for antennas far from the array center (corresponding to observations with small fringe spacings) they may occur every second. We presently attempt to remove these jumps by recording when the delay line steps occur and simply removing any jump in the data at that time (the voltage jumps seem to have a time constant of about 1 second, so they affect several integrations). This can be adequate in compact array configurations, but when the antennas are far from the array center this method can result in flares which appear to go down in total power, since part of the voltage jump which has been removed as instrumental is actually due to the flare. Fortunately, the delay line jumps in total power do not affect the correlated amplitudes and phases.

For observations in compact arrays, the remaining problem with total power data is that it suffers the problems of single–dish data at millimeter wavelengths referred to above: atmospheric effects are very important, and introduce variability in the total power which must be distinguished from flares. The Sun itself can in principle be used as a calibration source, by assuming a brightness temperature for the solar disk. We expect that more use will be made of the total power data in the future.

As an example of the data which we obtain with the present system, Figure 3 presents a summary of observations during an active period in 1991 June 8 – 13. During this period BIMA was in its most compact configuration, with fringe spacings ranging from $30''$ EW to $45''$ NS; such large fringe spacings should guarantee that the correlated amplitude corresponds to the total flux of the source, at the expense of information about small spatial scales. Figure 3 shows the correlated amplitude on a single baseline for the whole period of observation with the BIMA array in 1991 June. Most of the larger events can be seen: about 30 radio bursts in total were detected by BIMA and some of the smaller ones do not show up well on this plot. Two timescales of emission are readily seen in this figure: a rapid rise and fall component which appears as sharp spikes, and a much less rapidly varying component which often follows an impulsive spike. We now have clear evidence that the impulsive component corresponds to nonthermal gyrosynchrotron emission from energetic electrons as expected (Kundu *et al.* 1994), while the long-duration events are produced by thermal bremsstrahlung in the hot dense plasma seen in the corona in the decay phase of solar flares. In general we seem to see a signature at 86 GHz of every flare large enough to be given a GOES classification, and most of them appear to show impulsive phase emission. Two of the giant (> GOES class X9) flares from the June period (early UT on June 9 and 11, here shown at the end of June 8 and 10) saturated the telescope receivers shortly after flare onset.

Two contrasting examples of the types of time profiles exhibited by solar flares are shown at 0.4 second time resolution in Figure 4. The first has a classic impulsive profile, with a linear rise to a well-defined maximum in 6 seconds, followed by an exponential delay of time constant about 30 seconds. The second burst is a long-duration event, lasting over 15 minutes and having a generally smooth profile with a number of small peaks superimposed on it. The two bursts are plotted to the same timescale in order to facilitate comparison. Although the first burst has the classic appearance of the prototypical impulsive nonthermal solar radio burst, on analysis it was found to have a very flat radio spectrum and to be hard to explain with any of the usual radio burst models (White *et al.* 1992).

7. Closure Phase

The use of closure phase is an old technique in interferometry (Jennison 1958) which largely disappeared in the 1960's because it was no longer needed. It was subsequently "rediscovered" by the VLBI community (Rogers *et al.* 1974) and is now an integral part of VLBI analysis; it is also extensively used at millimeter arrays, and at any array with a small number of dishes (such as the Owens Valley frequency-agile interferometer: e.g., Gary and Hurford 1990). Generally it is used to improve the quality of the data; however, we use it to obtain some structural information, since with only three baselines and with the relative calibration of the baselines not well determined we otherwise have only limited information.

As described above, in interferometry signals are collected at two telescopes, detected by a receiver, and then combined in a correlator. The outcome of the procedure is a complex number, which ideally is the one-dimensional spatial Fourier transform of the brightness distribution on the sky within the beam of the telescope (see the discussion in section 6). The receivers at each telescope are complicated and sensitive devices, which cannot easily be calibrated absolutely. The phase and amplitude scale at each receiver will in general drift independently, and much of the effort in interferometry goes into determining these scales. This procedure is usually called "calibration" in radio astronomy, and is carried out by frequent observation of a strong celestial source for which we already know (from repeated previous observations) the expected interferometer response. The phase and amplitude scales at each antenna can be determined by choosing them to produce the expected response, and this is done frequently enough that any changes in these scales are small and can be determined by interpolation.

However even with these scales determined, phase errors may also be introduced by the atmosphere, by the path between the receiver and the correlator (which includes amplification, downshifting in frequency and other processes), and by the correlator itself. Any phase errors present will degrade the observations, as discussed above. However, there is one quantity which is independent of most of the phase errors, and that is the closure phase.

The closure phase can be formed for any group of three antennas in an interferometer. Suppose the source is a point source; then the radiation arriving at the telescopes can be regarded as a plane wave which will have a well-defined phase at each telescope. Let θ_1, θ_2, and θ_3 be the phases which each telescope would measure for a given point source if they were ideal instruments (i.e., perfectly calibrated and error-free). Now let ϕ_1, ϕ_2, and ϕ_3 be the phase errors at each antenna: these will be a combination of errors introduced by the atmosphere, as well as any drift in the receiver phases at each antenna. Thus the actual phase measured at antenna i is $\psi_i = \theta_i + \phi_i$.

When the signals from antennas i and j are correlated, the interferometer phase measured is just $\psi_{ij} = \psi_j - \psi_i$. The closure phase is the sum $\eta = \psi_{12} + \psi_{23} + \psi_{31}$, and it is clear from the above that $\eta = 0$ for a point source. More generally, it may be shown that no matter what the source distribution is, the antenna-based errors (due either to the atmosphere or imperfect calibration) cancel when the closure phase is formed. The properties of closure phase are often used at millimeter telescopes to determine the phase corresponding to a (point) calibrator source on a long baseline: on long baselines the phase noise is often so large as to prevent the actual measurement of the phase. Instead, the phases on two shorter baselines with lower noise can be measured, and closure is used to determine the phase on the third baseline.

It is widely known amongst radio astronomers that the closure phase is zero for a point (i.e., unresolved) source. It is probably less widely known that the closure phase will be zero for a gaussian brightness distribution of the form (7), even when the source is resolved and a significant fraction of the flux of the source is not observed on any given baseline (although the flux from the source must be the dominant contribution to the correlated amplitude on each baseline, so that it determines the measured phases). The more general statement of this result is that the closure phase should be zero for any brightness distribution which has a well-defined peak, such that on any cross-section through the peak the brightness distribution is symmetric on either side of the peak.

This principle can be used to gain some information about source structure: when closure phase is zero, then the source structure is either simple and symmetric, or else the source appears unresolved with the available fringe spacings.

In Figure 5 we present an example of closure phase formation which demonstrates both important aspects of phase closure: its independence of antenna-based phase errors; and its value for a simple source. The four panels of this figure show the measured phase as a function of time (seconds) on each of baselines 12, 23, and 31, as well as the closure phase, for a radio burst observed on 1991 March 7 (Lim *et al.* 1992). The time profile of the correlated emission on baseline 23 is plotted over the closure phase: the profile was similar on all three baselines, and reached peak fluxes of 6, 8 and 6 sfu (with estimated uncertainties in the amplitude calibration of 30% on each baseline). The fringe spacings at the time of the burst were $5''$, $10''$ and $4.4''$ on the three baselines, respectively, with different orientations for each. The data have been background-subtracted, so that prior to the burst there is effectively only noise on each baseline, and the random nature of the phases on each baseline demonstrates this. During the burst each baseline shows a well-defined phase which has both a steady drift and a fluctuation of up to $40°$ on a timescale of less than 20 seconds. However, the closure phase is steady with time during the burst and does not show the short-term fluctuations, which we attribute to the atmosphere. The width of the closure phase line during the burst, which represents the noise in this technique, is much less than $10°$. The fact that the fluxes were similar on all three baselines despite different fringe spacings suggests that the source was nearly unresolved, and the constancy of closure phase with time appears to confirm this. We have a number of examples of bursts from the same time period which showed different fluxes and time profiles on the different baselines, and for which the closure phase, while usually well-behaved, was not constant in time (these are discussed by Lim *et al.* 1992).

However, it is apparent that the value of the closure phase in Figure 5 is not zero. This is due to known errors in the analog (wideband) correlator at BIMA which we use for solar observations. The closure phase is affected by correlator errors even though it is independent of antenna-based errors. The correlator errors also lead to different values for the closure phase on the upper and lower sidebands (in our observations, 86 and 89 GHz), which we also see both in the solar observations and in the observations of strong unresolved calibrator sources. Nonetheless, it is difficult in principle to imagine a situation in which the closure phase in the absence of correlator errors was constant in time at some value different from zero, because we expect that the combination of source time variation and asymmetry will lead to variation in the value of the closure phase. Thus, when we see a closure phase constant in time but nonzero, we can safely assume that it is actually zero but with correlator errors imposed. This gives us a measure of source size/morphology even with just three baselines.

8. Current Developments

Millimeter-wavelength interferometry is currently one of the most exciting areas of radio astronomy world-wide. The U.S. National Radio Astronomy Observatory plans to build a Millimeter Array consisting of 40 8–m dishes as its next major instrument. The existing facilities at Hat Creek (BIMA) and Owens Valley (Caltech) are both being expanded with funds from the National Science Foundation. The arrays at Nobeyama (Japan) and IRAM (Spain) are also being expanded. An array operating at submillimeter wavelengths is also being planned in the US by the Smithsonian Institution. We can expect that these facilities will provide the opportunity for arcsecond imaging at high time resolution, ideal for the study of the most energetic component of solar flares, as we have emphasized here. They will also allow the study of chromospheric structures, which will complement the use of microwave arrays to study coronal structures.

There has been renewed interest in the millimeter domain by solar radio astronomers in line with the recent technical improvements in the available facilities. As an example of this, we mention that during the 1991 July 13 solar eclipse at least four milimeter/submillimeter facilities were used simultaneously for solar studies: the BIMA array (Maryland group), the Owens Valley array (Caltech), the James Clerk Maxwell telescope (Hawaii group), and the Caltech Submillimeter telescope (Caltech/NRAO). In addition several other major telescopes on Hawaii were used for infrared observations. We regard this as an indication of the likely level of interest in solar astronomy at millimeter/submillimeter wavelengths in the next decade.

Acknowledgements

The work of N. Gopalswamy and J. Lim from the solar group at Maryland has been essential for the success of solar observations at BIMA. Use of the BIMA telescope for solar research would not be possible without the efforts of many people, amongst whom we need to mention by name John Bieging, Leo Blitz, Rick Forster, Dick Plambeck, Stuart Vogel, Jack Welch, and Mel Wright. We are particularly grateful for the flexible scheduling of solar observations allowed by the BIMA board which, due to the unpredictable nature of the Sun, is essential for the success of solar research with a non-solar-dedicated telescope. We thank the Commitee of European Solar Radio Astronomers for the opportunity to attend the Fourth *CESRA* Workshop. Research at U. Md was supported by NSF grant ATM 90–19893 and NASA grants NAG-5–1540, NAG-W-1541 and NAG-W-2172. The use of BIMA for scientific research at U. Md. is supported by NSF grant AST 91–00306.

Bibliography

Abramov-Maksimov, V. E., and Gelfreikh, G. B.: 1983, *Sov. Astron. Lett.* **9**, 132.

Akabane, K., Nakajima, K., Ohki, K., Moriyama, F., and Miyaji, T.: 1973, *Solar Phys.* **33**, 431.

Alissandrakis, C. E., and Kundu, M. R.: 1978, *Astrophys. J.* **222**, 342.

Archer, J. W.: 1977, *Instruments and Techniques for High Resolution Solar Astronomy at 3 mm*, Ph.D. thesis, University of Sydney, unpublished.

Avrett, E. H.: 1992, in D. Rabin (ed.), *IAU Symposium 154: Infrared Solar Physics*, Kluwer Academic Publishers, Dordrecht, in press.

Babin, A. N., Gopasyuk, S. I., Efanov, V. A., Kotov, V. A., Moiseev, I. G., Nesterov, N. S., and Tsap, T. T.: 1976, *Izv. Krymsk. Astrofiz. Obs.* **55**, 3.

Bai, T., and Sturrock, P. A.: 1989, *Ann. Rev. Astron. Astrophys.* **27**, 421.

Bruggman, G., Benz, A. O., Magun, A., and Stehling, W.: 1990, *Astron. Astrophys.* **240**, 506.

Coates, R. J.: 1958, *Proc. IRE* **46**, 122.

Cogdell, J. R.: 1972, *Solar Phys.* **22**, 147.

Costa, J. E. R., Homor, J. L., and Kaufmann, P.: 1986, in E. Tandberg-Hanssen, R. M. Wilson, and H. S. Hudson (eds.), *Solar Flares and Coronal Physics Using P/OF as a Research Tool*, NASA CP 2421, p. 201.

Croom, D. L.: 1970, *Solar Phys.* **15**, 414.

Croom, D. L., and Powell, R. J.: 1969, *Nature* **221**, 945.

de Jager, C., Kuijpers, J., Correia, E., and Kaufmann, P.: 1987, *Solar Phys.* **110**, 317.

Dulk, G. A.: 1985, *Ann. Rev. Astron. Astrophys.* **23**, 169.

Dulk, G. A., Bastian, T. S., Gary, D. E., Hjellming, R. M., and Kundu, M. R.: 1990, in *Science with the Millimeter Array: Working Group Report on the Sun and the Stars*, NRAO.

Dulk, G. A., and Marsh, K. A.: 1982, *Astrophys. J.* **259**, 350.

Feix, G.: 1970, *J. Geophys. Res.* **75**, 211.

Gary, D. E., and Hurford, G. J.: 1990, *Astrophys. J.* **361**, 290.

Hagen, J. P.: 1957, in H. C. Van de Hulst (ed.), *Radio Astronomy,* I.A.U. Symposium No. 4, Cambridge University Press, Cambridge, p. 263.

Holt, S. S., and Ramaty, R.: 1969, *Solar Phys.* **8**, 119.

Hoyng, P., Marsh, K. A., Zirin, H., and Dennis, B. R.: 1983, *Astrophys. J.* **268**, 865.

Hudson, H. S., and Ohki, K.: 1972, *Solar Phys.* **23**, 155.

Jennison, R. C.: 1958, *Mon. Not. Roy. Astron. Soc.* **118**, 276.

Kahler, S. W.: 1982, *J. Geophys. Res.* **87**, 3439.

Kaufmann, P., Correia, E., Costa, J. E. R., and Vaz, A. M. Z.: 1986, *Astron. Astrophys.* **157**, 11.

Kaufmann, P., Correia, E., Costa, J. E. R., Vaz, A. M. Z., and Dennis, B. R.: 1985, *Nature* **313**, 380.

Kaufmann, P., Strauss, F. M., Schaal, R. E., and Laporte, C.: 1982, *Solar Phys.* **78**, 389.

Kawabata, K., and Ogawa, H.: 1989, in B. M. Haisch and M. Rodonò (eds.), *IAU Colloquium 104: Solar and Stellar Flares*, Catania Astrophysical Observatory, Catania, p. 191.

Kawabata, K., Ogawa, H., Sofue, Y., and Suzuki, I.: 1974, *Publ. Astron. Soc. Japan* **26**, 387.

Kawabata, K., Ogawa, H., Takakura, T., Tsuneta, S., Ohki, K., Yoshimori, M., Okudaira, K., Hirashima, Y., and Kondo, I.: 1982, in *HINOTORI Symposium on Solar Flares*, Institute of Space and Astronautical Sciences, Tokyo, p. 168.

Kosugi, T., Ishiguro, M., and Shibasaki, K.: 1986, *Publ. Astron. Soc. Japan* **38**, 1.

Krüger, A., and Hildebrandt, J.: 1988, *Astron. Nachr.* **309**, 43.

Kundu, M. R.: 1965, *Solar Radio Astronomy*, Interscience Publishers, New York (First edition).

Kundu, M. R.: 1970, *Solar Phys.* **13**, 348.

Kundu, M. R.: 1972, *Solar Phys.* **25**, 108.

Kundu, M. R.: 1982, *Rep. Prog. Phys.* **45**, 1435.

Kundu, M. R., and Gergely, T. E.: 1973, *Solar Phys.* **31**, 461.

Kundu, M. R., Velusamy, T., and White, S. M.: 1987, *Astrophys. J.* **321**, 593.

Kundu, M. R., White, S. M., Gopalswamy, N., Bieging, J. H., and Hurford, G. J.: 1990, *Astrophys. J. (Lett.)* **358**, L69.

Kundu, M. R., White, S. M., Gopalswamy, N., and Lim, J.: 1994, *Astrophys. J. Supp.* **90**, 599.

Kundu, M. R., White, S. M., Welch, W. J., and Bieging, J. H.: 1991, *Adv. Space Res.* **11**, (5)91.

Labrum, N. R.: 1978, *Proc. Astron. Soc. Aust.* **3**, 256.

Labrum, N. R., Archer, J. W., and Smith, C. J.: 1978, *Solar Phys.* **59**, 331.

Lim, J., White, S. M., Kundu, M. R., and Gary, D. E.: 1992, *Solar Phys.* **140**, 343.

Lin, R. P., and Schwartz, R. A.: 1987, *Astrophys. J.* **312**, 462.

Lin, R. P., Schwartz, R. A., Pelling, R. M., and Hurley, K. C.: 1981, *Astrophys. J. (Lett.)* **251**, L109.

Lindsey, C. A., and Jefferies, J. T.: 1991, *Astrophys. J.* **383**, 443.

Lindsey, C. A., and Roellig, T. L.: 1991, *Astrophys. J.* **375**, 414.

Lindsey, C. A., Yee, S., Roellig, T. L., Hills, R., Brock, D., Duncan, W., Watt, G., Webster, A., and Jefferies, J. T.: 1990, *Astrophys. J. (Lett.)* **353**, L53.

Mal'tsev, V. A., Rassadovskii, V. A., Serov, N. V., Chernyshev, V. I., and Shabanova, G. A.: 1989, *Sov. Astron. Lett.* **15(5)**, 408.

Marsh, K. A., and Hurford, G. J.: 1980, *Astrophys. J. (Lett.)* **240**, L111.

Mayfield, E. B., Higman, J., and Samson, C.: 1970, *Solar Phys.* **13**, 372.

Nagnibeda, V. G., and Piotrovich, V. V.: 1990, *Astron. Nachr.* **311**, 413.

Nagnibeda, V. G., Piotrovich, V. V., Ivanov, V. N., Rozanov, B. A., and Solovev, G. N.: 1990, *Astron. Nachr.* **311**, 417.

Nakajima, H., Sekiguchi, H., Sawa, M., Kai, K., Kawashima, S., Kosugi, T., Shibuya, N., Shinohara, N., and Shiomi, Y.: 1985, *Publ. Astron. Soc. Japan* **37**, 163.

Naylor, D. A., Clark, T. A., Schultz, A. A., and Davis, G. R.: 1991, *Mon. Not. Roy. Astron. Soc.* **251**, 199.

Pelyushenko, S. A.: 1985, *Sov. Astron.* **29**, 219.

Pelyushenko, S. A., and Chernyshev, V. I.: 1983, *Sov. Astron.* **27**, 340.

Petrosian, V.: 1982, *Astrophys. J. (Lett.)* **255**, L85.

Ramaty, R.: 1969, *Astrophys. J.* **158**, 753.

Ramaty, R., and Petrosian, V.: 1972, *Astrophys. J.* **178**, 241.

Rogers, A. E., Hinteregger, H. F., Whitney, A. R., Counselman, C. C., Shapiro, I. I., Wittels, J. J., Klemperer, W. K., Warnock, W. W., Clark, T. A., Hutton, L. K., Marandino, G. E., Rönnäng, B. O., Rydbeck, O. E. H., and Niell, A. E.: 1974, *Astrophys. J.* **193**, 293.

Shibasaki, K.: 1992, in D. Rabin (ed.), *IAU Symposium 154: Infrared Solar Physics*, Kluwer Academic Publishers, Dordrecht.

Shimabukuro, F. I.: 1972, *Solar Phys.* **23**, 169.

Shimabukuro, F. L.: 1970, *Solar Phys.* **15**, 424.

Takakura, T.: 1972, *Solar Phys.* **26**, 151.

Thomas, R. J., Starr, R., and Crannell, C.-J.: 1985, *Solar Phys.* **95**, 323.

Thompson, A. R., Moran, J. M., and Swenson, Jr., G. W.: 1986, *Interferometry and Synthesis in Radio Astronomy*, Wiley, New York.

Urpo, S., Hildebrandt, J., and Krüger, A.: 1987, *Solar Phys.* **112**, 119.

Urpo, S., Teräsranta, H., Krüger, A., Hildebrandt, J., and Ruždjak, V.: 1989, *Astron. Nachr.* **310**, 423.

Vernazza, J. E., Avrett, E. H., and Loeser, R.: 1976, *Astrophys. J. Supp.* **30**, 1.

Vernazza, J. E., Avrett, E. H., and Loeser, R.: 1981, *Astrophys. J. Supp.* **45**, 635.

Wang, J. L., Takakura, T., Liu, J. Q., Ohki, K., Kawabata, K., and Ogawa, H.: 1987, *Solar Phys.* **108**, 169.

White, S. M., Kundu, M. R., Bastian, T. S., Gary, D. E., Hurford, G. J., Kucera, T., and Bieging, J. H.: 1992, *Astrophys. J.* **384**, 656.

White, S. M., Kundu, M. R., and Gopalswamy, N.: 1991, *Astrophys. J. (Lett.)* **366**, L43.

Willson, R. F.: 1983, *Solar Phys.* **83**, 285.

Zlotnik, E. Y.: 1987, *Sov. Astron.* **31**, 569.

Zodi, A. M., Correia, E., Costa, J. E. R., Kaufmann, P., Martin, S. F., and Kundu, M. R.: 1988, *Solar Phys.* **116**, 83.

Figure Captions

Figure 1. A plot of nonthermal gyrosynchrotron flux spectra as a function of frequency for different values of the low-energy cutoff. The spectral index is 4, the magnetic field 300 G, the high-energy cutoff is 10 MeV and the angle between the line of sight and the magnetic field is 45°. The flux scale is in sfu (the same values for the source dimensions were used in all cases); the logarithn of the flux is plotted. The curves are labelled according to the low-energy cutoff. The actual values of the frequency plotted are 1.5, 1.89, 2.37, 2.98, 3.75, 4.72, 5.93, 7.46, 9.38, 11.79, 14.82, 18.54, 35.0, and 86.0 GHz.

Figure 2. Contour plots of the ratio of the gyrosynchrotron flux in a homogeneous magnetic field at 86 GHz for a nonthermal energy distribution with a low-energy cutoff at 1 Mev to the corrresponding flux for the same distribution with a cutoff at 10 keV, as a function of energy spectral index and magnetic field strength. Plots are given for θ = 30° (left panel) and θ = 70° (right panel). The contours plotted are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

Figure 3. A plot of correlated amplitude versus time (UT hours) summarizing the BIMA 86 GHz observations during the 1991 June GRO solar campaign. This was a particularly active period for flares, and the flaring region was sufficiently small that virtually all the flares fell within our field of view.

Figure 4. Two examples of the contrasting types of behaviour exhibited by solar flares at millimeter wavelengths, both observed in June 1989. The left panel shows a simple impulsive event with a sharp linear rise and a smooth exponential decay; this was a flat–spectrum burst with peculiar properties. The right panel, which is at the same time scale as the left panel, shows an event with a complicated non-impulsive profile, lasting for over 10 minutes.

Figure 5. The four panels in this figure show the phases on each of the three baselines, plus the closure phase (the sum of the phases on the three baselines), as a function of time during a burst on 1991 March 7. The time profile of the correlated amplitude for this burst is also shown plotted as a continuous line plotted with the closure phase on the fourth panel. The phases on baselines 12, 23, and 31 all show short-timescale variability which we attribute to the atmosphere. The closure phase is constant during the burst and does not show variability due to the atmosphere; the offset of closure phase from zero is due to known correlator errors.